

# Blatt 4

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$$\text{Mittelwerte: } \vec{\mu}_j = \frac{1}{n_j} \begin{pmatrix} \sum x_{j,1,i} \\ \vdots \\ \sum x_{j,n,i} \end{pmatrix}$$

$$\vec{\mu}_0 = \frac{1}{6} \left( \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 1,5 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ 3 \end{pmatrix} \right) = \frac{1}{6} \begin{pmatrix} 11,5 \\ 12 \end{pmatrix} = \begin{pmatrix} 23/12 \\ 2 \end{pmatrix}$$

$$\vec{\mu}_1 = \frac{1}{6} \left( \begin{pmatrix} 1,5 \\ 1 \end{pmatrix} + \begin{pmatrix} 2,5 \\ 1 \end{pmatrix} + \begin{pmatrix} 3,5 \\ 1 \end{pmatrix} + \begin{pmatrix} 2,5 \\ 2 \end{pmatrix} + \begin{pmatrix} 3,5 \\ 2 \end{pmatrix} + \begin{pmatrix} 4,5 \\ 2 \end{pmatrix} \right) = \frac{1}{6} \begin{pmatrix} 18 \\ 9 \end{pmatrix} = \begin{pmatrix} 3 \\ 3/2 \end{pmatrix}$$

Streuematrizen:

$$S_w = \sum S_j$$

$$S_j = \sum_i \frac{n_j}{n_j} (\vec{x}_i - \vec{\mu}_j) (\vec{x}_i - \vec{\mu}_j)^T$$

$$S_0 = \begin{pmatrix} -1/12 \\ -1 \end{pmatrix} \begin{pmatrix} -11/12 & -1 \end{pmatrix} + \begin{pmatrix} 1/12 \\ -1 \end{pmatrix} \begin{pmatrix} 1/12 & -1 \end{pmatrix} + \begin{pmatrix} -5/12 \\ 0 \end{pmatrix} \begin{pmatrix} -5/12 & 0 \end{pmatrix}$$

$$+ \begin{pmatrix} 1/12 \\ 0 \end{pmatrix} \begin{pmatrix} 1/12 & 0 \end{pmatrix} + \begin{pmatrix} 1/12 \\ 1 \end{pmatrix} \begin{pmatrix} 1/12 & 1 \end{pmatrix} + \begin{pmatrix} 13/12 \\ 1 \end{pmatrix} \begin{pmatrix} 13/12 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{121}{144} & \frac{11}{12} \\ \frac{11}{12} & 1 \end{pmatrix} + \begin{pmatrix} \frac{1}{144} & -\frac{1}{12} \\ -\frac{1}{12} & 1 \end{pmatrix} + \begin{pmatrix} \frac{25}{144} & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} \frac{1}{144} & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} \frac{1}{144} & \frac{1}{12} \\ \frac{1}{12} & 1 \end{pmatrix}$$

$$+ \begin{pmatrix} \frac{169}{144} & \frac{13}{12} \\ \frac{13}{12} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{53}{24} & 2 \\ 2 & 4 \end{pmatrix}$$

$$\begin{aligned}
 S_1 &= \begin{pmatrix} -3/2 \\ -1/2 \end{pmatrix} \begin{pmatrix} -3 & -1 \end{pmatrix} + \begin{pmatrix} -1/2 \\ -1/2 \end{pmatrix} \begin{pmatrix} -1 & -1 \end{pmatrix} + \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix} \begin{pmatrix} 1 & -1 \end{pmatrix} \\
 &+ \begin{pmatrix} -1/2 \\ 1/2 \end{pmatrix} \begin{pmatrix} -1 & 1 \end{pmatrix} + \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix} \begin{pmatrix} 1 & 1 \end{pmatrix} + \begin{pmatrix} 3/2 \\ 1/2 \end{pmatrix} \begin{pmatrix} 3 & 1 \end{pmatrix} \\
 &= \begin{pmatrix} 9/4 & 3/4 \\ 3/4 & 1/4 \end{pmatrix} + \begin{pmatrix} 1/4 & 1/4 \\ 1/4 & 1/4 \end{pmatrix} + \begin{pmatrix} 1/4 & -1/4 \\ -1/4 & 1/4 \end{pmatrix} + \begin{pmatrix} 1/4 & -1/4 \\ -1/4 & 1/4 \end{pmatrix} + \begin{pmatrix} 1/4 & 1/4 \\ 1/4 & 1/4 \end{pmatrix} + \begin{pmatrix} 9/4 & 3/4 \\ 3/4 & 1/4 \end{pmatrix} \\
 &= \begin{pmatrix} 11/2 & 3/2 \\ 3/2 & 3/2 \end{pmatrix}
 \end{aligned}$$

