Assignment 1: Solution

1. Prove that '$X \to Y$' is equivalent to '$\sim (X \wedge \sim Y)$'.

Truth Table:

X	Y	$X \to Y$	$\sim Y$	$X \wedge \sim Y$	$\sim(X \wedge \sim Y)$
T	T	T	F	F	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	F	T

2. (De Morgan's Law)

(a) Prove that '$\sim (X \wedge Y)$' is equivalent to '$(\sim X) \vee (\sim Y)$'.

Truth Table:

X	Y	$X \wedge Y$	$\sim(X \wedge Y)$	$\sim X$	$\sim Y$	$(\sim X) \vee (\sim Y)$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

2. (De Morgan's Law)

(a) Prove that '$\sim (X \wedge Y)$' is equivalent to '$(\sim X) \vee (\sim Y)$'.

(b) Use (a) to prove that '$\sim (X \vee Y)$' is equivalent to '$(\sim X) \wedge (\sim Y)$'.

$$(\sim X') \vee (\sim Y') \quad \Leftrightarrow \quad \sim (X' \wedge Y')$$

$$\sim((\sim X') \vee (\sim Y')) \quad \Leftrightarrow \quad X' \wedge Y'$$

Let $X = \sim X'$ and $Y = \sim Y'$

$$\sim(X \vee Y) \quad \Leftrightarrow \quad (\sim X) \wedge (\sim Y)$$

3. Verify the following set identities by using the Venn diagram. Pay attention to the meaning of these identities.

   (a) $(A - B) - C = A - (B \cup C)$

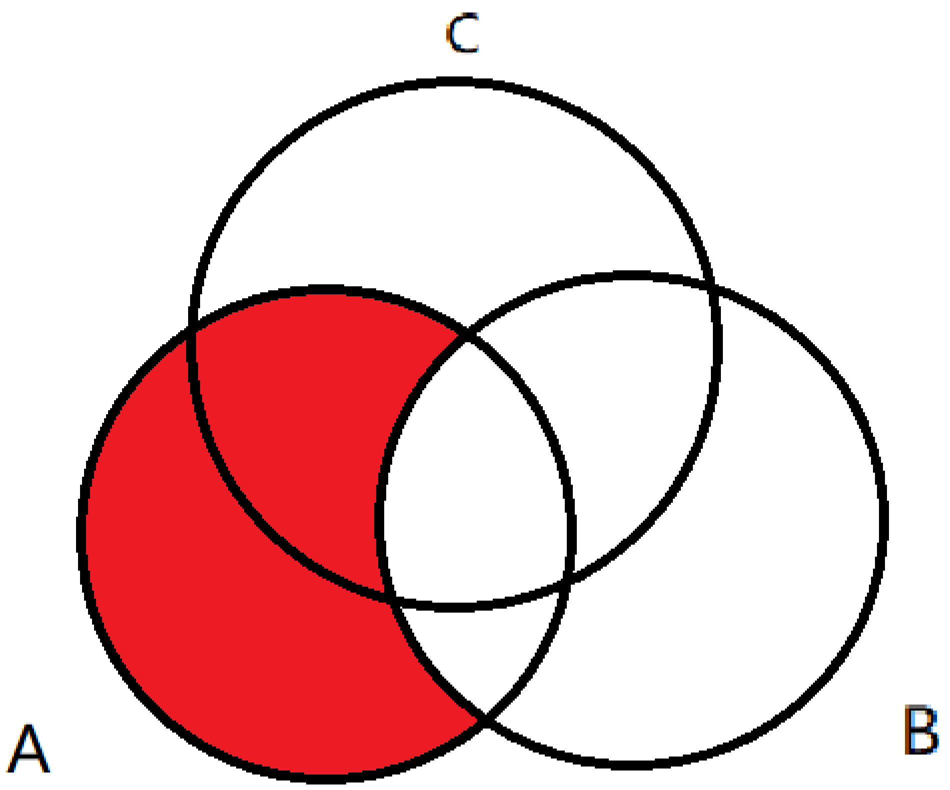
   (b) $(A - B) - C = (A - C) - B$

   (c) $(A \cup B) - C = (A - C) \cup (B - C)$

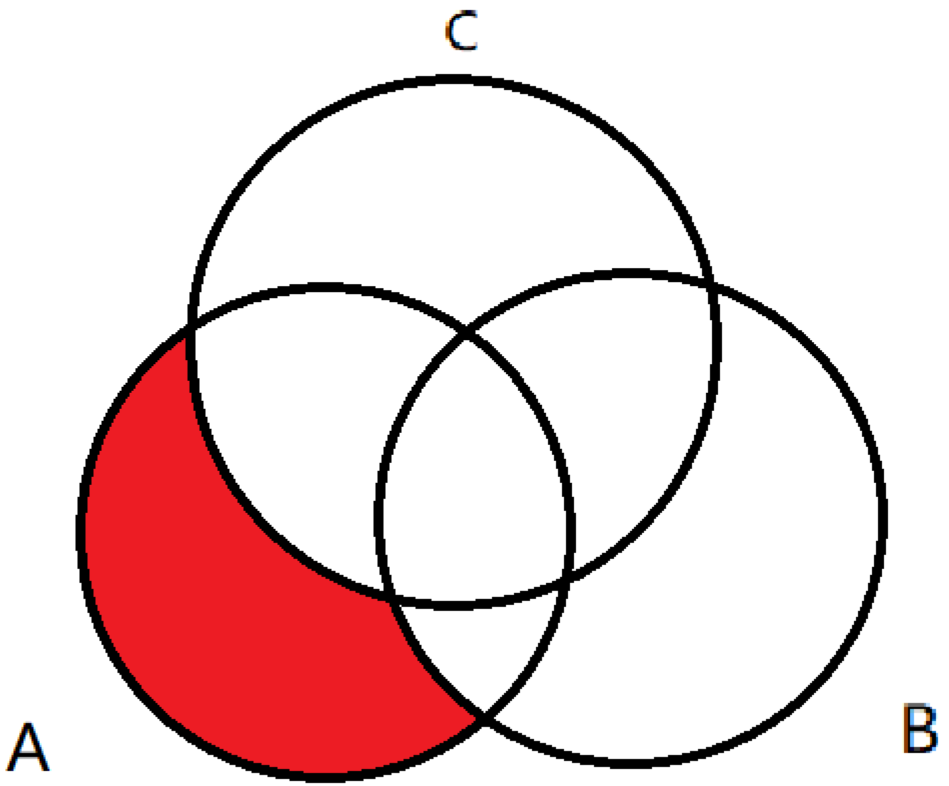
   (d) $(A \cap B) - C = (A - C) \cap (B - C)$

