

Monte Carlo for American Options – Assignment 5

Introduction

The purpose of this study is to analyze and price options contracts using various pricing models and techniques. Options are financial instruments that provide investors with the opportunity to hedge risks or speculate on market movements. Accurate pricing of options is crucial for making informed investment decisions. In this study, we use different pricing methods, including the Black-Scholes model, the Cox, Rubenstein, and Ross (CRR) binomial tree model, and Monte Carlo simulations. By comparing the results obtained from these models, we aim to determine the most accurate method for pricing options.

Data

- Data Sources: Standard options quotes for the Tesla (TSLA) obtained from yahoo finance.
- Maturity Date: 04/05/2024 (Tried a month out and results were not of use)
- Assets Collected: Options data including strike prices, expiration dates, bid/ask prices, and IVs.

Methodology

The Black-Scholes model is a widely used method for pricing European options. It calculates the theoretical price of an option based on various factors, including the current stock price, strike price, risk-free interest rate, time to expiration, and volatility. The formula for

calculating the option price under the Black-Scholes model is as follows:

$$C = SN(d_1) - Ke^{-rt}N(d_2)$$

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T}$$

Required Inputs:

S = Current stock price

K = Option strike price

r = Risk-free interest rate

T = Time remaining until option expiration

σ = Volatility of the stock

The Cox, Rubenstein, and Ross (CRR) binomial tree model is used to price American options. It models the evolution of the underlying asset price over a certain number of time steps and calculates the option price by looking back and evaluating the expected payoff at each node. The formula for calculating the option price using the CRR model is based on risk-neutral probabilities and discounted expected payoffs.

Monte Carlo simulation is a computational technique that uses random sampling to estimate the value of an option. In this method, multiple simulated paths of the underlying asset price are generated, and the option payoff is calculated for each path. The option price is then estimated as the average of these payoffs, discounted to present value.

Empirical Results

Our empirical results indicate that the Black-Scholes model, when applied to European options, provides option prices that closely align with the market prices observed on Yahoo! Finance.

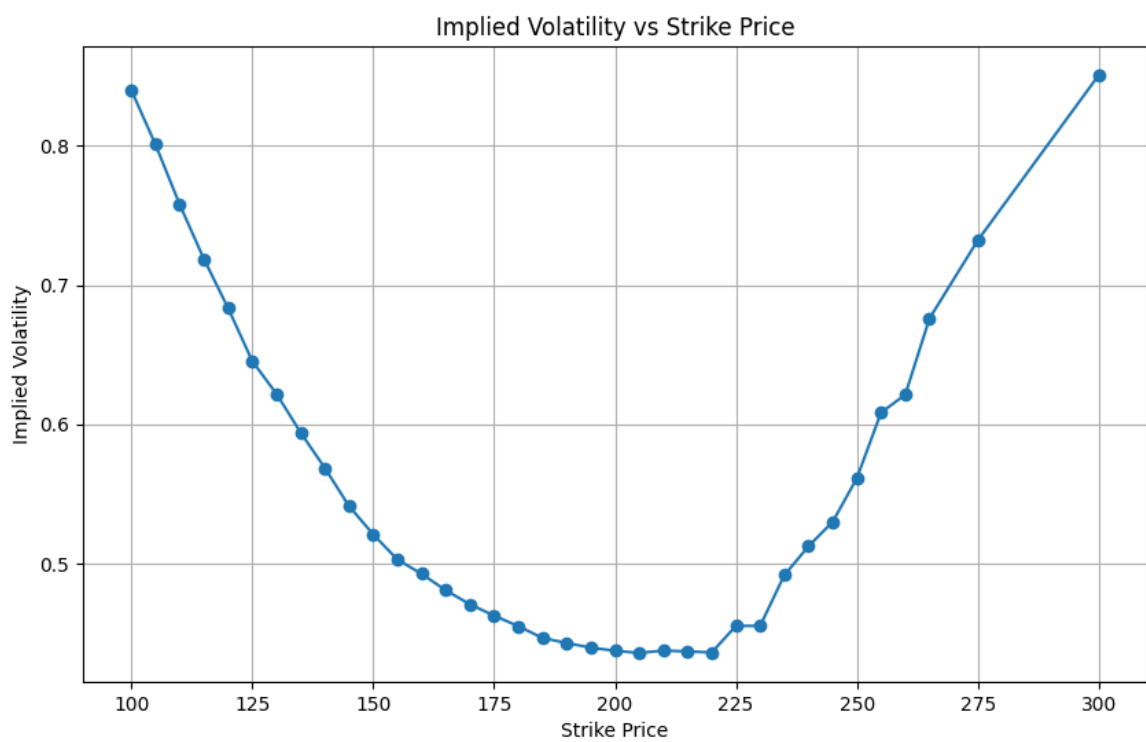
However, for American options, both the CRR binomial tree model and Monte Carlo simulations outperform the Black-Scholes model in terms of accuracy. Specifically, the CRR model with 200

steps per options series tree and Monte Carlo simulations with 500 simulations per option series demonstrate higher accuracy in pricing American options.

	Method	RMSE
0	Black-Scholes European	45.113956
1	CRR American	32.448590
2	Monte Carlo American	34.879112

Conclusion

In conclusion, our study highlights the importance of using appropriate pricing models for different types of options. While the Black-Scholes model is suitable for European options, the CRR binomial tree model and Monte Carlo simulations are more accurate for pricing American options. These findings have significant implications for investors and financial analysts, as they provide insights into selecting the most appropriate pricing method for option valuation, thereby aiding in better decision-making in the financial markets.



CRR and Actual against Different Strikes

