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Decay of the Z^0-Boson Into Lepton Pairs*

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1. INTRODUCTION

The Z^0-boson is an elementary particle that is commonly produced in proton-proton interactions. It's a neutral carrier of the *weak* force. The Z^0 is unstable and eventually decays; 10% of the time, it will decay into pair of charged leptons ($Z^0 \to l\bar{l}$). Due to the conservation of energy, the sum of the energy stored in the two leptons must sum to the mass of the Z^0. Thus, a peak/maximum frequency of double-lepton detections should be present at the mass of the Z^0.

In this report, we investigate the distribution of double-lepton events using real world ATLAS (A Toroidal LHC Apparatus) data. We also explore the theoretical distribution of decays by comparing this data with the Breit-Wigner peak, which utilizes two fitting parameters. We visualize the joint probability space to see the confidence of this 2D fit.

Calculations of the succeeding sections made use of graphical software composed of arrays and mesh grids, which will be explained in later sections. Significance tests utilized fitting and statistical python packages cited below.

2. THE INVARIANT MASS DISTRIBUTION

The *four momentum* of a particle, p, can be represented as the following vector: $p = (E, p_x, p_y, p_z)$, where component momentum's are described by:

$$p_x = p_T \cos\phi, \quad p_y = p_T \cos\phi, \quad p_x = p_T \sinh\eta \tag{1}$$

Where p_T is the transverse-momentum, η is the pseudo rapidity (angle at which the particle makes w.r.t the beamline detector), and ϕ is the azimuthal angle about the beam.

The difference between the three-momentum and the energy is the particle's **invariant mass**:

$$M = \sqrt{E^2 - \left(p_x^2 + p_y^2 + p_z^2\right)} \tag{2}$$

Meaning, that if you have two particles, such as in the double-lepton case, the sum of the four momenta is simply $p_{tot} = p_1 + p_2$.

Five-thousand 2020 ATLAS open dataset events were allocated into numpy arrays from a .csv file via numpy's array generation function. The first two columns of data in the .csv file contained the transverse-momentum p_T (in GeV) for the two leptons; the next two contained η, the next two contained ϕ (radians), and the last two contained energy E (GeV^2).

With the data now loaded, we made a histogram of the calculated invariant mass, with masses ranging from 80 to 100 GeV and 41 bins. We assumed Poisson error (Where $\sigma = \sqrt{N}$) for the binning frequencies.

We know that, due to scattering theory, the distribution of decays $\mathcal{D}$ follows the Breit-Wigner peak. This distribution is dependent on the true rest-mass of the Z^0 (m), m_0, and the 'width' parameter Γ:

$$\mathcal{D}(m; m_0, \Gamma) = \frac{1}{\pi} \frac{\Gamma/2}{(m - m_0)^2 + (\Gamma/2)^2} \tag{3}$$

The Breit-Wigner function can now be applied to the mass-distribution by fitting the data with $5000/2 \times \mathcal{D}$ (or $\frac{5000*10^9}{2} \times \mathcal{D}$ if working with just eV). In order to make uniform grading possible, we did an isolated fit using only data with invariant masses between 87-93 GeV. Using scipy.optimize.curve_fit, we were able to determine the following values, as well as their uncertainties:

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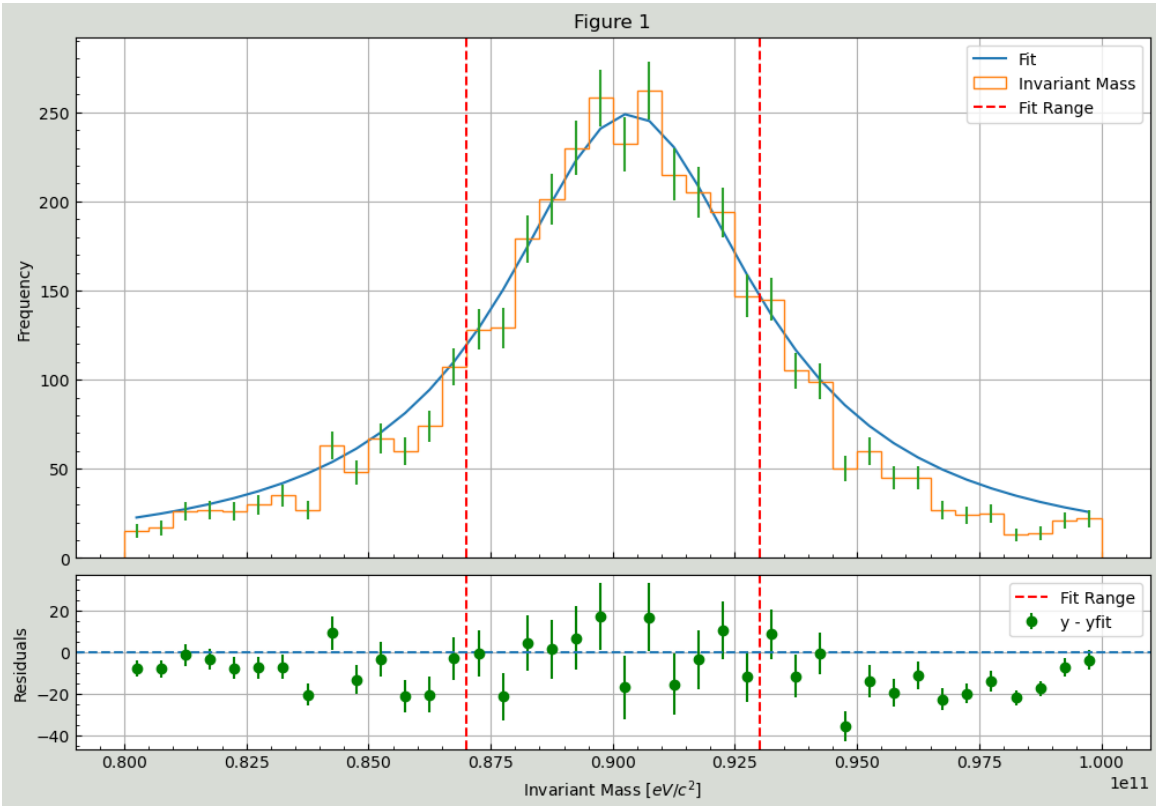

Figure 1. This histogram graphs the frequency of double lepton events vs. the total invariant mass of the decay. The blue line is the Breit-Wigner peak. The horizontal dashed lines mark the region where the fitting was done. The lower plot graphs the residuals (y-yth) of the measured data vs. theory, with the dotted blue line representing perfect agreement.

Table 1. Fitted Values

m_0 (GeV)	Γ_0 (GeV)	χ^2	n DoF	Reduced χ^2	p value
90.3 ± 0.1	6.4 ± 0.2	10	10	1.0	0.4

From Figure 1 as well as Table 1, we can see that the ATLAS data does agree with theory, with $p > 0.5$. If we were to expand the fitting range to the lower and higher extremes, this agreement will weaken, however. We only had 12 data points in the specified range, with a DoF of 10. We can also see, qualitatively, that the Breit-Wigner function repeatedly overestimates the values at the extremes, but this could be due to sensor limitations under representing this. Nevertheless, both data and function indicate statistical agreement.

3. 2D PARAMETER SCAN

Since Z^0 and Γ cannot be determined independently, we must look at the joint-probability space.

To do so, we performed a 2D chi-square scan of the mass-width parameter space, with a mass range of 83-91 GeV, and a width scanning range of 5-8 GeV. We decided to use 300 bins along each dimension for optimal resolution. Using the mapping space for both Z^0 and Γ, we calculated the χ^2 value for each value combination in the mesh space. Logically, the best fit value that was found by scipy.optimize.curve_fit should be where χ^2 is at its minimum.

For better visualization, $\Delta\chi^2$ ($\chi^2 - \chi^2_{min}$) was plotted in a contour plot. This was done by subtracting the minimum χ^2 value of the entire mesh grid from the coordinate χ^2 value. For a free parameter N=2 case, the 1σ and 2σ uncertainties in $\Delta\chi^2$ is located where $\Delta\chi^2$ is equal to 2.30 and 4.61, respectively.

We can see that the 1σ and 2σ confidence intervals narrow down the values of Γ_0 and m_0 quite well, resulting in the small uncertainties present in Table 1.