(a) $(A-B)-C = A-(B\cup C)$

B

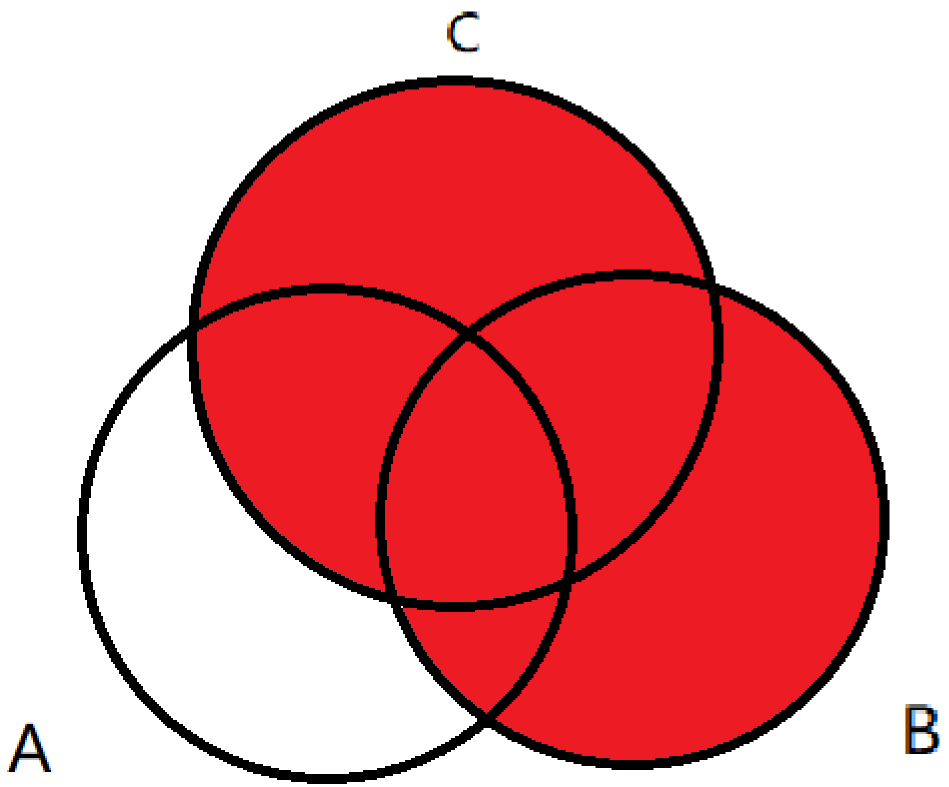


(A-B)-C

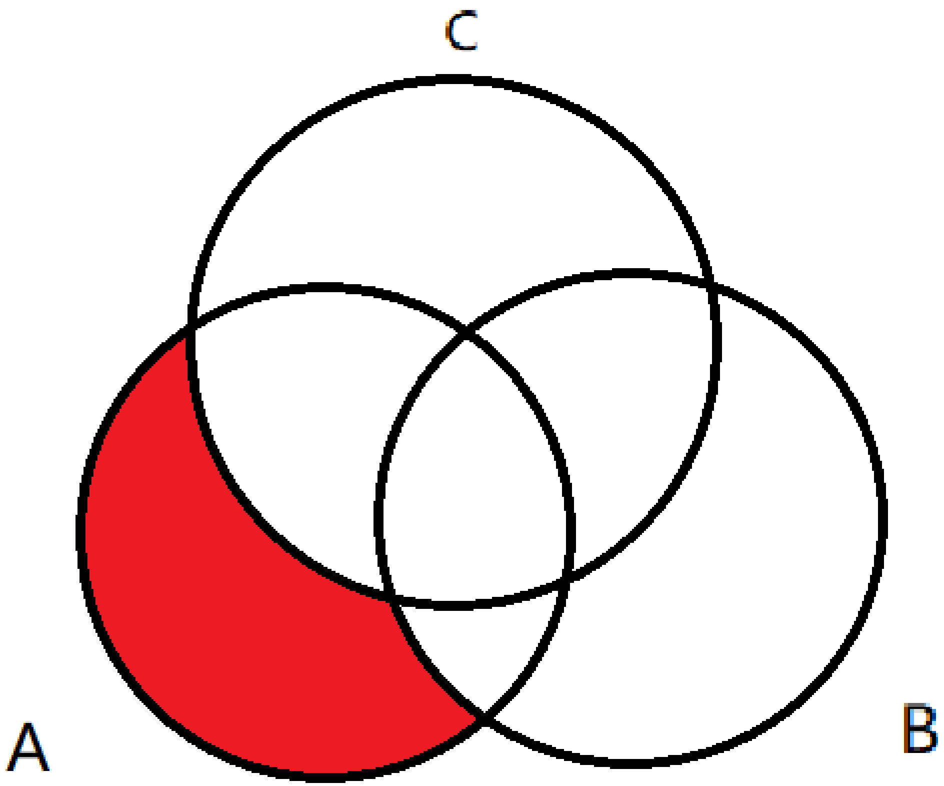


