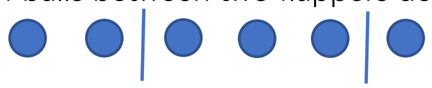
ENGG 2470A Assignment 3 Solution

1. Consider that n+m balls in a line are split by m-1 flapper, where flappers cannot be inserted at the begin and end, and two flappers cannot be inserted at the same place. The number of balls between two flappers denotes the nonnegative integers+1.



1+2+0=3;

Hence, we have $\binom{n+m-1}{m-1}$ ways to split the balls and $\binom{n+m-1}{m-1}$ ways for m nonnegative integers to add up to n.

2.

$$P(F = NR) = 1 - P(F = R) = 0.4$$

$$P(W = R|F = R) = 1 - P(W = NR|F = R) = 0.7$$

a) $P(W = R) = P(W = R|F = R)P(F = R) + P(W = R|F = NR)P(F = NR) = 0.7 * 0.6 + 0.3 * 0.4 = 0.54$;

b) $P(F = NR|W = R) = \frac{P(W=R|F=NR)P(F=NR)}{P(W=R)} = \frac{0.3*0.4}{0.54} = \frac{2}{9}$.

3. 1.5.1

a. $P[H_0] = P[LH_0] + P[BH_0] = 0.5$;

b. $P[B] = P[BH_0] + P[BH_1] + P[BH_2] = 0.6$;

c. $P[L \cup H2] = P[LH_0] + P[LH_1] + P[LH_2] + P[BH_2] = 0.5$.

4. 1.6.3

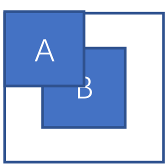
Let A and B denote Apricot and Banana

$$P(A) + P(B) = 1;\ P(A) = 2P(B);$$

$$P(A) = 2/3, P(B) = 1/3.$$

$$P(AA \cup BB) = P(A)^2 + P(B)^2 = 5/9.$$

5. 1.6.4



$$P(A \cap B) = P(A)P(B);$$

$$P(A) = P(B) = 1/4$$

$$P(AB) = P(A)P(B) = 1/16$$

6. 1.6.5

a) $P(A \cap B) = 0$; $P(A \cup B) = P(A) + P(B) = \frac{1}{4} + \frac{1}{8} = \frac{3}{8}$;

$P(A \cap B^c) = \frac{1}{4}$; $P(A \cup B^c) = \frac{7}{8}$

b) $P(A \cap B) \neq P(A) * P(B)$. Hence, A and B are dependent.

7. 1.6.6

   a) $P(C \cap D) = \frac{5}{8} * \frac{3}{8} = \frac{15}{64}$; $P(C \cap D^c) = 1 - \frac{3}{8} = \frac{5}{8}$; $P(C^c \cap D^c) = \frac{15}{64}$;

   b) $P(C^c \cap D^c) = P(C^c)P(D^c)$, and $P(C \cap D) = P(C)P(D)$. Hence, C and D are independent.

8. 1.6.9

   Consider the events

$$A_1 = \{1,2\}, A_2 = \{1,3\}, A_3 = \{2,3\}.$$

$$P(A_1A_2) = P(A_1A_3) = P(A_2A_3) = \frac{1}{4}.$$

$$P(A_1A_2A_3) = 0 \neq P(A_1)P(A_2)P(A_3).$$

9. 1.6.11(a)

$$\mathrm{P(AB) = P(A)P(B)}.$$

$$\mathrm{P(AB^c) = P(A) - P(AB) = P(A) - P(A)P(B) = P(A)[1 - P(B)]} = P(A)P(B^c).$$

10. 3.2.1

   a) $c + \frac{c}{2} + \frac{c}{4} = 1; c = \frac{4}{7}$

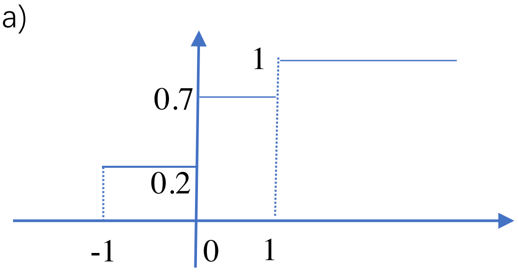
   b) $P(N \leq 1) = \frac{4}{7} + \frac{2}{7} = \frac{6}{7}$

11. 3.3.5

$$P(x) = \begin{cases} \binom{m}{k} h^k (1-h)^{m-k}, & k = 0, \dots, m \\ 0, & otherwise \end{cases}$$

12. 3.4.2

   a)



   b) $P(x) = \begin{cases} 0.2, & x = -1 \\ 0.5, & x = 0 \\ 0.3, & x = 1 \\ 0, & else \end{cases}$