Homework 4

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1.

a.

```
set.seed(1)
x1 <- runif(100)
x2 <- 0.5 * x1 + rnorm(100) / 10
y <- 2 + 2 * x2 + rnorm(100)
```

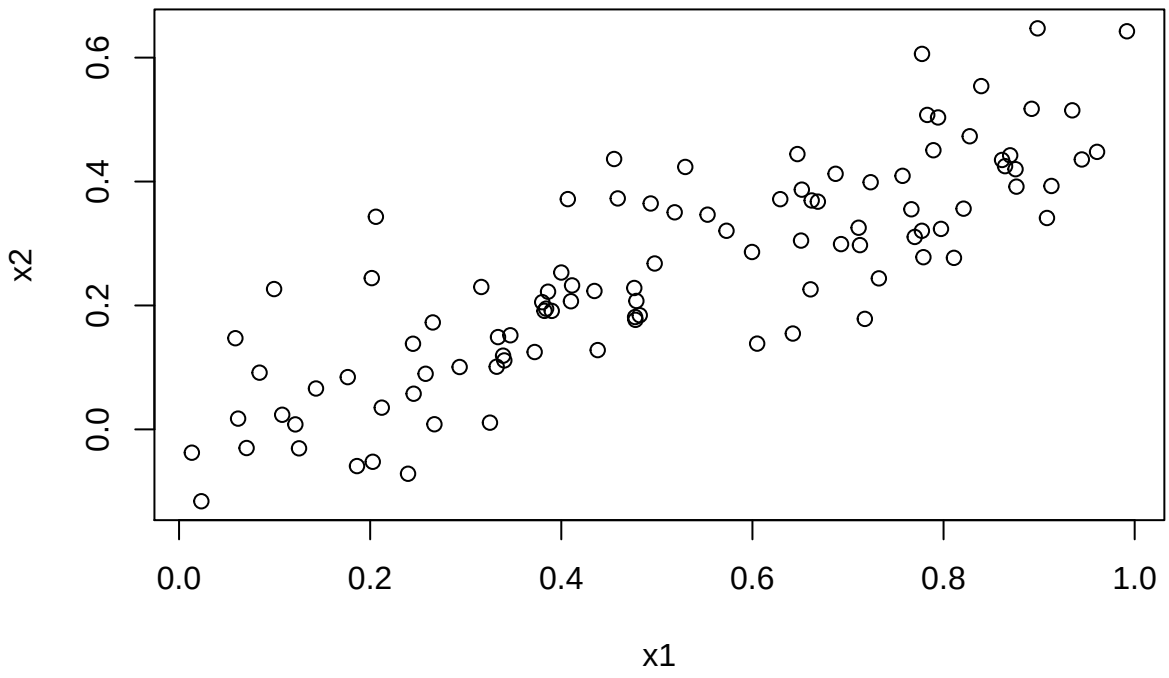
$Y = 2 + 2X_1 + 0.3X_2 + \epsilon$

b.

```
cor(x1, x2)
```

```
## [1] 0.8351212
```

```
plot(x1, x2)
```



c.

```
lm.fit <- lm(y ~ x1 + x2)
summary(lm.fit)
```

```
##
## Call:
## lm(formula = y ~ x1 + x2)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.8311 -0.7273 -0.0537  0.6338  2.3359
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.1305     0.2319   9.188 7.61e-15 ***
## x1           -0.5604     0.7212  -0.777   0.4390
## x2            2.7097     1.1337   2.390   0.0188 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.056 on 97 degrees of freedom
## Multiple R-squared:  0.0987, Adjusted R-squared:  0.08012
## F-statistic: 5.311 on 2 and 97 DF,  p-value: 0.006473
```

We can reject the null hypothesis that $\beta_2 = 0$ based on a p-value of 0.0188. However, with this current model, we fail to reject the null hypothesis that $\beta_1 = 0$ based on a p-value of 0.4390.

