$F_Z(z) = \iint_{\{(x,y): x+y<z\}} f_{XY}(x,y)\,dx\,dy$ CDF Method

$= \int_{-\infty}^{\infty} \int_{-\infty}^{z-y} f_{XY}(x,y)\,dx\,dy$

$f_Z(z) = \frac{d}{dz}(F_Z(z)) = \int_{-\infty}^{\infty} \frac{d}{dz} \int_{-\infty}^{z-y} f_{XY}(x,y)\,dx\,dy$

$= \int_{-\infty}^{\infty} f_{XY}(z-y,y)\,dy$

$f_Z(z)\cdot dz = P[z \in [z, z+dz]] = P[X+Y \in [z, z+dz]]$

$= \iint_{\{(x,y): x+y \in [z, z+dz)\}} f_{XY}(x,y)\,dx\,dy$

ENGG 2470A **Assignment 6** **Spring 21**

1. Let $(X, Y) \sim f_{XY}(x, y)$ and $Z = X + Y$. Show that

$Y = Z - X$

$$f_Z(z) = \int_{-\infty}^{\infty} f_{XY}(x, z - x)\,dx = \int_{-\infty}^{\infty} f_{XY}(z - y, y)\,dy.$$

2. According to the statistics, among the students coming back from UK and USA, 1% are infected by COVID-19. They all take the saliva test with accuracy 99%. Suppose a particular student's saliva test is negative. A layman makes the following interpretation. The probability that the student is infected in the first place is 0.01 (i.e., 1%). Since the test independently indicates that he is not infected and the error probability of the test is 0.01 (i.e., 1%), the actual probability of the student being infected is $0.01 \times 0.01 = 0.01\%$. Comment on his interpretation.

$P(A=1) = 0.01$

$P(B=0|A=1) = 0.01 = P(B=1|A=0)$

$P(B=0) = P(B=0|A=1)\cdot P(A=1) + P(B=0|A=0)\cdot P(A=0) = 0.01 \times 0.01 + 0.99 \times 0.99$

A: student actual situation $\{0,1\}$, 0 for negative, 1 for positive.

B: test result of this student. $\{0,1\}$

we want: $P(A=1|B=0) = \frac{P(B=0|A=1)\cdot P(A=1)}{P(B=0)}$

$= \frac{0.01 \times 0.01}{0.01 \times 0.01 + 0.99 \times 0.99}$

$\approx 0.0102\%$

Textbook Problems 5.5.5, 6.5.4, 7.1.1, 7.1.5, 7.2.1, 7.2.3, 7.3.3, 7.4.3, 7.4.8, 7.5.7, 7.6.1.

Notes Work out Problem 6.5.4 by two methods:

1. Take advantage of the observation that X and Y are independent random variables and apply the result in Example 4.17.

2. Use the result in the supplementary problem above.