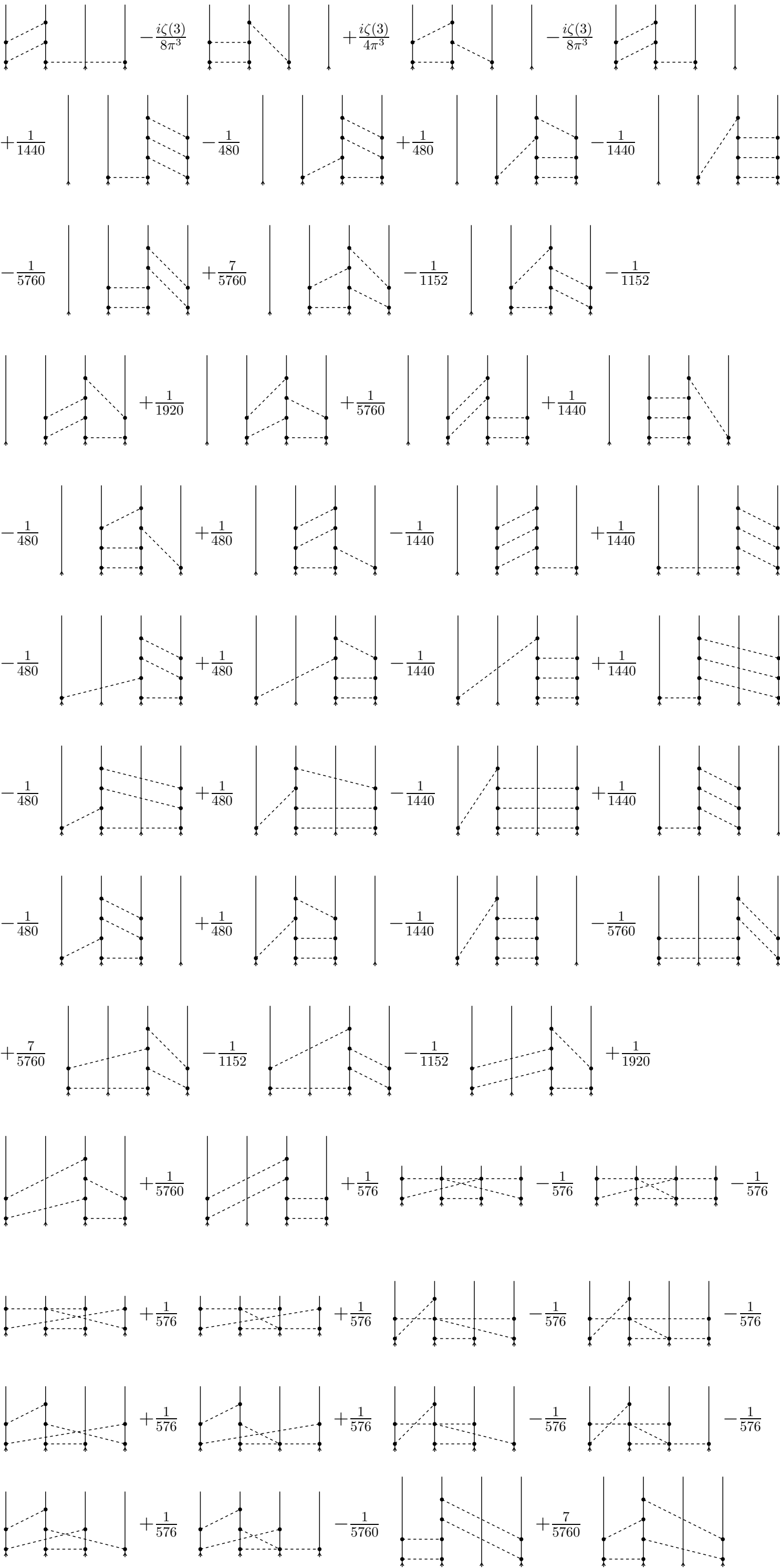
