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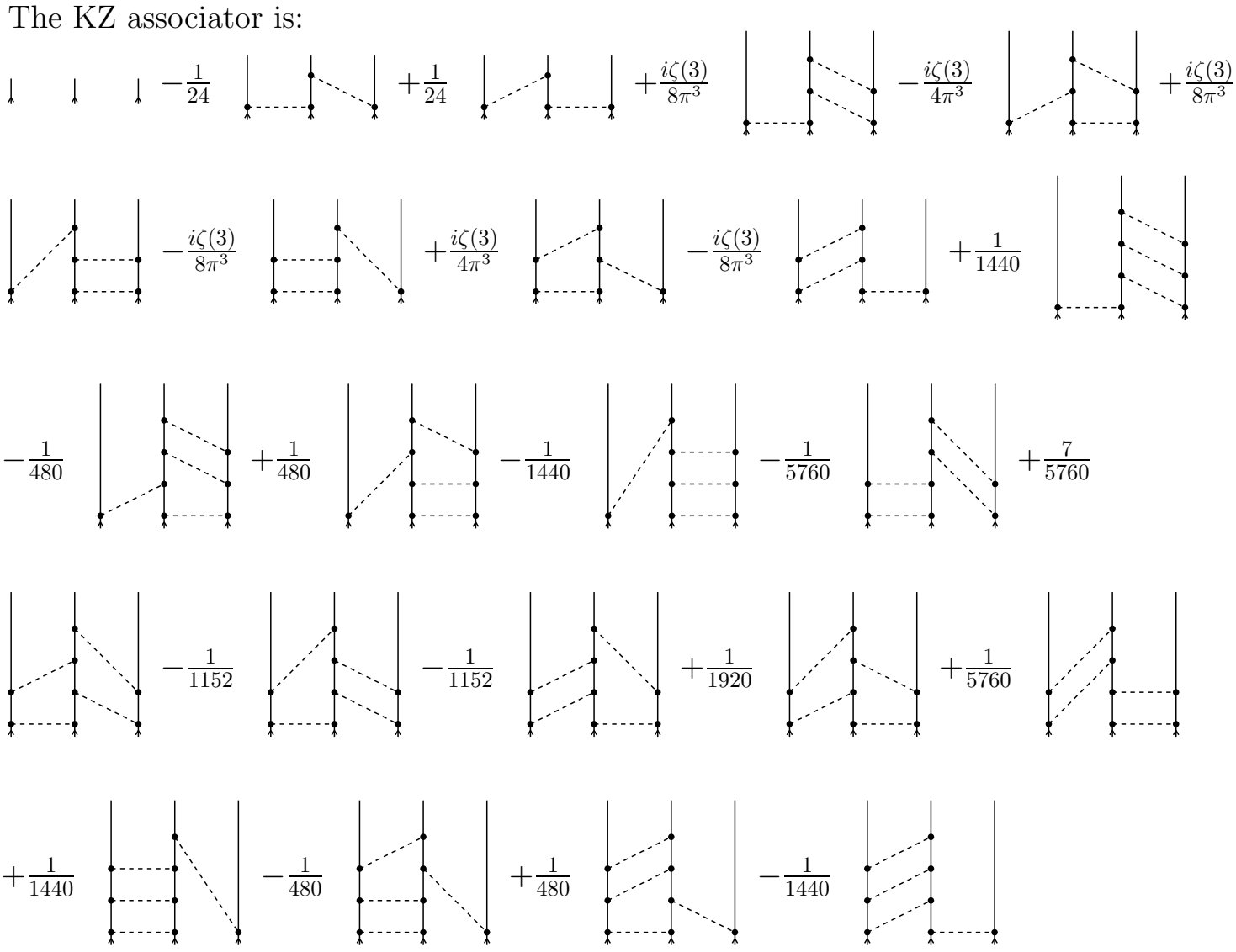
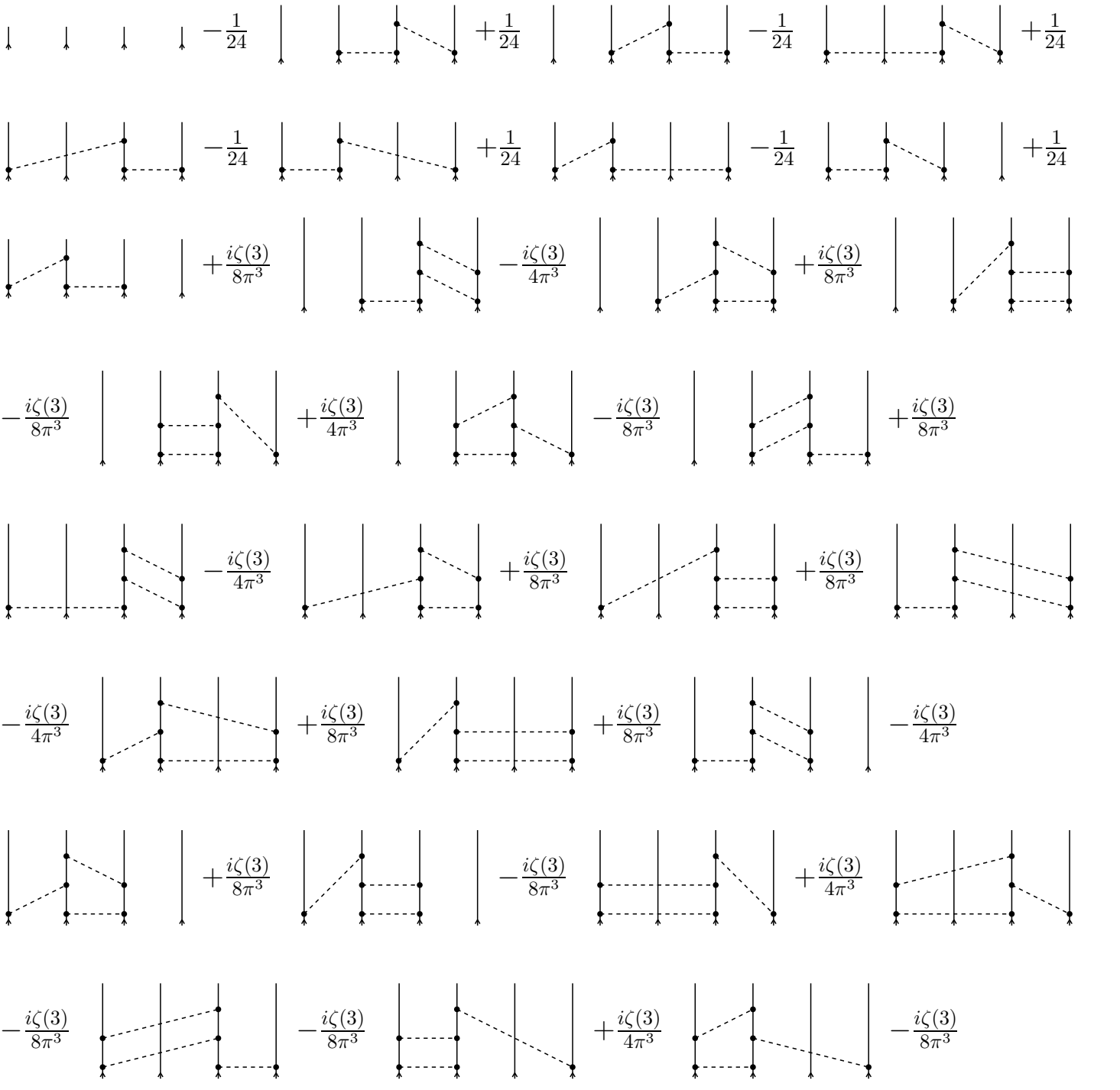
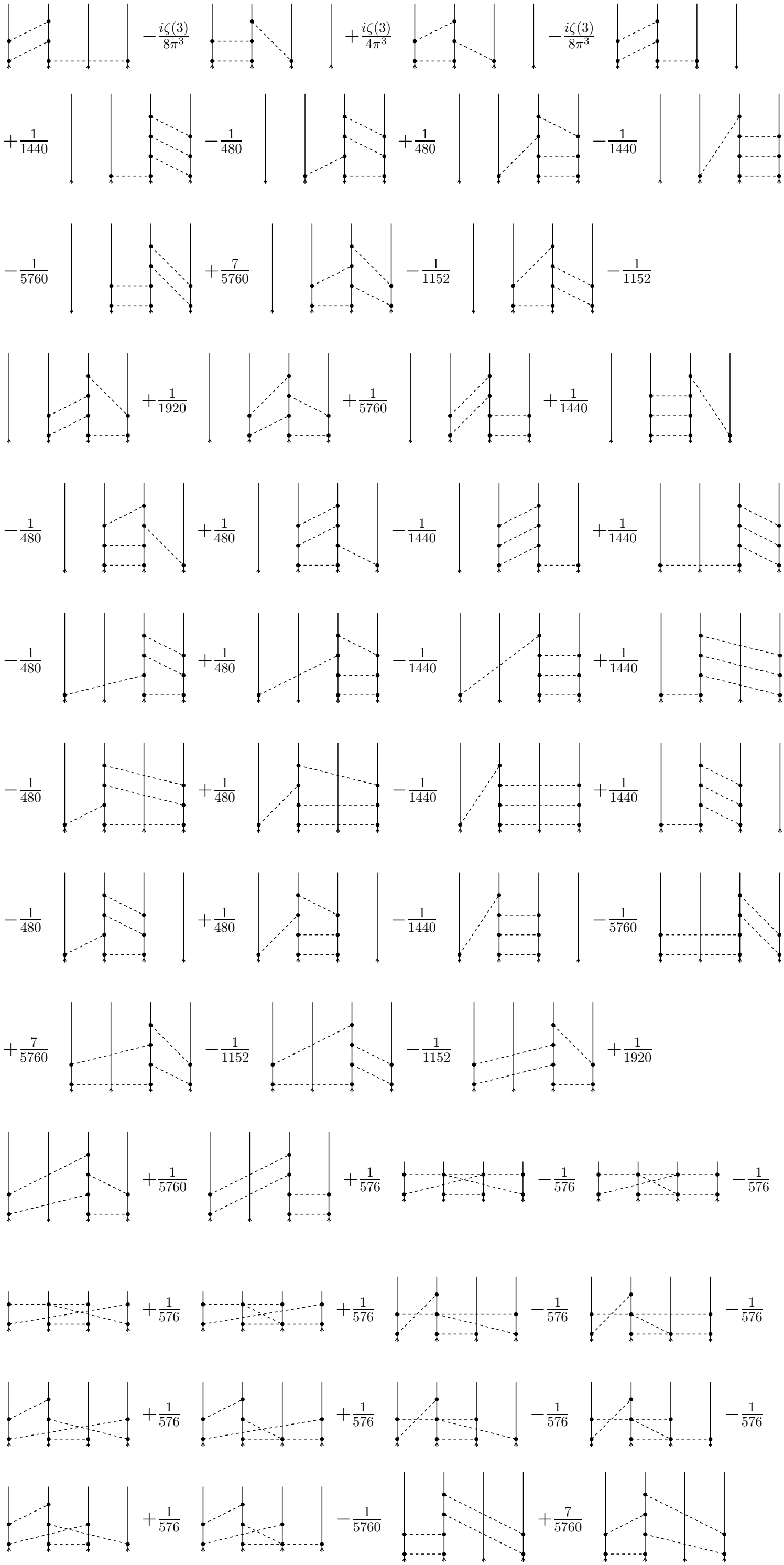


Pentagon up to order 4

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$+\frac{i\zeta(3)}{4\pi^3}$
$-\frac{i\zeta(3)}{8\pi^3}$
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$-\frac{1}{480}$
$+\frac{1}{480}$
$-\frac{1}{1440}$
$-\frac{1}{5760}$
$+\frac{7}{5760}$
$-\frac{1}{1152}$
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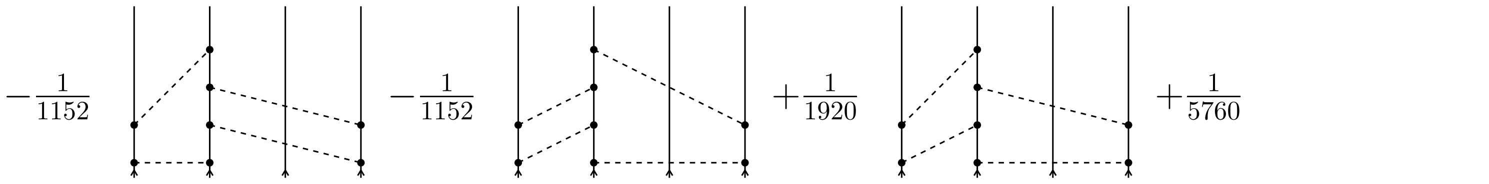

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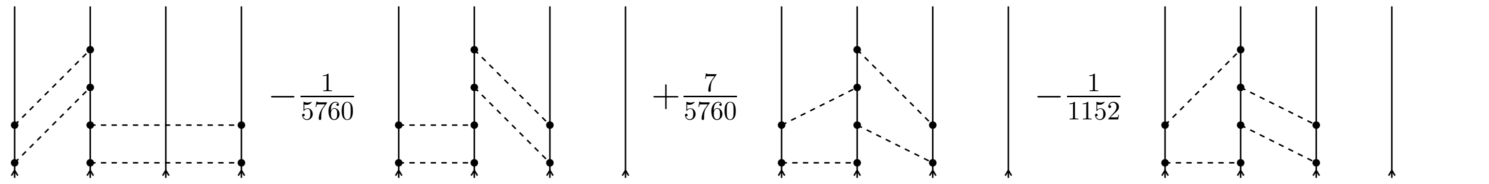

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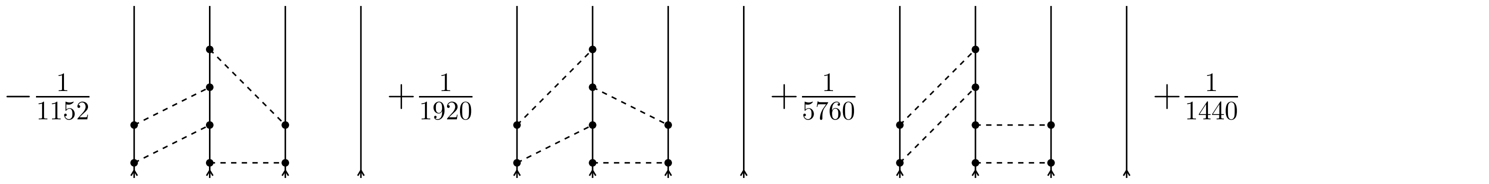

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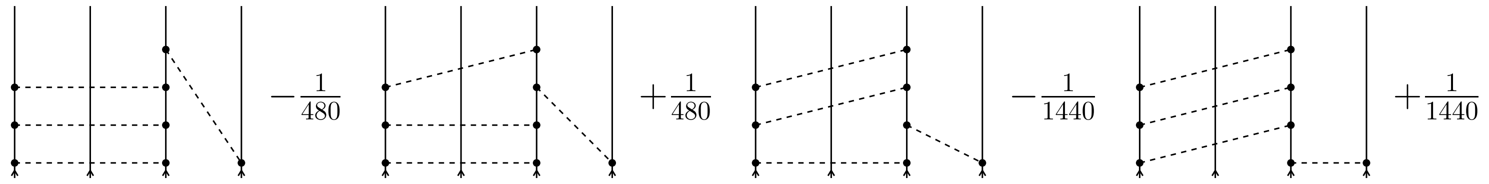

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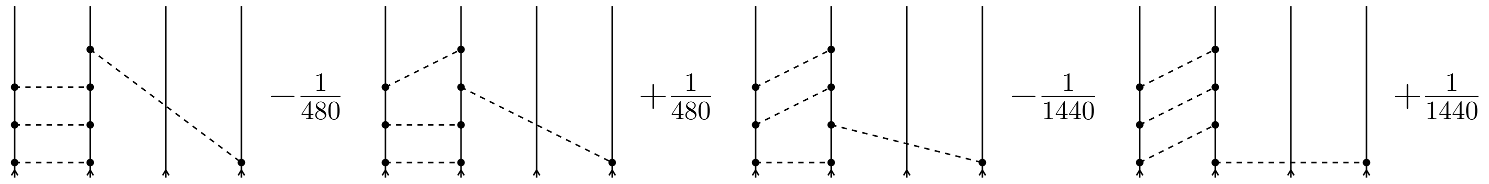

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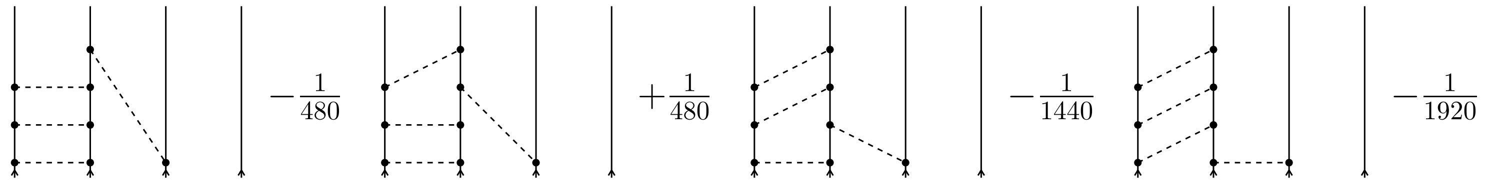

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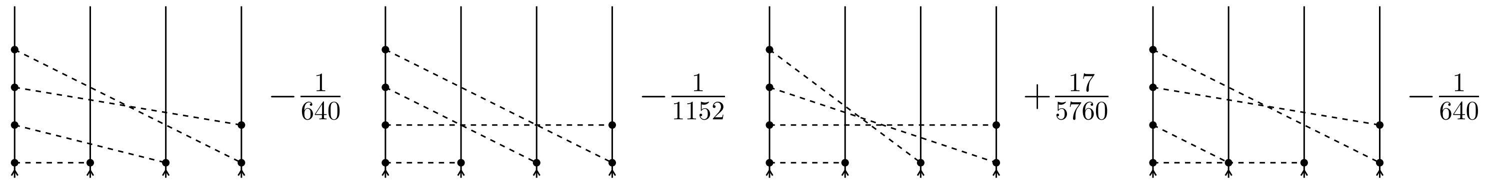

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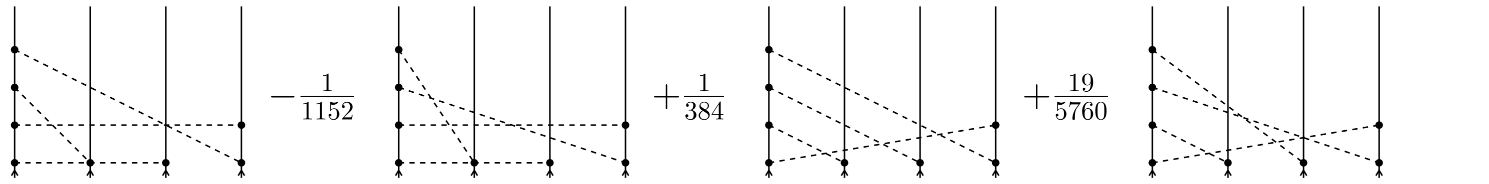

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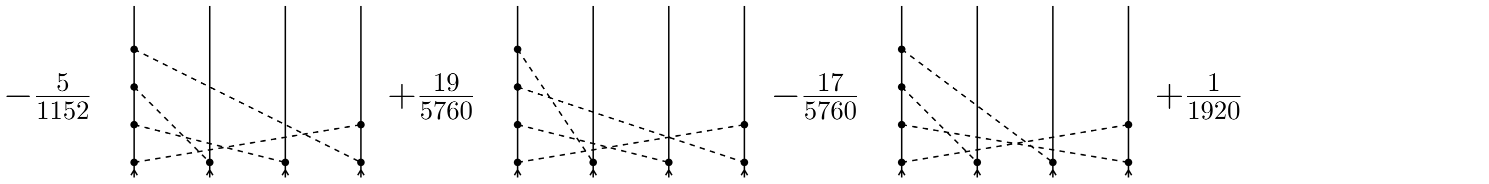

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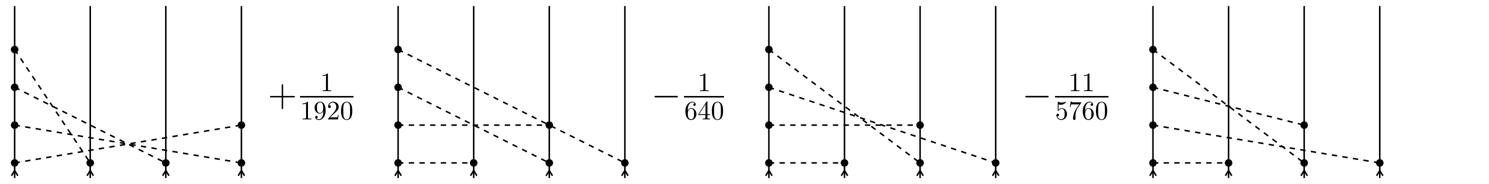

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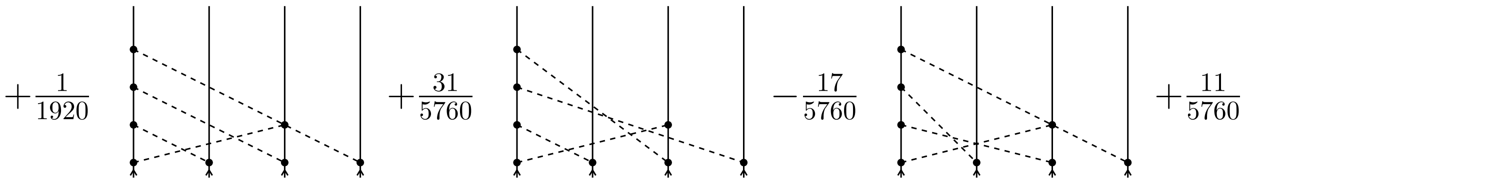
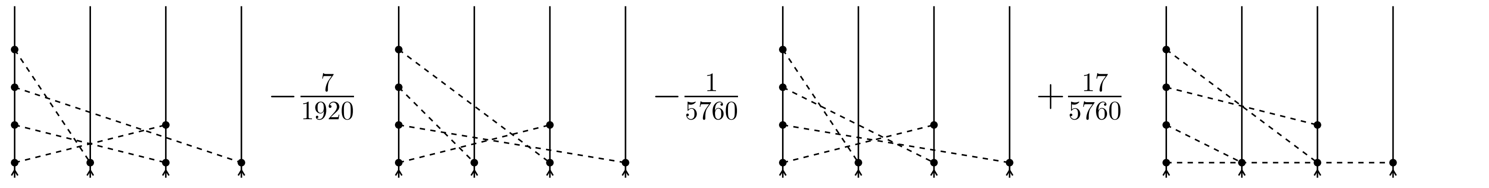
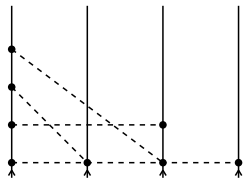
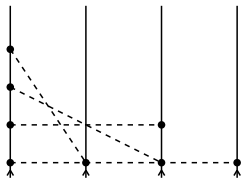
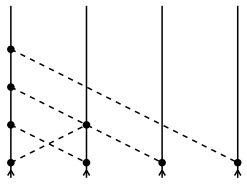
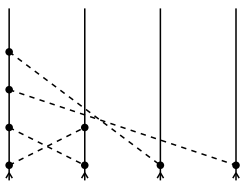
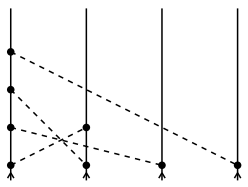
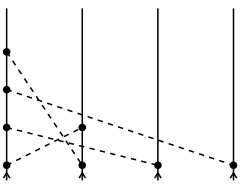
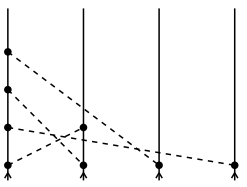


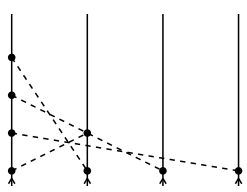
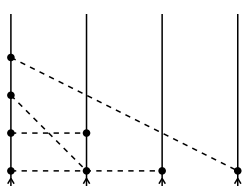
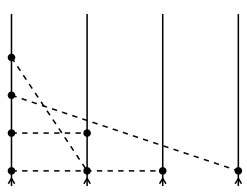
$+\frac{1}{1920}$ $+\frac{31}{5760}$ $-\frac{17}{5760}$ $+\frac{11}{5760}$

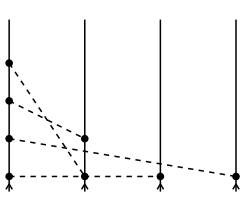
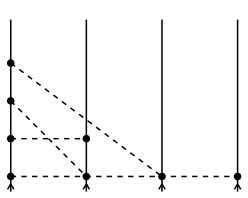
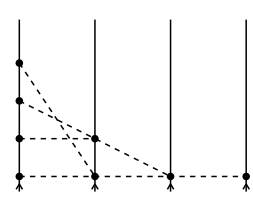
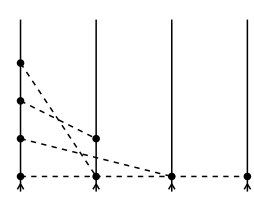


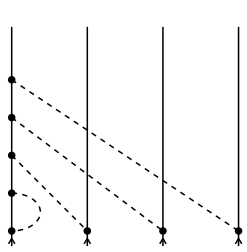
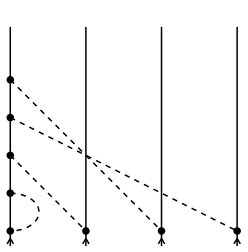
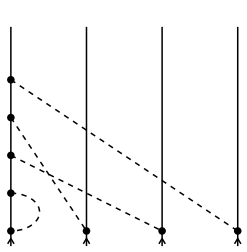
$-\frac{7}{1920}$ $-\frac{1}{5760}$ $+\frac{17}{5760}$

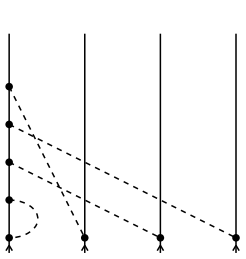
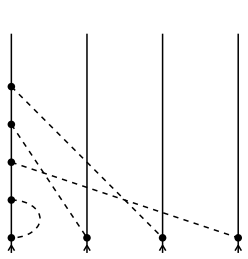
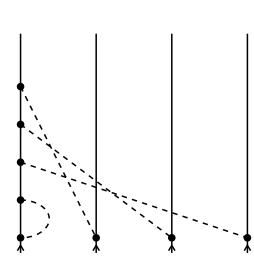
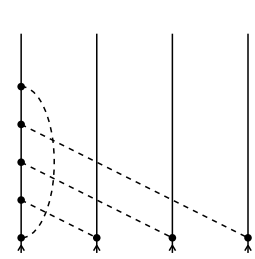
$-\frac{13}{5760}$  $+\frac{7}{5760}$  $+\frac{1}{1920}$  $+\frac{11}{5760}$

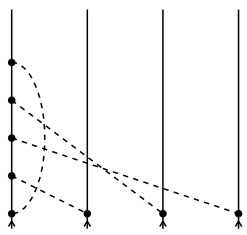
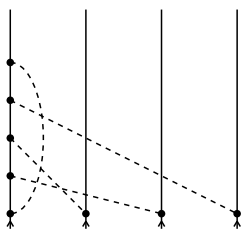
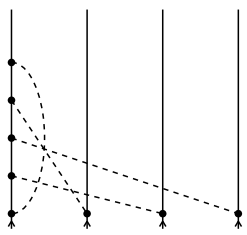
 $+\frac{1}{1920}$  $-\frac{1}{640}$  $-\frac{7}{1920}$ 

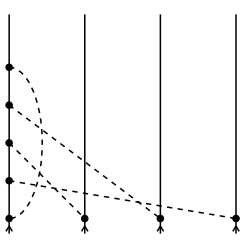
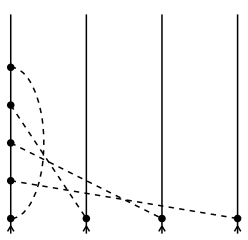
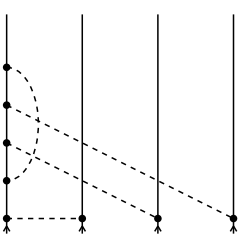
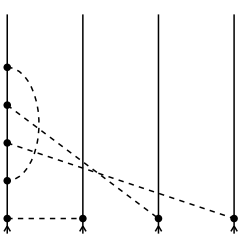
$-\frac{1}{5760}$  $-\frac{17}{5760}$  $+\frac{31}{5760}$  $-\frac{11}{5760}$

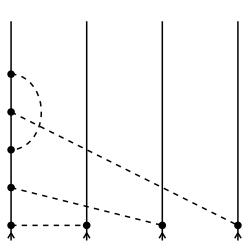
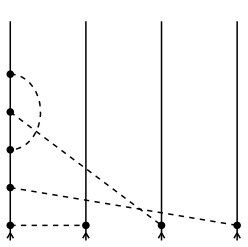
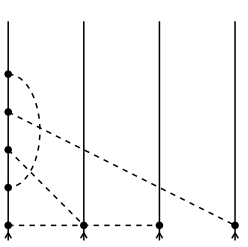
 $+\frac{7}{5760}$  $+\frac{7}{5760}$  $-\frac{1}{1920}$ 

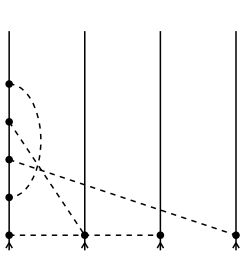
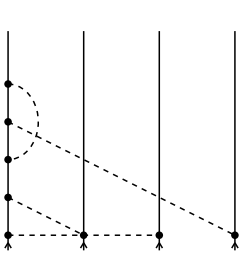
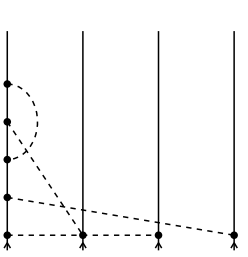
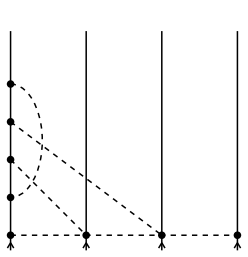
$-\frac{1}{1920}$  $+\frac{1}{1152}$  $+\frac{17}{5760}$  $+\frac{1}{1152}$

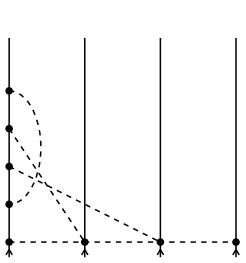
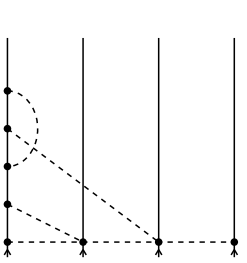
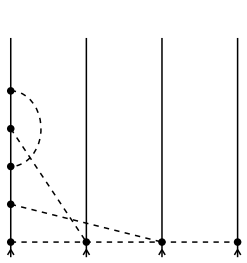
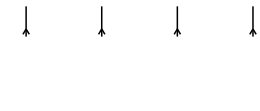
 $-\frac{7}{5760}$  $-\frac{7}{5760}$  $+\frac{7}{5760}$ 

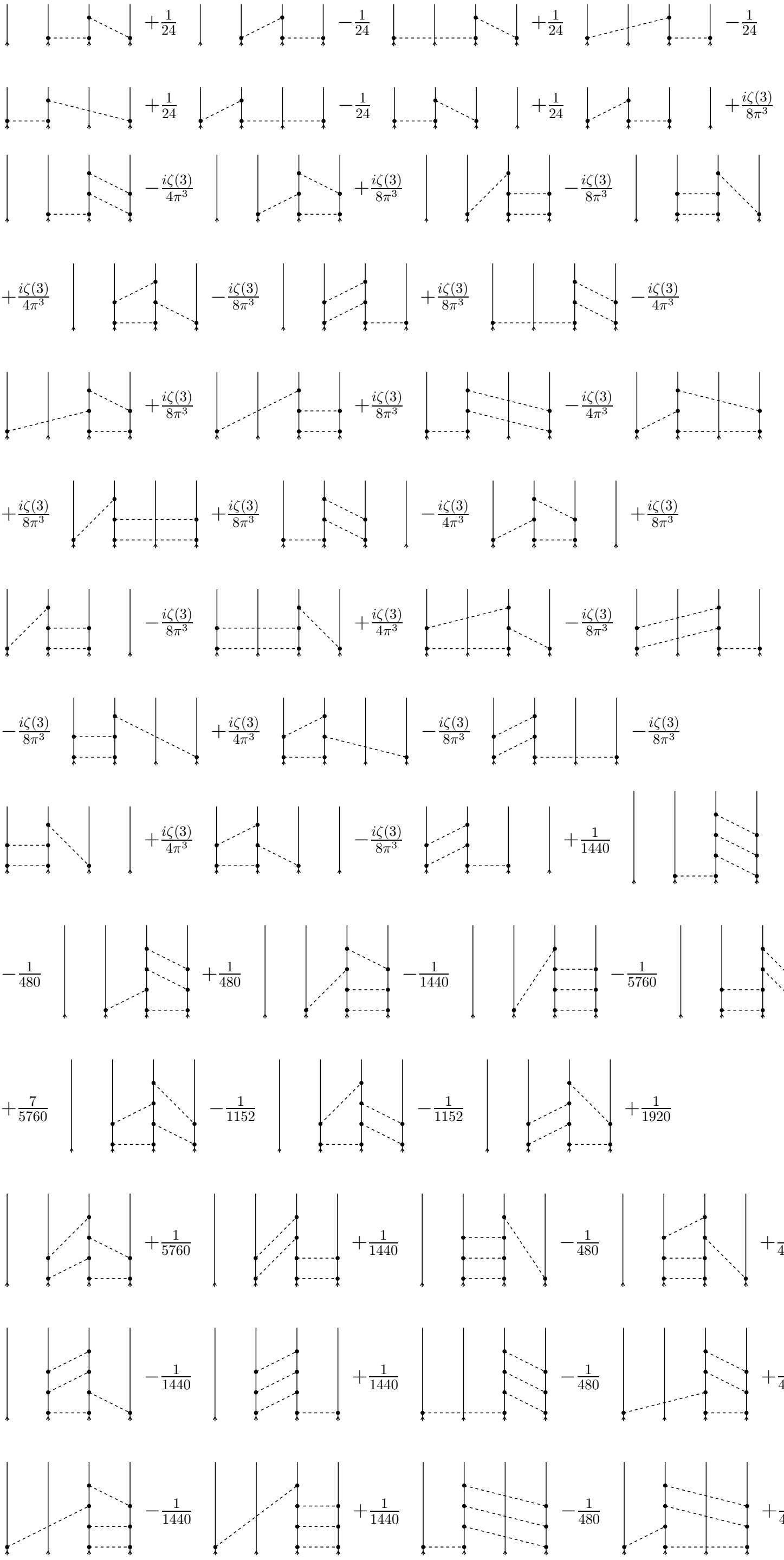
$-\frac{1}{1152}$  $+\frac{7}{5760}$  $-\frac{1}{1152}$  $-\frac{17}{5760}$

 $+\frac{1}{1920}$  $-\frac{1}{640}$  $+\frac{19}{5760}$ 

$+\frac{1}{1152}$  $-\frac{19}{5760}$  $-\frac{1}{640}$  $+\frac{19}{5760}$

 $-\frac{1}{384}$  $-\frac{19}{5760}$  $+\frac{1}{384}$  $-\frac{1}{1152}$

 $+\frac{1}{640}$  $+\frac{1}{640}$  $-($  $-\frac{1}{24}$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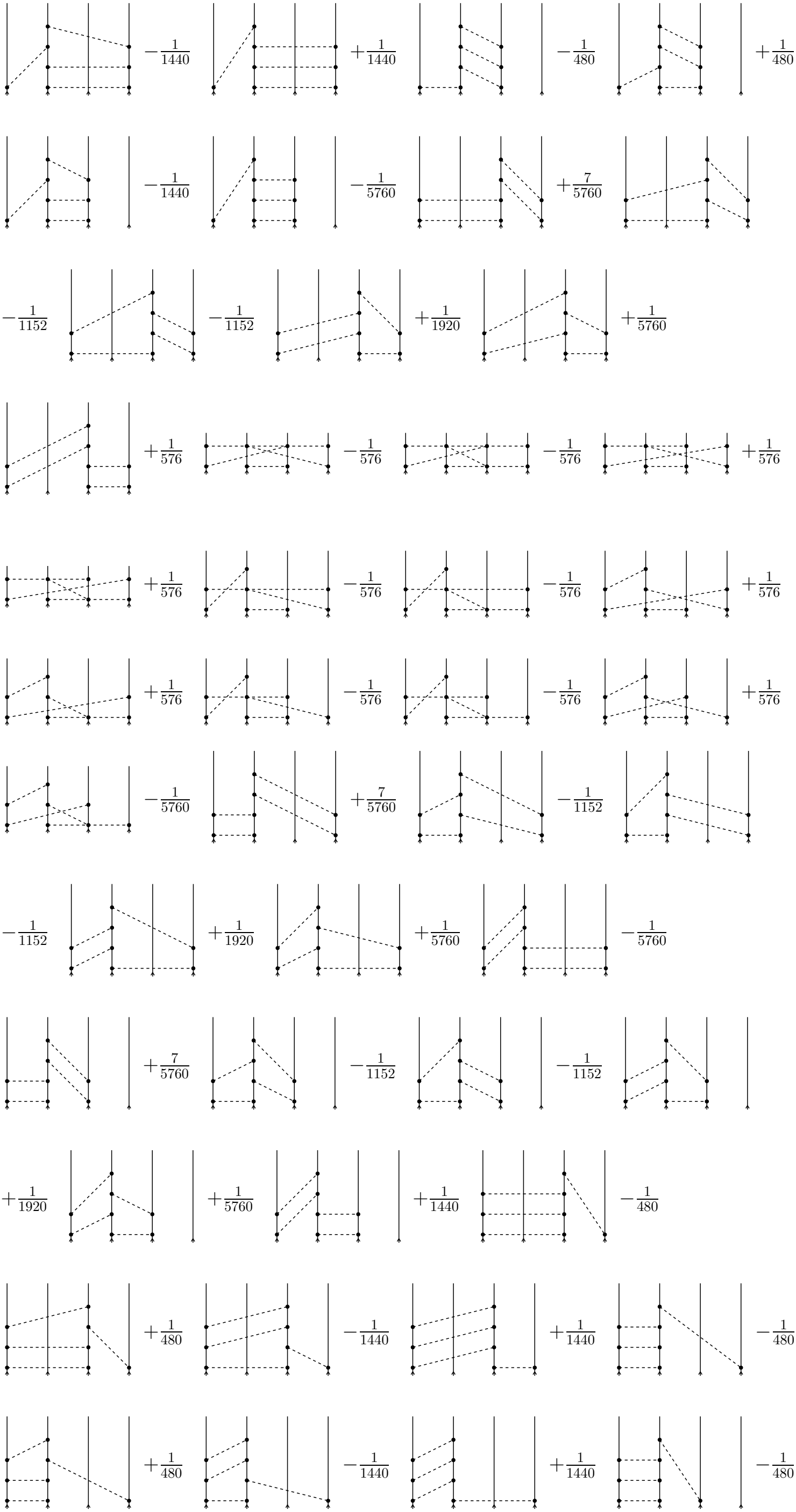
+iζ(3)/8π³
+iζ(3)/8π³
+iζ(3)/8π³
−iζ(3)/4π³
+iζ(3)/8π³
−iζ(3)/8π³
+iζ(3)/4π³
−iζ(3)/8π³
−iζ(3)/8π³
+iζ(3)/4π³
−iζ(3)/8π³
−iζ(3)/8π³
+iζ(3)/4π³
−iζ(3)/8π³
+1/1440
−1/480
+1/480
−1/1440
−1/5760
+7/5760
−1/1152
−1/1152
+1/1920
+1/5760
+1/1440
−1/480
+1/480
−1/1440
+1/1440
−1/480
+1/480
−1/1440
+1/1440
−1/480
+1/480


$-\frac{1}{1440}$
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$-\frac{1}{1152}$
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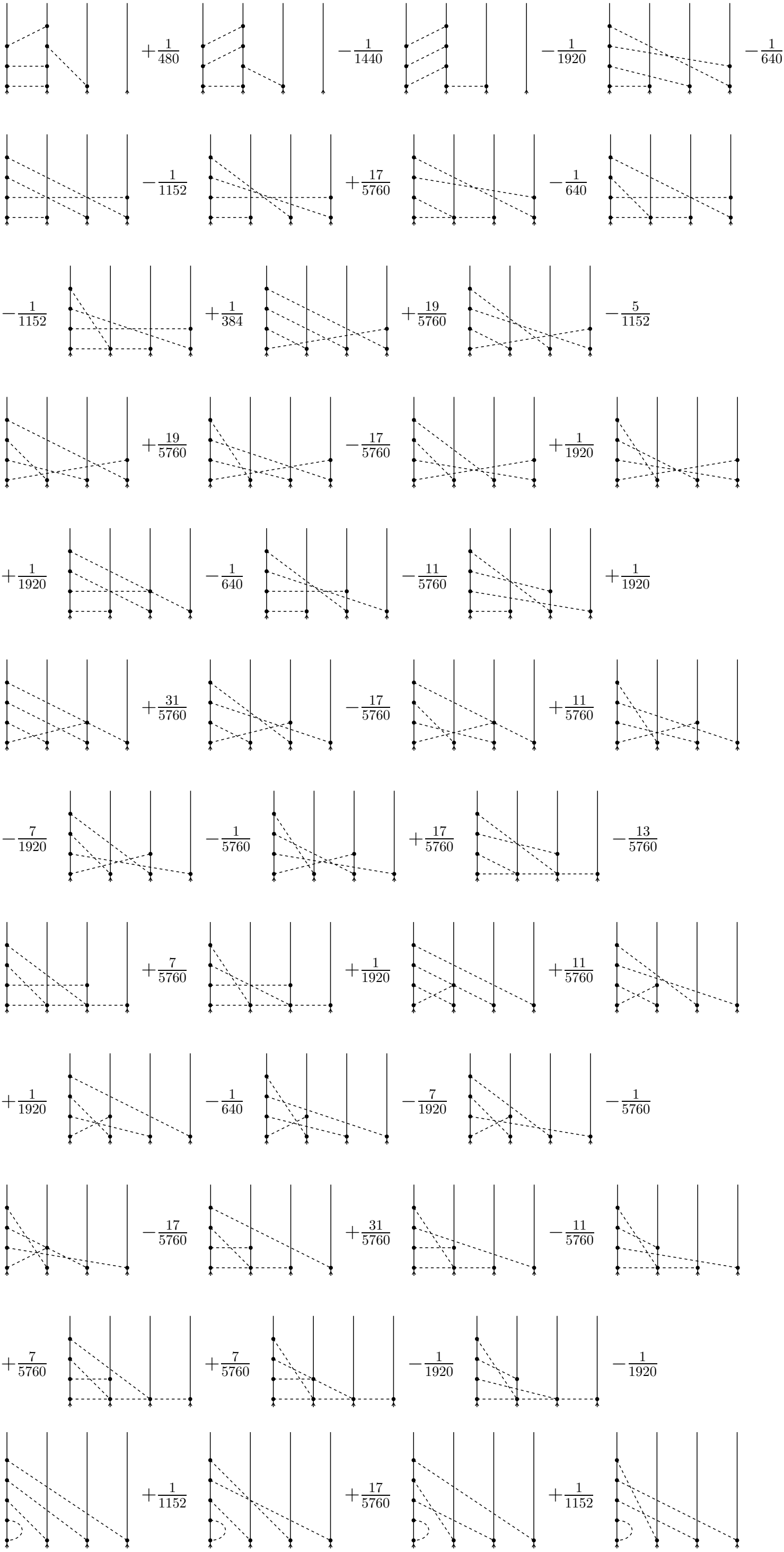
$-\frac{1}{480}$


+1/480
−1/1440
−1/1920
−1/640
−1/1152
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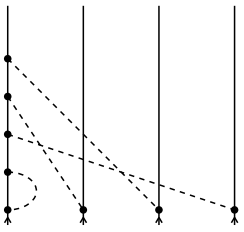
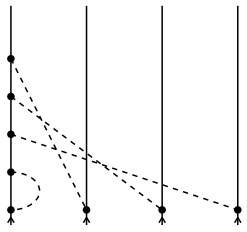
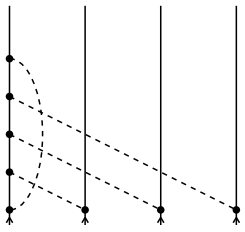
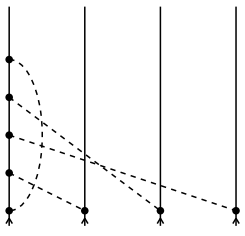
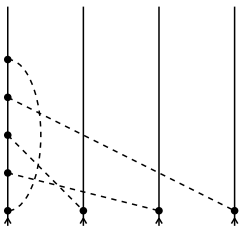
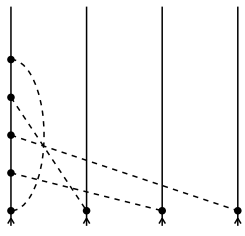
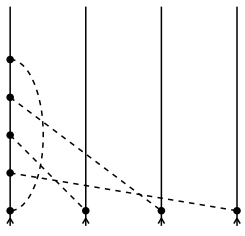
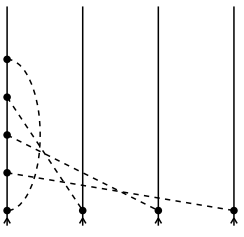
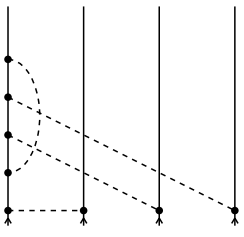
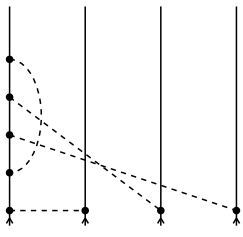
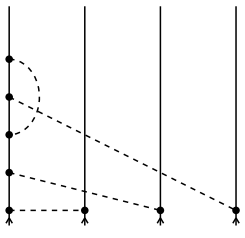
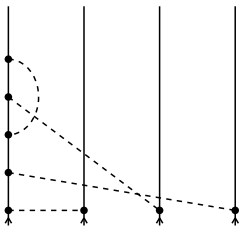
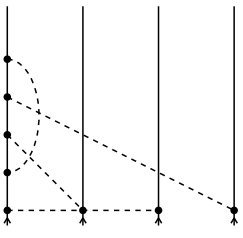
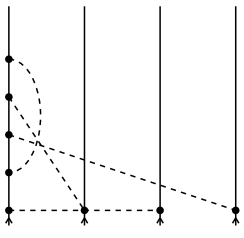
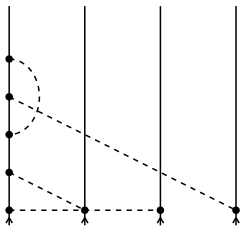
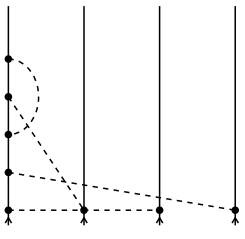
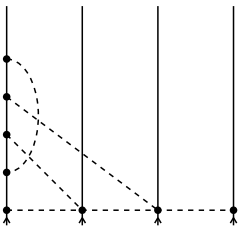
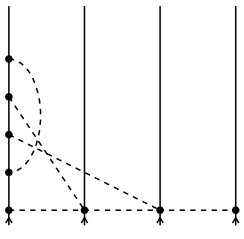
−1/1920
+1/1152
+17/5760
+1/1152

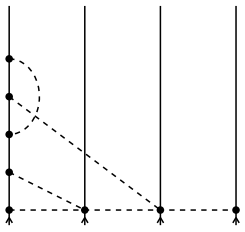
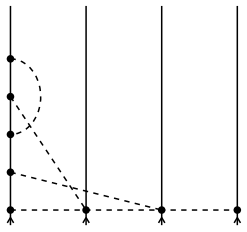
$-\frac{7}{5760}$  $-\frac{7}{5760}$  $+\frac{7}{5760}$  $-\frac{1}{1152}$

 $+\frac{7}{5760}$  $-\frac{1}{1152}$  $-\frac{17}{5760}$ 

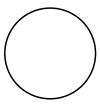
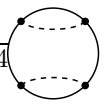
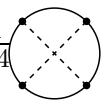
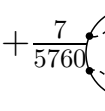
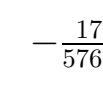
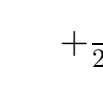
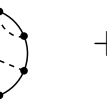
$+\frac{1}{1920}$  $-\frac{1}{640}$  $+\frac{19}{5760}$  $+\frac{1}{1152}$

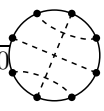
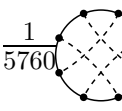
 $-\frac{19}{5760}$  $-\frac{1}{640}$  $+\frac{19}{5760}$  $-\frac{1}{384}$

 $-\frac{19}{5760}$  $+\frac{1}{384}$  $-\frac{1}{1152}$  $+\frac{1}{640}$

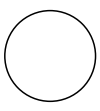
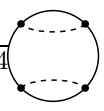
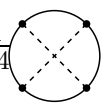
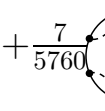
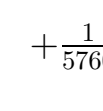
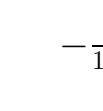
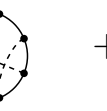
 $+\frac{1}{640}$  $) = \quad 0$

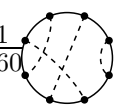
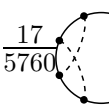
ν^{-1} = from the [CMD] book on page 345 is

 $-\frac{1}{24}$  $+\frac{1}{24}$  $+\frac{7}{5760}$  $-\frac{17}{5760}$  $+\frac{7}{2880}$  $+\frac{1}{1920}$ 

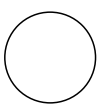
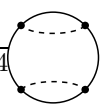
$-\frac{1}{720}$  $+\frac{1}{5760}$ 

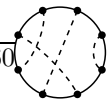
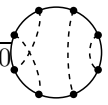
the same element, written in our basis, is

 $-\frac{1}{24}$  $+\frac{1}{24}$  $+\frac{7}{5760}$  $+\frac{1}{5760}$  $-\frac{1}{1152}$  $+\frac{1}{1920}$ 

$+\frac{11}{5760}$  $-\frac{17}{5760}$ 

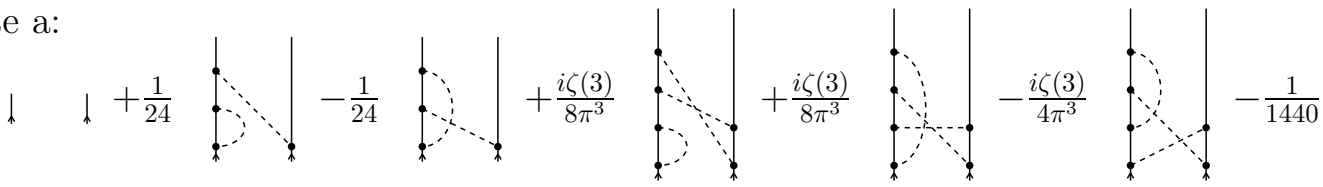
ν^{-1} = from the the KZ associator

 $-\frac{1}{24}$  $+\frac{1}{24}$ $+\frac{7}{5760}$ $+\frac{1}{5760}$ $-\frac{1}{1152}$ $+\frac{1}{1920}$

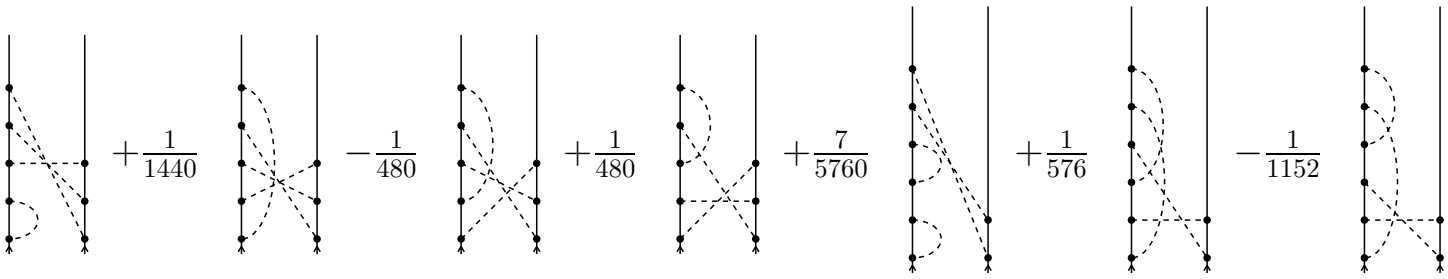
$+\frac{11}{5760}$  $-\frac{17}{5760}$ 

Their difference: 0

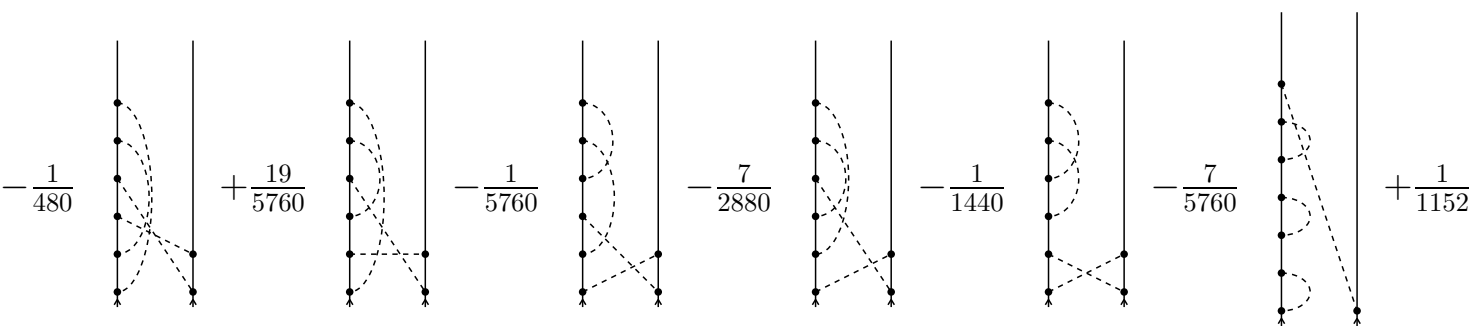
Calculate a:

a = 

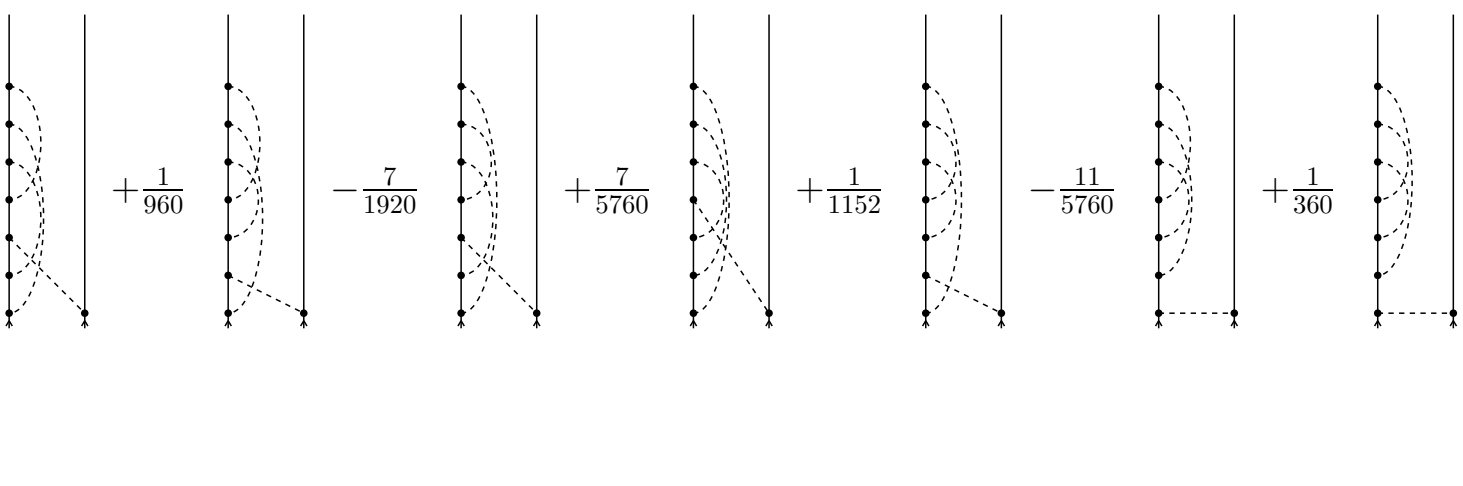
$+\frac{1}{24}$ $-\frac{1}{24}$ $+\frac{i\zeta(3)}{8\pi^3}$ $+\frac{i\zeta(3)}{8\pi^3}$ $-\frac{i\zeta(3)}{4\pi^3}$ $-\frac{1}{1440}$



$+\frac{1}{1440}$ $-\frac{1}{480}$ $+\frac{1}{480}$ $+\frac{7}{5760}$ $+\frac{1}{576}$ $-\frac{1}{1152}$

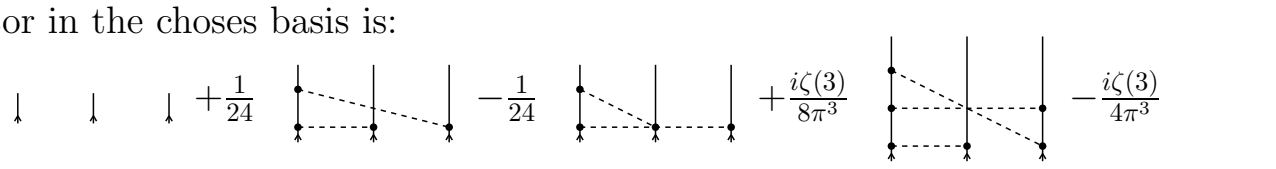


$-\frac{1}{480}$ $+\frac{19}{5760}$ $-\frac{1}{5760}$ $-\frac{7}{2880}$ $-\frac{1}{1440}$ $-\frac{7}{5760}$ $+\frac{1}{1152}$

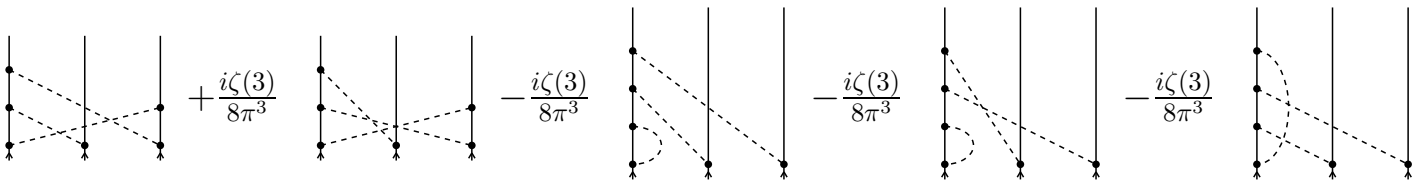


$+\frac{1}{960}$ $-\frac{7}{1920}$ $+\frac{7}{5760}$ $+\frac{1}{1152}$ $-\frac{11}{5760}$ $+\frac{1}{360}$

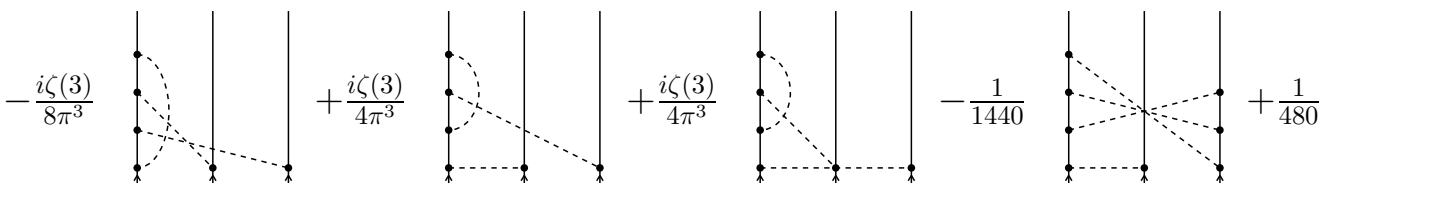
Associator in the choses basis is:

Φ = 

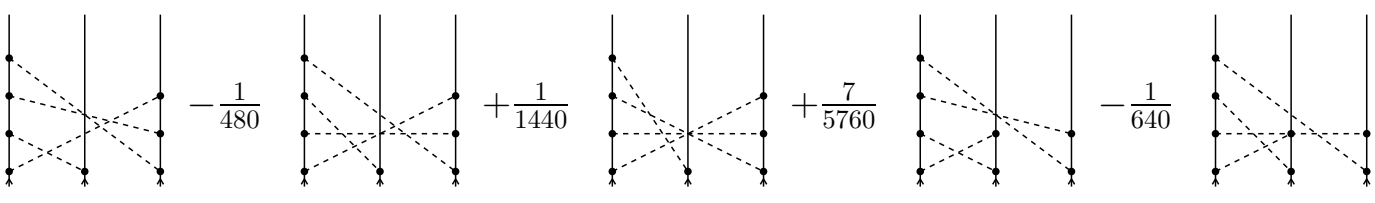
$+\frac{1}{24}$ $-\frac{1}{24}$ $+\frac{i\zeta(3)}{8\pi^3}$ $-\frac{i\zeta(3)}{4\pi^3}$



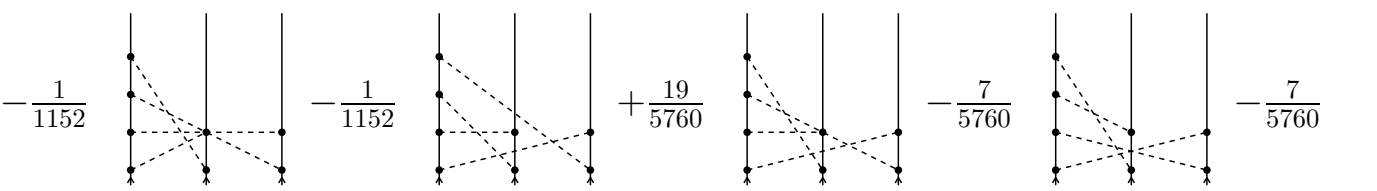
$+\frac{i\zeta(3)}{8\pi^3}$ $-\frac{i\zeta(3)}{8\pi^3}$ $-\frac{i\zeta(3)}{8\pi^3}$ $-\frac{i\zeta(3)}{8\pi^3}$



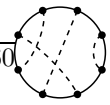
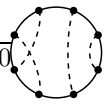
$-\frac{i\zeta(3)}{8\pi^3}$ $+\frac{i\zeta(3)}{4\pi^3}$ $+\frac{i\zeta(3)}{4\pi^3}$ $-\frac{1}{1440}$ $+\frac{1}{480}$



$-\frac{1}{480}$ $+\frac{1}{1440}$ $+\frac{7}{5760}$ $-\frac{1}{640}$

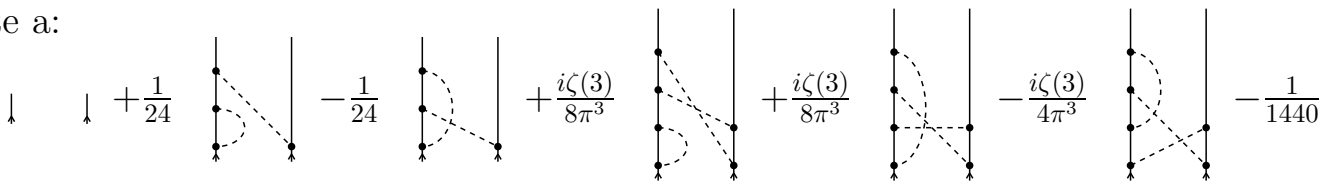


$-\frac{1}{1152}$ $-\frac{1}{1152}$ $+\frac{19}{5760}$ $-\frac{7}{5760}$ $-\frac{7}{5760}$

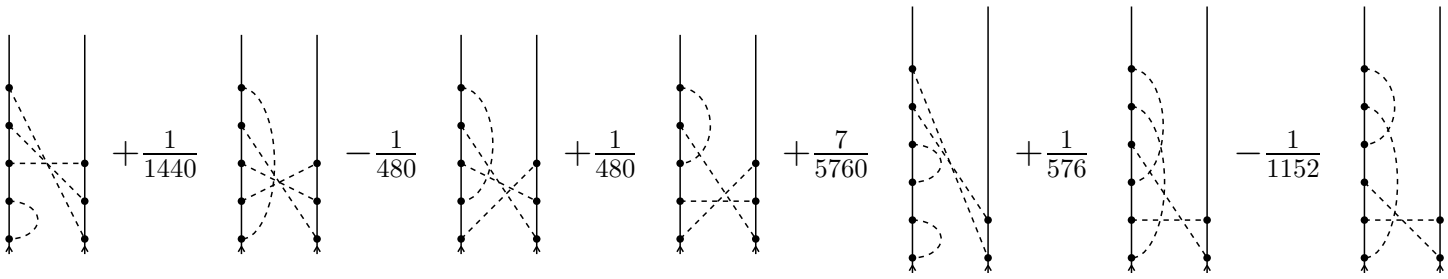
$+\frac{11}{5760}$  $-\frac{17}{5760}$ 

Their difference: 0

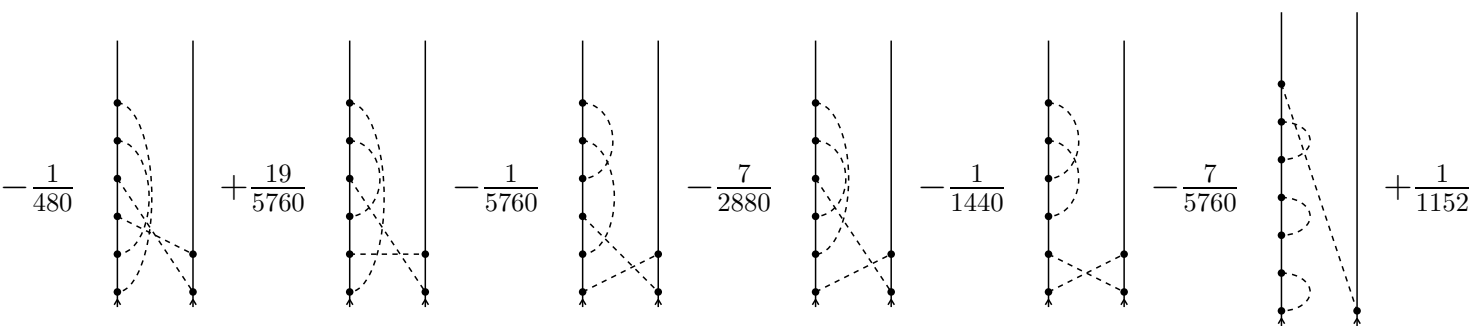
Calculate a:

a = 

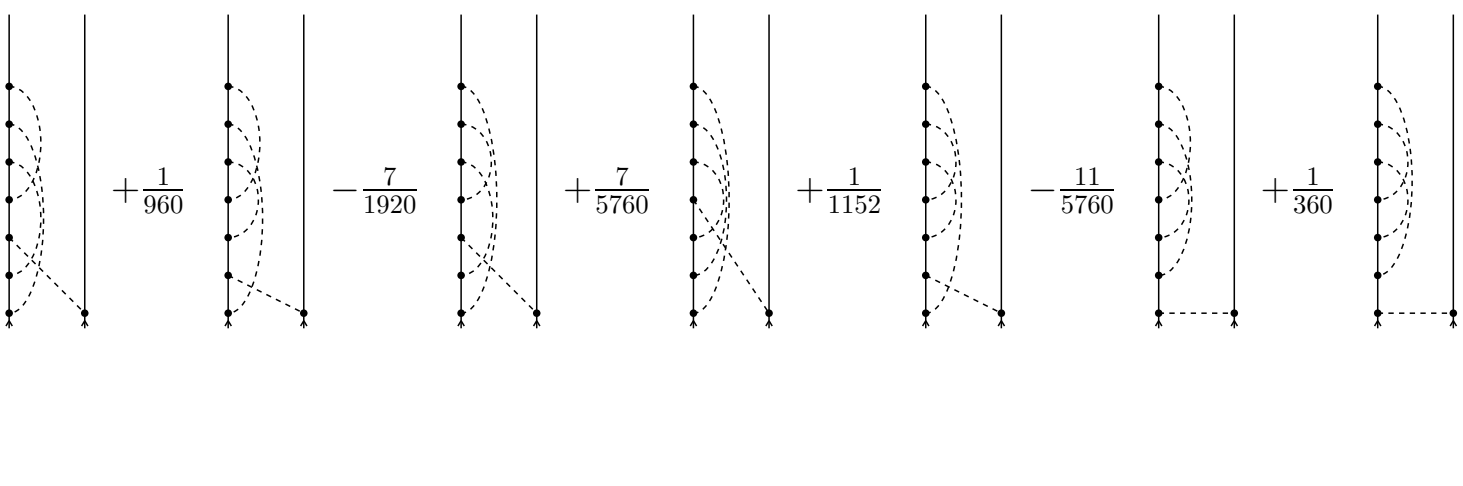
$+\frac{1}{24}$ $-\frac{1}{24}$ $+\frac{i\zeta(3)}{8\pi^3}$ $+\frac{i\zeta(3)}{8\pi^3}$ $-\frac{i\zeta(3)}{4\pi^3}$ $-\frac{1}{1440}$



$+\frac{1}{1440}$ $-\frac{1}{480}$ $+\frac{1}{480}$ $+\frac{7}{5760}$ $+\frac{1}{576}$ $-\frac{1}{1152}$

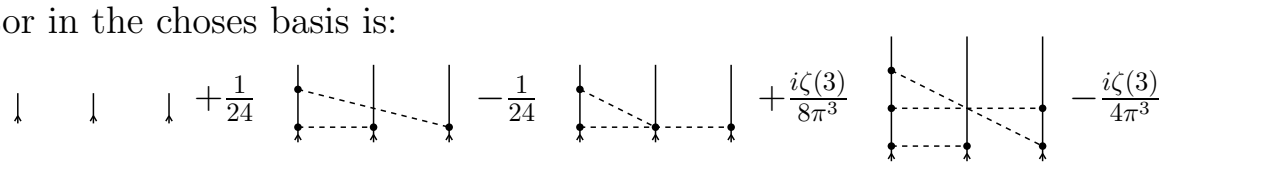


$-\frac{1}{480}$ $+\frac{19}{5760}$ $-\frac{1}{5760}$ $-\frac{7}{2880}$ $-\frac{1}{1440}$ $-\frac{7}{5760}$ $+\frac{1}{1152}$

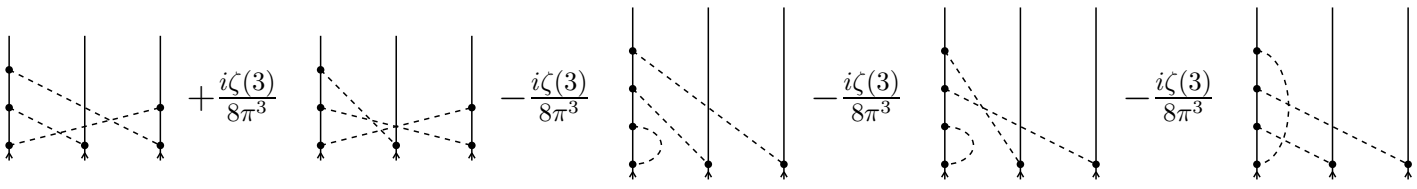


$+\frac{1}{960}$ $-\frac{7}{1920}$ $+\frac{7}{5760}$ $+\frac{1}{1152}$ $-\frac{11}{5760}$ $+\frac{1}{360}$

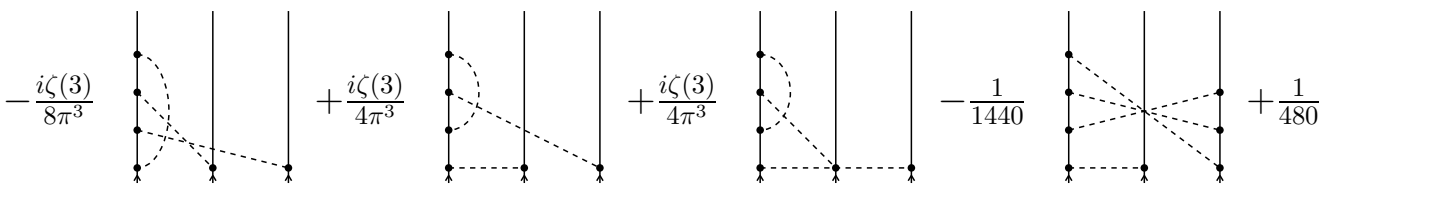
Associator in the choses basis is:

Φ = 

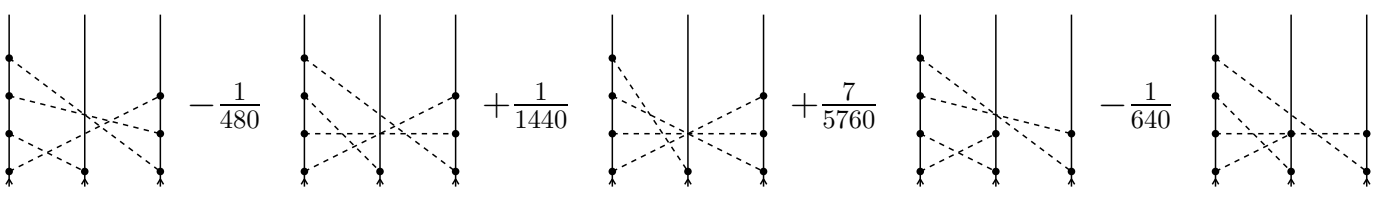
$+\frac{1}{24}$ $-\frac{1}{24}$ $+\frac{i\zeta(3)}{8\pi^3}$ $-\frac{i\zeta(3)}{4\pi^3}$



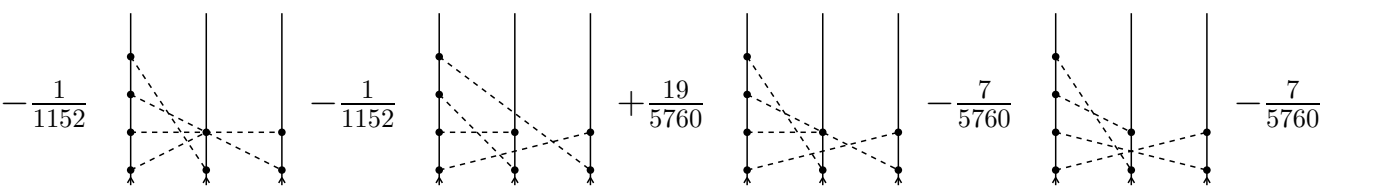
$+\frac{i\zeta(3)}{8\pi^3}$ $-\frac{i\zeta(3)}{8\pi^3}$ $-\frac{i\zeta(3)}{8\pi^3}$ $-\frac{i\zeta(3)}{8\pi^3}$



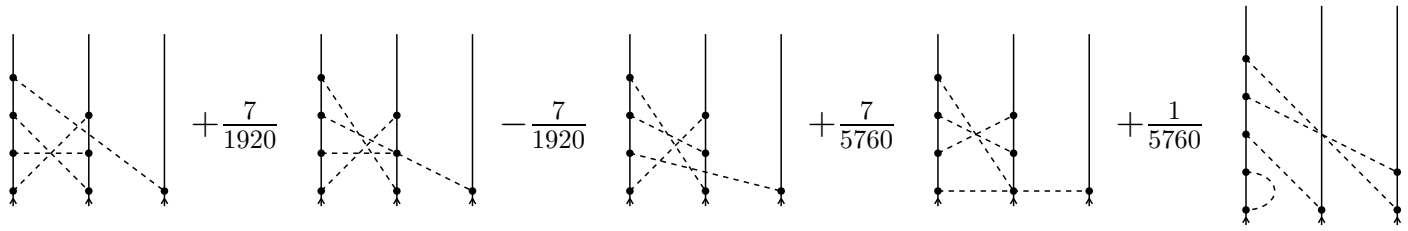
$-\frac{i\zeta(3)}{8\pi^3}$ $+\frac{i\zeta(3)}{4\pi^3}$ $+\frac{i\zeta(3)}{4\pi^3}$ $-\frac{1}{1440}$ $+\frac{1}{480}$

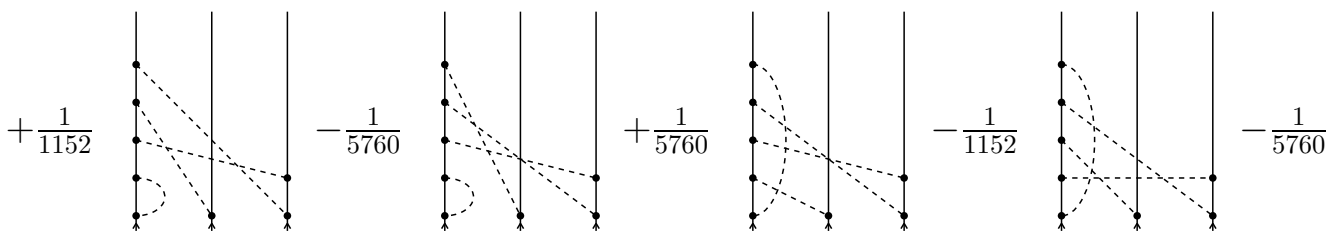


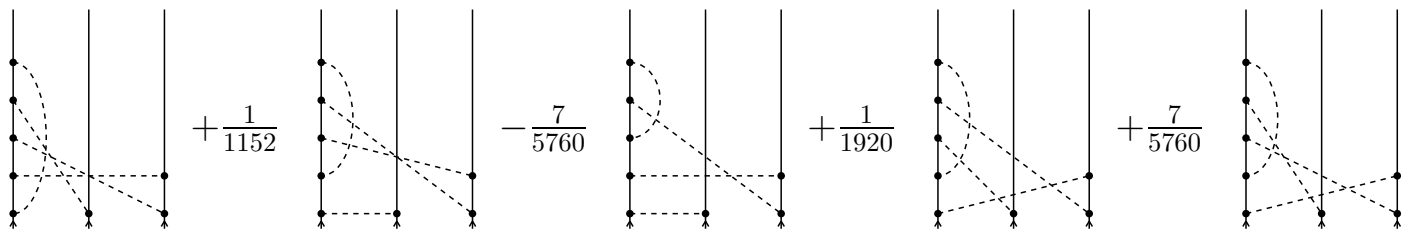
$-\frac{1}{480}$ $+\frac{1}{1440}$ $+\frac{7}{5760}$ $-\frac{1}{640}$

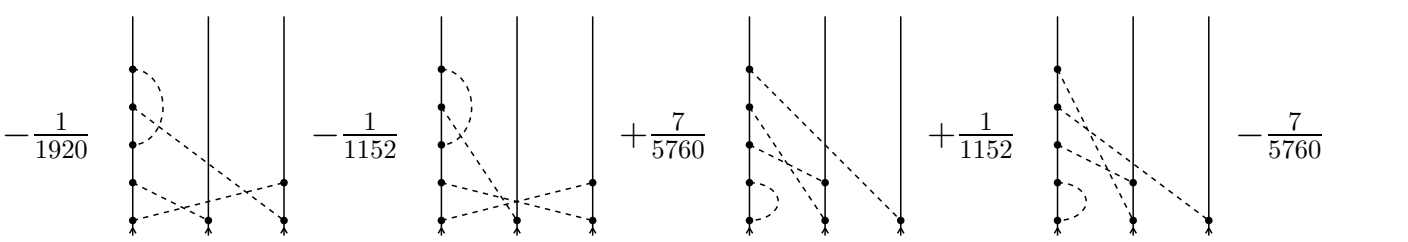


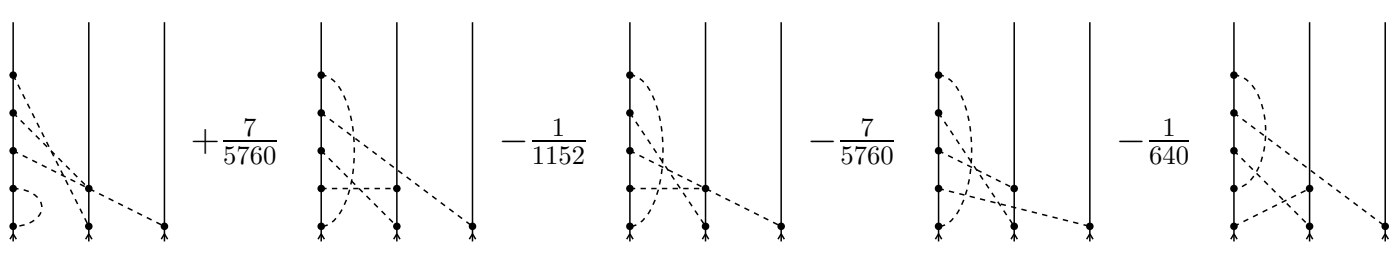
$-\frac{1}{1152}$ $-\frac{1}{1152}$ $+\frac{19}{5760}$ $-\frac{7}{5760}$ $-\frac{7}{5760}$

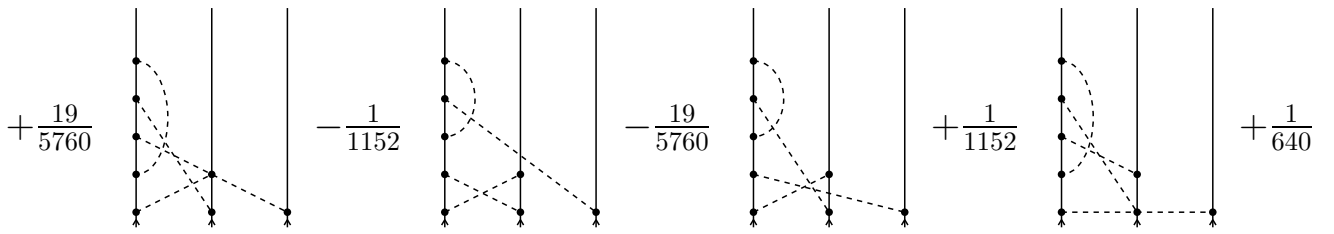


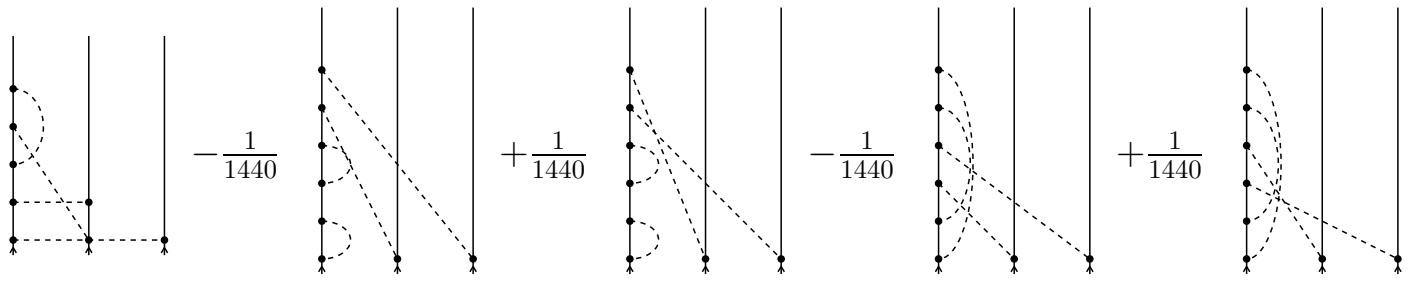


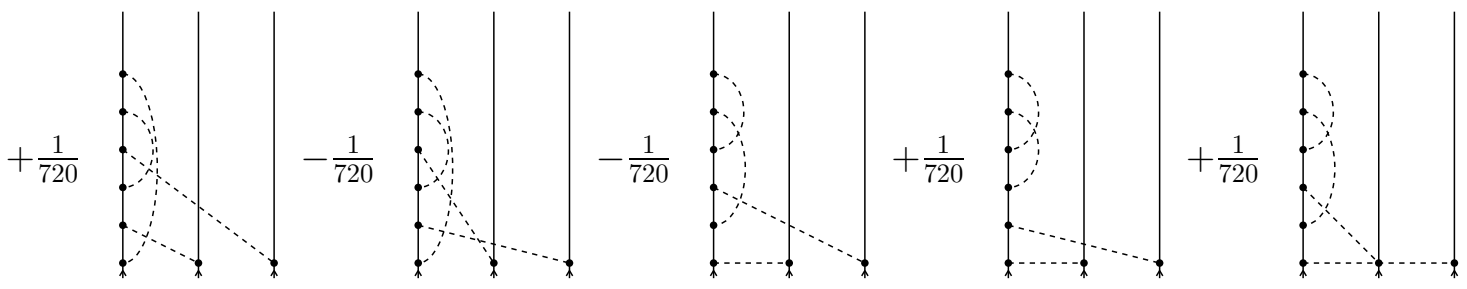


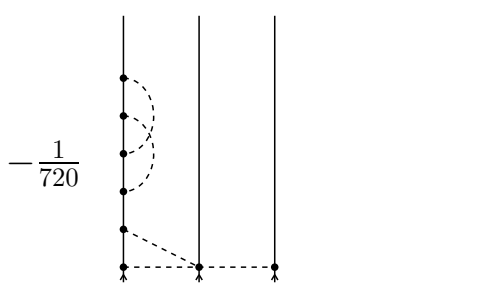






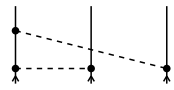
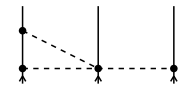
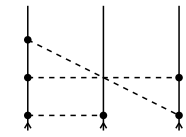


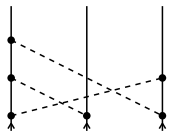
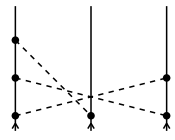
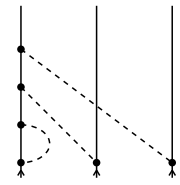
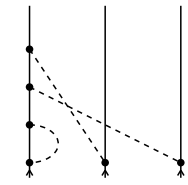
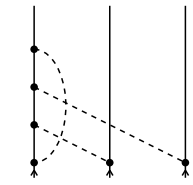


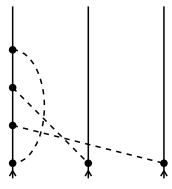
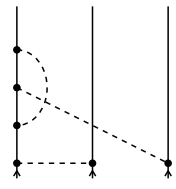
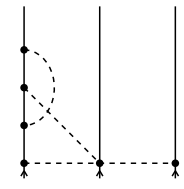
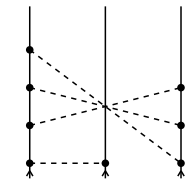


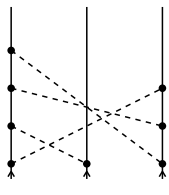
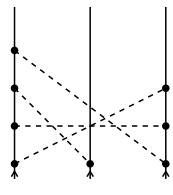
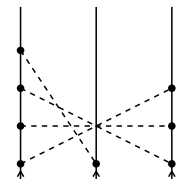
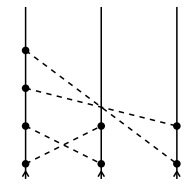
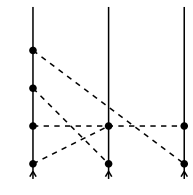
Associator twisted by a, in the choses basis, is:

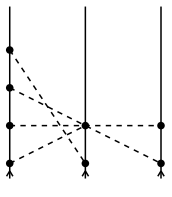
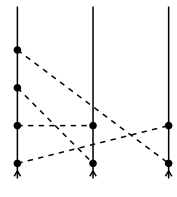
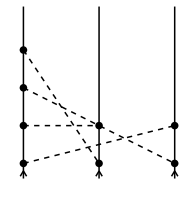
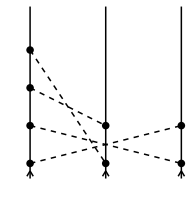
Φ^a =

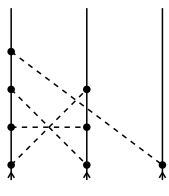
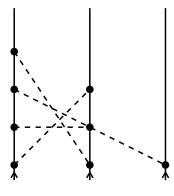
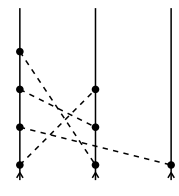
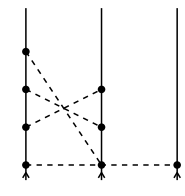
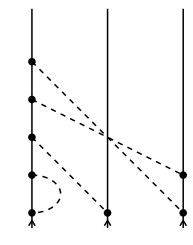
$+\frac{1}{24}$

$-\frac{1}{24}$

$-\frac{i\zeta(3)}{8\pi^3}$

$+\frac{i\zeta(3)}{4\pi^3}$

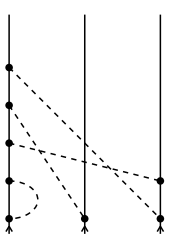
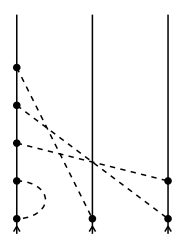
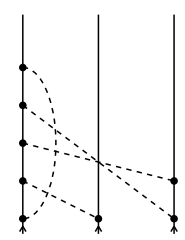
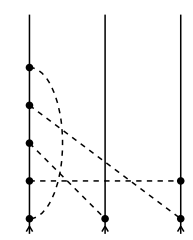

$-\frac{i\zeta(3)}{8\pi^3}$

$+\frac{i\zeta(3)}{8\pi^3}$

$+\frac{i\zeta(3)}{8\pi^3}$

$+\frac{i\zeta(3)}{8\pi^3}$


$+\frac{i\zeta(3)}{8\pi^3}$

$-\frac{i\zeta(3)}{4\pi^3}$

$-\frac{i\zeta(3)}{4\pi^3}$

$-\frac{1}{1440}$

$+\frac{1}{480}$


$-\frac{1}{480}$

$+\frac{1}{1440}$

$+\frac{7}{5760}$

$-\frac{1}{640}$


$-\frac{1}{1152}$

$-\frac{1}{1152}$

$+\frac{19}{5760}$

$-\frac{7}{5760}$

$-\frac{7}{5760}$


$+\frac{7}{1920}$

$-\frac{7}{1920}$

$+\frac{7}{5760}$

$+\frac{1}{5760}$


$+\frac{1}{1152}$

$-\frac{1}{5760}$

$+\frac{1}{5760}$

$-\frac{1}{1152}$

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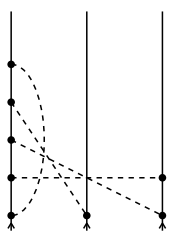
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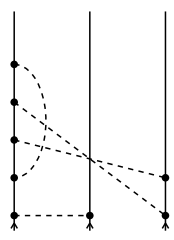
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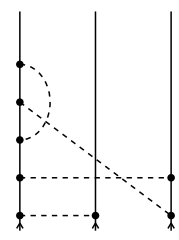
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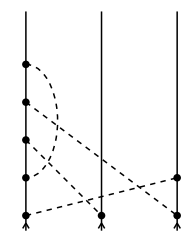
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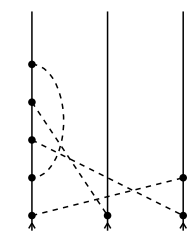

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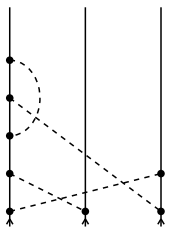
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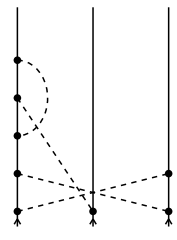
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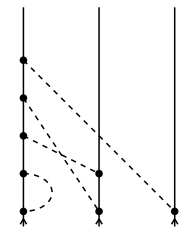
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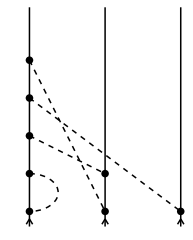
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$+\frac{7}{5760}$

$-\frac{1}{1152}$

$-\frac{7}{5760}$

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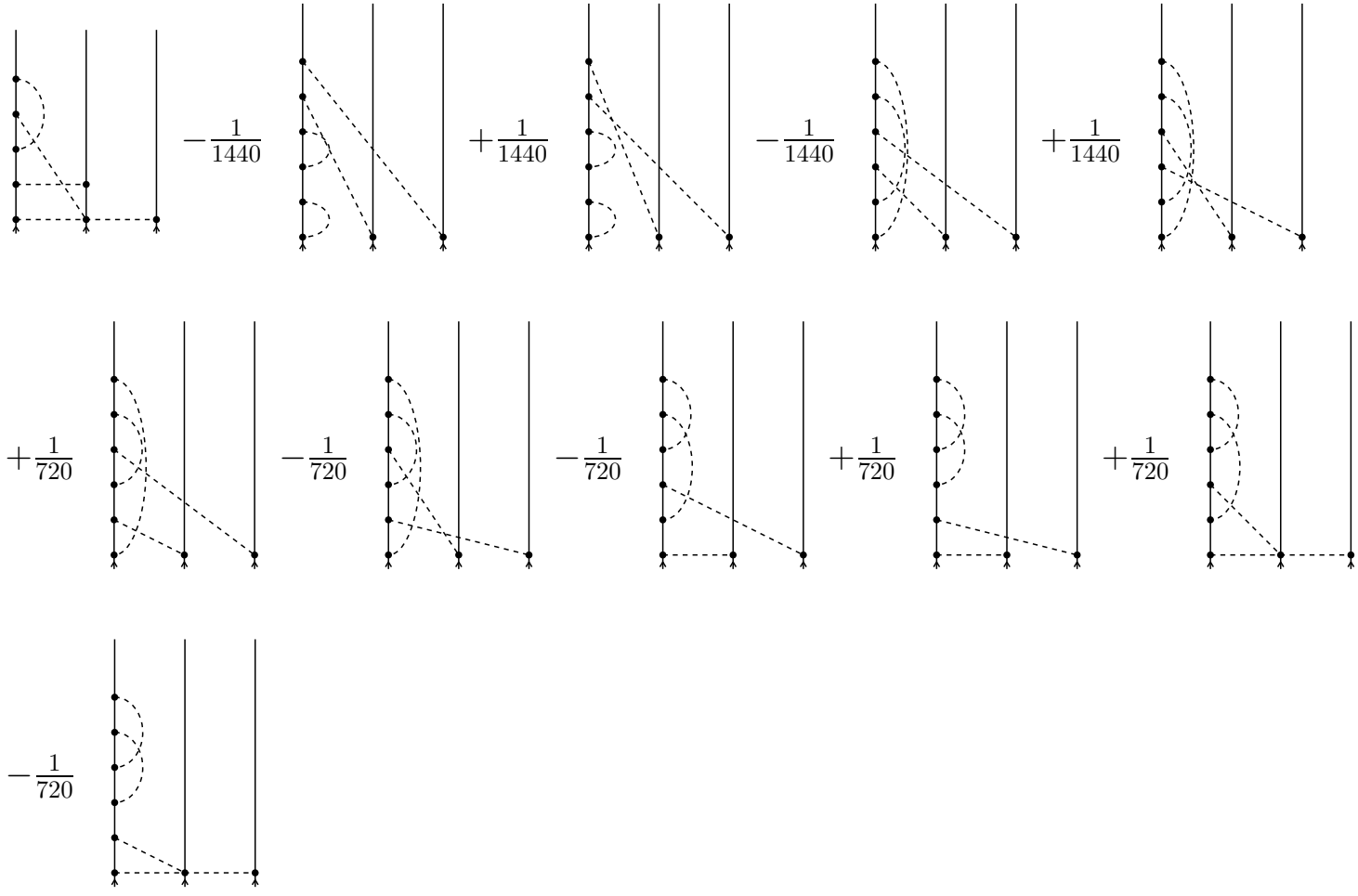
$+\frac{1}{1152}$

$+\frac{1}{640}$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