d.

```
lm.fit.x1<- lm(y ~ x1)
summary(lm.fit.x1)
```

```
##
## Call:
## lm(formula = y ~ x1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -3.00241 -0.66755 -0.09282  0.71984  2.78124
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.0819     0.2365   8.804 4.75e-14 ***
## x1            0.8790     0.4061   2.164   0.0329 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.081 on 98 degrees of freedom
## Multiple R-squared:  0.04562,    Adjusted R-squared:  0.03588
## F-statistic: 4.685 on 1 and 98 DF,  p-value: 0.03286
```

We can reject the null hypothesis that $\beta_1 = 0$ based on a p-value of 0.0329 for a model that does not contain x2.

e.

```
lm.fit.x2 <- lm(y ~ x2)
summary(lm.fit.x2)
```

```
##
## Call:
## lm(formula = y ~ x2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.91065 -0.65771 -0.06083  0.65167  2.47408
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.0295     0.1916  10.590  < 2e-16 ***
## x2            1.9739     0.6224   3.172  0.00202 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Residual standard error: 1.054 on 98 degrees of freedom
## Multiple R-squared:  0.09309,    Adjusted R-squared:  0.08384
## F-statistic: 10.06 on 1 and 98 DF,  p-value: 0.002024
```

In this model, we can reject the null hypothesis that $\beta_1 = 0$ based on a p-value of 0.00202.

f.

No the results do not contradict each other because multicollinearity exists between `x1` and `x2`, making it difficult to distinguish their effects on `y`.

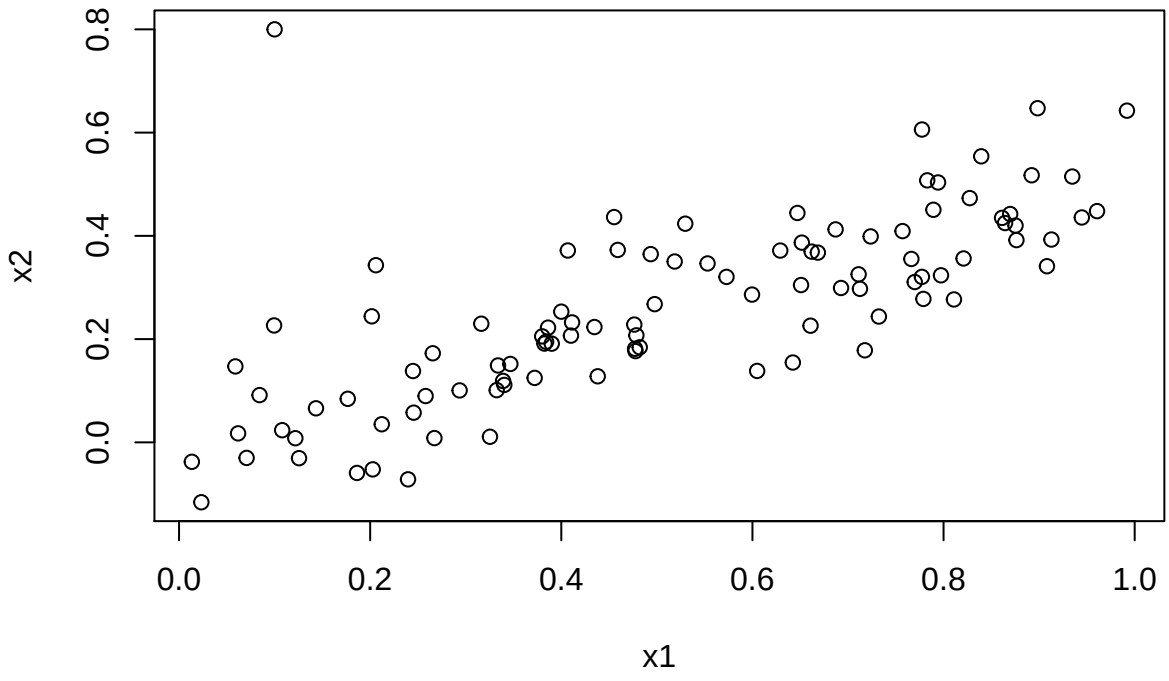
g.

```
x1 <- c(x1, 0.1)
x2 <- c(x2, 0.8)
y <- c(y, 6)
```

```
cor(x1, x2)
```

```
## [1] 0.7392279
```

```
plot(x1, x2)
```