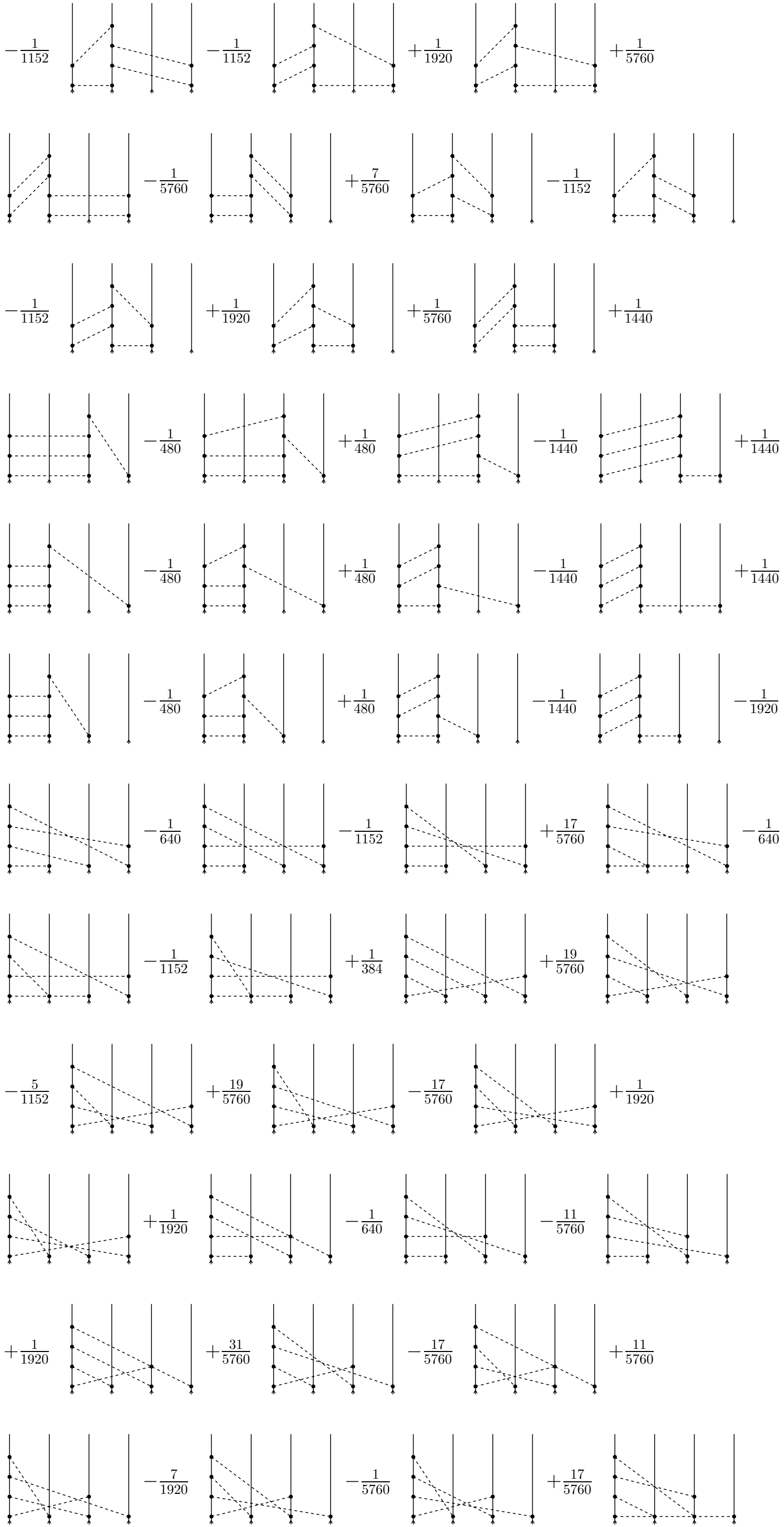
