PreCalculus Honors
Unit 11 Days 4-5 Hyperbolas

Date Key 2021

Warm Up:

Graph each of the following, finding all relevant information, including eccentricity for ellipses.

1. $x = \frac{1}{8}(y-2)^2 - 4$ Parabola

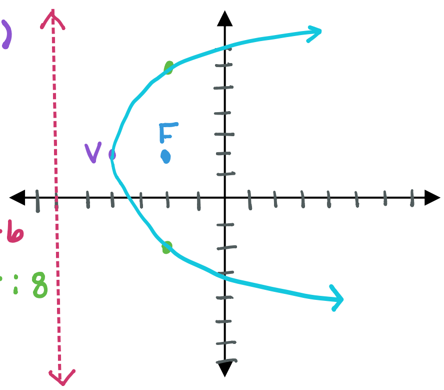
vertex: $(-4, 2)$

$\frac{1}{4p} = \frac{1}{8}$ ∴ $p = 2$

focus: $(-2, 2)$

directrix: $x = -6$

focal diameter: 8



2. $\frac{(x-2)^2}{9} + \frac{(y+1)^2}{16} = 1$ Ellipse

$b = 3$ $\quad$ $a = 4$

Center: $(2, -1)$

vertices:

$(2, -5)(2, 3)$

$(-1, -1)(5, -1)$

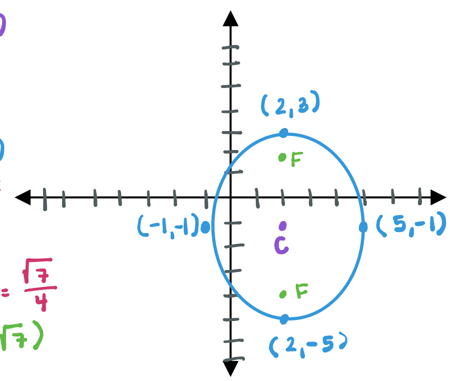
eccentricity:

$c^2 = a^2 - b^2$

$c^2 = 16 - 9$

$c = \sqrt{7}$ ∴ $e = \frac{\sqrt{7}}{4}$

foci: $(2, -1 \pm \sqrt{7})$



Standard 1: Create a Mathematical Representation

- To sketch the graphs of parabolas, circles, ellipses, hyperbolas, and semi-conics.
- To locate foci, directrix, asymptotes, and/or find the eccentricity of conics.
- To write equations of conic sections or semi-sections of conics.

GEOMETRIC DEFINITION OF A HYPERBOLA

A **hyperbola** is the set of all points in the plane, the difference of whose distances from two fixed points F_1 and F_2 is a constant. (See Figure 1.) These two fixed points are the **foci** of the hyperbola.

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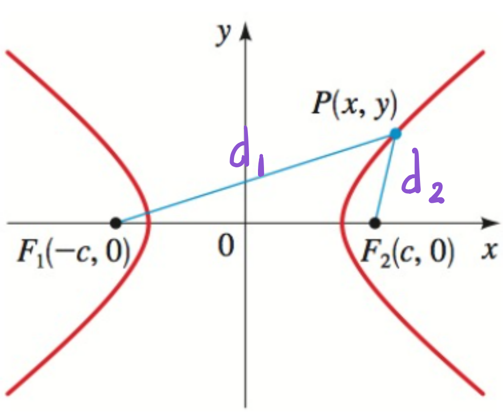
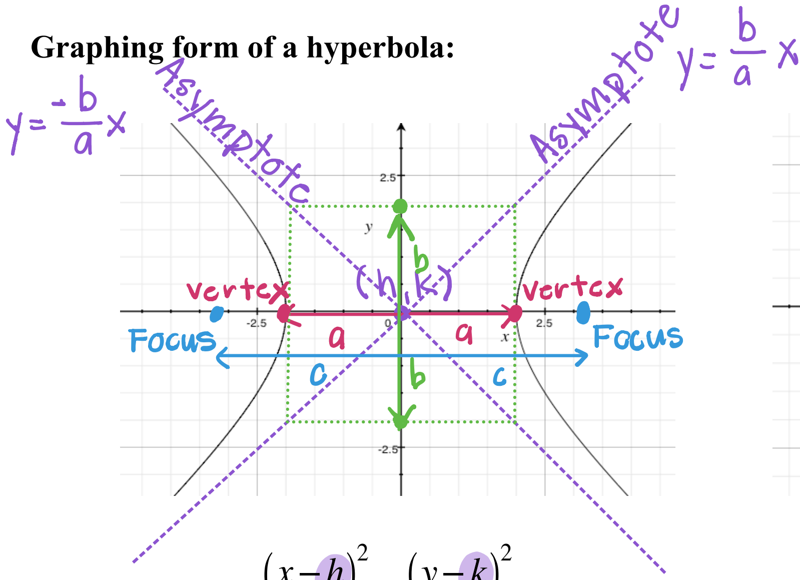


