Signals & Systems EGE 311-01

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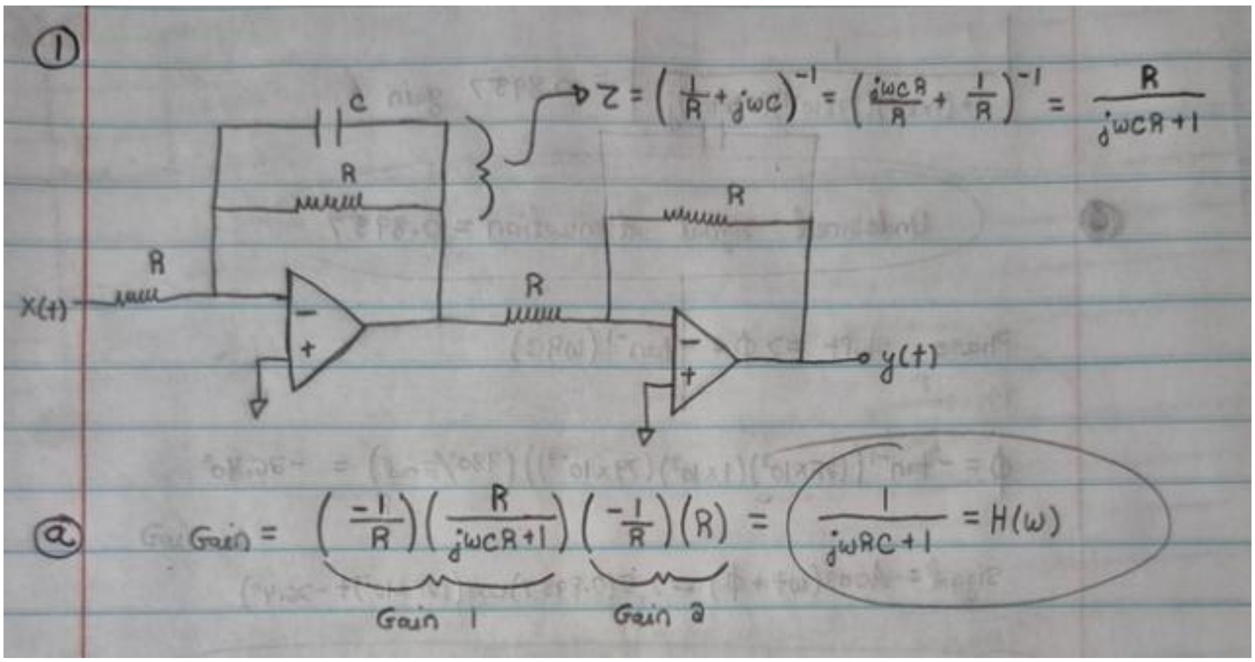
<u>Design objective:</u> To design a filter whose output signal will have a desired signal to noise ratio of 50 / 1.

Part 1: First-Order Filter:

Step 1: Finding the transfer function $H(\omega) = \frac{Y(\omega)}{X(\omega)}$



Step 2: Design the filter so that it attenuates the noise by a factor of 10

(b) Noise

Gain = 0.1 $\quad \omega = 2\pi(20 \times 10^3) = 4\pi \times 10^4$ rad/sec

$$\left| \frac{1}{j\omega RC + 1} \right| = \frac{1}{\sqrt{(\omega RC)^2 + 1}} \qquad 0.1 = \frac{1}{\sqrt{(\omega RC)^2 + 1}}$$

$$\frac{1}{0.1} = \sqrt{(\omega RC)^2 + 1} \implies (\omega RC)^2 + 1 = 100 \implies \omega RC = \sqrt{99}$$

$$RC = \frac{\sqrt{99}}{4\pi \times 10^4} = 7.918 \times 10^{-5} \text{ sec}$$

$$R = 1\ k\Omega \implies C = \frac{7.918 \times 10^{-5}}{1 \times 10^3} = 79.18\ nF$$

$R = 1\ k\Omega$, $C = 79\ nF$

Step 3: After simulating the designed circuit, we can see that the filter does severely affect the noise. However, there are some side effects such as a phase shift and signal attenuation

