

The KZ associator is:

THE RZ ASSOCIATOR IS:

$$\begin{aligned}
 & \begin{array}{c} | \\ \hline | \\ \hline | \end{array} - \frac{1}{24} \begin{array}{c} | \quad | \\ \hline | \quad \bullet \\ \hline | \quad | \end{array} + \frac{1}{24} \begin{array}{c} | \quad | \\ \hline \bullet \quad | \\ \hline | \quad | \end{array} + \frac{i\zeta(3)}{8\pi^3} \begin{array}{c} | \quad | \\ \hline | \quad \bullet \\ \hline | \quad | \end{array} - \frac{i\zeta(3)}{4\pi^3} \begin{array}{c} | \quad | \\ \hline | \quad \bullet \\ \hline | \quad | \end{array} + \frac{i\zeta(3)}{8\pi^3} \begin{array}{c} | \quad | \\ \hline | \quad \bullet \\ \hline | \quad | \end{array} \\
 & - \frac{i\zeta(3)}{8\pi^3} \begin{array}{c} | \quad | \\ \hline | \quad \bullet \\ \hline | \quad | \end{array} + \frac{i\zeta(3)}{4\pi^3} \begin{array}{c} | \quad | \\ \hline | \quad \bullet \\ \hline | \quad | \end{array} - \frac{i\zeta(3)}{8\pi^3} \begin{array}{c} | \quad | \\ \hline | \quad \bullet \\ \hline | \quad | \end{array}
 \end{aligned}$$

### Pentagon up to order 3

Figure 1 displays a collection of 48 Feynman diagrams, arranged in 8 rows of 6 columns. Each diagram represents a different topological configuration of particle interactions, with vertices marked by dots and lines representing particles. The diagrams are associated with various mathematical coefficients, including  $\frac{1}{24}$ ,  $\frac{i\zeta(3)}{8\pi^3}$ , and  $\frac{i\zeta(3)}{4\pi^3}$ , which are placed next to them. The diagrams show various combinations of solid and dashed lines, and some include a minus sign in parentheses.

$$\begin{aligned}
& +\frac{i\zeta(3)}{8\pi^3} \left( \text{diagram 1} \right) - \frac{i\zeta(3)}{4\pi^3} \left( \text{diagram 2} \right) + \frac{i\zeta(3)}{8\pi^3} \left( \text{diagram 3} \right) + \frac{i\zeta(3)}{8\pi^3} \left( \text{diagram 4} \right) \\
& - \frac{i\zeta(3)}{4\pi^3} \left( \text{diagram 5} \right) - \frac{i\zeta(3)}{8\pi^3} \left( \text{diagram 6} \right) + \frac{i\zeta(3)}{8\pi^3} \left( \text{diagram 7} \right) + \frac{i\zeta(3)}{8\pi^3} \left( \text{diagram 8} \right) \\
& - \frac{i\zeta(3)}{4\pi^3} \left( \text{diagram 9} \right) + \frac{i\zeta(3)}{8\pi^3} \left( \text{diagram 10} \right) - \frac{i\zeta(3)}{8\pi^3} \left( \text{diagram 11} \right) + \frac{i\zeta(3)}{4\pi^3} \left( \text{diagram 12} \right) \\
& - \frac{i\zeta(3)}{8\pi^3} \left( \text{diagram 13} \right) - \frac{i\zeta(3)}{8\pi^3} \left( \text{diagram 14} \right) - \frac{i\zeta(3)}{8\pi^3} \left( \text{diagram 15} \right) + \frac{i\zeta(3)}{4\pi^3} \left( \text{diagram 16} \right) \\
& - \frac{i\zeta(3)}{8\pi^3} \left( \text{diagram 17} \right) - \frac{i\zeta(3)}{8\pi^3} \left( \text{diagram 18} \right) + \frac{i\zeta(3)}{4\pi^3} \left( \text{diagram 19} \right) - \frac{i\zeta(3)}{8\pi^3} \left( \text{diagram 20} \right) \\
& \left( \text{diagram 21} \right) = 0
\end{aligned}$$

$\nu^{-1}$  = from the [CMD] book on page 345 is

$$\text{diagram 1} - \frac{1}{24} \text{diagram 2} + \frac{1}{24} \text{diagram 3}$$

$\nu^{-1}$  = from the the KZ associator

$$\text{diagram 1} - \frac{1}{24} \text{diagram 2} + \frac{1}{24} \text{diagram 3}$$