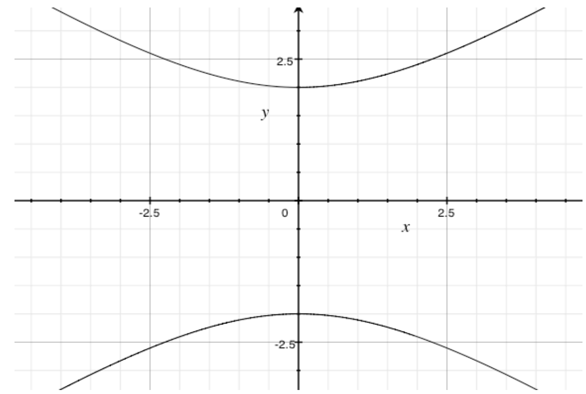
$d_1 - d_2 = $ constant #

for all points on the hyperbola

Remember: For an Ellipse, $d_1 + d_2 = $ constant

Graphing form of a hyperbola:





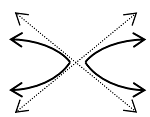
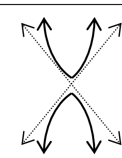
$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1 \qquad \frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$

a = distance from center to vertex

c = distance from center to foci

The Hyperbola

$a^2+b^2=c^2$

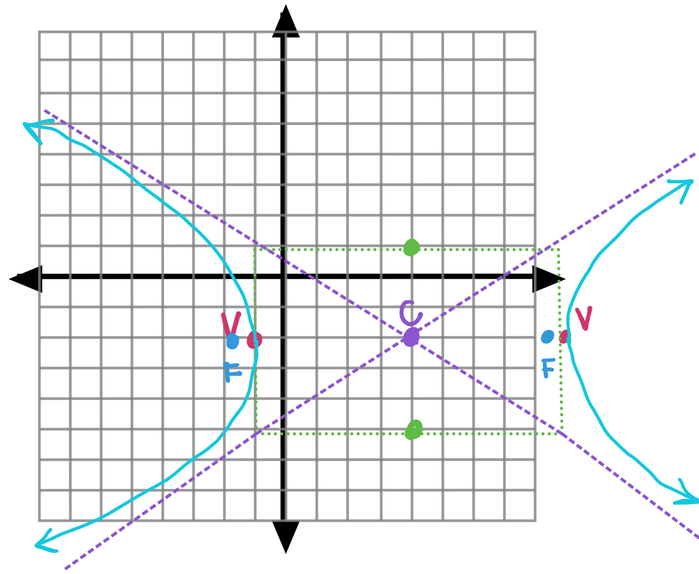
	
Transverse Axis: Horizontal	Transverse Axis: Vertical
x^2 is positive, opens left and right $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$	y^2 is positive, opens up and down $\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$
Center: (h,k)	Center: (h,k)
Foci: $(h+c,k)$, $(h-c,k)$	Foci: $(h,k+c)$, $(h,k-c)$
Vertices: $(h+a,k)$, $(h-a,k)$	Vertices: $(h,k+a)$, $(h,k-a)$
Asymptotes: $y=\pm\frac{b}{a}(x-h)+k$	Asymptotes: $y=\pm\frac{a}{b}(x-h)+k$
eccentricity: $e=\frac{c}{a}$	eccentricity: $e=\frac{c}{a}$