(c) Signal

$$\left|\frac{1}{1+(1\times10^3)(79\times10^{-9})(2\pi\times10^3)j}\right| = 0.8957 \text{ gain}$$

Undesired signal attenuation = 0.8957

Phase Shift => $\phi = -\tan^{-1}(wRC)$

$$\phi = -\tan^{-1}\left((2\pi\times10^3)(1\times10^3)(79\times10^{-9})\right)\left(180°/\pi \text{ rad}\right) = -26.40°$$

$$\text{Signal} = A\cos(wt+\phi) \Rightarrow 5(0.8957)\cos\left((2\pi\times10^3)t + -26.4°\right)$$

The signal output lags behind the signal input by 26.40°

$$\left|\frac{1}{1+j(1\times10^3)(79\times10^{-9})(4\pi\times10^4)j}\right| = 0.1002$$

$$\frac{S}{N} = \frac{5(0.8957)}{1(0.1002)} = 44.7$$

Output signal to noise ratio = 44.7

$$\%\ \text{Error} = \left(\frac{50-44.7}{50}\right)\times100 = 10.6\ \%$$

The calculated (S/N) ratio is very close to the desired (S/N) ratio. There is a % error of 10.6%

Undesired Attenuation: 0.8957
Signal to Noise Ratio (S/N) : 44.7
Phase Shift: 26.4°

<u>NOTE:</u> The signal to noise ratio is 10.6% lower than our desired ratio of 50.

Figure 1: Input and output voltages of the pure signal

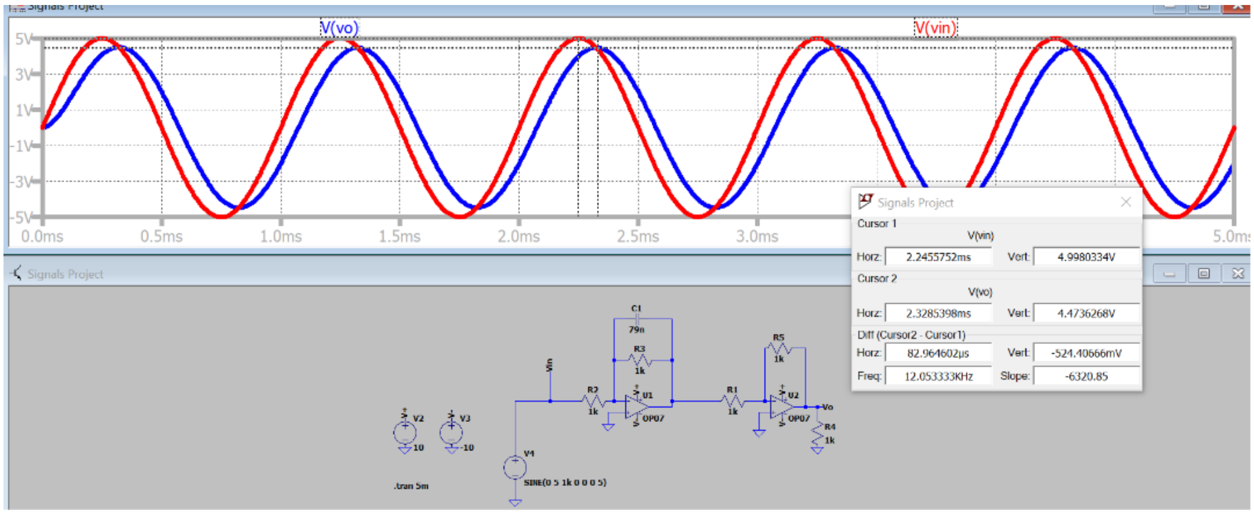


Figure 1: Input and output voltages of the pure signal

NOTE: The simulation demonstrates the slight phase 26.4° and the signal attenuation of 0.8957.