(a) $(A - B) - C = A - (B \cup C)$

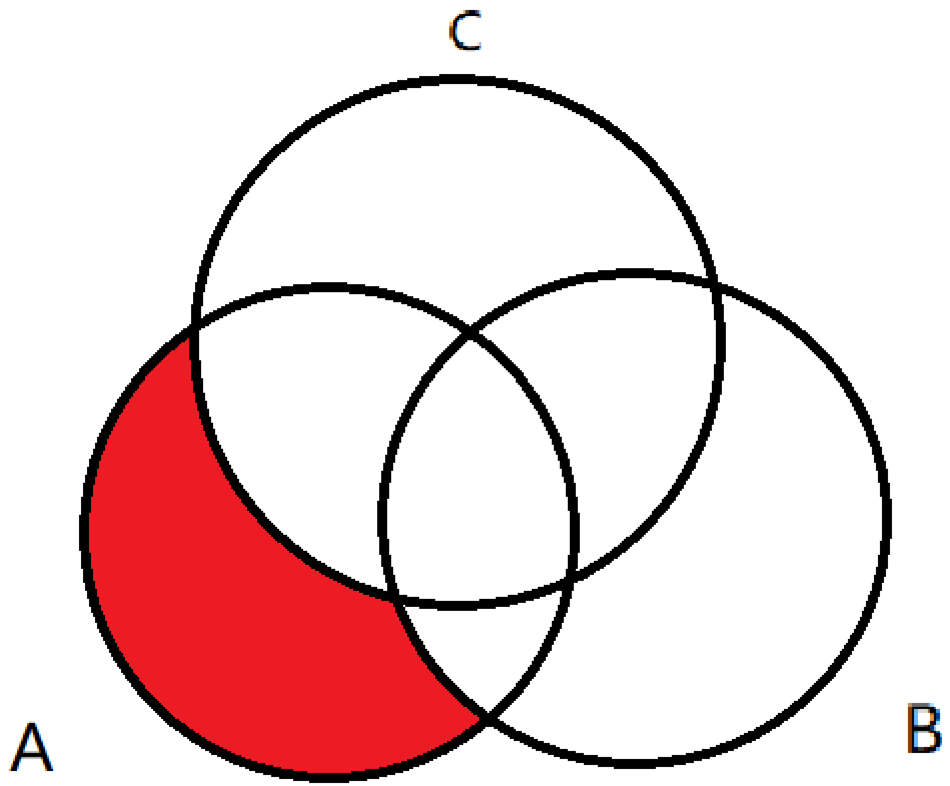
BUC



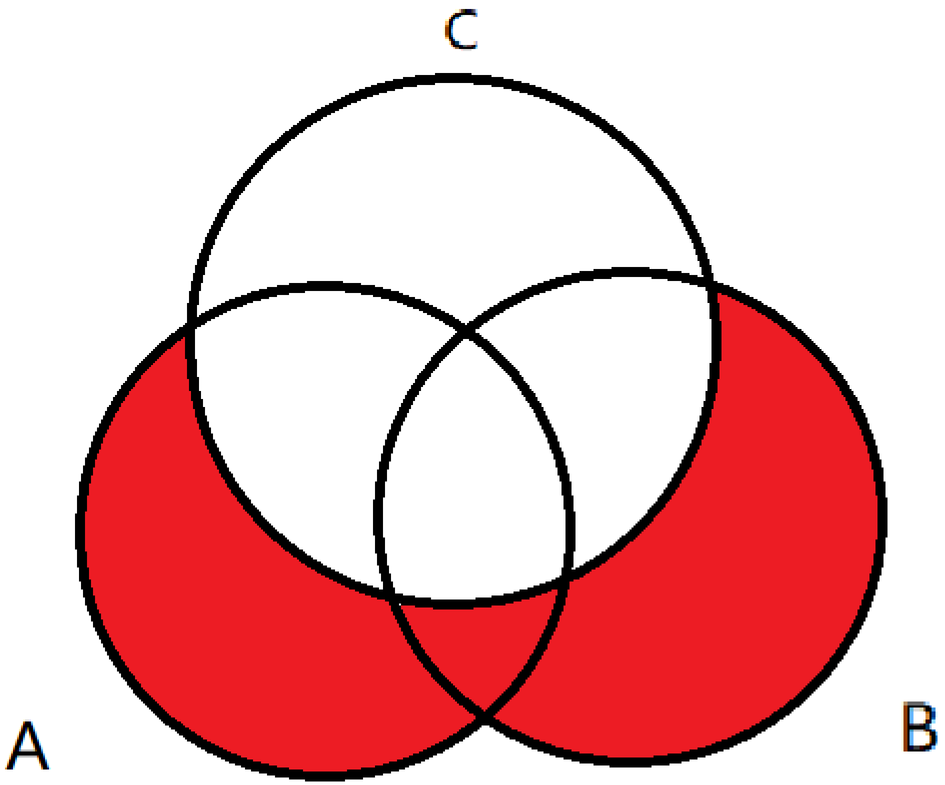
A-(BUC)