$c>a$ so $e>1$ for hyperbolas

Ex. 1: Graph $\frac{(x-4)^2}{25}-\frac{(y+2)^2}{9}=1$

h k

$a=5$ $b=3$



Center: $(4,-2)$

Vertices: $(-1,-2)(9,-2)$

Foci: $(4\pm\sqrt{34},-2)$

$c^2=a^2+b^2$
$c^2=25+9$
$c=\sqrt{34}$

Asymptotes: $y=\pm\frac{3}{5}(x-4)-2$

Eccentricity: $e=\frac{c}{a}=\frac{\sqrt{34}}{5}$

Ex. 2: Graph. $9y^2-x^2+2x+54y+71=0$

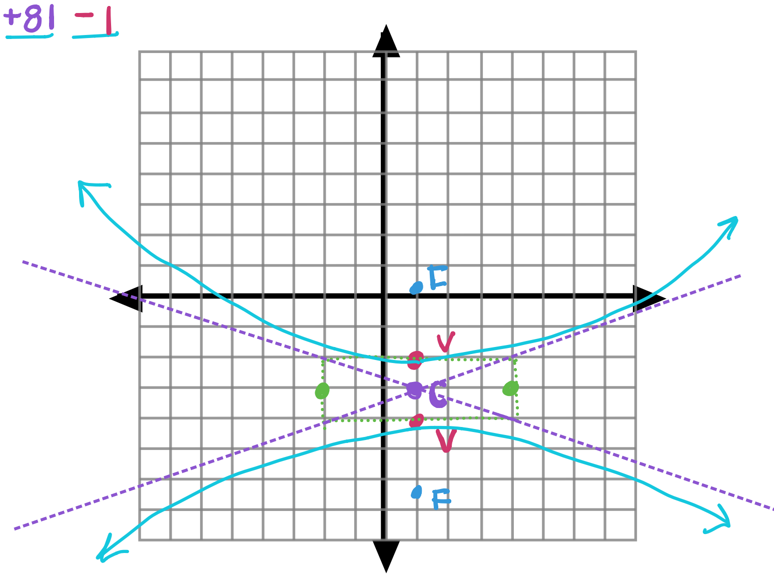
$9y^2+54y-x^2+2x=-71$

$9(y^2+6y+9)-(x^2-2x+1)=-71+81-1$

$9(y+3)^2-(x-1)^2=9$

$(y+3)^2-\frac{(x-1)^2}{9}=1$

$a=1$ $b=3$



Center: $(1,-3)$

Vertices: $(1,-4)(1,-2)$

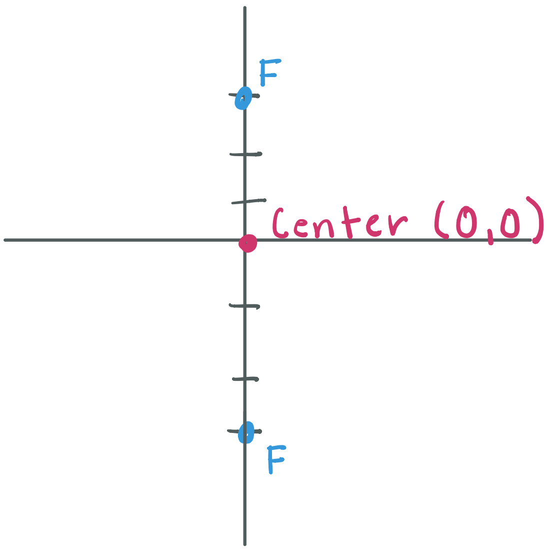
Foci: $(1,-3\pm\sqrt{10})$

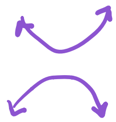
$c^2=a^2+b^2$
$c^2=9+1$
$c=\sqrt{10}$

Asymptotes: $y=\pm\frac{1}{3}(x-1)-3$

Eccentricity: $e=\frac{c}{a}=\frac{\sqrt{10}}{1}$

Ex. 3: Write the equation of the hyperbola with Foci $(0,\pm 3)$ and asymptotes $y=\pm\frac{1}{2}x$.





