Blatt 4

11 a

Mittelwerte: $\vec{\mu}_j = \frac{1}{N_j} \begin{pmatrix} \sum x_{j,1,i} \\ \vdots \\ \sum x_{j,n,i} \end{pmatrix}$

$$\vec{\mu}_0 = \frac{1}{6}\left(\begin{pmatrix}1\\1\end{pmatrix}+\begin{pmatrix}2\\1\end{pmatrix}+\begin{pmatrix}1{,}5\\2\end{pmatrix}+\begin{pmatrix}2\\2\end{pmatrix}+\begin{pmatrix}2\\3\end{pmatrix}+\begin{pmatrix}3\\3\end{pmatrix}\right) = \frac{1}{6}\begin{pmatrix}11{,}5\\12\end{pmatrix} = \begin{pmatrix}23/12\\2\end{pmatrix}$$

$$\vec{\mu}_1 = \frac{1}{6}\left(\begin{pmatrix}1{,}5\\1\end{pmatrix}+\begin{pmatrix}2{,}5\\1\end{pmatrix}+\begin{pmatrix}3{,}5\\1\end{pmatrix}+\begin{pmatrix}2{,}5\\2\end{pmatrix}+\begin{pmatrix}3{,}5\\2\end{pmatrix}+\begin{pmatrix}4{,}5\\2\end{pmatrix}\right) = \frac{1}{6}\begin{pmatrix}18\\9\end{pmatrix} = \begin{pmatrix}3\\3/2\end{pmatrix}$$

Streumatrizen:

$$S_w = \sum S_j \qquad S_j = \sum_i^{n_j} (\vec{x}_i - \vec{\mu}_j)(\vec{x}_i - \vec{\mu}_j)^T$$

$$S_0 = \begin{pmatrix}-11/12\\-1\end{pmatrix}\begin{pmatrix}-\frac{11}{12} & -1\end{pmatrix} + \begin{pmatrix}1/12\\-1\end{pmatrix}\begin{pmatrix}\frac{1}{12} & -1\end{pmatrix} + \begin{pmatrix}-5/12\\0\end{pmatrix}\begin{pmatrix}-\frac{5}{12} & 0\end{pmatrix}$$

$$+ \begin{pmatrix}\frac{1}{12}\\0\end{pmatrix}\begin{pmatrix}\frac{1}{12} & 0\end{pmatrix} + \begin{pmatrix}\frac{1}{12}\\1\end{pmatrix}\begin{pmatrix}\frac{1}{12} & 1\end{pmatrix} + \begin{pmatrix}\frac{13}{12}\\1\end{pmatrix}\begin{pmatrix}\frac{13}{12} & 1\end{pmatrix}$$

$$= \begin{pmatrix}\frac{121}{144} & \frac{11}{12}\\ \frac{11}{12} & 1\end{pmatrix} + \begin{pmatrix}\frac{1}{144} & -\frac{1}{12}\\ -\frac{1}{12} & 1\end{pmatrix} + \begin{pmatrix}\frac{25}{144} & 0\\ 0 & 0\end{pmatrix} + \begin{pmatrix}\frac{1}{144} & 0\\ 0 & 0\end{pmatrix} + \begin{pmatrix}\frac{1}{144} & \frac{1}{12}\\ \frac{1}{12} & 1\end{pmatrix}$$

$$+ \begin{pmatrix}\frac{169}{144} & \frac{13}{12}\\ \frac{13}{12} & 1\end{pmatrix}$$

$$= \begin{pmatrix}\frac{53}{24} & 2\\ 2 & 4\end{pmatrix}$$

$$S_1 = \begin{pmatrix} -3/2 \\ -\frac{1}{2} \end{pmatrix} \begin{pmatrix} -\frac{3}{2} & -\frac{1}{2} \end{pmatrix} + \begin{pmatrix} -\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} \end{pmatrix} + \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$$

$$+ \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} \end{pmatrix} + \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix} + \begin{pmatrix} 3/2 \\ 1/2 \end{pmatrix} \begin{pmatrix} 3/2 & \frac{1}{2} \end{pmatrix}$$

