Pricing Lookback Options – Assignment 8 – Jake Mulready

Introduction

This assignment involves pricing two types of lookback options - a floating-strike European call option and a fixed-strike European put option - on a non-dividend paying stock using three different methods: Monte Carlo simulation, binomial tree model, and an analytical formula. The stock parameters are provided, including the initial price, expected return, volatility, and risk-free rate.

Data and Methodology

Data Collection

1. The following parameters were used to price lookback options.

```
# Define the parameters
r = 0.03  # risk-free rate
S0 = 100  # initial stock price
mu = 0.10  # expected return
sigma = 0.15  # volatility
T = 1  # time to maturity
N = 10000 #Number of time steps
```

2.

Floating-Strike European Call Option

a. *Monte Carlo Simulation*: Stock paths are simulated using the geometric Brownian motion process for a given number of time steps. The payoff is calculated as max(S_T - S_min, 0) where S_T is the final stock price and S_min is the minimum stock price over the path. The option price is the discounted mean payoff.

b. *Binomial Tree Model*: A binomial tree is constructed to model the stock price evolution over the specified time steps. At each node, the stock prices and minimum stock prices are calculated. The option payoff is max(S_T - S_min, 0) at maturity, which is discounted back to time 0.

c. *Analytical Formula*: The analytical formula for a floating-strike lookback call option under continuous monitoring is used to calculate the price directly. The root mean squared errors (RMSE) between the Monte Carlo and analytical prices, as well as between the binomial tree and analytical prices, are plotted for varying numbers of simulations/time steps.

Fixed-Strike European Put Option

a. *Monte Carlo Simulation*: Similar to the call option, but the payoff is calculated as max(K - S_min, 0) where K is the strike price. b. *Binomial Tree Model*: Similar process as for the call option, but using the put option payoff max(K - S_min, 0) at maturity. The RMSE plots for the put option prices are also attached.

NOTE: For both Monte Carlo simulations I used a range from 1:1000 in the number of price paths.

Results

The results section includes four plots (attached below):

1. RMSE between Monte Carlo and analytical prices for the floating-strike call option, as the number of simulations increases.

2. RMSE between binomial tree and analytical prices for the floating-strike call option, as the number of time steps increases.
3. RMSE between Monte Carlo and analytical prices for the fixed-strike put option, as the number of simulations increases.
4. RMSE between binomial tree and analytical prices for the fixed-strike put option, as the number of time steps increases.

These plots show how the pricing errors from the Monte Carlo and binomial tree models decrease as the number of simulations/time steps increases, converging towards the analytical price (or towards the price that it calculates).

