ENGG 2470A **Assignment 6** **Spring 21**

1. Let $(X, Y) \sim f_{XY}(x, y)$ and $Z = X + Y$. Show that

$$f_Z(z) = \int_{-\infty}^{\infty} f_{XY}(x, z - x)\, dx = \int_{-\infty}^{\infty} f_{XY}(z - y, y)\, dy.$$

2. According to the statistics, among the students coming back from UK and USA, 1% are infected by COVID-19. They all take the saliva test with accuracy 99%. Suppose a particular student's saliva test is negative. A layman makes the following interpretation. The probability that the student is infected in the first place is 0.01 (i.e., 1%). Since the test independently indicates that he is not infected and the error probability of the test is 0.01 (i.e., 1%), the actual probability of the student being infected is $0.01 \times 0.01 = 0.01\%$. Comment on his interpretation.

Textbook Problems 5.5.5, 6.5.4, 7.1.1, 7.1.5, 7.2.1, 7.2.3, 7.3.3, 7.4.3, 7.4.8, 7.5.7, 7.6.1.

Notes Work out Problem 6.5.4 by two methods:

1. Take advantage of the observation that X and Y are independent random variables and apply the result in Example 4.17.

2. Use the result in the supplementary problem above.