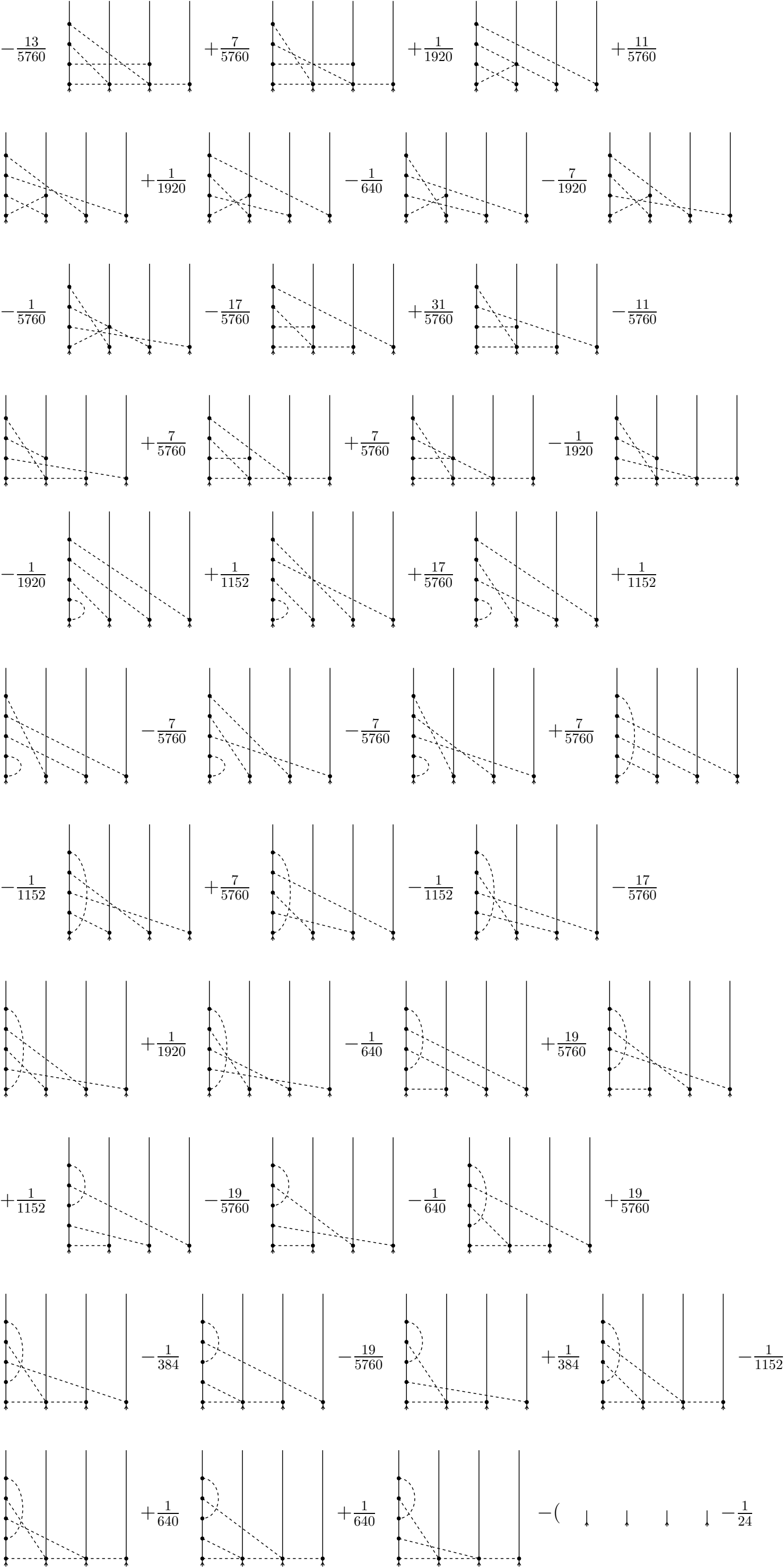