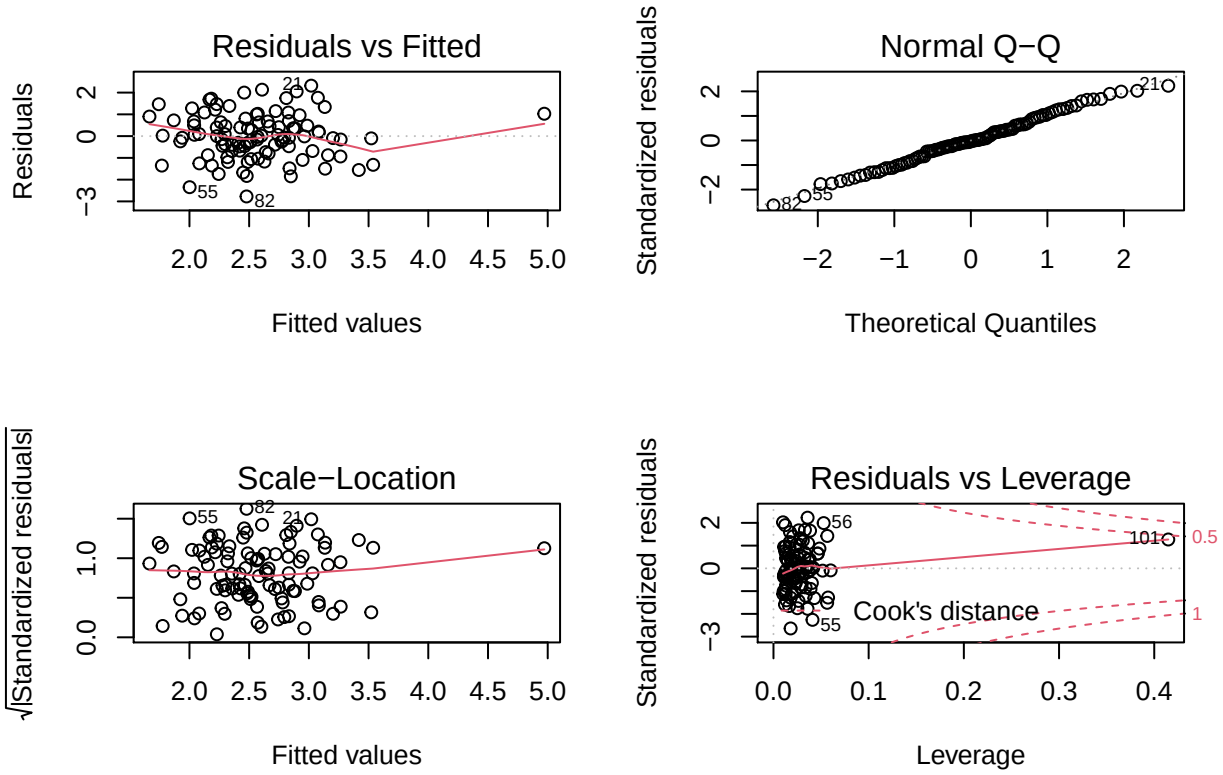


With the new observations, we see a lower correlation between `x1` and `x2`. It also looks like there's a serious outlier towards the top-left of the plot above, which corresponds to our new value.

```
lm.fit2 <- lm(y ~ x1 + x2)
summary(lm.fit2)
```

```
##
## Call:
## lm(formula = y ~ x1 + x2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.77230 -0.68497 -0.03604  0.67478  2.31801
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.1884     0.2281   9.595 9.18e-16 ***
## x1           -1.1027     0.5838  -1.889   0.0619 .
## x2            3.6163     0.8850   4.086 8.98e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.06 on 98 degrees of freedom
## Multiple R-squared:  0.1661, Adjusted R-squared:  0.1491
## F-statistic: 9.761 on 2 and 98 DF,  p-value: 0.0001363
```

```
par(mfrow = c(2, 2))
plot(lm.fit2)
```