$$= \begin{pmatrix} 9/4 & 3/4 \\ 3/4 & 1/4 \end{pmatrix} + \begin{pmatrix} 1/4 & 1/4 \\ 1/4 & 1/4 \end{pmatrix} + \begin{pmatrix} 1/4 & -1/4 \\ -1/4 & 1/4 \end{pmatrix} + \begin{pmatrix} 1/4 & -1/4 \\ -1/4 & 1/4 \end{pmatrix} + \begin{pmatrix} 1/4 & 1/4 \\ 1/4 & 1/4 \end{pmatrix} + \begin{pmatrix} 9/4 & 3/4 \\ 3/4 & 1/4 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{11}{2} & 3/2 \\ 3/2 & 3/2 \end{pmatrix}$$

$$S_w = S_0 + S_1 = \begin{pmatrix} \frac{185}{24} & \frac{7}{2} \\ \frac{7}{2} & \frac{11}{2} \end{pmatrix}$$

$$S_B = (\vec{\mu}_0 - \vec{\mu}_1)(\mu_0 - \vec{\mu}_1)^T = \begin{pmatrix} -\frac{13}{12} \\ \frac{1}{2} \end{pmatrix} \begin{pmatrix} -\frac{13}{12} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{169}{144} & -\frac{13}{24} \\ -\frac{13}{24} & 1/4 \end{pmatrix}$$

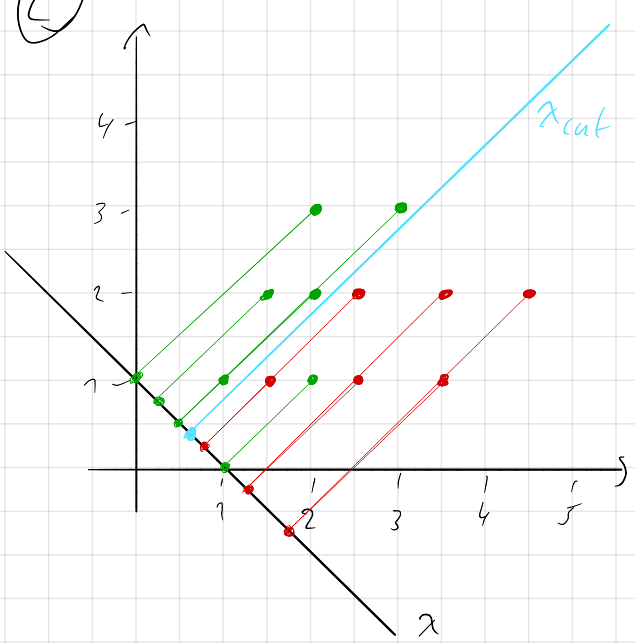
(b)

$$\vec{\lambda}^* = S_w^{-1} (\vec{\mu}_0 - \vec{\mu}_1)$$

$$S_w^{-1} = \frac{1}{\frac{185}{24} \cdot \frac{11}{2} - \frac{49}{4}} \begin{pmatrix} \frac{11}{2} & -\frac{7}{2} \\ -\frac{7}{2} & \frac{185}{24} \end{pmatrix} = \frac{48}{1447} \begin{pmatrix} \frac{11}{2} & -\frac{7}{2} \\ -\frac{7}{2} & \frac{185}{24} \end{pmatrix}$$

$$\vec{\lambda}^* = \frac{48}{1447} \begin{pmatrix} \frac{11}{2} & -\frac{7}{2} \\ -\frac{7}{2} & \frac{185}{24} \end{pmatrix} \begin{pmatrix} -\frac{13}{12} \\ 1/2 \end{pmatrix} = \frac{48}{1447} \begin{pmatrix} -\frac{143}{24} - \frac{7}{4} \\ \frac{91}{24} + \frac{185}{48} \end{pmatrix} = \begin{pmatrix} -\frac{370}{1447} \\ \frac{367}{1447} \end{pmatrix}$$

(c)



Population 0

Population 1

x_{cut} muss so gewählt werden, dass möglichst wenig werte falsch zugeordnet sind.

$$\lambda(x) = -\frac{367}{370} x + 1$$

(e)

Reinheit bzgl. Pop. 0: $R_0 = \frac{5}{5} = 1$

Reinheit bzgl. Pop. 1: $R_1 = \frac{6}{7}$

Effizienz bzgl. Pop. 0: $E_0 = \frac{5}{6}$

Effizienz bzgl. Pop. 1: $E_1 = \frac{6}{6} = 1$