Figure 10 displays 48 Feynman diagrams arranged in a 6x8 grid, representing the two-loop contribution to the four-point function. Each diagram is a combination of solid and dashed lines with external legs marked by arrows. The diagrams are grouped into four sets of three rows each, each set associated with a specific coefficient:

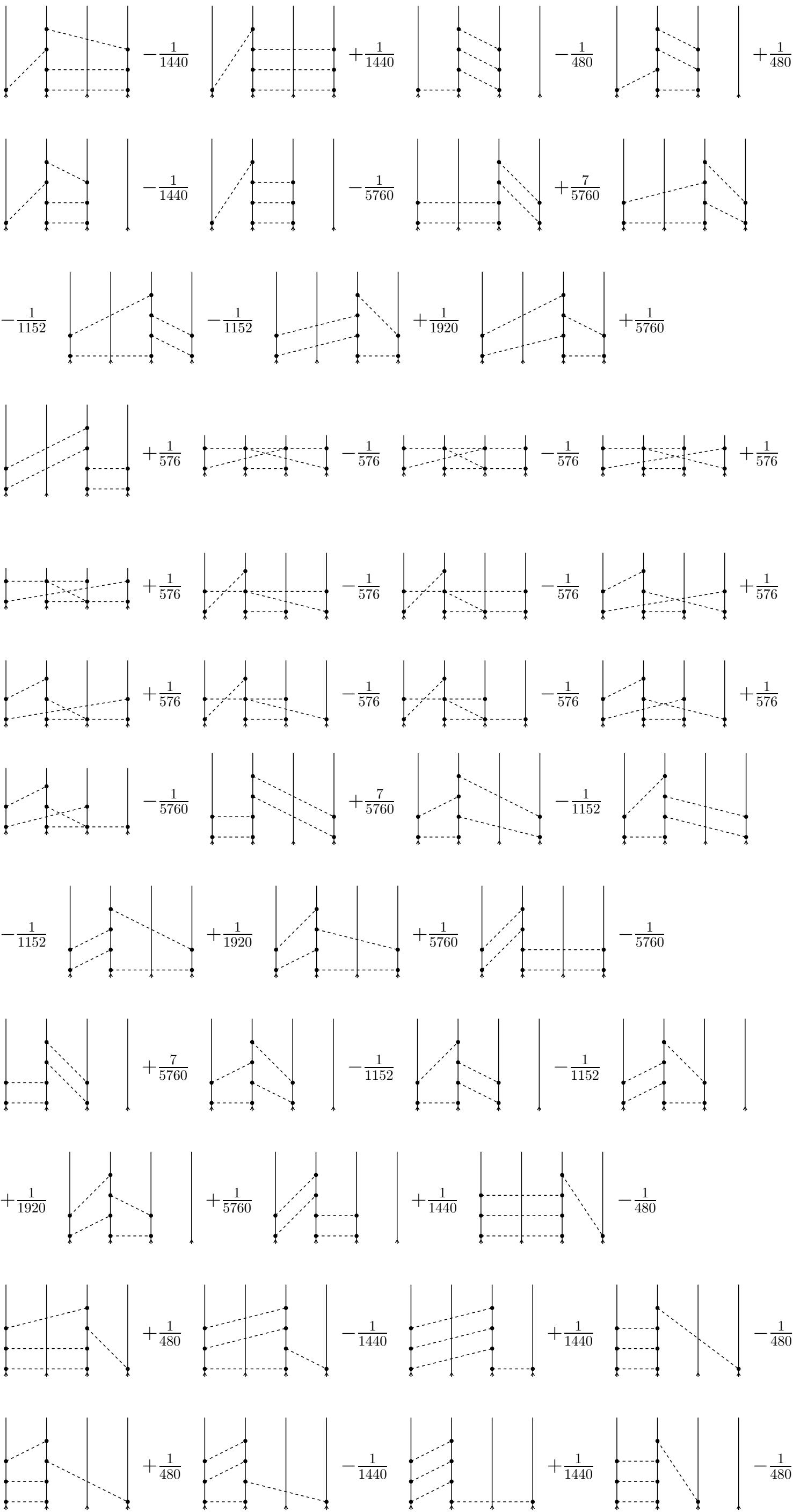
- Row 1:  $-\frac{1}{24}$
- Row 2:  $+\frac{1}{24}$
- Row 3:  $-\frac{1}{24}$
- Row 4:  $+\frac{1}{24}$
- Row 5:  $+\frac{i\zeta(3)}{8\pi^3}$
- Row 6:  $-\frac{i\zeta(3)}{4\pi^3}$
- Row 7:  $+\frac{i\zeta(3)}{8\pi^3}$
- Row 8:  $-\frac{i\zeta(3)}{8\pi^3}$
- Row 9:  $+\frac{i\zeta(3)}{4\pi^3}$
- Row 10:  $-\frac{i\zeta(3)}{8\pi^3}$
- Row 11:  $+\frac{i\zeta(3)}{8\pi^3}$
- Row 12:  $+\frac{i\zeta(3)}{8\pi^3}$
- Row 13:  $-\frac{i\zeta(3)}{4\pi^3}$
- Row 14:  $+\frac{i\zeta(3)}{8\pi^3}$
- Row 15:  $+\frac{i\zeta(3)}{8\pi^3}$
- Row 16:  $-\frac{i\zeta(3)}{4\pi^3}$
- Row 17:  $+\frac{i\zeta(3)}{8\pi^3}$
- Row 18:  $-\frac{i\zeta(3)}{8\pi^3}$
- Row 19:  $+\frac{i\zeta(3)}{8\pi^3}$
- Row 20:  $-\frac{i\zeta(3)}{8\pi^3}$