Their difference: 0

Calculate a:

$$a = \text{diagram 1} + \frac{1}{24} \left( \text{diagram 2} - \text{diagram 3} + \frac{i\zeta(3)}{8\pi^3} \text{diagram 4} + \frac{i\zeta(3)}{8\pi^3} \text{diagram 5} - \frac{i\zeta(3)}{4\pi^3} \text{diagram 6} \right)$$

Associator in the choses basis is:

$$\begin{aligned}
\Phi = & \text{diagram 1} + \frac{1}{24} \left( \text{diagram 2} - \text{diagram 3} + \frac{i\zeta(3)}{8\pi^3} \text{diagram 4} - \frac{i\zeta(3)}{4\pi^3} \text{diagram 5} \right) \\
& + \frac{i\zeta(3)}{8\pi^3} \text{diagram 6} - \frac{i\zeta(3)}{8\pi^3} \text{diagram 7} - \frac{i\zeta(3)}{8\pi^3} \text{diagram 8} \\
& - \frac{i\zeta(3)}{8\pi^3} \text{diagram 9} + \frac{i\zeta(3)}{4\pi^3} \text{diagram 10} + \frac{i\zeta(3)}{4\pi^3} \text{diagram 11}
\end{aligned}$$

Associator twisted by  $a$ , in the chosés basis, is:

$$\begin{aligned}
 \Phi^a = & \text{Diagram 1} + \frac{1}{24} \text{Diagram 2} - \frac{1}{24} \text{Diagram 3} - \frac{i\zeta(3)}{8\pi^3} \text{Diagram 4} + \frac{i\zeta(3)}{4\pi^3} \text{Diagram 5} \\
 & - \frac{i\zeta(3)}{8\pi^3} \text{Diagram 6} + \frac{i\zeta(3)}{8\pi^3} \text{Diagram 7} + \frac{i\zeta(3)}{8\pi^3} \text{Diagram 8} + \frac{i\zeta(3)}{8\pi^3} \text{Diagram 9} \\
 & + \frac{i\zeta(3)}{8\pi^3} \text{Diagram 10} - \frac{i\zeta(3)}{4\pi^3} \text{Diagram 11} - \frac{i\zeta(3)}{4\pi^3} \text{Diagram 12}
 \end{aligned}$$

The diagrams are represented as follows:

- Diagram 1:** Three vertical lines with dots at the top and bottom. The top dots are connected by a dashed line, and the bottom dots are connected by a dashed line.
- Diagram 2:** Three vertical lines with dots at the top and bottom. The top dots are connected by a dashed line, and the bottom dots are connected by a dashed line. The top dots are also connected to the bottom dots by dashed lines.
- Diagram 3:** Three vertical lines with dots at the top and bottom. The top dots are connected by a dashed line, and the bottom dots are connected by a dashed line. The top dots are also connected to the bottom dots by dashed lines.
- Diagram 4:** Three vertical lines with dots at the top and bottom. The top dots are connected by a dashed line, and the bottom dots are connected by a dashed line. The top dots are also connected to the bottom dots by dashed lines.
- Diagram 5:** Three vertical lines with dots at the top and bottom. The top dots are connected by a dashed line, and the bottom dots are connected by a dashed line. The top dots are also connected to the bottom dots by dashed lines.
- Diagram 6:** Three vertical lines with dots at the top and bottom. The top dots are connected by a dashed line, and the bottom dots are connected by a dashed line. The top dots are also connected to the bottom dots by dashed lines.
- Diagram 7:** Three vertical lines with dots at the top and bottom. The top dots are connected by a dashed line, and the bottom dots are connected by a dashed line. The top dots are also connected to the bottom dots by dashed lines.
- Diagram 8:** Three vertical lines with dots at the top and bottom. The top dots are connected by a dashed line, and the bottom dots are connected by a dashed line. The top dots are also connected to the bottom dots by dashed lines.
- Diagram 9:** Three vertical lines with dots at the top and bottom. The top dots are connected by a dashed line, and the bottom dots are connected by a dashed line. The top dots are also connected to the bottom dots by dashed lines.
- Diagram 10:** Three vertical lines with dots at the top and bottom. The top dots are connected by a dashed line, and the bottom dots are connected by a dashed line. The top dots are also connected to the bottom dots by dashed lines.
- Diagram 11:** Three vertical lines with dots at the top and bottom. The top dots are connected by a dashed line, and the bottom dots are connected by a dashed line. The top dots are also connected to the bottom dots by dashed lines.
- Diagram 12:** Three vertical lines with dots at the top and bottom. The top dots are connected by a dashed line, and the bottom dots are connected by a dashed line. The top dots are also connected to the bottom dots by dashed lines.