Let $x = r\cos\theta$ $dx\,dy = r\,dr\,d\theta$
$y = r\sin\theta$

$$I^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{2\pi} e^{-\frac{1}{2}(x^2+y^2)}\,dx\,dy = \int_0^{2\pi}\int_0^{\infty} \frac{1}{2\pi} e^{-\frac{1}{2}r^2} \cdot r\,dr\,d\theta$$

$$= \int_0^{2\pi} \frac{1}{2\pi}\,d\theta \int_0^{\infty} r e^{-\frac{1}{2}r^2}\,dr.$$

1. The pdf of the Gaussian distribution $\mathcal{N}(0,1)$ is given by

$$I = \int \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \cdot dx$$

   Show that $\int_{-\infty}^{\infty} f(x)dx = 1$. Hint: Use the polar coordinates.

2. Show that the expectation of the exponential distribution with parameter λ is $1/\lambda$. $f(x) = \lambda e^{-\lambda x}$, $x \geq 0$

   $E[X] = \int_0^{\infty} x \cdot \lambda e^{-\lambda x}\,dx$ integration by parts

3. The pmf of a geometric distribution with parameter p is given by

$$p_n = \begin{cases} p(1-p)^n & n = 0, 1, 2, \ldots \\ 0 & \text{otherwise.} \end{cases}$$

   $\sum^{\infty} np(1-p)^n$

   $p(1-p)\underline{\sum n(1-p)^{n-1}}$

   $\frac{1}{p^2}$

   Show that the expectation is $(1-p)/p$.

Textbook Problems 3.6.1, 3.6.7, 3.7.6, 3.8.5, 4.2.2, 4.3.2, 4.3.4, 6.2.3, 6.2.5.

Notes

1. In Problem 6.2.3, *Tour de France* is the world's most prestigious bicyle race.

2. In Problem 6.2.5, in addition to using the method described in Section 3.4 in the lecture notes, also obtain $f_W(w)$ by differentiating $F_W(w)$.