4. DISCUSSION AND FUTURE WORK

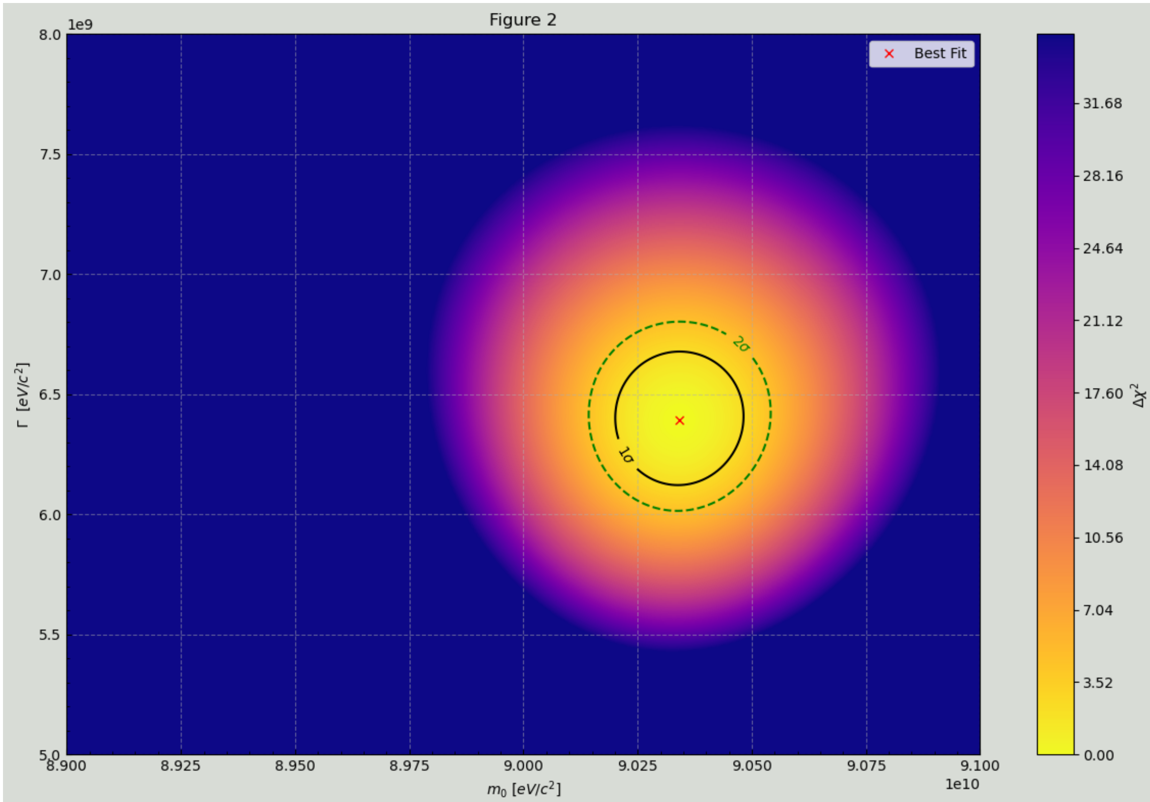

Figure 2. This is a contour plot of $\Delta\chi^2 = \chi^2 - \chi^2_{min}$. $\Delta\chi^2$ was clipped at 35 units to make visualization of the plot easier. 1σ and 2σ confidence labels are outlined via a solid and dashed line. The best fit location made up of the two free parameters from last section is indicated via an 'x'.

From the 2020 ATLAS open data set, we found the value of m_{Z^0} of 90.3 ± 0.1 GeV$/c^2$, as well as the value of Γ_0 to be 6.4 ± 0.2 GeV$/c^2$. We found that the data agreeed with the Breit-Wigner peak function, with $0.5 < p < 0.95$.

Unfortunately, this does not agree with the latest accepted value of 91.1880 ± 0.0020 GeV$/c^2$. We made a number of assumptions and approximations with the data that could explain this disparity.

First of all, we assumed Poisson error distribution, where $\sigma = \sqrt{N}$, where N is the counts of data. This approximation becomes more unreliable with higher N values (such as the 250 we encounter at the peak). We also had a low bin count of 40, meaning that this error could be misrepresenting 'bundled up' data. Additionally, due to limitations of the physical world, detectors can only measure the width Γ subject to experimental uncertainties, meaning that $\Gamma_{exp} > \Gamma_0$. We also only fitted a small fraction of data, relying on the clean 'peak' range (87 to 93 GeV). This means our parameters do not necessarily represent the extreme ends of our measured data. Critically, our fit does not include systematic uncertainties, or the resolution of the ATLAS detector.

In order to address these limitations, we can utilize a better approximation for uncertainties (Gaussian distribution or Bayesian statistics) for a more sound χ^2 and fitting analysis. Fitting the entirety of our data is a little more tricky. This would be affected by our measuring instrument's resolution at those invariant masses, and would require a whole recalibration. Further work will need to include these equipment resolutions/uncertainties to garner a more accurate m_{Z^0} value.

Software: numpy, scipy.stats, scipy.optimize.curve_fit, matplotlib.pyplot

REFERENCES