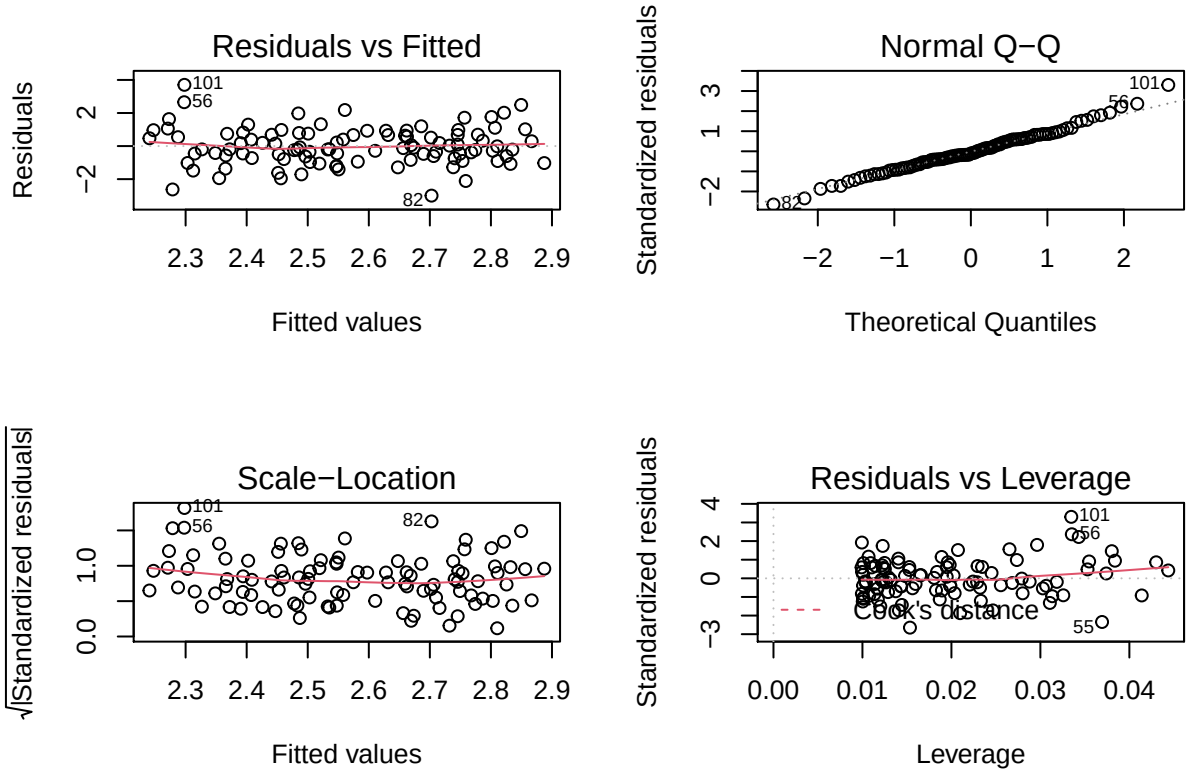


Our full model now shows that we fail to reject that $\beta_1 = 0$ with a p-value of 0.0619. We can still conclude that $\beta_2 \neq 0$ in this current model. The leverage plot suggests that our 101st observation, or our new value, acts as a high leverage point, but it does not exceed the 2 outlier threshold in regards to the Studentized residuals.

```
lm.fit3 <- lm(y ~ x1)
summary(lm.fit3)
```

```
##
## Call:
## lm(formula = y ~ x1)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.9970 -0.7260 -0.1236  0.6885  3.7020
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.2319     0.2452   9.101 9.99e-15 ***
## x1            0.6608     0.4232   1.561    0.122
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.14 on 99 degrees of freedom
## Multiple R-squared:  0.02403,    Adjusted R-squared:  0.01418
## F-statistic: 2.438 on 1 and 99 DF,  p-value: 0.1216
```

```
par(mfrow = c(2, 2))
plot(lm.fit3)
```



