ENGG 2470A **Assignment 2** **Spring 21**

1. (Binomial Formula)

   $$\frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!} = \frac{(n+1)!}{r!(n-r+1)}$$

   (a) Prove that for $n \geq 1$ and $1 \leq r \leq n$,

   $$\binom{n}{r} + \binom{n}{r-1} = \binom{n+1}{r}$$

   $$\frac{n!(n-r+1)}{r!(n-r+1)!} \quad \frac{n! \cdot r}{r!(n-r+1)!}$$

   (b) Use this formula to explain why Pascal's triangle works. You do not have to submit your work for this part.

   (c) Prove the binomial formula by mathematical induction.

2. (Horses again!) Someone claims to have proved by mathematical induction that all the horses in the world have the same color. Here is his formulation. Let n be the number of horses in any group of horses. For all $n \geq 1$, the proposition is that all the horses in the group have the same color. Here is the proof:

   (a) The proposition is obviously true for $n = 1$.

   (b) Assume that the proposition is true for some $n \geq 1$. Now consider any group of $n+1$ horses. By the induction hypothesis, Horses 1 to n, which is a group of n horses, have the same color. Again by the induction hypothesis, Horses 2 to $n+1$ have the same color. Since Horses 2 to n are common to both groups of n horses, the two groups of n horses must have the same color. Therefore, we conclude that Horses 1 to $n+1$ all have the same color.

   Are you convinced?

Textbook Problems 1.2.1, 1.2.2, 1.3.2, 1.3.4, 1.3.5, 1.4.1, 1.4.2, 1.4.5, 1.4.6.