$c^2 = a^2 + b^2$

$9 = a^2 + (2a)^2$

$9 = a^2 + 4a^2$

$9 = 5a^2$

$\frac{9}{5} = a^2$

$a = \frac{3}{\sqrt{5}}$ or $\frac{3\sqrt{5}}{5}$

$\therefore 9 = \frac{9}{5} + b^2$

$\frac{45}{5} - \frac{9}{5} = b^2$

$\frac{36}{5} = b^2$

$$\frac{(y-0)^2}{a^2} - \frac{(x-0)^2}{b^2} = 1$$

$$\frac{5y^2}{9} - \frac{5x^2}{36} = 1$$

Day 2 Exit Slip

Quick Formative

S1, S3

1. Given: $36x^2 - 4y^2 - 36x - 8y = 31$

A. What type of conic? How do you know? Hyperbola because signs different on x and y.

B. Now, graph the conic section. Find and label all appropriate key features.

$36x^2 - 36x \_\_\_ - 4y^2 - 8y \_\_\_ = 31 \_\_\_ \_\_\_$

$36(x^2 - x + \frac{1}{4}) - 4(y^2 - 2y + 1) = 31 + 9 - 4$

$\frac{36(x-\frac{1}{2})^2}{36} - \frac{4(y+1)^2}{36} = \frac{36}{36}$

$(x-\frac{1}{2})^2 - \frac{(y-1)^2}{9} = 1$

$a = 1$ $b = 3$

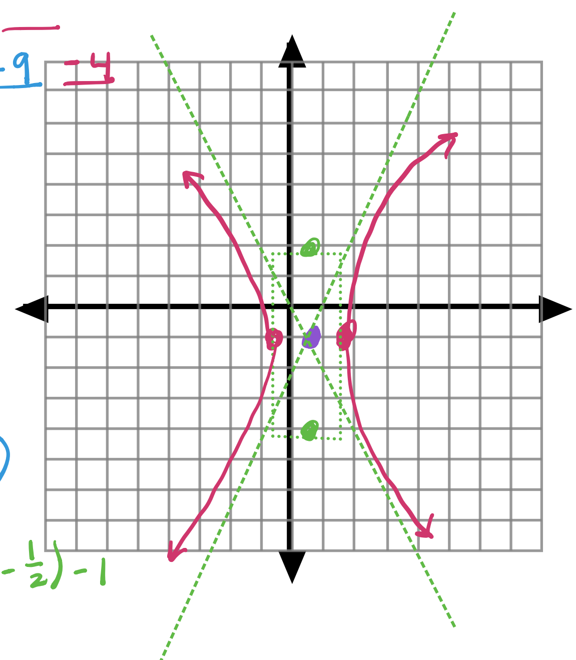
Center: $(\frac{1}{2}, -1)$

Vertices: $(\frac{3}{2}, -1)(-\frac{1}{2}, -1)$

Foci: $(\frac{1}{2} \pm \sqrt{10})$

Asymptotes:

$y = \pm 3(x - \frac{1}{2}) - 1$



2. Given: $x^2 + 4y^2 = 4x + 12$

A. What type of conic? How do you know?

B. Now, graph the conic section. Find and label all appropriate key features.

$x^2 - 4x + 4 + 4y^2 = 12 + 4$

$\frac{(x-2)^2}{16} + \frac{4y^2}{16} = \frac{16}{16}$

$\frac{(x-2)^2}{16} + \frac{y^2}{4} = 1$

$a = 4$ $b = 2$

center: $(2, 0)$

vertices: $(-2, 0)(6, 0)(2, 2)(2, -2)$

foci: $(2 \pm 2\sqrt{3}, 0)$

$c^2 = a^2 - b^2$

$c^2 = 16 - 4$

$c^2 = 12$

$c = 2\sqrt{3}$