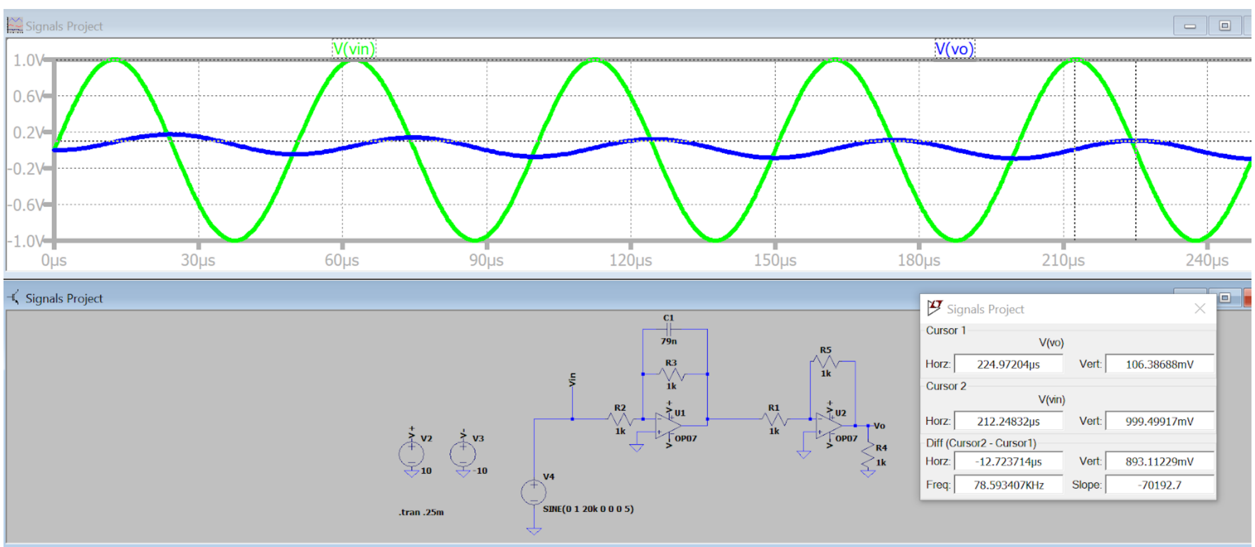


Figure 2: Input and output voltages of the pure noise

NOTE: The simulation demonstrates the filter's ability to reduce the noise by a factor of nearly 10.

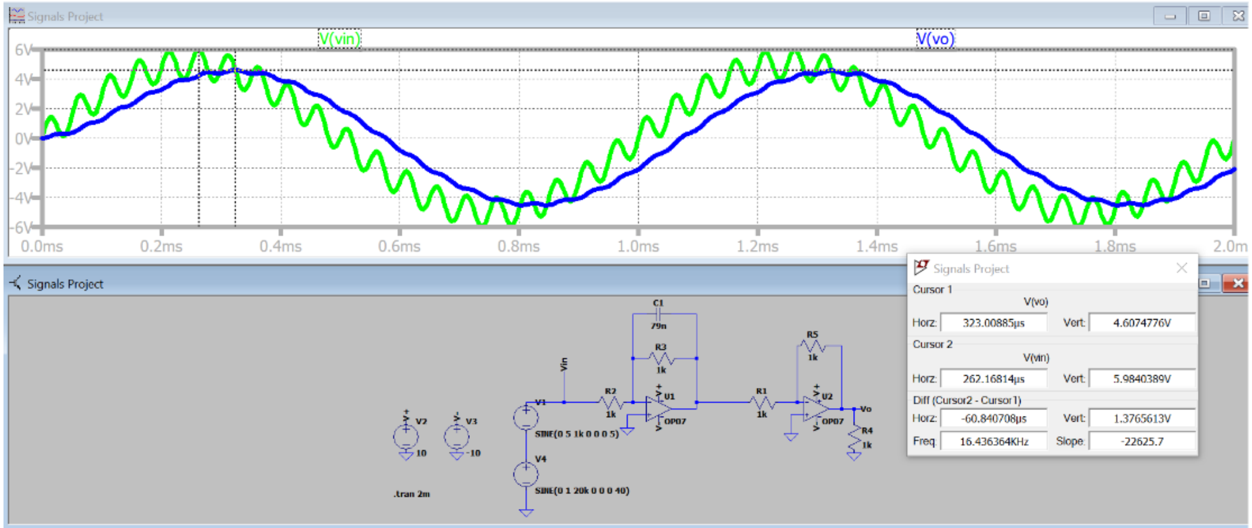
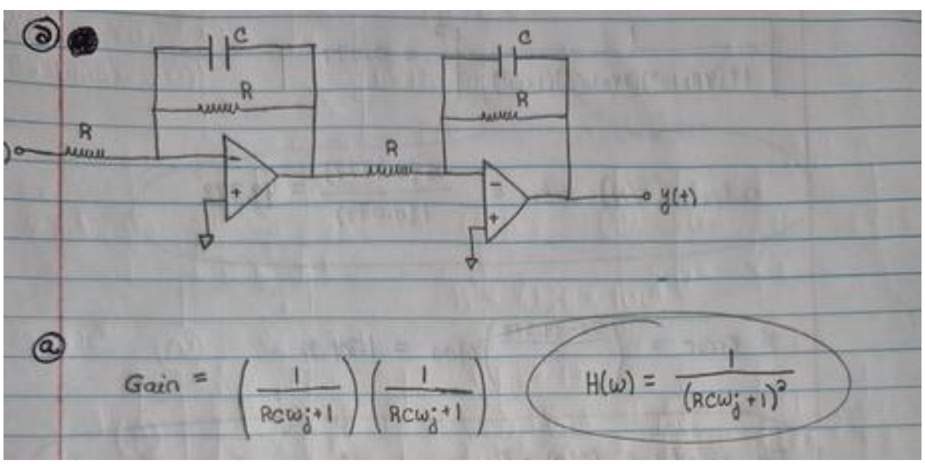


