ENGG 2470A **Assignment 7** **Spring 21**

Textbook Problems 8.4.10, 9.1.1, 9.1.2, 9.1.4, 9.1.5, 9.3.1, 9.3.2.

Notes Work out Problem 9.1.5 by two methods:

1. Use $\operatorname{var} W = \operatorname{var} X + \operatorname{var} Y + 2\operatorname{cov}(X, Y)$.
2. First determine $f_W(w)$.

Supplementary Problems

1. Let $\mathbf{Y} = [\,Y_1\ Y_2\,]^\top$ and $\mathbf{X} = [\,X_1\ X_2\,]^\top$ be two random vectors such that $\mathbf{Y} = A\,\mathbf{X}$, where A is a 2×2 *invertible* real matrix. Determine $f_{\mathbf{Y}}$ in terms of $f_{\mathbf{X}}$.

2. Show that
$$\operatorname{var}\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n}\sum_{j=1}^{n} \operatorname{cov}(X_i, X_j).$$
Compare this formula with $\operatorname{var}(X+Y) = \operatorname{var} X + \operatorname{var} Y + 2\operatorname{cov}(X, Y)$.

3. The expectation operator E can be applied to a random vector. Specifically, if $\mathbf{X} = [\,X_1\ X_2\ \cdots\ X_n\,]^\top$, then $E\,\mathbf{X} = [\,EX_1\ EX_2\ \cdots\ EX_n\,]^\top$. Show that
$$E[(\mathbf{X} - E\mathbf{X})(\mathbf{X} - E\mathbf{X})^\top] = [\,\operatorname{cov}(X_i, X_j)]_{i,j=1}^{n}\,.$$
This matrix is called the covariance matrix of the random vector $\mathbf{X}$.

4. Let $X_1, X_2, \cdots$ be i.i.d. random variables $\sim X$, and let N be random variable taking values in $\{0, 1, 2, \cdots\}$.
   (a) Show that $E[X_1 + X_2 + \cdots + X_N] = (EN)(EX)$ if N is independent of $X_1, X_2, \cdots$.
   (b) Is it true in general that $E[X_1 + X_2 + \cdots + X_N] = (EN)(EX)$?