- Row 21:  $+\frac{i\zeta(3)}{8\pi^3}$
- Row 22:  $-\frac{i\zeta(3)}{8\pi^3}$
- Row 23:  $+\frac{i\zeta(3)}{8\pi^3}$
- Row 24:  $-\frac{i\zeta(3)}{8\pi^3}$
- Row 25:  $+\frac{i\zeta(3)}{8\pi^3}$
- Row 26:  $-\frac{i\zeta(3)}{8\pi^3}$
- Row 27:  $+\frac{i\zeta(3)}{8\pi^3}$
- Row 28:  $-\frac{i\zeta(3)}{8\pi^3}$
- Row 29:  $+\frac{i\zeta(3)}{8\pi^3}$
- Row 30:  $-\frac{i\zeta(3)}{8\pi^3}$
- Row 31:  $+\frac{i\zeta(3)}{8\pi^3}$
- Row 32:  $-\frac{i\zeta(3)}{8\pi^3}$
- Row 33:  $+\frac{i\zeta(3)}{8\pi^3}$
- Row 34:  $-\frac{i\zeta(3)}{8\pi^3}$
- Row 35:  $+\frac{i\zeta(3)}{8\pi^3}$
- Row 36:  $-\frac{i\zeta(3)}{8\pi^3}$
- Row 37:  $+\frac{i\zeta(3)}{8\pi^3}$
- Row 38:  $-\frac{i\zeta(3)}{8\pi^3}$
- Row 39:  $+\frac{i\zeta(3)}{8\pi^3}$
- Row 40:  $-\frac{i\zeta(3)}{8\pi^3}$
- Row 41:  $+\frac{i\zeta(3)}{8\pi^3}$
- Row 42:  $-\frac{i\zeta(3)}{8\pi^3}$
- Row 43:  $+\frac{i\zeta(3)}{8\pi^3}$
- Row 44:  $-\frac{i\zeta(3)}{8\pi^3}$
- Row 45:  $+\frac{i\zeta(3)}{8\pi^3}$
- Row 46:  $-\frac{i\zeta(3)}{8\pi^3}$
- Row 47:  $+\frac{i\zeta(3)}{8\pi^3}$
- Row 48:  $-\frac{i\zeta(3)}{8\pi^3}$











