

1. Prove that ' $X \rightarrow Y$ ' is equivalent to ' $\sim (X \wedge \sim Y)$ '.
2. (De Morgan's Law)
 - (a) Prove that ' $\sim (X \wedge Y)$ ' is equivalent to ' $(\sim X) \vee (\sim Y)$ '.
 - (b) Use (a) to prove that ' $\sim (X \vee Y)$ ' is equivalent to ' $(\sim X) \wedge (\sim Y)$ '.
3. Verify the following set identities by using the Venn diagram. Pay attention to the meaning of these identities.
 - (a) $(A - B) - C = A - (B \cup C)$
 - (b) $(A - B) - C = (A - C) - B$
 - (c) $(A \cup B) - C = (A - C) \cup (B - C)$
 - (d) $(A \cap B) - C = (A - C) \cap (B - C)$
4. (a) Show the union of the following sets in a Venn diagram:
 $(A \cap B) - C, (B \cap C) - D, (C \cap D) - A, (A \cap D) - B.$
 Are these sets disjoint?
 (b) Show the union of the following sets in a Venn diagram:
 $(C \cap D) - B, (A \cap D) - C, (A \cap B) - D, (B \cap C) - A.$
 Are these sets disjoint?
 (c) Are the unions in (a) and (b) equivalent?
5. For simplicity, assume that CUHK has only four colleges: Chung Chi, New Asia, United, and Shaw. Let C, N, U , and S be the sets of all Chung Chi students, New Asia students, United students, and Shaw students, respectively. Let M be the set of all Music Major students. Let Y be the sets of all students living in Ying Lin Tang (a Chung Chi hostel). Let W be the set of all students who attended the Wei Lun Lecture.
 - (a) Express the following in set-theoretic terms:
 - i. a Music Major student must belong to Chung Chi College (which is true)
 - ii. Music Major students living in Ying Lin Tang

- iii. students belonging to neither United College nor Shaw College who did not attend the 2015 Wei Lun Lecture
 - iv. a student cannot belong to more than one college
 - v. no students from Shaw attended the Wei Lun Lecture
 - vi. Chung Chi Music Major students who attended the 2015 Wei Lun Lecture
 - vii. New Asia students who did not attend the 2015 Wei Lun Lecture.
- (b) Show that for a student not belonging to Category vi in (a), if the student is a Music Major, then at least one of the following is true:
- i. belongs to New Asia, United, or Shaw
 - ii. did not attend the Wei Lun Lecture.
- (c) Show that for a student not belonging to Category vii in (a), if the student did not attend the 2015 Wei Lun Lecture, then the student does not belong to New Asia.