Draft version March 10, 2025 $\cdot 10^{-3}$
Typeset using LaTeX **twocolumn** style in AASTeX631

Implementation of the Saturn V Rocket Platform for Future Apollo Missions*

Jeremy Vo[1] and Gene Kranz †[2]

[1] *University of Maryland, College Park*
[2] *NASA*

Keywords: Saturn V, Gravitational Potential, Gravitational Force, ΔV

1. INTRODUCTION

The Saturn V rocket system is a promising platform that will allow American astronauts unprecedented access to the Earth-Moon system, compared to previous Saturn I and IB platforms. It will have the largest payload potential of any American rocket to date and will be the ultimate nail in the coffin for this space race. The five F1 solid rocket design of the first stage will ensure proper penetration of the atmosphere of the Earth, with approximately 7.6 million pounds of thrust (SFC 1968).

In this report, we explore three critical areas of investigation that emphasize the need for such a drastic change in our rocket design, and thus funding for implementation: the gravitational potential of the Earth-Moon system, the gravitational force of the Earth-Moon system, and the projected performance of Stage 1 of the Saturn V rocket design.

Calculations of all three fields made use of graphical software composed of arrays and mesh grids, which will be explained in later sections.

2. THE GRAVITATIONAL POTENTIAL OF THE EARTH-MOON SYSTEM

Gravitational potential is the amount of work, or transfer of energy, per unit mass at a certain point in space (having units of J/kg). Calculating gravitational potential is critical to understanding the amount of kinetic energy per unit mass required to break low-earth orbit (LEO).

* Released on May, 9th, 2025

The gravitational potential at distance r by a mass M is expressed by the following equation:

$$\phi(r) = -\frac{GM}{r} \tag{1}$$

Where G is the gravitational constant.

To visualize the gravitational potential of the Earth-Moon system, a mesh grid with dimensions $1.5d_{moon} \times 1.5d_{moon}$ was used, where d_{moon} is the distance to the moon. Gravitational potential at coordinate points were then evaluated with (1).

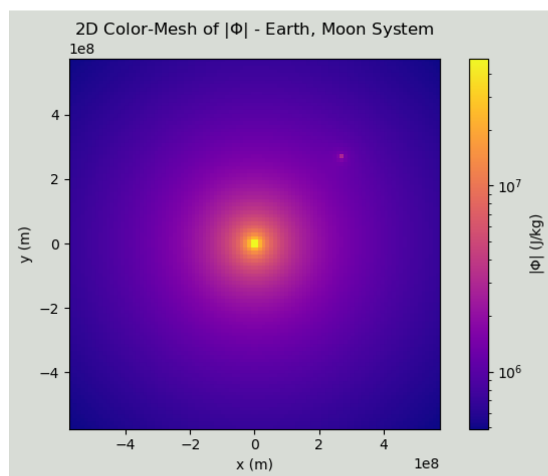

Figure 1. This mesh plot represents the combined gravitational potential of the Earth-Moon system. The Earth's gravitational potential dominates the environment, with the moon only affecting a small portion of the upper right. Brighter colors represent larger gravitational potential. The color bar is logarithmic, meaning actual potential drops off much quicker.

From Figure 1, it's apparent that the gravitational potential of Earth has a massive area of influence, but quickly drops off in intensity with increasing distance.

Most of the gravitational potential is located in LEO, meaning that the power of the first stage of our rocket design is critical.

The newly-designed first stage of the Saturn V system, fitted with 5 F-1 rocket engines, will ensure that Apollo missions have enough energy to overcome the dense gravitational potential of LEO (SFC 1968).

3. THE GRAVITATIONAL FORCE OF THE EARTH-MOON SYSTEM

Gravitational force an attractive force caused by the existence of mass, with the Earth and Moon being the two most relevant objects of mass to our mission. Gravitational force results in attractive acceleration, meaning that an object will accelerate toward that mass, and that mass will accelerate towards the object.

This is what allows the Moon to orbit the Earth. The Earth exhibits centripetal (perpendicular to path of motion, pointed inwards) force on the moon, resulting in a curved, circular path of motion.

Calculating the gravitational force is critical for the Apollo program, as it will highlight the varying forces our rocket will experience during its course to the Moon and back.

The gravitational force $\vec{F}$ that a mass M_1 exerts on m_2 is given by the following equation:

$$\vec{F} = -\frac{GM_1 m_2}{|\vec{r}_{21}|^2}\hat{r}_{21} \tag{2}$$

Where M_1 is the mass of the Earth, m_2 is the mass of the command module, and $\vec{r}_{21}$ 21 is the displacement vector from M_1 to m_2

In order to visualize the direction and magnitude of these forces, a stream plot was used. The same coordinate parameters were used for this stream plot for consistency ($1.5d_{moon} \times 1.5d_{moon}$).

From Figure 2, it's apparent that for most of the journey, the gravitational force the command module will experience will be almost analogous to a point-source, meaning that only the Earth's gravitational force will have a significant impact on the mission. The Moon will only start to significantly affect the net force when the command module is extremely close to the moon. Due to most of the mass of the Saturn V rocket being part of stages 1 and 2, the command module's much smaller mass will allow for more-responsive insertion into lunar orbit.