$+\frac{1}{480}$ 
 $-\frac{1}{1440}$ 
 $-\frac{1}{1920}$ 
 $-\frac{1}{640}$

$-\frac{1}{1152}$ 
 $+\frac{17}{5760}$ 
 $-\frac{1}{640}$

$-\frac{1}{1152}$ 
 $+\frac{1}{384}$ 
 $+\frac{19}{5760}$ 
 $-\frac{5}{1152}$

$+\frac{19}{5760}$ 
 $-\frac{17}{5760}$ 
 $+\frac{1}{1920}$

$+\frac{1}{1920}$ 
 $-\frac{1}{640}$ 
 $-\frac{11}{5760}$ 
 $+\frac{1}{1920}$

$+\frac{31}{5760}$ 
 $-\frac{17}{5760}$ 
 $+\frac{11}{5760}$

$-\frac{7}{1920}$ 
 $-\frac{1}{5760}$ 
 $+\frac{17}{5760}$ 
 $-\frac{13}{5760}$

$+\frac{7}{5760}$ 
 $+\frac{1}{1920}$ 
 $+\frac{11}{5760}$

$+\frac{1}{1920}$ 
 $-\frac{1}{640}$ 
 $-\frac{7}{1920}$ 
 $-\frac{1}{5760}$

$-\frac{17}{5760}$ 
 $+\frac{31}{5760}$ 
 $-\frac{11}{5760}$

$+\frac{7}{5760}$ 
 $+\frac{7}{5760}$ 
 $-\frac{1}{1920}$ 
 $-\frac{1}{1920}$

$+\frac{1}{1152}$ 
 $+\frac{17}{5760}$ 
 $+\frac{1}{1152}$

$$\begin{aligned}
& -\frac{7}{5760} \quad \text{[diagram]} \quad -\frac{7}{5760} \quad \text{[diagram]} \quad +\frac{7}{5760} \quad \text{[diagram]} \quad -\frac{1}{1152} \quad \text{[diagram]} \\
& \text{[diagram]} \quad +\frac{7}{5760} \quad \text{[diagram]} \quad -\frac{1}{1152} \quad \text{[diagram]} \quad -\frac{17}{5760} \quad \text{[diagram]} \\
& +\frac{1}{1920} \quad \text{[diagram]} \quad -\frac{1}{640} \quad \text{[diagram]} \quad +\frac{19}{5760} \quad \text{[diagram]} \quad +\frac{1}{1152} \quad \text{[diagram]} \\
& \text{[diagram]} \quad -\frac{19}{5760} \quad \text{[diagram]} \quad -\frac{1}{640} \quad \text{[diagram]} \quad +\frac{19}{5760} \quad \text{[diagram]} \quad -\frac{1}{384} \quad \text{[diagram]} \\
& \text{[diagram]} \quad -\frac{19}{5760} \quad \text{[diagram]} \quad +\frac{1}{384} \quad \text{[diagram]} \quad -\frac{1}{1152} \quad \text{[diagram]} \quad +\frac{1}{640} \quad \text{[diagram]} \\
& \text{[diagram]} \quad +\frac{1}{640} \quad \text{[diagram]} \quad ) = 0
\end{aligned}$$

$\nu^{-1}$  = from the [CMD] book on page 345 is

$$\begin{aligned}
& \text{[circle]} \quad -\frac{1}{24} \text{[diagram]} \quad +\frac{1}{24} \text{[diagram]} \quad +\frac{7}{5760} \text{[diagram]} \quad -\frac{17}{5760} \text{[diagram]} \quad +\frac{7}{2880} \text{[diagram]} \quad +\frac{1}{1920} \text{[diagram]} \\
& -\frac{1}{720} \text{[diagram]} \quad +\frac{1}{5760} \text{[diagram]}
\end{aligned}$$

the same element, written in our basis, is

$$\begin{aligned}
& \text{[circle]} \quad -\frac{1}{24} \text{[diagram]} \quad +\frac{1}{24} \text{[diagram]} \quad +\frac{7}{5760} \text{[diagram]} \quad +\frac{1}{5760} \text{[diagram]} \quad -\frac{1}{1152} \text{[diagram]} \quad +\frac{1}{1920} \text{[diagram]} \\
& +\frac{11}{5760} \text{[diagram]} \quad -\frac{17}{5760} \text{[diagram]}
\end{aligned}$$

$\nu^{-1}$  = from the the KZ associator

$$\begin{aligned}
& \text{[circle]} \quad -\frac{1}{24} \text{[diagram]} \quad +\frac{1}{24} \text{[diagram]} \quad +\frac{7}{5760} \text{[diagram]} \quad +\frac{1}{5760} \text{[diagram]} \quad -\frac{1}{1152} \text{[diagram]} \quad +\frac{1}{1920} \text{[diagram]}
\end{aligned}$$



$$+\frac{11}{5760}\text{diagram}_1 - \frac{17}{5760}\text{diagram}_2$$

Their difference: 0

Calculate a:

$$a = \text{diagram}_1 + \frac{1}{24} \text{diagram}_2 - \frac{1}{24} \text{diagram}_3 + \frac{i\zeta(3)}{8\pi^3} \text{diagram}_4 + \frac{i\zeta(3)}{8\pi^3} \text{diagram}_5 - \frac{i\zeta(3)}{4\pi^3} \text{diagram}_6 - \frac{1}{1440} \text{diagram}_7 + \frac{1}{1440} \text{diagram}_8 - \frac{1}{480} \text{diagram}_9 + \frac{1}{480} \text{diagram}_{10} + \frac{7}{5760} \text{diagram}_{11} + \frac{1}{576} \text{diagram}_{12} - \frac{1}{1152} \text{diagram}_{13} - \frac{1}{480} \text{diagram}_{14} + \frac{19}{5760} \text{diagram}_{15} - \frac{1}{5760} \text{diagram}_{16} - \frac{7}{2880} \text{diagram}_{17} - \frac{1}{1440} \text{diagram}_{18} - \frac{7}{5760} \text{diagram}_{19} + \frac{1}{1152} \text{diagram}_{20} + \frac{1}{960} \text{diagram}_{21} - \frac{7}{1920} \text{diagram}_{22} + \frac{7}{5760} \text{diagram}_{23} + \frac{1}{1152} \text{diagram}_{24} - \frac{11}{5760} \text{diagram}_{25} + \frac{1}{360} \text{diagram}_{26}$$