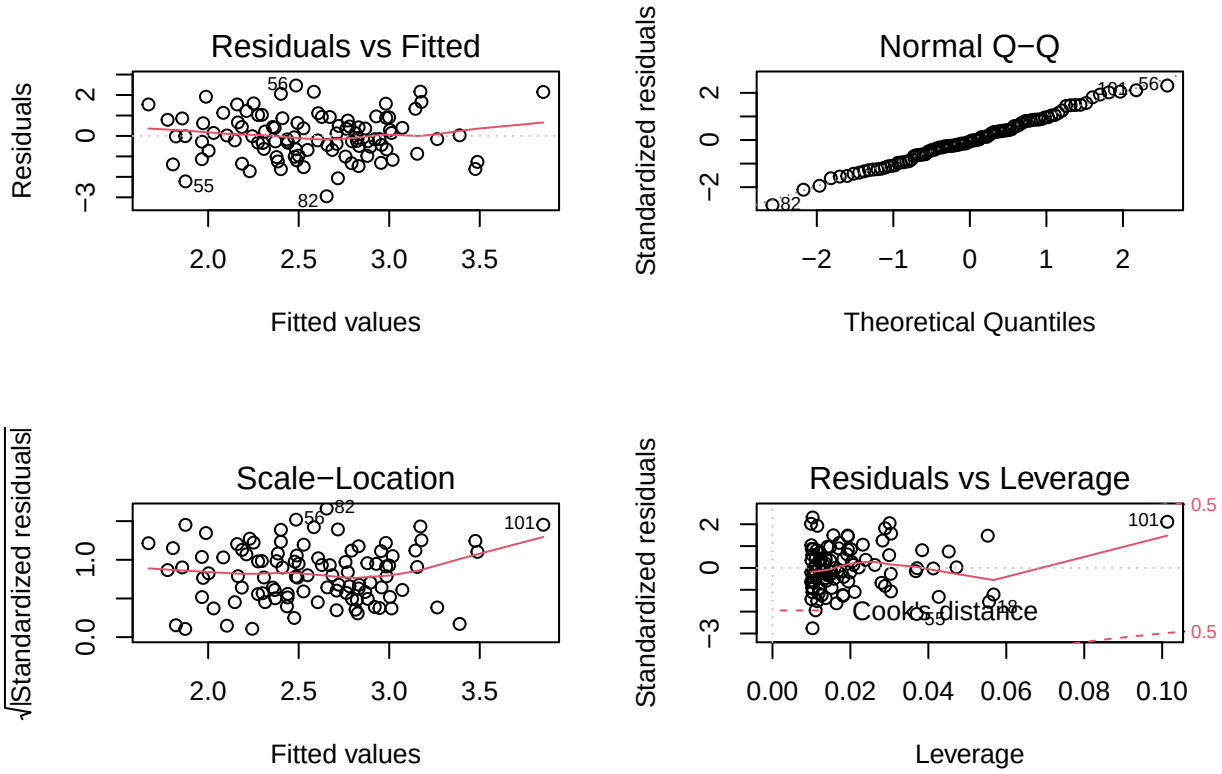
x1 alone no longer appears to have a relationship with y based on a p-value of 0.122. For this model, observation 101 acts as a both a serious outlier above a Studentized residual value of 2 and it also serves as a high leverage point according the leverage plot above.

```
lm.fit4 <- lm(y ~ x2)
summary(lm.fit4)
```

```
##
## Call:
## lm(formula = y ~ x2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.94849 -0.68322 -0.06569  0.75209  2.46508
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   1.9464     0.1911  10.185  < 2e-16 ***
## x2            2.3806     0.6037   3.943  0.00015 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.073 on 99 degrees of freedom
```

```
## Multiple R-squared:  0.1358, Adjusted R-squared:  0.127
## F-statistic: 15.55 on 1 and 99 DF,  p-value: 0.00015
```

```
par(mfrow = c(2, 2))
plot(lm.fit4)
```



`x2` continues to show that a relationship exists with `y` based on a p-value of 0.00015. Just like the previous `lm.fit3` model, observation 101 is both a high leverage point and outlier.

