Pricing Asian Options – Assignment 7 – Jake Mulready

Introduction

The purpose of this study is to price Asian call options on the West Texas Intermediate (WTI) crude oil using various methods, including the Black-Scholes model, bootstrapping simulation, and the binomial tree method. The goal is to compare the performance of these different approaches and analyze the characteristics of the implied volatility surface.

Data and Methodology

Data Collection

1. We collected call and put options data for the WTI average price options from the CME Group website as of 03/26/24. The data includes strike prices, option prices, and percentage changes.

2. We ignored storage costs and convenience yields and assumed that the average price for the Asian option is the geometric average.

Black-Scholes Model for Asian Options

1. We used the Black-Scholes model to back out the implied volatilities for the call options and put options.

2. We plotted the call option implied volatility curve and the put option implied volatility curve.

3. We fitted a spline model to the call implied volatility curve to obtain a smooth surface.

Pricing Grid

1. We created a grid of moneyness (K/S(0)) ranging from 0.9 to 1.1 with a step size of 0.005.

2. We used the spline model to fill in the grid of implied volatilities as a function of moneyness.

3. We then calculated the Black-Scholes Asian call option prices for each point in the grid using the corresponding implied volatilities.

Bootstrapping Simulation

1. We obtained daily WTI price data for the past year from the yfinance.

2. We calculated the log returns of the daily prices and used them to simulate 1,000 paths for the underlying asset.

3. For each strike price in the grid, we calculated the Asian call option prices based on the simulated paths and the current WTI price.

4. We calculated the Root Mean Squared Error (RMSE) of 1.8795 between the bootstrapped prices and the Black-Scholes prices. The RMSE provides a measure of the average difference between the two sets of prices, with a lower RMSE indicating a better fit.

Edgeworth Expansion

1. We estimated the cumulant coefficients (first 4 moments) of the simulated average prices using the skewness and kurtosis functions from scipy.stats.

- Cumulant 1: 82.6581 - This represents the mean of the simulated average prices.

- Cumulant 2: 56.1539 - This represents the variance of the simulated average prices.

- Cumulant 3: 15.5028 - This represents the skewness of the simulated average prices.

- Cumulant 4: 9320.1029 - This represents the kurtosis of the simulated average prices.

The cumulant coefficients provide insights into the higher-order moments of the underlying asset distribution, which can be useful for pricing and risk management.

Binomial Tree Method

1. We implemented the Asian call option pricing using the binomial tree method.

2. We calculated the Asian call option prices for each strike price in the grid and compared the RMSE of 3.5884 to the Black-Scholes prices.

Results

1. The call option implied volatility curve was plotted, and a spline model was fitted to the call implied volatility curve.

2. The Black-Scholes Asian call option prices were calculated for the grid of moneyness and plotted.

3. The bootstrapping simulation resulted in an RMSE of 1.8795 relative to the Black-Scholes prices. This RMSE value indicates that the bootstrapped prices deviate, on average, by 1.8795 from the Black-Scholes prices, which suggests a reasonably good fit.

4. The cumulant coefficients estimated via OLS for the Asian call options derived from bootstrapping are:

- Cumulant 1: 82.6581
- Cumulant 2: 56.1539
- Cumulant 3: 15.5028
- Cumulant 4: 9320.1029

These cumulant coefficients provide insights into the higher-order moments of the simulated average prices, which can be used to refine the pricing model or understand the risk characteristics of the Asian call options.

5. The RMSE of the Asian call option prices calculated using the binomial tree method relative to the Black-Scholes prices is 3.5884. This higher RMSE value suggests that the binomial tree method may not be as accurate in capturing the dynamics of the Asian call options compared to the Black-Scholes model for the given parameters.

