ENGG 2470A **Assignment 5** **Spring 21**

$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}$

1. Consider the Gaussian distribution $\mathcal{N}(m, \sigma^2)$.

   $\frac{1}{\sigma\sqrt{2\pi}} \int e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2} dx$ Polar cooridnates + I^2

   (a) Show that the pdf integrates to 1.

   $E(X) = \int x f(x)\, dx = \frac{1}{\sigma\sqrt{2\pi}} \int x e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2} dx$

   (b) Show that the mean is m and the variance is σ^2.

2. Work out Example 4.16 in the lecture notes by using the pdf method.

3. Work out Example 4.18 in the lecture notes by using the CDF method.

4. (Optional) For $i = 1, 2$, let X_i be a binary random variable, i.e., $\mathcal{X}_i = \{0, 1\}$, and

   $$\mathrm{P}(X_i = 0) = \mathrm{P}(X_i = 1) = 0.5.$$

   X_i is called a *fair bit.* Now let $X_3 = X_1 \oplus X_2$, where

   $$i \oplus j = \begin{cases} 0 & \text{if } i = j \\ 1 & \text{if } i \neq j. \end{cases}$$

   Here, $\oplus$ denotes the *binary sum*, which is also known as XOR (Exclusive OR). Show that the random variables X_1, X_2, and X_3 are pairwise independent but not mutually independent.

Textbook Problems 5.1.3, 5.1.4, 5.2.4, 5.2.6, 5.3.2, 5.4.2, 5.6.7,