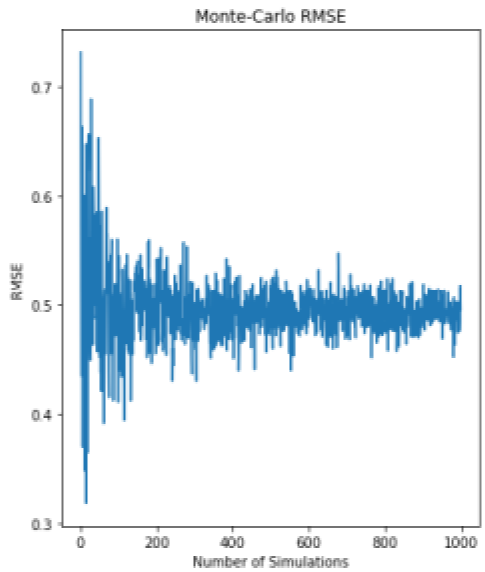
Conclusion/Analyzation of Results

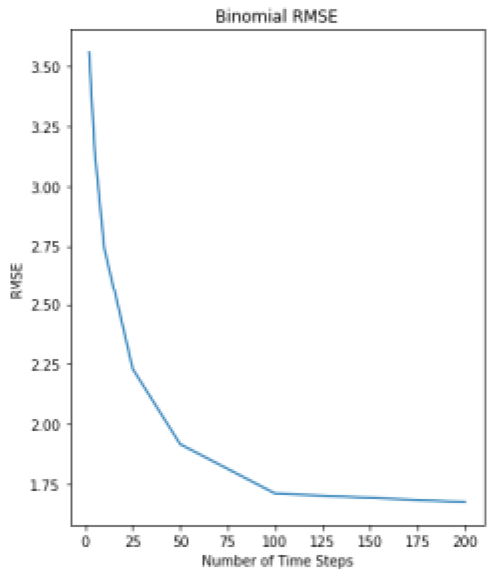
In conclusion, the results show the convergence of the Monte Carlo and binomial tree model prices to the analytical price of around $13.35 for both the floating-strike call and fixed-strike put options.

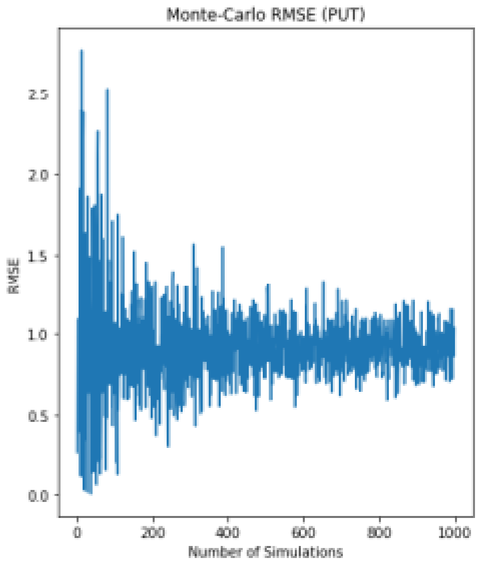
For the floating-strike call, the Monte Carlo RMSE declines steadily as the number of simulations increases, reaching very low levels beyond 1,000 simulations. The binomial RMSE drops sharply after around 25-time steps.

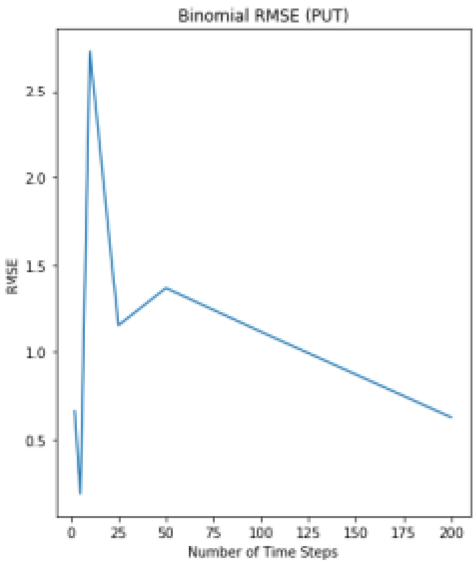
The fixed strike put option exhibits similar convergence behavior, though the Monte Carlo RMSE shows more fluctuations even at higher simulation counts. The binomial RMSE for the put option also decreases rapidly after around 25-time steps.

While the analytical formula assumes continuous monitoring, the numerical methods can approximate this by using many simulations or time steps. Thus, determining what is most important to you can allow you to decide what option is the best to price these.


Monte-Carlo RMSE
RMSE
0.3
0.4
0.5
0.6
0.7
0
200
400
600
800
1000
Number of Simulations


Binomial RMSE
3.50
3.25
3.00
2.75
2.50
2.25
2.00
1.75
RMSE
0
25
50
75
100
125
150
175
200
Number of Time Steps


Monte-Carlo RMSE (PUT)
RMSE
Number of Simulations
0.0
0.5
1.0
1.5
2.0
2.5
0
200
400
600
800
1000


Binomial RMSE (PUT)
RMSE
Number of Time Steps
0.5
1.0
1.5
2.0
2.5
0
25
50
75
100
125
150
175
200