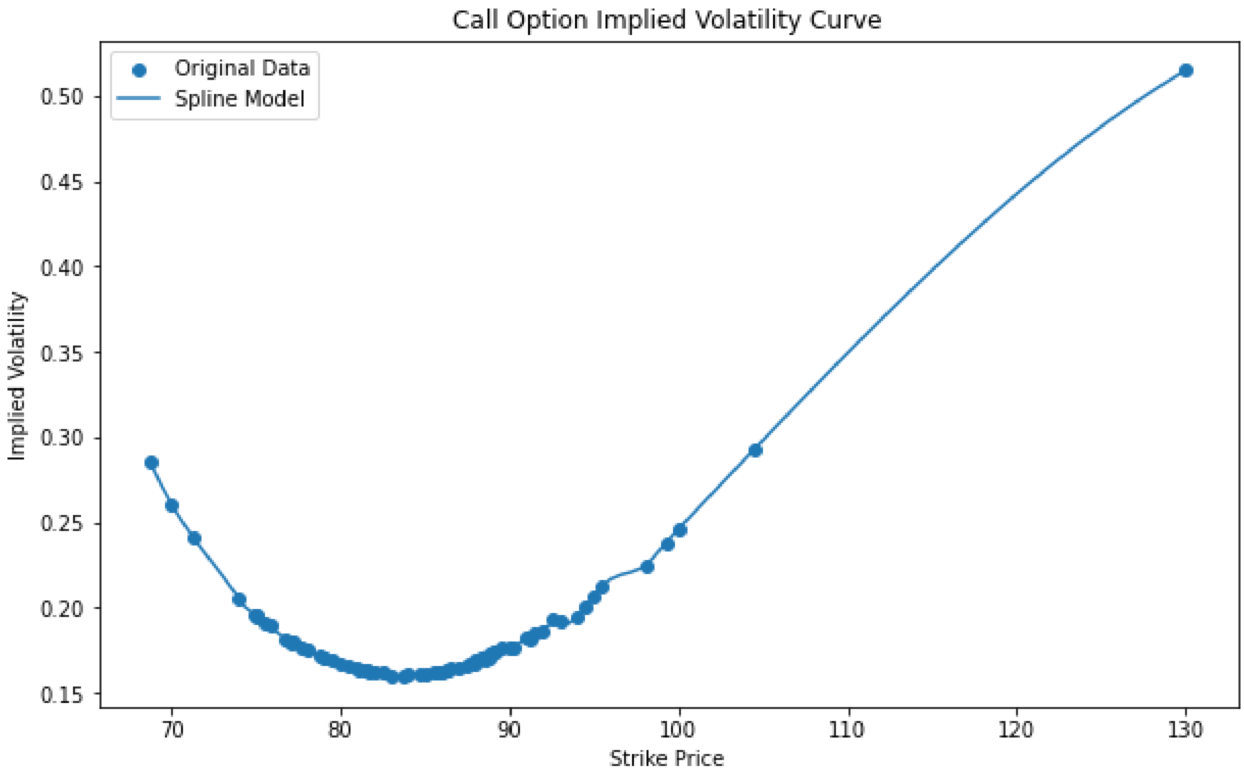
Conclusion

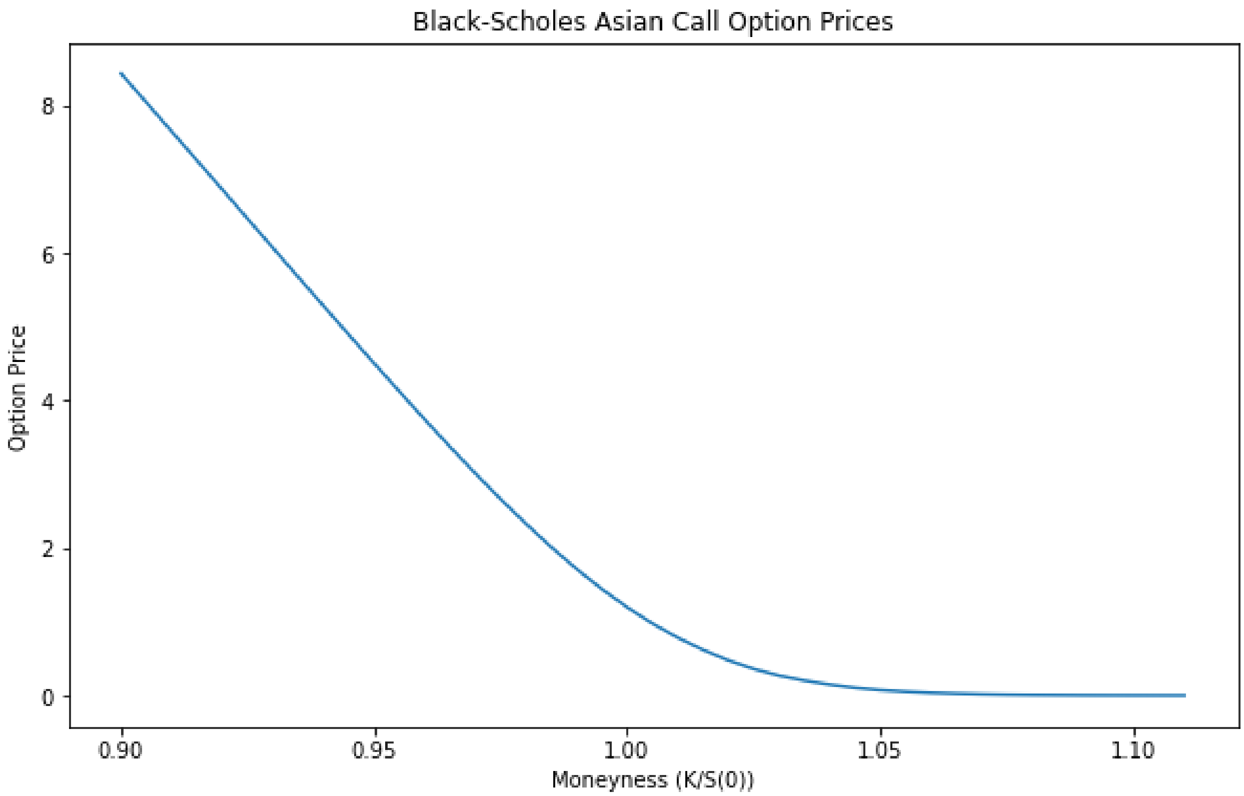
This study demonstrates the application of various techniques for pricing Asian call options on the WTI crude oil. The Black-Scholes model, bootstrapping simulation, Edgeworth expansion, and binomial tree method were used to analyze the option pricing dynamics.

The results show that the bootstrapping simulation provides a reasonably good fit to the Black-Scholes prices, with an RMSE of 1.8795. The Edgeworth expansion provides insights into the higher-order moments of the simulated average prices, which can be useful for more advanced pricing and risk management.

However, the binomial tree method shows a higher RMSE of 3.5884 compared to the Black-Scholes prices. This suggests that the binomial tree approach may not be as accurate in capturing the dynamics of the Asian call options for the given parameters.

Overall, the study provides a comprehensive analysis of the pricing of Asian call options and highlights the strengths and limitations of the different methodologies. Further research could explore the impact of various model assumptions, the use of alternative simulation techniques, and the incorporation of additional market factors to enhance the accuracy and reliability of the option pricing framework.


Call Option Implied Volatility Curve
Original Data
Spline Model
Implied Volatility
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
70
80
90
100
110
120
130
Strike Price



Black-Scholes Asian Call Option Prices
Option Price
8
6
4
2
0
0.90
0.95
1.00
1.05
1.10
Moneyness (K/S(0))

Smoothed Black-Scholes Implied Volatility Surface