Figure 3: Input and output voltages of the signal plus the noise

NOTE: The simulation shows the filter's ability to maintain the desired signal and filter out the high frequency noise.

Part 2: Second-Order Filter

Step 1: Finding the transfer function $H(\omega) = \frac{Y(\omega)}{X(\omega)}$



Step 2: Designing the filter to attenuate the noise by a factor of 10

(b)

$$\left|\frac{1}{RC\omega j+1}\right|^2 = \left(\frac{1}{\sqrt{1+(\omega CR)^2}}\right)\left(\frac{1}{\sqrt{1+(\omega CR)^2}}\right) = \frac{1}{1+(\omega CR)^2}$$

Noise

$$0.1 = \frac{1}{1+(\omega CR)^2} \Rightarrow 10 = 1+(\omega CR)^2 \Rightarrow 9 = (\omega CR)^2$$

$$3 = \omega CR \Rightarrow CR = \frac{3}{(4\pi \times 10^4)} = 2.387 \times 10^{-5} \text{ sec}$$

$$R = 1\,k\Omega \Rightarrow C = \frac{2.387 \times 10^{-5}}{1 \times 10^3} = 23.87 \text{ nF}$$

$$R = 1\,K\Omega, \quad C = 24 \text{ nF}$$

Step 3: Calculating the side effects of the filter

(c)

$$\left|\frac{1}{1+j(2\pi \times 10^3)(1\,k\Omega)(24 \times 10^{-9})}\right|^2 = 0.9778$$

Undesired signal attenuation = 0.9778

$$\left|\frac{1}{1+(4\pi\times10^{4})(24\times10^{-9})(1\times10^{3})j}\right|^2 = 0.099$$

Output (S/N) ratio = $\frac{5(0.9777)}{1(0.099)} = 49.378$

% Error = $\left(\frac{50-49.378}{50}\right)\times 100 = 1.24\ \%$

The calculated (S/N) ratio is extremely close to the desired (S/N) ratio. There is a % error of 1.24 %

$$\phi = -\tan^{-1}\left((2\pi\times10^{3})(1\times10^{3})(24\times10^{-9})\right)\left(180^\circ/\pi\ \text{rad}\right) = -8.57^\circ$$

The signal output lags behind the signal input by 8.57°

Undesired Attenuation: 0.9778
Signal to Noise Ratio (S/N) : 49.378
Phase Shift: 8.57°

NOTE: The calculated Signal to Noise ratio (S/N) is extremely close to the desired ratio of 50. There is a percent error of 1.24% between the desired and calculated error.