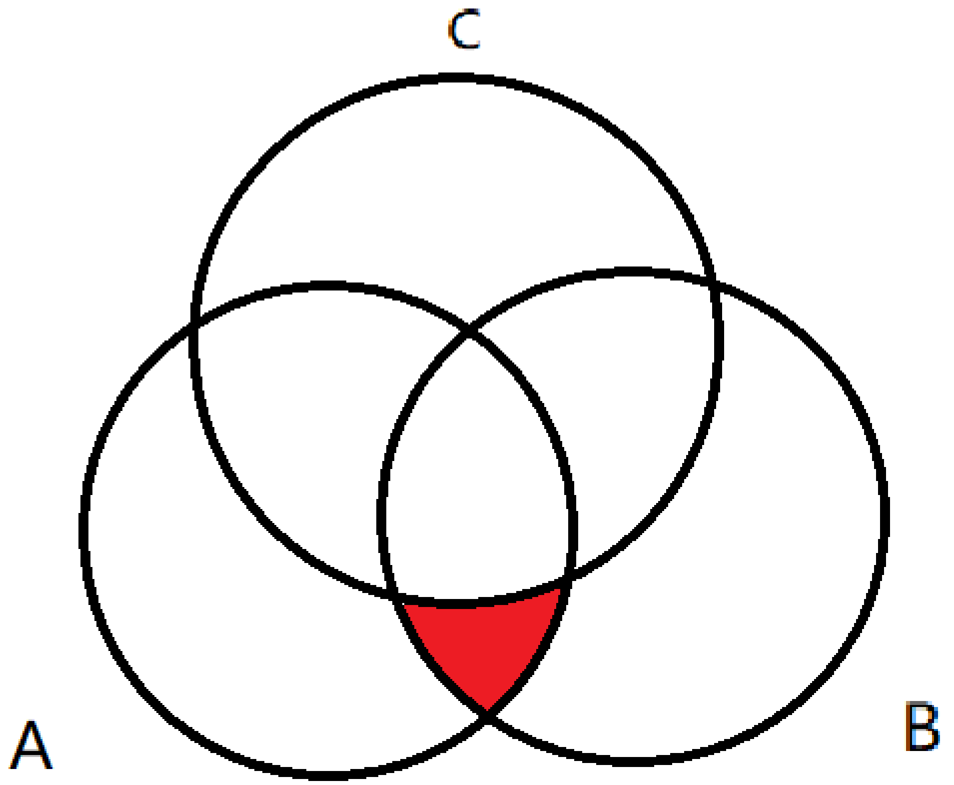
(b) $(A - B) - C = (A - C) - B$



(c) $(A \cup B) - C = (A - C) \cup (B - C)$



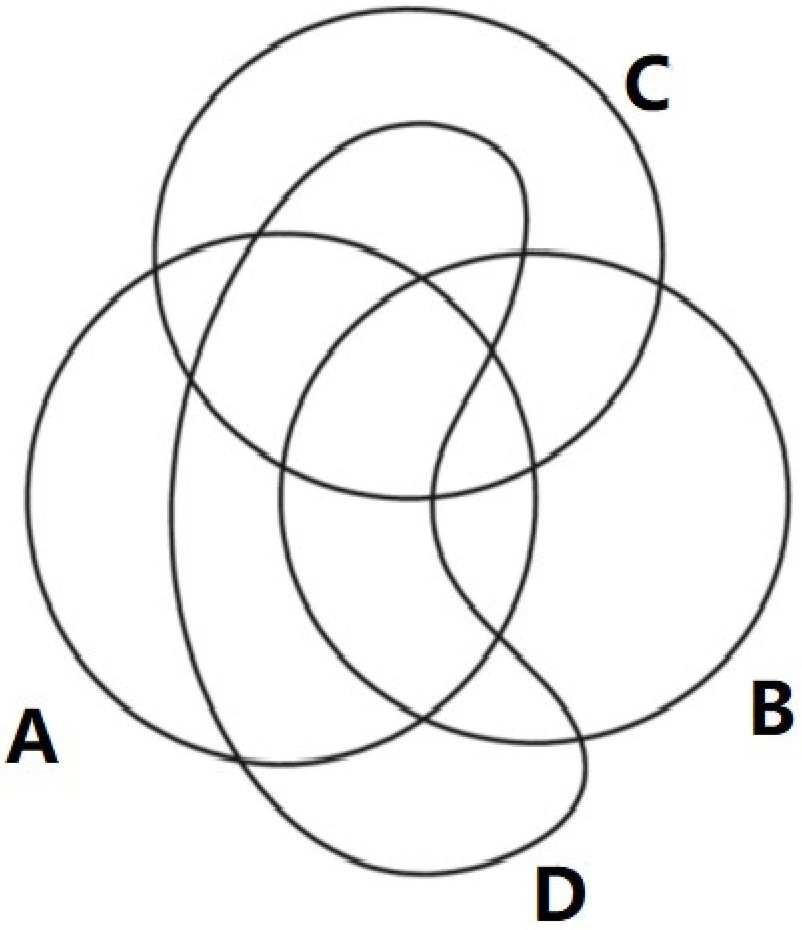
(d) $(A \cap B) - C = (A - C) \cap (B - C)$



4. (a) Show the union of the following sets in a Venn diagram:

$$(A \cap B) - C,\ (B \cap C) - D,\ (C \cap D) - A,\ (A \cap D) - B.$$

Are these sets disjoint?



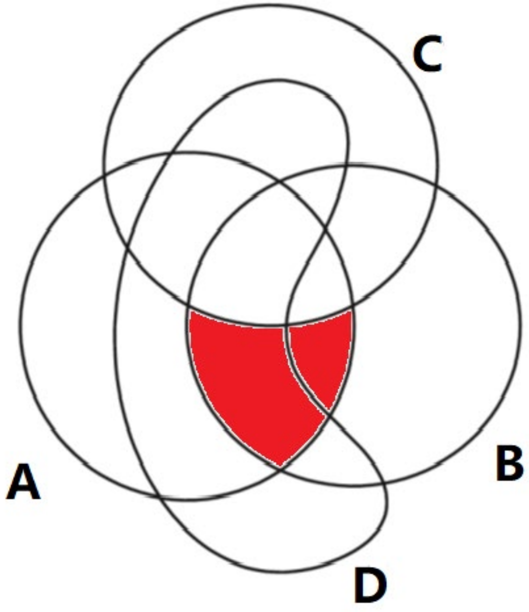
4. (a) Show the union of the following sets in a Venn diagram:

$$(A \cap B) - C,\ (B \cap C) - D,\ (C \cap D) - A,\ (A \cap D) - B.$$

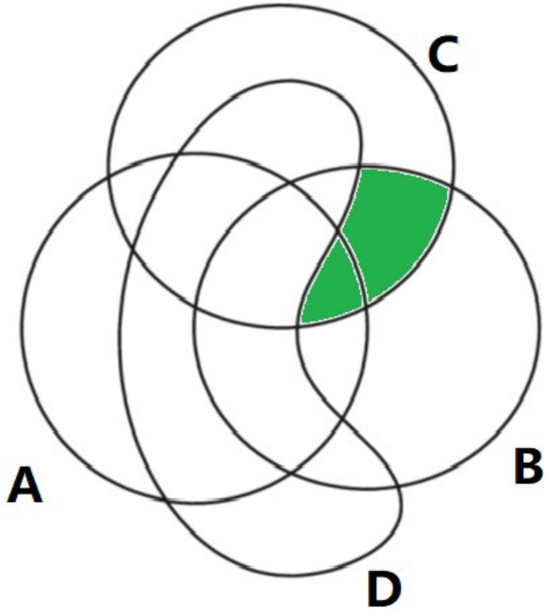
Are these sets disjoint?