Associator in the choses basis is:

$$\Phi = \text{diagram}_1 + \frac{1}{24} \text{diagram}_2 - \frac{1}{24} \text{diagram}_3 + \frac{i\zeta(3)}{8\pi^3} \text{diagram}_4 - \frac{i\zeta(3)}{4\pi^3} \text{diagram}_5 + \frac{i\zeta(3)}{8\pi^3} \text{diagram}_6 - \frac{i\zeta(3)}{8\pi^3} \text{diagram}_7 - \frac{i\zeta(3)}{8\pi^3} \text{diagram}_8 + \frac{i\zeta(3)}{4\pi^3} \text{diagram}_9 + \frac{i\zeta(3)}{4\pi^3} \text{diagram}_{10} - \frac{1}{1440} \text{diagram}_{11} + \frac{1}{480} \text{diagram}_{12} - \frac{1}{480} \text{diagram}_{13} + \frac{1}{1440} \text{diagram}_{14} + \frac{7}{5760} \text{diagram}_{15} - \frac{1}{640} \text{diagram}_{16} - \frac{1}{1152} \text{diagram}_{17} - \frac{1}{1152} \text{diagram}_{18} + \frac{19}{5760} \text{diagram}_{19} - \frac{7}{5760} \text{diagram}_{20} - \frac{7}{5760} \text{diagram}_{21}$$

$$+\frac{11}{5760} \text{ (diagram)} - \frac{17}{5760} \text{ (diagram)}$$

Their difference: 0

Calculate a:

$$a = \text{diagram} + \frac{1}{24} \text{ (diagram)} - \frac{1}{24} \text{ (diagram)} + \frac{i\zeta(3)}{8\pi^3} \text{ (diagram)} + \frac{i\zeta(3)}{8\pi^3} \text{ (diagram)} - \frac{i\zeta(3)}{4\pi^3} \text{ (diagram)} - \frac{1}{1440} \text{ (diagram)}$$

$$+ \frac{1}{1440} \text{ (diagram)} - \frac{1}{480} \text{ (diagram)} + \frac{1}{480} \text{ (diagram)} + \frac{7}{5760} \text{ (diagram)} + \frac{1}{576} \text{ (diagram)} - \frac{1}{1152} \text{ (diagram)}$$

$$- \frac{1}{480} \text{ (diagram)} + \frac{19}{5760} \text{ (diagram)} - \frac{1}{5760} \text{ (diagram)} - \frac{7}{2880} \text{ (diagram)} - \frac{1}{1440} \text{ (diagram)} - \frac{7}{5760} \text{ (diagram)} + \frac{1}{1152} \text{ (diagram)}$$

$$+ \frac{1}{960} \text{ (diagram)} - \frac{7}{1920} \text{ (diagram)} + \frac{7}{5760} \text{ (diagram)} + \frac{1}{1152} \text{ (diagram)} - \frac{11}{5760} \text{ (diagram)} + \frac{1}{360} \text{ (diagram)}$$

Associator in the choses basis is:

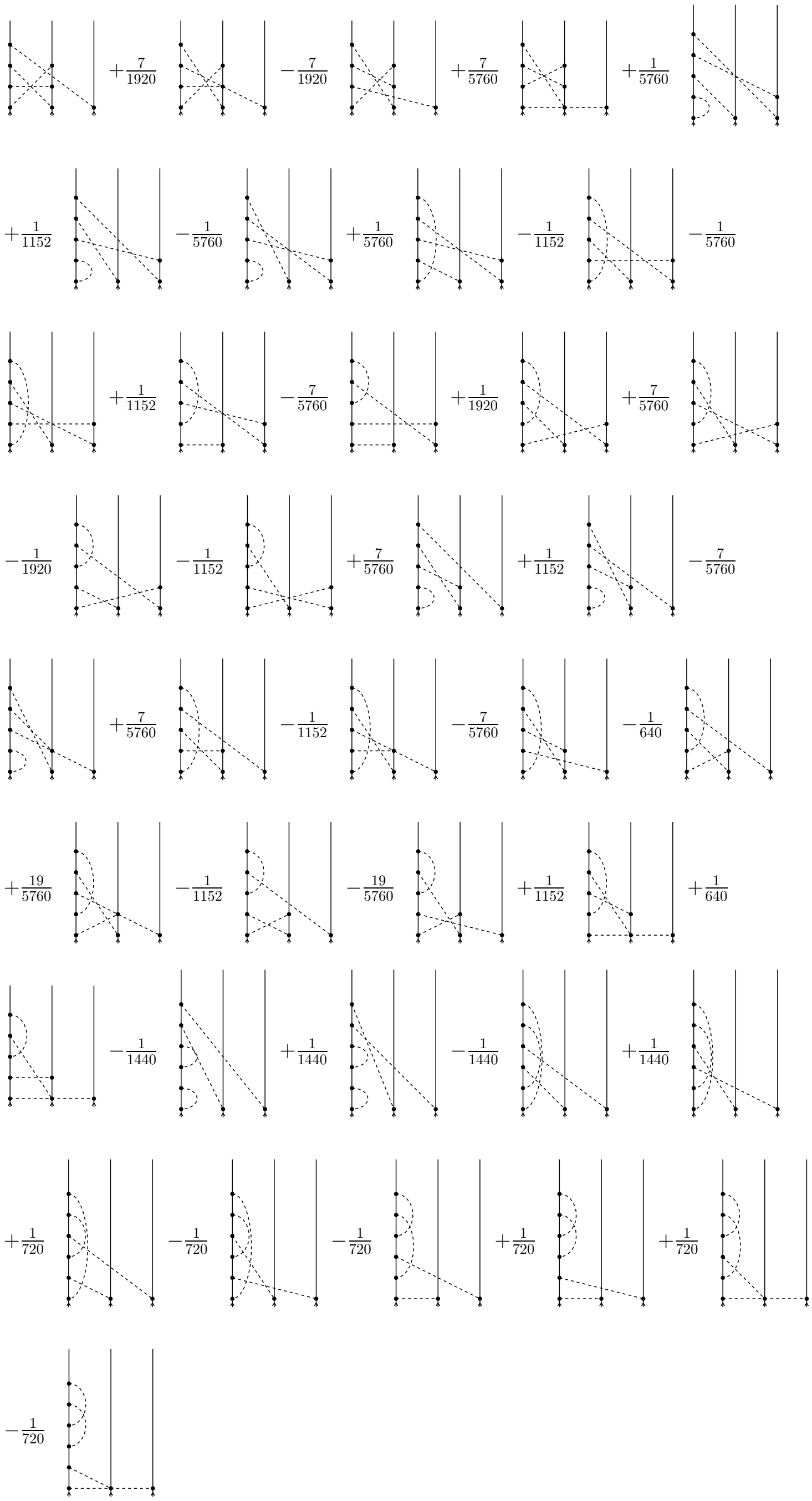
$$\Phi = \text{diagram} + \frac{1}{24} \text{ (diagram)} - \frac{1}{24} \text{ (diagram)} + \frac{i\zeta(3)}{8\pi^3} \text{ (diagram)} - \frac{i\zeta(3)}{4\pi^3} \text{ (diagram)}$$

$$+ \frac{i\zeta(3)}{8\pi^3} \text{ (diagram)} - \frac{i\zeta(3)}{8\pi^3} \text{ (diagram)} - \frac{i\zeta(3)}{8\pi^3} \text{ (diagram)} - \frac{i\zeta(3)}{8\pi^3} \text{ (diagram)}$$

$$- \frac{i\zeta(3)}{8\pi^3} \text{ (diagram)} + \frac{i\zeta(3)}{4\pi^3} \text{ (diagram)} + \frac{i\zeta(3)}{4\pi^3} \text{ (diagram)} - \frac{1}{1440} \text{ (diagram)} + \frac{1}{480} \text{ (diagram)}$$

$$- \frac{1}{480} \text{ (diagram)} + \frac{1}{1440} \text{ (diagram)} + \frac{7}{5760} \text{ (diagram)} - \frac{1}{640} \text{ (diagram)}$$

$$- \frac{1}{1152} \text{ (diagram)} - \frac{1}{1152} \text{ (diagram)} + \frac{19}{5760} \text{ (diagram)} - \frac{7}{5760} \text{ (diagram)} - \frac{7}{5760} \text{ (diagram)}$$



Associator twisted by  $a$ , in the chosses basis, is:



