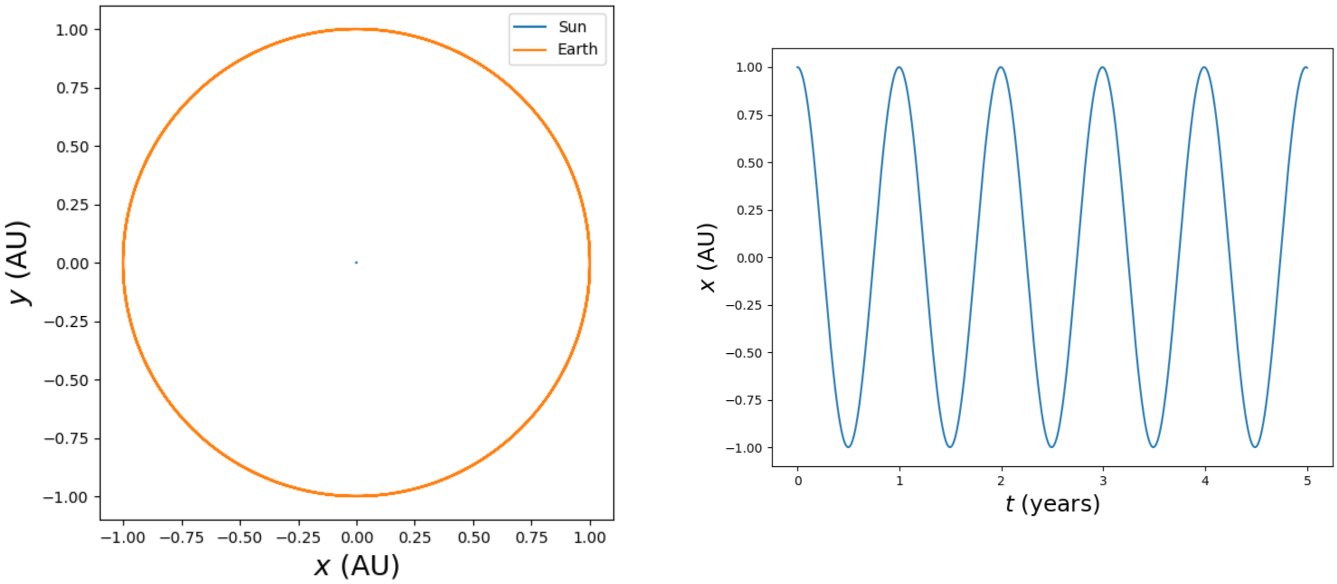

Figure 1: the 2-body system of the Earth orbiting the Sun over a period of 5 Earth years. The plot is an inertial frame of reference in space, within which both the Earth and Sun are permitted to move. The initial conditions posit the Earth at the co-ordinates (1AU,0) and the Sun at (0,0). Whilst the Sun is given no initial velocity in the x or y-directions, the Earth is given an initial velocity in the positive y-direction given by the Keplerian orbital velocity equation, $\sqrt{\frac{GM_{sun}}{1AU}} \approx 29{,}900ms^{-1}$. The plot was created by providing 5,000 time steps, running from 0 to 5 Earth years, to `odeint` with a function to solve the system of differential equations. The script proceeded to solve the system of equations for the motion of the bodies, producing the figures above; this formed the basis of all the simulations run and presented in this report. The small dot in the centre of the figure is the Sun's motion in the positive y-direction, since linear momentum is not conserved in this orbital system. The right-hand plot shows the periodic oscillation of the Earth's position between ±1AU over 1 year, which is an expected result. This is evidence that the code is operating correctly, passing the basic tests.

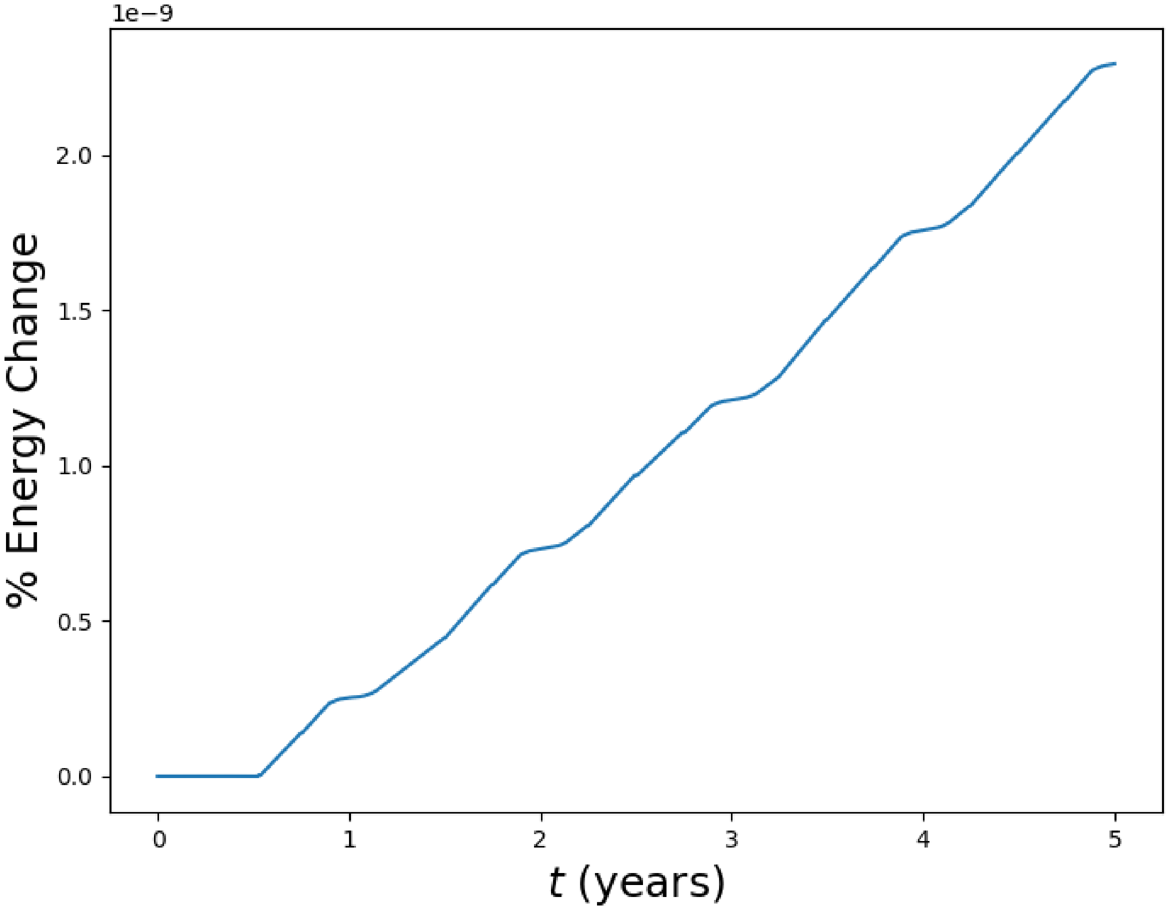

Figure 2: the percentage energy change over time for the 2-body system of the Earth and Sun, demonstrating energy conservation. By calculating the initial kinetic and potential energy of the system, a comparison can be made between the energy at time t $E(t)$ and initial energy. The above plot is the equation $\frac{E_{initial}-E(t)}{E_{initial}}$ and corresponds to the percentage change in energy at time t, with the energy at time t calculated using a self-written module `energy_calculator.py` imported into the script. Although the plot initially seems to suggest that energy is not conserved, the percentage change is on order of 10^{-9} % over 5 Earth years, which is within reasonable numerical accuracy over an extended period. Therefore, this plot is further evidence that the code works properly in 2-body systems.

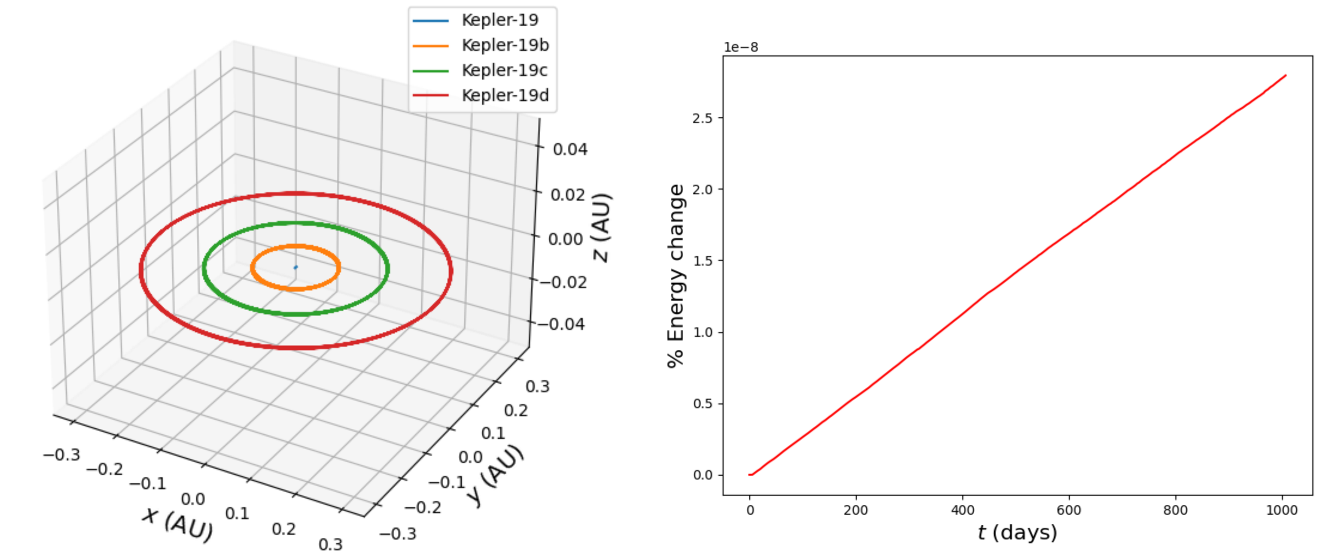

Figure 3: the Kepler-19 system over 16 periods of Kepler-19d (1007 Earth days). The plot shows that the system is stable, and the planets do not deviate from their orbits. Again, the star Kepler-19 has some linear momentum in the y direction due to the initial velocities being directed in this direction The parameters and initial conditions were retrieved from NASA's Exoplanet Archive[1], with 10,000 time steps provided to the simulation. The planets were assumed to be in Keplerian orbits in a flat plane, and their initial x-positions were assumed to be the semi-major axis of their orbits, which were determined through Kepler's Third Law $T^2 = \frac{4\pi^2}{GM_{Kepler-19}} a^3$ for orbital period T. Their initial velocities were set to be non-zero in only the positive y-direction and the planets were placed along the x-axis at y=0, z=0. This code took approximately 17 seconds to run. The right-hand plot demonstrates energy conservation being within negligible over long timescales, of order 10^{-8} % over 1000 days. This confirms that this system is both stable and must be physically correct, further illustrating the simulation's capability of more than 2-body solutions.

[1] Nasa Exoplanet Archive, Kepler-19 Overview. Available at: https://exoplanetarchive.ipac.caltech.edu/overview/Kepler-19%20b#planet_Kepler-19-b_collapsible [Accessed 17/12/21]

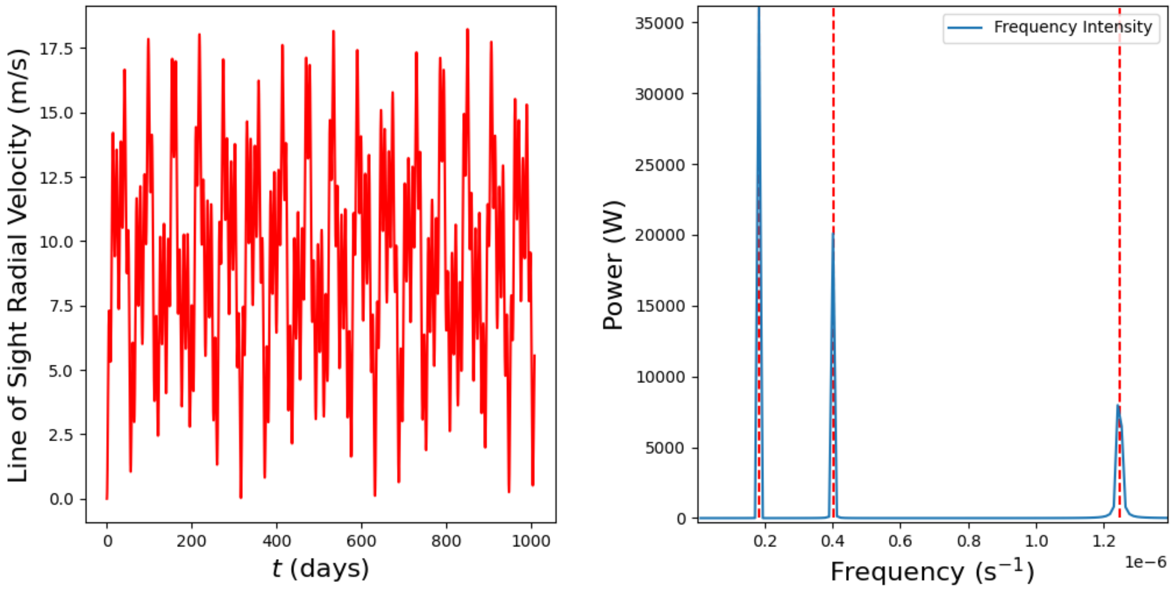

Figure 4: the radial velocity curve of Kepler-19 (left) and its Fourier transform (right). This is over a period of 16 Kepler-19d periods (1007 Earth days), produced from the same simulation Figure 4 was made in. With no orbiting bodies, a flat radial velocity curve is expected; and with a single body, the star's radial velocity would oscillate with the period of the planet's orbit, producing a sinusoidal curve. Therefore, with multiple planets it must oscillate sinusoidally with a period linked to the orbiting bodies' periods. A self-written module `fourier_transforms.py`, containing pre-written fast Fourier transforms from the `numpy` library, was imported; when the radial velocity curve for the star is Fourier transformed, there are three distinct peaks. The power spectrum is plotted in blue. The vertical red dashed lines correspond to the frequencies of the planet orbits (1/period) that were provided to simulate the system, which line up exactly with the power spectrum. The Fourier transform code exactly reproduces the periods provided, indicating that both the simulation and Fourier transform are self-consistent and must indeed be physically correct. A known physical number was provided and recovered from the data.

