Assignment #1

Statistically equivalent portfolios

Required materials: Python

Directions:

1. Collect returns data for 4 (or more) different assets, which have in T observations, where T is 5 years or more.
2. Estimate the sample mean vector μ_T and the sample covariance matrix Σ_T.
3. Calculate the global minimum variance (GMV) portfolio weights, the GMV portfolio's expected return, and the GMV portfolio's standard deviation.
4. Calculate the maximum Sharpe ratio (MSR) portfolio weights, the MSR portfolio's expected return, and the MSR portfolio's standard deviation.
5. Assume that asset returns conform to a multivariate normal distribution, with mean and covariance matrix equal to the sample ones, which were estimated in Step 2.
6. Simulate 1,000 independent samples for each asset from the multivariate normal distribution with mean μ_T and covariance matrix Σ_T, with each draw consisting of T returns.
7. For each drawn sample:
   a. Estimate the GMV portfolio's weights.
      i. Use these weights, but μ_T to calculate the GMV portfolio expected return, and use these weights, but Σ_T to calculate the GMV portfolio's standard deviation.
   b. Estimate the MSR portfolio's portfolio weights.
      i. Use these weights, but μ_T to calculate the MSR portfolio expected return, and use these weights, but Σ_T to calculate the MSR portfolio's standard deviation.
      ii.
8. Plot the 1,000 GMV and MSR (on a separate figure each for the GMV and MSR portfolios) portfolios' expected returns and standard deviations in mean-standard deviation space (plot the points $\left(\sigma_{gmv}^{(i)}, \mu_{gmv}^{(i)}\right), \left(\sigma_{msr}^{(i)}, \mu_{msr}^{(i)}\right)$).
   a. Include the original sample's GMV and MSR portfolios on the plots
9. Make a table with the summary statistics for the re-sampled GMV portfolio expected returns, standard deviations, and portfolio weights (number of simulations, mean, standard deviation, min, P25, P75, max)
10. Make a table with the summary statistics for the re-sampled MSR portfolio expected returns, standard deviations, and portfolio weights (number of simulations, mean, standard deviation, min, P25, P75, max)
11. Make a table with the summary statistics for the re-sampled Sharpe ratios (number of simulations, mean, standard deviation, min, P25, P75, max)