

1. Consider the Gaussian distribution $\mathcal{N}(m, \sigma^2)$.
 - (a) Show that the pdf integrates to 1.
 - (b) Show that the mean is m and the variance is σ^2 .
2. Work out Example 4.16 in the lecture notes by using the pdf method.
3. Work out Example 4.18 in the lecture notes by using the CDF method.
4. (Optional) For $i = 1, 2$, let X_i be a binary random variable, i.e., $\mathcal{X}_i = \{0, 1\}$, and

$$P(X_i = 0) = P(X_i = 1) = 0.5.$$

X_i is called a *fair bit*. Now let $X_3 = X_1 \oplus X_2$, where

$$i \oplus j = \begin{cases} 0 & \text{if } i = j \\ 1 & \text{if } i \neq j. \end{cases}$$

Here, \oplus denotes the *binary sum*, which is also known as XOR (Exclusive OR). Show that the random variables X_1 , X_2 , and X_3 are pairwise independent but not mutually independent.

Textbook Problems 5.1.3, 5.1.4, 5.2.4, 5.2.6, 5.3.2, 5.4.2, 5.6.7,