Chapter 1 The Role of Algorithms in Computing

1.1 Algorithms

Exercise 1.1-1

Give a real-world example that requires sorting or a real-world example that requires computing a convex hull.

Exercise 1.1-2

Other than speed, what other measures of efficiency might one use in a real-world setting?

Exercise 1.1-3

Select a data structure that you have seen previously, and discuss its strengths and limitations.

Exercise 1.1-4

How are the shortest-path and traveling-salesman problems given above similar? How are they different?

Exercise 1.1-5

Come up with a real-world problem in which only the best solution will do. Then come up with one in which a solution that is "approximately" the best is good enough.

1.2 Algorithms as a technology

Exercise 1.2-1

Give an example of an application that requires algorithmic content at the application level, and discuss the function of the algorithms involved.

Exercise 1.2-2

Suppose we are comparing implementations of insertion sort and merge sort on the same machine. For inputs of size n, insertion sort runs in $8n^2$ steps, while merge sort runs in $64n \lg n$ steps. For which values of n does insertion sort beat merge sort?

Answer:

$$\begin{aligned} 8n^2 &< 64n \lg n \\ n - 8 \lg n &< 0 \\ n &\leq 43 \end{aligned}$$

Exercise 1.2-3

What is the smallest value of n such that an algorithm whose running time is $100n^2$ runs faster than an algorithm whose running time is 2^n on the same machine?

Answer:

$$\begin{aligned} 100n^2 &< 2^n \\ n - 2 \lg n - 2 \lg 5 - 2 &> 0 \\ n &\geq 15 \end{aligned}$$

Problems

Problem 1-1 Comparison of running times

For each function $f(n)$ and time t in the following table, determine the largest size n of a problem that can be solved in time t, assuming that the algorithm to solve the problem takes $f(n)$ microseconds.

Answer:

	1 second	1 minute	1 hour	1 day	1 month	1 year	1 century
$\lg n$	2^{10^6}	$2^{6\times10^7}$	$2^{3.6\times10^9}$	$2^{8.64\times10^{10}}$	$2^{2.592\times10^{12}}$	$2^{3.153\times10^{13}}$	$2^{3.153\times10^{15}}$
$\sqrt{n}$	10^{12}	3.6×10^{15}	1.296×10^{19}	7.464×10^{21}	6.718×10^{24}	9.944×10^{26}	9.944×10^{30}
n	10^6	6×10^7	3.6×10^9	8.64×10^{10}	2.592×10^{12}	3.153×10^{13}	3.153×10^{15}
$n\lg n$	62746	2.801×10^6	1.333×10^8	2.755×10^9	7.187×10^{10}	7.976×10^{11}	6.861×10^{13}
n^2	1000	7745	60000	2.939×10^5	1.609×10^6	5.615×10^6	5.615×10^7
n^3	100	391	1532	4420	13736	31593	1.466×10^5
2^n	19	25	31	36	41	44	51
$n!$	9	11	12	13	15	16	17