Disjoint?

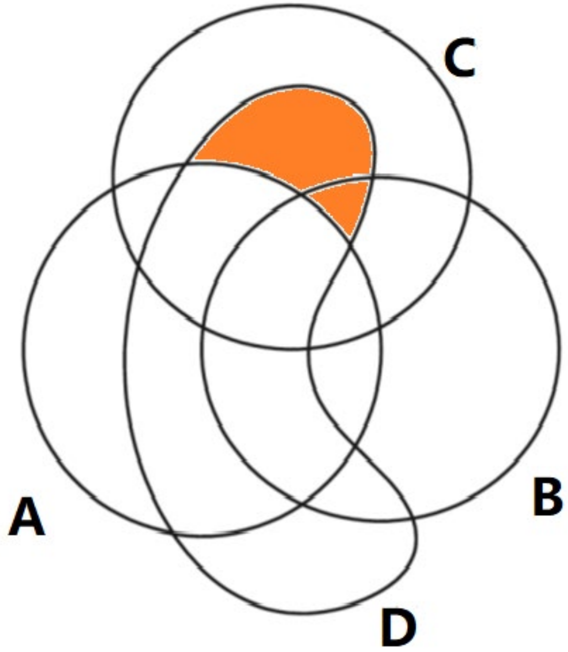
$(A \cap B) - C$



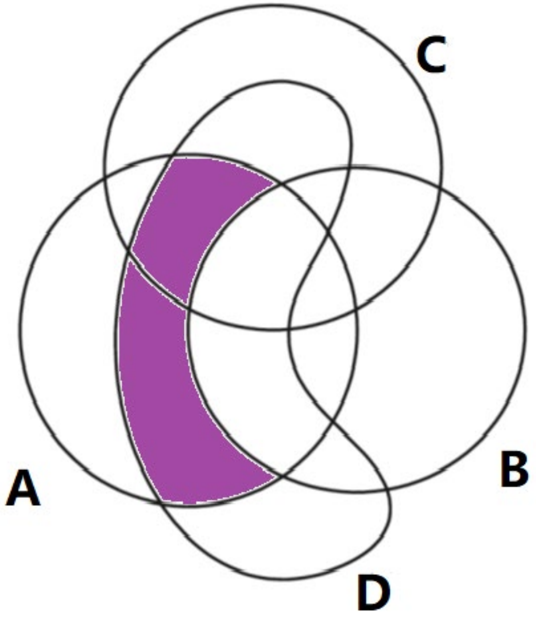
$(B \cap C) - D$



$(C \cap D) - A$



$(A \cap D) - B$



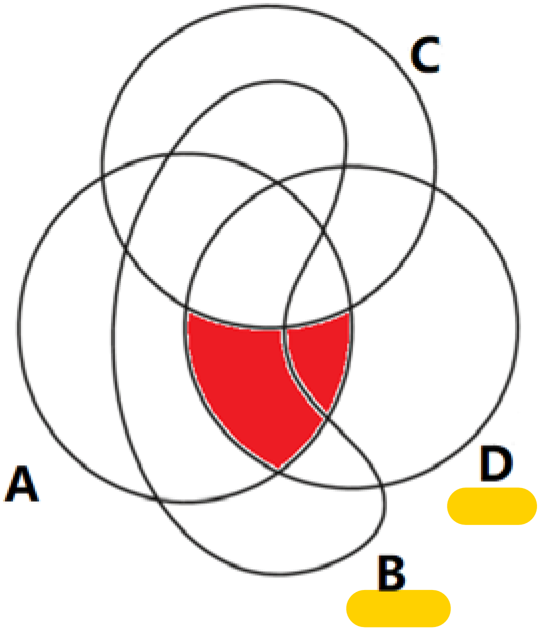
(b) Show the union of the following sets in a Venn diagram:

$$(C \cap D) - B,\ (A \cap D) - C,\ (A \cap B) - D,\ (B \cap C) - A.$$

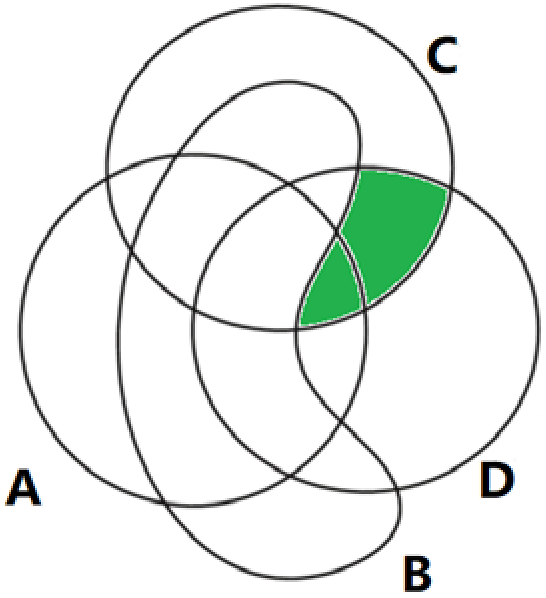
Are these sets disjoint?