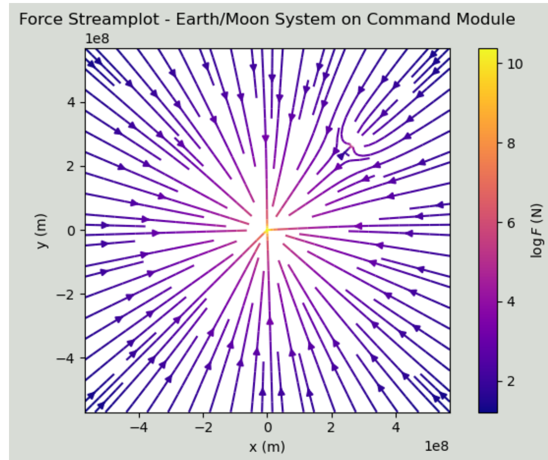

Figure 2. This stream plot represents the path and magnitude of gravitational forces that the Apollo command module will experience in the Earth-Moon system. Once again, the Earth dominates most of the spacial environment, with only a slight perturbation in the upper left caused by the moon's weak gravitational field.

4. PROJECTED PERFORMANCE OF THE SATURN V STAGE 1

Rockets function via the conservation of momentum. The ejection of propellant 'backwards' propels the rockets 'forwards'. The effective change in velocity of a rocket can be quantitatively measured via ΔV.

ΔV as a function of time t can be calculated by Tsiolkovsky's rocket equation:

$$\Delta v(t) = v_e \ln \frac{m_0}{m(t)} - gt \tag{3}$$

Where m_0 is the initial wet mass (fuel, rocket structure, and payload of the system), $m_(t) = m_0 - \dot{m}t$ is the mass at time t, $\dot{m}$ is the fuel burn rate, v_e is the fuel exhaust velocity, and g is gravitational acceleration.

The change in distance is simply the integral of ΔV, where:

$$h = \int_0^T \Delta v(t)dt \tag{4}$$

Where T is the total burn time of the rocket, which can be computed via:

$$T = \frac{m_0 - m_f}{\dot{m}} \tag{5}$$

Where m_f is the dry mass (no fuel) of the rocket.

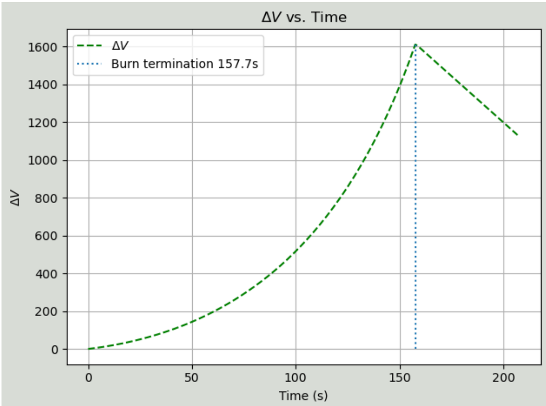

Figure 3. ΔV of the first stage of the Saturn V rocket. The first stage ΔV is ~ 1.6 km/s, reaching a height of ~ 74 km.

Calculations show that Stage 1 of Saturn V has approximately 157.7 seconds of burn time, and reaches a height of approximately 74 kilometers. This will enable the system to punch through the stratosphere and troposphere (0-50 km), the two most dense layers of the atmosphere.

5. DISCUSSION AND FUTURE WORK

Many approximations were used to simplify the calculations that were undertaken in this investigation, and also to make findings more comprehensive for the general public. These are simply preliminary calculations to show how the Saturn V platform makes it possible for a round-trip to the Moon.

A more comprehensive series of calculations will be undertaken once funding for future Apollo missions is secured. Air resistance is a critical factor that needs to be accounted for, as test results from the first prototype of Saturn V have found a practical burn time of $T = 160$ seconds, as well as an altitude of $h = 70$km. 4 kilometers are lost due to air resistance and other friction variables competing against the motion of the rocket. We also assumed that g was constant, when in reality it decreases as the system gets higher in altitude. An extra 2.3 seconds of burn time are also required, due to the fact that burn rate $\dot{m}$ is not constant, like we assumed earlier. We also need to account for the fact that the Earth and Moon are not static objects. NASA teams will have to calculate optimal launch dates to have an efficient flight path.

Ultimately, the Saturn V rocket platform is a powerful system that will enable US astronauts to break low-Earth orbit and land on the moon. Future calculations will require a more in-depth analysis of the gravitational and atmospheric variables present in the Earth-Moon system to find an optimal flight path. With the Saturn V system, we are confident that we can achieve a moon landing by 1968-1970.

Software: numpy, scipy, matplotlib

REFERENCES

SFC, G. C. M. 1968, MSFC-Man-503. https://www.nasa.gov/wp-content/uploads/static/history/afj/ap08fj/pdf/sa503-flightmanual.pdf