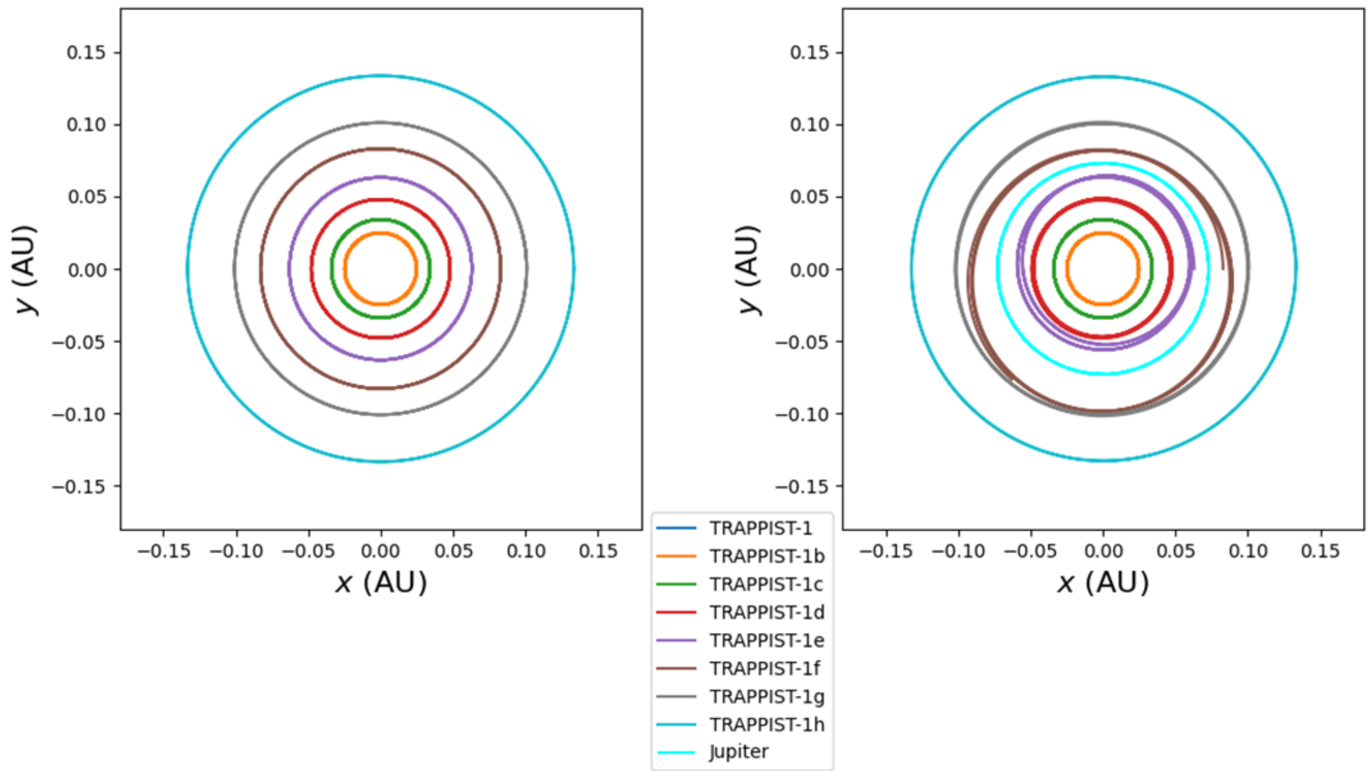

Figure 5: the TRAPPIST system simulated over 2 orbital periods of TRAPPIST-1h (37 days), with 100,000 time steps (left); and the same system with an interloping Jupiter mass between TRAPPIST-1e and 1f (right). This was simulated for the investigation using the tool I have built, `N_body_simulator.`py. Each body starts along the x-axis at y=0, z=0 with an initial velocity given by the Keplerian orbital velocity equation. The star TRAPPIST-1 was positioned at (0,0,0) at rest and the system was allowed to evolve. The initial conditions were from the NASA Exoplanet Archive[2]. The undisturbed system is clearly stable, although TRAPPIST-1e and TRAPPIST-1f were notably disrupted with an interloper. Each body is initially being attracted towards Jupiter before TRAPPIST-1e accelerated away. This explains the elongation of its orbit towards the top-right, whereas TRAPPIST1-f was accelerated towards Jupiter – giving it additional kinetic energy and throwing it further out into the system; this is what the brown orbit shows. Clearly, a Jupiter sized mass causes major disruption to the existing orbital resonances, meaning it could not exist within that region of the system for a long period of time without significant disruption. This shows the code can simulate complicated multi-body interactions.

[2] Nasa Exoplanet Archive, TRAPPIST-1 Overview. Available at: https://exoplanetarchive.ipac.caltech.edu/overview/TRAPPIST-1 [accessed 17/12/21]

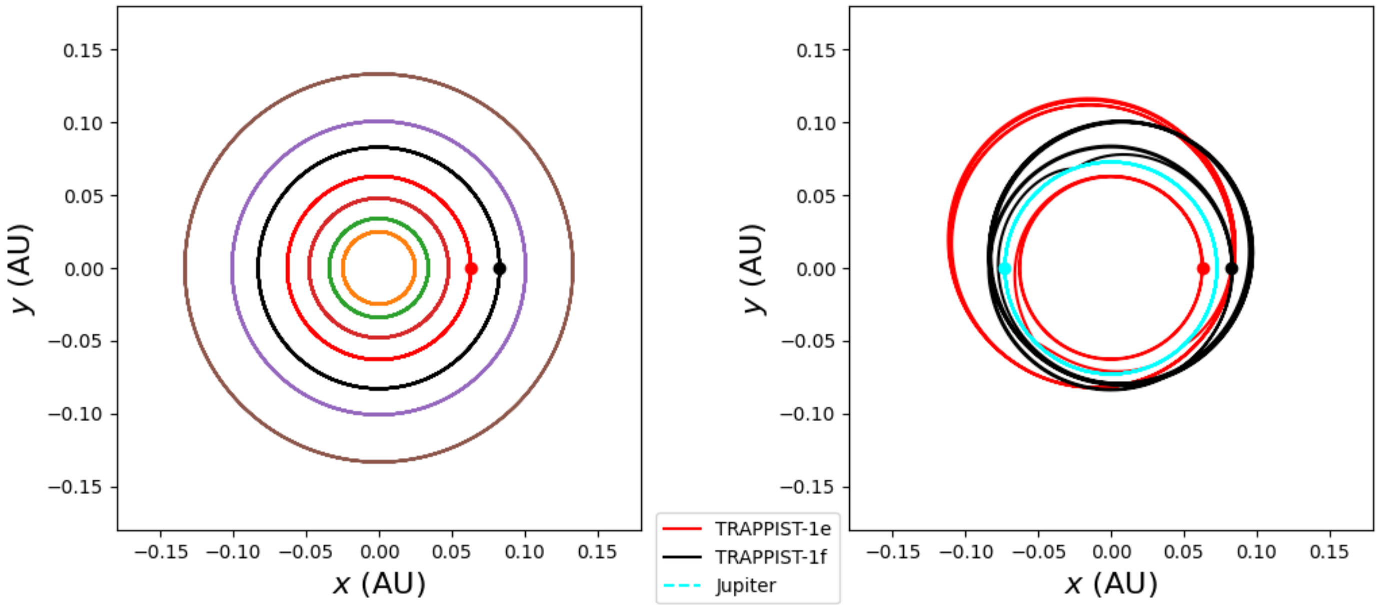

Figure 6: the TRAPPIST system over 6 TRAPPIST-1g orbital periods (74 days) with 50,000 points(left); TRAPPIST-1e, 1f and Jupiter's orbits in the disturbed system (right). Only these were plotted for visual clarity. Jupiter was moved to the left-side of the system, with dots representing initial positions. This was to prevent initial close encounters. The bodies had the same initial velocities as the undisturbed system and were such that they would orbit counter-clockwise. Here, Jupiter attracts TRAPPIST-1f into a close encounter as it nears (-0.02, 0.06), giving TRAPPIST-1f kinetic energy and launching it into an elliptical orbit. TRAPPIST-1e also undergoes an orbit before being ejected into the outer system due to a close encounter with Jupiter. This begins the study on Jupiter's influence on TRAPPIST-1e and 1f, as it demonstrates well that Jupiter, despite having been moved within the system, has a considerable influence on the orbit of nearby planets. By running the simulation for longer, this would reveal how Jupiter may chaotically disrupt TRAPPIST-1e and 1f, preventing them from ever settling into a stable orbit.

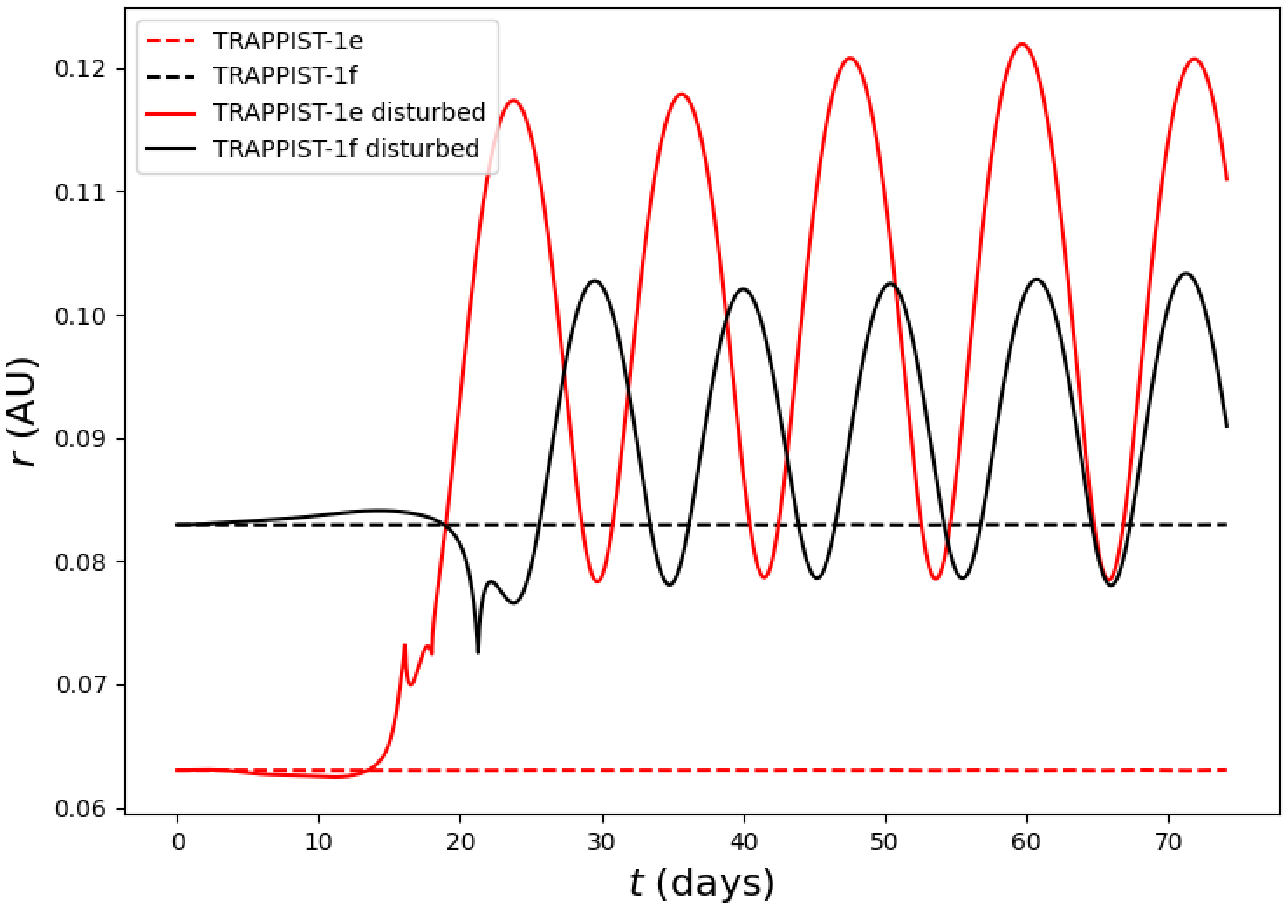

Figure 7: the separation r between TRAPPIST-1 and planets TRAPPIST-1e and 1f. The dashed lines show their separation when Jupiter is not present, for comparison. The solid lines are the system containing Jupiter. As expected, these are approximately flat with minor fluctuations due to the other planets within the system. TRAPPIST-1f's close encounter with Jupiter can be seen around t=21 days where the separation suddenly drops before increasing rapidly, showing a sudden, significant change to its momentum. TRAPPIST-1f then enters an approximately periodic elliptical orbit as evidenced by the sinusoidal separation curve in black. Similarly, TRAPPIST-1e is accelerated towards Jupiter suddenly around t=15 days before its orbit becomes highly elliptical, with significant variation in the separation distance. The varying peak heights show it is an unstable orbit. This graph explains the behaviour of the bodies over time in a clear, qualitative manner and further highlights Jupiter's influence on the system. The inconsistent apocentre of TRAPPIST-1e shows that its orbit has become unstable, highlighting the impossibility of a Jupiter-sized body existing within the interior TRAPPIST system – it is not stable over long periods.

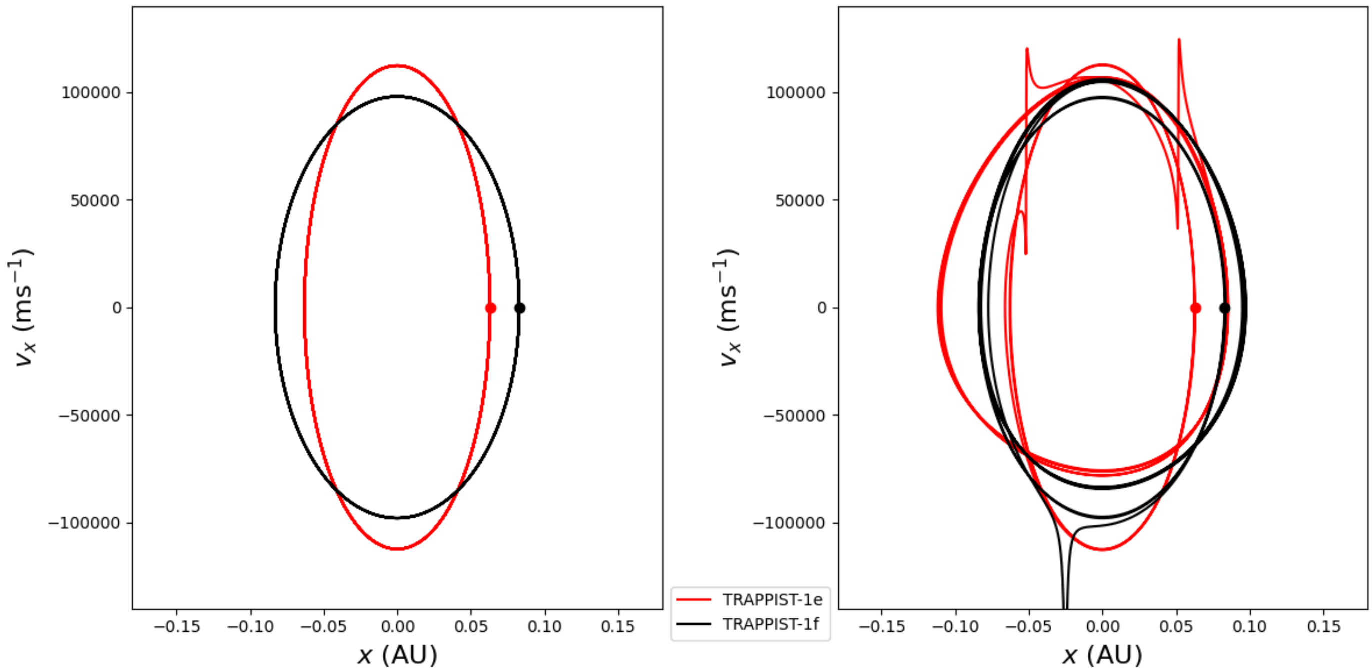

Figure 8: phase space diagram of the undisturbed TRAPPIST system (left); and the disturbed system (right). Further study of the system from the same simulation reveals the chaotic nature of the unstable orbits TRAPPIST-1e and 1f are sent onto by Jupiter. The stable orbits of TRAPPIST-1e and 1f can be seen as thin lines in the left plot, compared with the erratic, unstable and non-elliptical phase space curves in the right plot. The spikes indicate close encounters where Jupiter launches the planet outward where massive acceleration occurs. Observing phase space confirms that the system is indeed exhibiting chaotic behaviour; and periodically sampling Poincaré sections from phase space over a long period of time would produce a strange attractor, which would adequately demonstrate chaos. This was not possible since it would have taken an unreasonably long time to simulate for much longer time periods with this code – this is one limitation of the simulation coded.

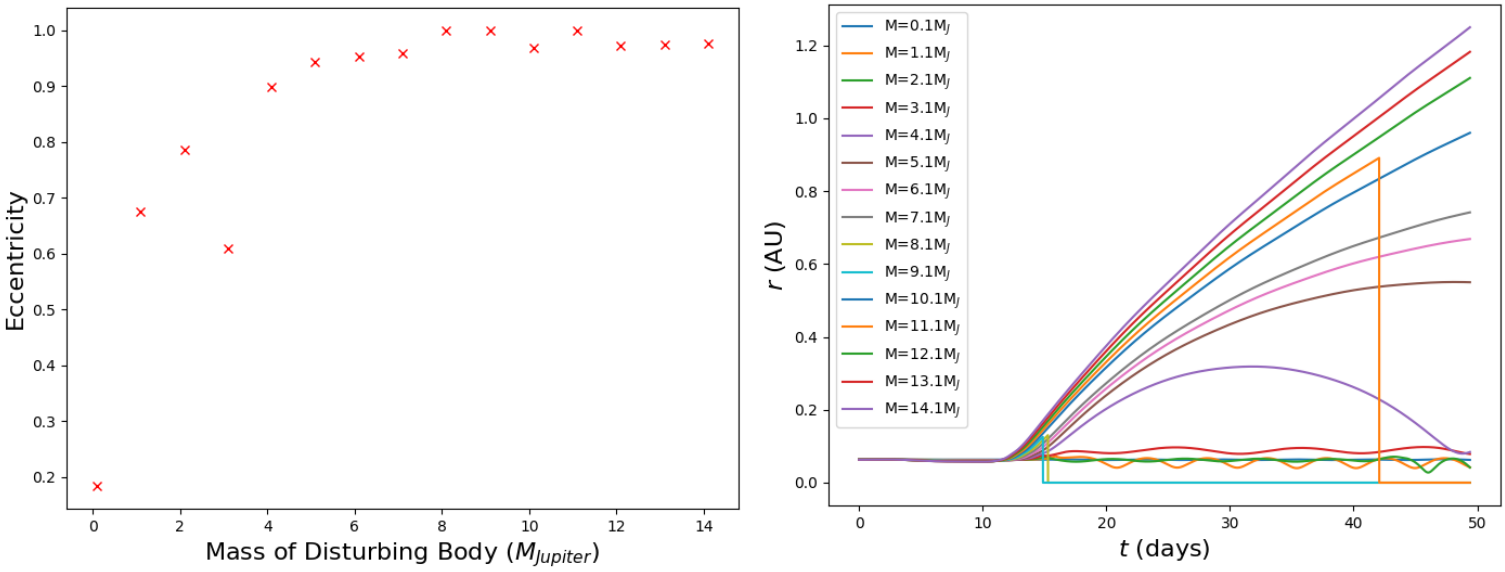

Figure 9: the eccentricity of TRAPPIST-1e's orbits (left); and the star-planet separation of TRAPPIST-1e over 74 days (right). As a parameter study, the mass of the disturbing body was varied between 0.1 and 14.1 Jupiter masses (M_J) and the eccentricity of TRAPPIST-1e's orbit calculated using the equation $e=\sqrt{1\pm\frac{b^2}{a^2}}$ (± depending on if $\frac{b^2}{a^2}>1$) for semi-major axis b and semi-minor axis a, assumed to be the apocentre and pericentre respectively. For low masses, there was some significant disruption to TRAPPIST-1e's orbit, making it highly elliptical quite quickly (1.1M_J). The ellipticity increases rapidly initially, but the orbit remains bound, as the right plot shows. At 4.1M_J, TRAPPIST-1e's orbit suddenly becomes much larger. Larger masses quickly send TRAPPIST-1e on hyperbolic trajectories – both visually evident in the separation over time as the curve straightens out, and the fact that the eccentricity rapidly converges to 1. On this timescale of 4 TRAPPIST-1g periods (49 days), TRAPPIST-1e remains bound to the system for interloping bodies of masses less than 6.1M_J, after which TRAPPIST-1e is quickly sent on a hyperbolic orbit, evidenced by the high ellipticities.

Finer study of the eccentricity and trajectories over longer time periods around this mass value would reveal what mass exactly causes TRAPPIST-1e to be ejected from the system on its first encounter. However, this was not possible on the timescale of this project, and the numerical integrator struggled with close encounters – this can be seen on the right plot where the separation plummets to 0 for 8.1M_J, 9.1M_J and 11.1M_J, and where the eccentricities appear anomalous. This introduced systematic errors since energy would not be conserved in these cases for time steps provided, making orbit predictions unreliable. This is discussed in the next figure. The eccentricity for 3.1M_J is anomalous, yet physical, since this appears to be a large, circular, stable orbit: it has escaped Jupiter's influence causing chaotic orbits. This can be seen as the red curve on the separation plot around 0.1AU.

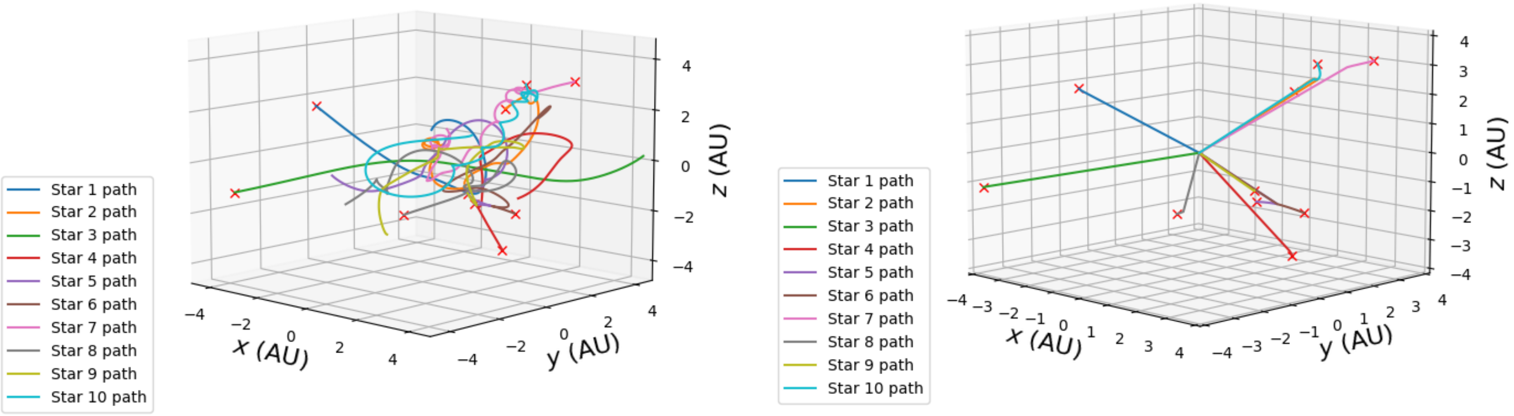

Figure 10: a randomly generated star cluster of 10 stars evolving over 10 Earth years with 50,000 time steps (left); and the same simulation with only 1,000 time steps (right), a clearly unphysical solution. The red crosses are the start positions of each star The simulator `N_body_solver` may not work in certain cases where insufficient time steps are provided to it. It may try to calculate a close encounter, where a body undergoes massive acceleration and energy conservation is briefly violated on a local level as a result. This makes future predictions in the orbital motions unreliable, or it may crash `odeint` altogether. This can occur in highly complicated systems and interactions. In the parameter study for $8.1M_J$, $9.1M_J$ and $11.1M_J$, the separation apparently suddenly became 0: there were not enough time steps to sufficiently calculate the dynamics accurately, and the code crashed. This is due to a close encounter causing problems for the numerical integrator. This provides a hidden systematic error within all simulations, since there could be close encounters causing inaccurate orbits later. The above figures are an illustration of a well-solved star cluster of 10 stars, compared to the same system with insufficient time steps such that it crashes instead. To improve upon this, the integrator must calculate the solution over either small or very large time steps to avoid close encounters where possible. Achieving this accuracy and consistency was not feasible in the time frame of this project, although offers possible extension to the investigation in the future.