$$S_w = S_0 + S_1 = \begin{pmatrix} 185/24 & 7/2 \\ 7/2 & 11/2 \end{pmatrix}$$

$$S_B = (\vec{\mu}_0 - \vec{\mu}_1)(\mu_0 - \vec{\mu}_1)^T = \begin{pmatrix} -13/12 \\ 1/2 \end{pmatrix} \begin{pmatrix} -13/12 & 1/2 \end{pmatrix} = \begin{pmatrix} 169/144 & -13/24 \\ -13/24 & 1/4 \end{pmatrix}$$

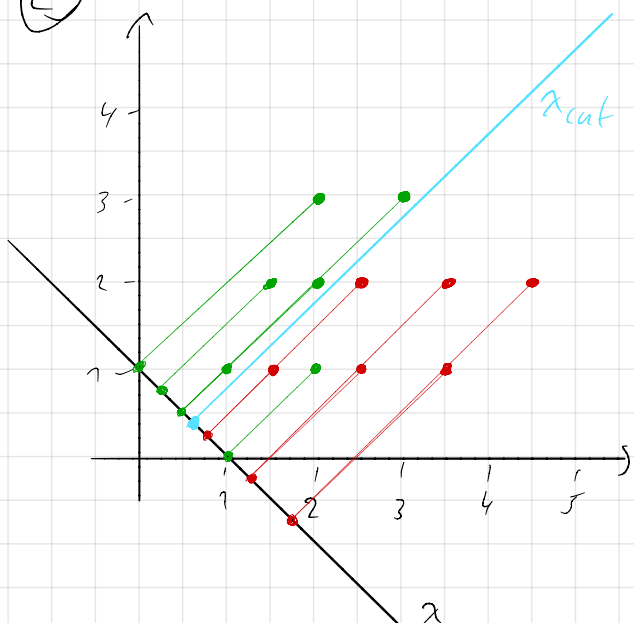
⑥

$$\vec{\lambda}^* = S_w^{-1} (\vec{\mu}_0 - \vec{\mu}_1)$$

$$S_w^{-1} = \frac{1}{\frac{185}{24} \cdot \frac{11}{2} - \frac{49}{4}} \begin{pmatrix} \frac{11}{2} & -\frac{7}{2} \\ -\frac{7}{2} & \frac{185}{24} \end{pmatrix} = \frac{48}{1447} \begin{pmatrix} \frac{11}{2} & -\frac{7}{2} \\ -\frac{7}{2} & \frac{185}{24} \end{pmatrix}$$

$$\vec{\lambda}^* = \frac{48}{1447} \begin{pmatrix} \frac{11}{2} & -\frac{7}{2} \\ -\frac{7}{2} & \frac{185}{24} \end{pmatrix} \begin{pmatrix} -13/12 \\ 1/2 \end{pmatrix} = \frac{48}{1447} \begin{pmatrix} -\frac{143}{24} - \frac{7}{4} \\ \frac{91}{24} + \frac{185}{48} \end{pmatrix} = \begin{pmatrix} -\frac{370}{1447} \\ \frac{367}{1447} \end{pmatrix}$$

⑦



Population 0

Population 1

$\lambda_{cat}$  muss so gewählt werden, dass möglichst wenig Werte falsch zugeordnet sind.

$$\lambda(x) = -\frac{367}{370}x + 1$$

e)

Reinheit bzgl. Pop. 0:  $R_0 = \frac{5}{5} = 1$

Reinheit bzgl. Pop. 1:  $R_1 = \frac{6}{7}$

Effizienz bzgl. Pop. 0:  $E_0 = \frac{5}{6}$

Effizienz bzgl. Pop. 1:  $E_1 = \frac{6}{6} = 1$