

# Principal Component Analysis of Treasury Yields - Assignment 9

## Introduction

This assignment involves performing principal component analysis (PCA) on the term structure of interest rates represented by various U.S. Treasury securities. The goal is to estimate the covariance matrix, calculate the first three principal components, create factor time series from these components, and simulate 5-year paths for each Treasury series using their factor loadings.

## Data and Methodology

### Data Collection

1. Collected the trailing 5 years of monthly data for the following Treasury securities (all from FRED api):
  - 1-month Treasury bill
  - 3-month Treasury bill
  - 2-year Treasury note
  - 5-year Treasury note
  - 7-year Treasury note
  - 10-year Treasury note
  - 30-year Treasury bond
2. To calculate the principal components, eigen decomposition was performed on the covariance matrix, obtaining the eigenvalues and eigenvectors. The eigenvalues were sorted in descending order, and the first three corresponding eigenvectors were selected as the principal components. The original Treasury yield data was then projected onto

these three principal components using matrix multiplication to create the factor time series (PC1, PC2, PC3) for plotting in Image 1.

3. For the simulated 5-year Treasury yield paths, random factor paths were generated for the three principal components using a suitable stochastic process, such as Geometric Brownian Motion. The factor loadings of each Treasury series on the three principal components were calculated using the eigenvectors from the previous step. The simulated 5-year Treasury yield paths were then reconstructed by combining the factor paths and factor loadings. These simulated paths were stored for plotting in Image 2.

## Results

The results section includes two plots (attached below):

1. Principal Component Factors (Image 1): A plot showing the three factor time series derived from the first three principal components.
2. Simulated 5-Year Treasury Yields (Image 2): A plot displaying the simulated 5-year paths for each Treasury series using their factor loadings on the three principal components.

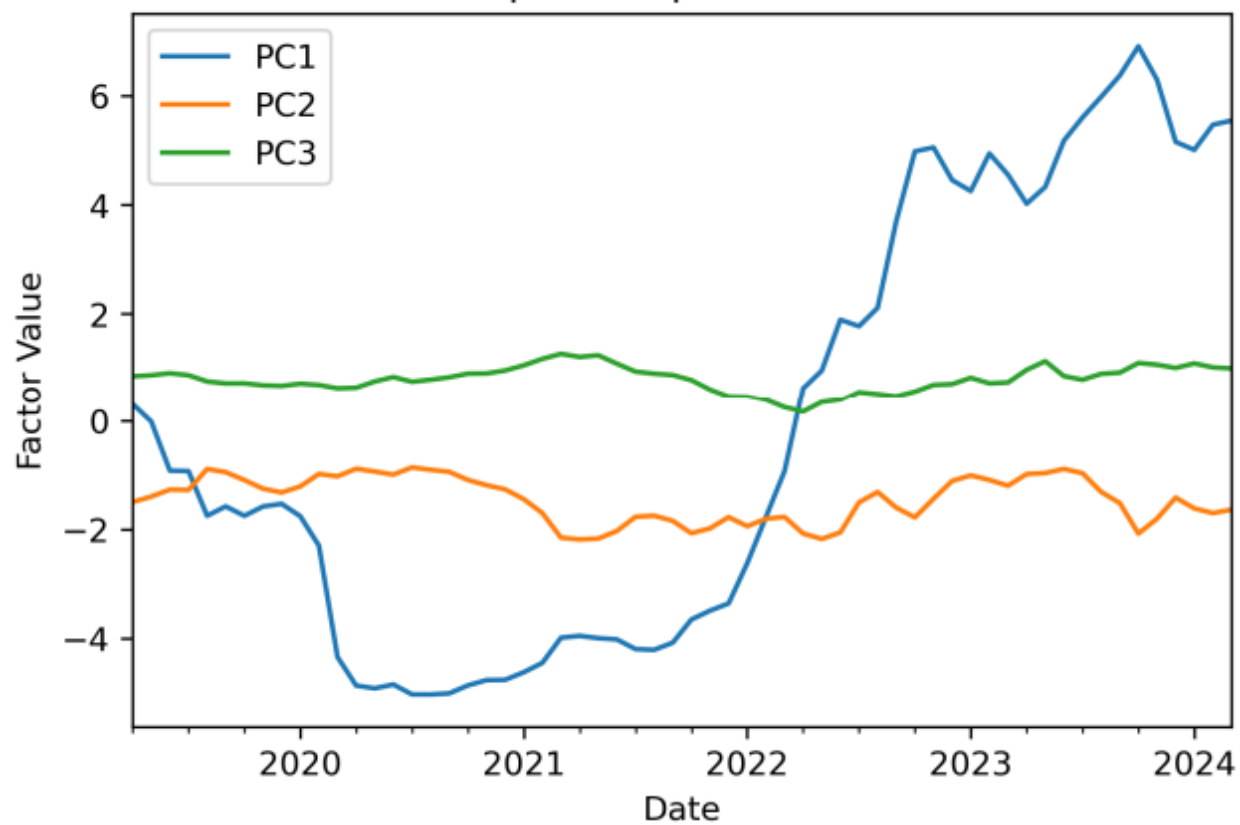
## Conclusion/Analysis of Results

The principal component analysis (PCA) decomposed the term structure of interest rates into three principal components. The first principal component (PC1) captures the overall level of interest rates, while the second (PC2) and third (PC3) components likely represent lesser variables such as the slope and curvature of the yield curve.

The time series plot (Image 1) shows the evolution of these three principal components over the 5-year period. The level factor (PC1) exhibits the most significant variations, with notable upward and downward movements. The slope (PC2) and curvature (PC3) factors display smaller fluctuations around a stable mean.

The simulated 5-year Treasury yields (Image 2) incorporate the factor loadings of each Treasury series on the three principal components. The simulated paths capture the dynamic behavior of interest rates over the 5-year horizon, with varying levels of volatility across different maturities. The shorter-term yields (GS1 and GS3) exhibit higher volatility, while the longer-term yields (GS10 and GS30) exhibit smoother movements.

Principal Component Factors



Simulated 5-Year Treasury Yields