Disjoint?

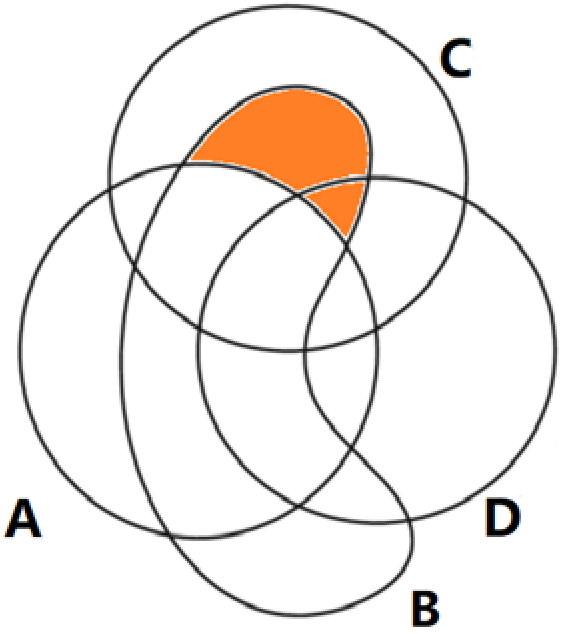
$(A \cap D) - C$



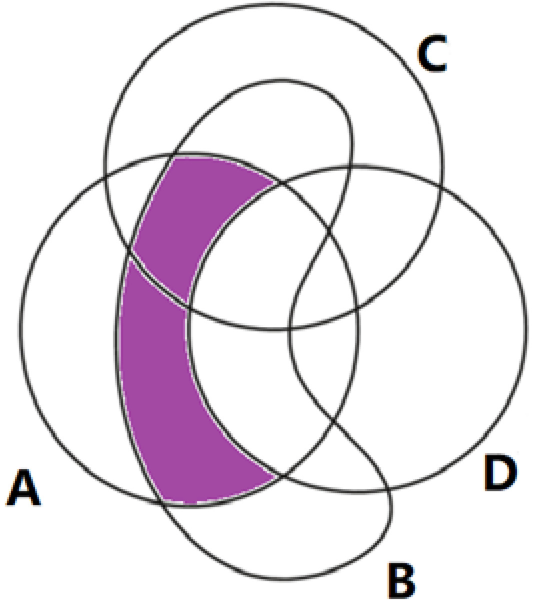
$(C \cap D) - B$



$(B \cap C) - A$



$(A \cap B) - D$



4. (a) Show the union of the following sets in a Venn diagram:

$$(A \cap B) - C,\ (B \cap C) - D,\ (C \cap D) - A,\ (A \cap D) - B.$$

Are these sets disjoint?