For `lm.fit3`, the slope of `x1` is reduced compared to the previous iteration. `lm.fit4` $beta_1$ estimate shows an increase of slope against `y`.

h.

Based on the outputs above:

- `lm.fit2`: 1.06
- `lm.fit3`: 1.14
- `lm.fit4`: 1.073

The full model, or `lm.fit2`, has the lowest standard error. This means that this model produces the most reliable estimates despite the lack of significance of `x1`.

i.

```
library(car)
```

```
vif(lm.fit)
```

```
##       x1       x2
## 3.304993 3.304993
```

```
vif(lm.fit2)
```

```
##       x1       x2
## 2.204867 2.204867
```

As we see from the VIF calculations, our model with the outlier has less multicollinearity than our model without the outlier. Our model with the outlier performed better because the lower multicollinearity among the predictors enabled us to better identify `x1` and `x2`'s effects on `y`.

Here's an example for the problem with multicollinearity. Let's assume that we wanted to guess a person's height based on their forearm length and their calf length. Let's also assume that both of these measurements are highly correlated. If we measured someone knowing that these are correlated, then why would we measure both? Instead, it would be better to choose another measurement that is not correlated with forearm length, such as neck length (this is just an example, I don't know if this is really the case, but it proves the point). We will have more accuracy predicting height in this case if we choose to measure something that's not correlated because we'll have more unique features to base our predictions on.

3.

a.

$p(X) = 0.37/1.37$

```
0.37/1.37
```

```
## [1] 0.270073
```

b.

$p(X) = 0.16/0.84$

```
0.16/0.84
```

```
## [1] 0.1904762
```

4.

β_0 = -6 $\beta_1 = 0.05$ $\beta_2 = 1$

a.

$e^-0.05/(1 + e^-0.05)$

```
exp(-0.5)/(1 + exp(-0.5))
```

```
## [1] 0.3775407
```

b.

$(log(1) + 2.5)/0.05$

```
(log(1) + 2.5)/0.05
```

```
## [1] 50
```

5.

a.

Distance formula:

$d(p, q) = \sqrt{(q_1 - p_1)^2 + \cdots + (q_n - p_n)^2}$

Observation distances:

1. 3
2. 2
3. 3.16
4. 2.23
5. 1.41
6. 1.73

b.

When $K = 1$, we would predict green because the Euclidean distance to observation 5 is the lowest at 1.41.

c.

When $K = 3$, we predict red because we have 2 red observations (distance of 1.73 and 2) and 1 green observation (distance of 1.41) within our boundary. The majority rule dictates that we predict red for our test observation.

d.

If the decision boundary is highly non-linear, then we would expect a K that's lower to work better than a higher value for K because a high value of K makes our model less-flexible. A lower value for K allows the model to be more flexible, which is better in the event that our Bayes decision boundary is non-linear.

6.

a.

QDA will perform better on the training set in this situation because it's more flexible than LDA. On the testing set, LDA will perform better because we reduce the chance of over-fitting our model.

b.

QDA will work better on both the training set and the test set because it's a more flexible method than LDA.

c.

QDA will perform better as n increases because QDA introduces higher variance to it's flexibility. Higher variance is not a major concern when dealing with a larger set of data. LDA might be too biased if the true Bayes decision boundary is not truly linear.

d.

False. LDA will better represent the true Bayes decision boundary and it will reduce our risk of overfitting. QDA will likely overfit to our data due to its higher flexibility.

7.

We can use the formula to obtain the posterior probability of x.

$p_1(4) = \frac{0.8e^{-(1/72)(4-10)^2}}{0.8e^{-(1/72)(4-10)^2}+0.2e^{-(1/72)(4-0)^2}} = 0.752$

Given a return of 4, the probability that a company issues a dividend is 0.752.