Step 4: Simulations

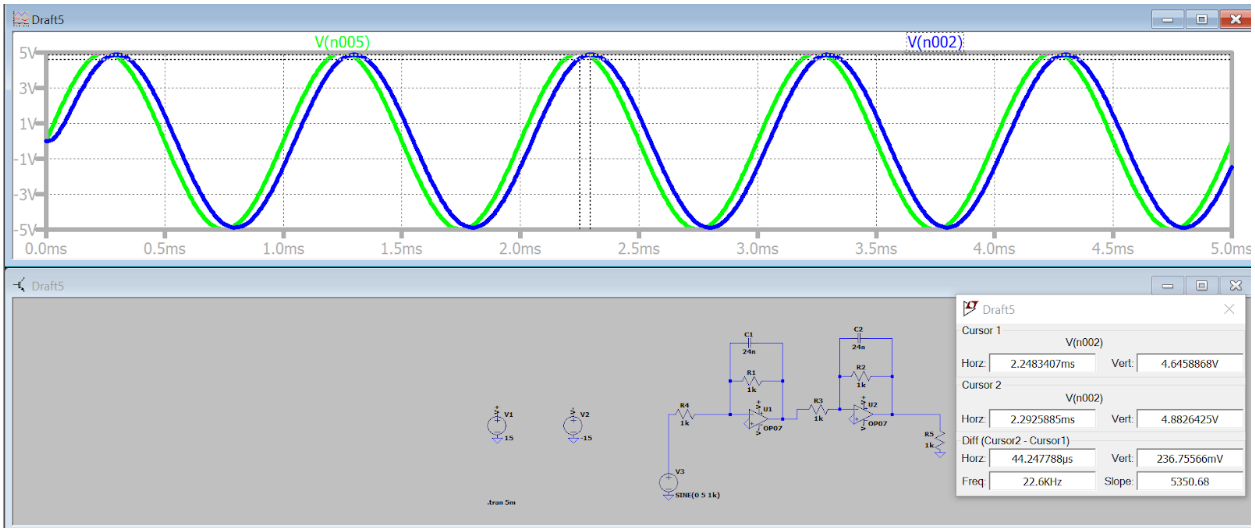


Figure 4: Input and output of the pure signal

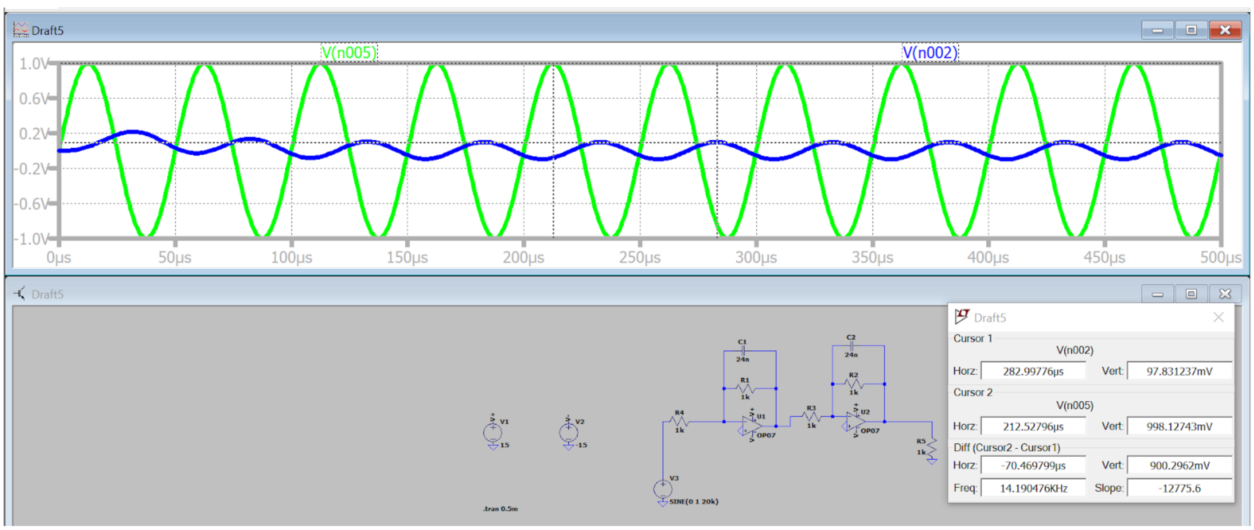


Figure 5: Input and output voltages of the pure noise

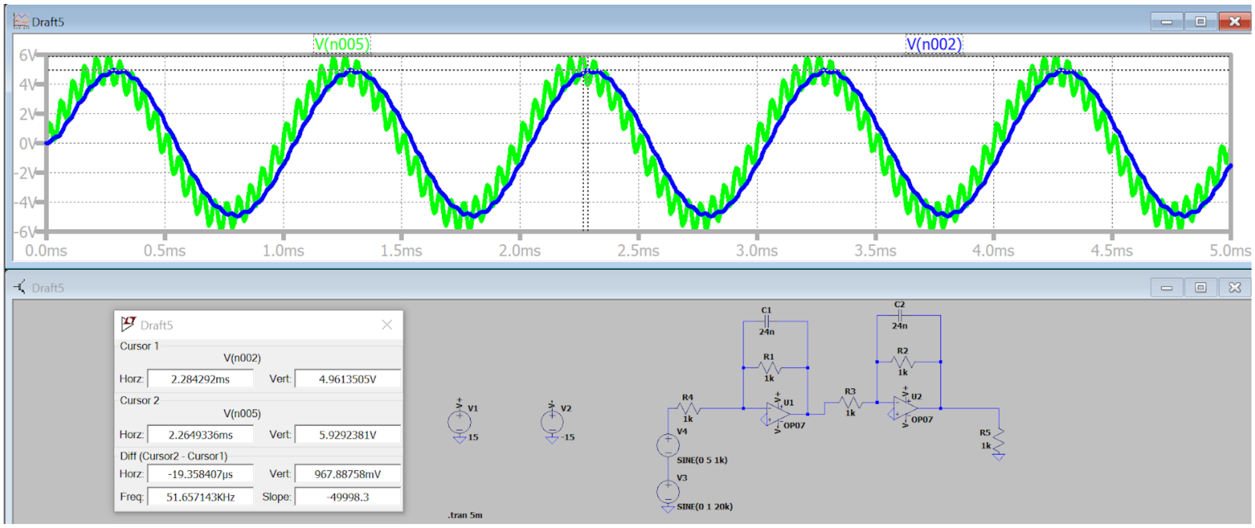


Figure 6: Input and output voltages of the signal plus the noise

Part 3: Comparison of the two filters

The first order-filter was able to achieve a signal to noise ratio (S/N) of 44.7. With a percent error of 10.6%. Our second-order filter, however achieved a signal to noise ratio (S/N) of 49.4. This was much closer to our goal of 50 with only a 1.24% percent error.

There are two main side effects of our filters. Undesired Attenuation and a phase shift of the output signal. The undesired attenuation for both signals is a slight, diminishing gain in the output. The closer the undesired attenuation is to 1 the better the filter.

The first-order filter had an undesired attenuation of 0.8957 and a phase shift of 26.4°. The undesired attenuation of our first-order filter created a noticeable change in amplitude to our output signal. The filter has a percent error of 0.209% compared to our goal of 1. Dependent on the application the attenuation may not be a problem. However, the resulting phase shift of the first-order filter is more noticeable, causing a substantial delay in the output. Limiting the filters application in applications that require faster responses.

The second-order filter had an undesired attenuation of 0.9788 and a phase shift of 8.57°. This undesired attenuation is much closer to 1 with a percent error of 0.044%. This second-order filter is much more exact with an undesired attenuation equal to 1 and a much smaller phase shift closer to 0.

In conclusion the second-order filter, when compared to the first-order filter, is much more exact. The second-order filter produces a signal to noise ratio closer to our desired 50/1, while also having an undesired attenuation closer to one and a phase shift closer to 0. The first-order filter however is still relevant as it only requires a single capacitor instead of two. Meaning the cost of production for the first-order filter should be lower. So, depending on your application, the first-order filter still has its place.