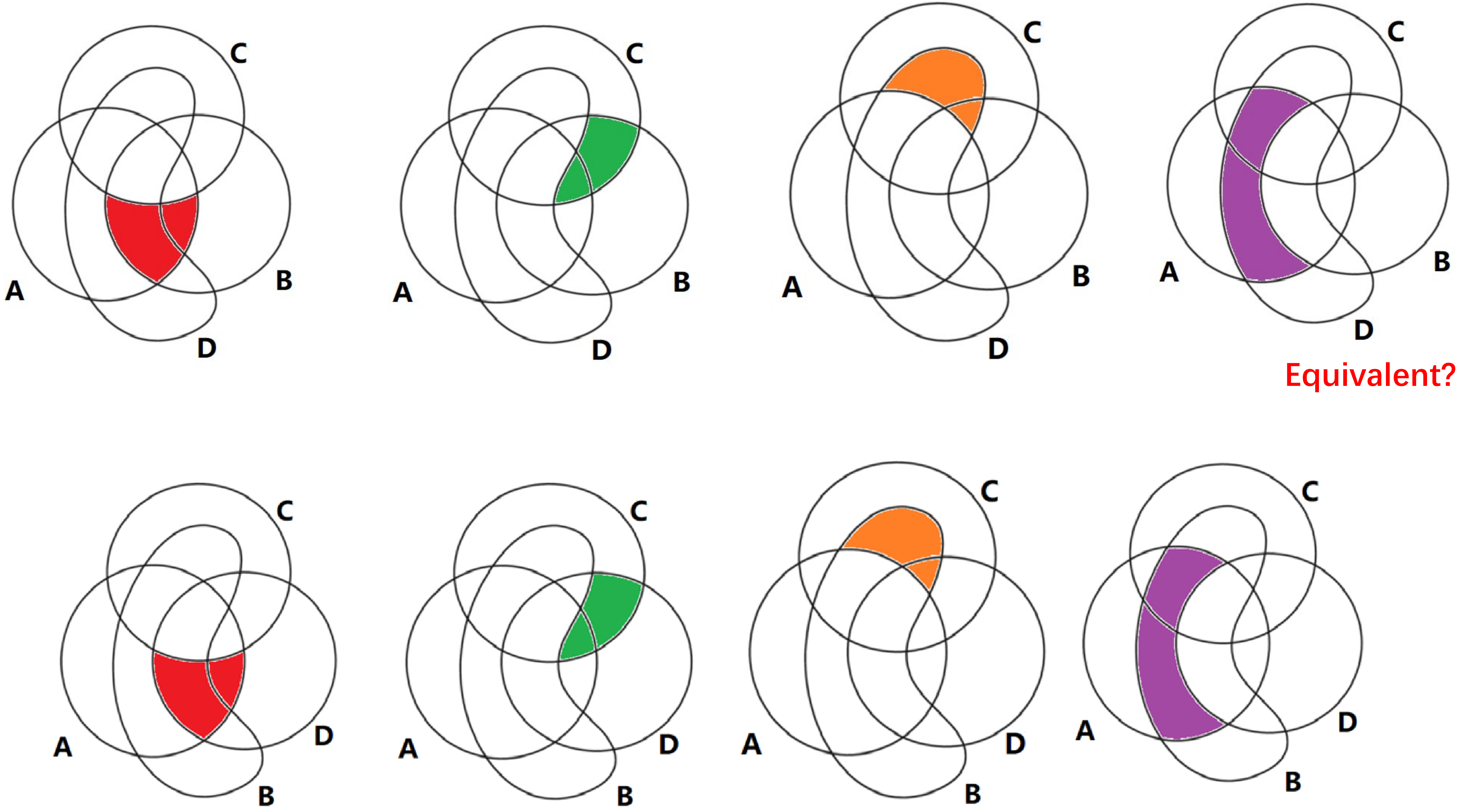
(b) Show the union of the following sets in a Venn diagram:

$$(C \cap D) - B,\ (A \cap D) - C,\ (A \cap B) - D,\ (B \cap C) - A.$$

Are these sets disjoint?

(c) Are the unions in (a) and (b) equivalent?

$(A \cap B) - C$ $(B \cap C) - D$ $(C \cap D) - A$ $(A \cap D) - B$



5. For simplicity, assume that CUHK has only four colleges: Chung Chi, New Asia, United, and Shaw. Let C, N, U, and S be the sets of all Chung Chi students, New Asia students, United students, and Shaw students, respectively. Let M be the set of all Music Major students. Let Y be the sets of all students living in Ying Lin Tang (a Chung Chi hostel). Let W be the set of all students who attended the Wei Lun Lecture.

Important!

1. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

2. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(a) Express the following in set-theoretic terms:

i. a Music Major student must belong to Chung Chi College (which is true) $M \subset C$

ii. Music Major students living in Ying Lin Tang $M \cap Y$

iii. students belonging to neither United College nor Shaw College who did not attend the 2015 Wei Lun Lecture $(U \cup S)^c \cap W^c$

iv. a student cannot belong to more than one college

$$C \cap N = C \cap U = C \cap S = N \cap U = N \cap S = U \cap S = \emptyset$$

v. no students from Shaw attended the Wei Lun Lecture $S \cap W = \emptyset$

vi. Chung Chi Music Major students who attended the 2015 Wei Lun Lecture $C \cap M \cap W$

vii. New Asia students who did not attend the 2015 Wei Lun Lecture. $N \cap W^c$

(b) Show that for a student not belonging to Category vi in (a), if the student is a Music Major, then at least one of the following is true:

i. belongs to New Asia, United, or Shaw

ii. did not attend the Wei Lun Lecture.

(a) vi: $C \cap M \cap W$

Not belonging vi: $(C \cap M \cap W)^c$

Music major: $(C \cap M \cap W)^c \cap M = (C^c \cup M^c \cup W^c) \cap M$

$$= (C^c \cap M) \cup (M^c \cap M) \cup (W^c \cap M)$$
$$= (C^c \cap M) \cup (W^c \cap M)$$
$$= (C^c \cup W^c) \cap M$$

Not belong to C → belong to N or U or S.

Not belong to W

(c) Show that for a student not belonging to Category vii in (a), if the student did not attend the 2015 Wei Lun Lecture, then the student does not belong to New Asia.

(a) vii: $N \cap W^c$

Not belonging vii: $(N \cap W^c)^c = N^c \cup W$

Did not attend W: $(N^c \cup W) \cap W^c$

$$= (N^c \cap W^c) \cup (W^c \cap W)$$
$$= N^c \cap W^c$$