Love ♡ Analysis

© Jingqi Sun*

February 14, 2023

Abstract

This article was written to illustrate a non-formal mathematical measurement and interpretation of the love between humanities. We define a prospective function to evaluate the love of arbitrary humanity element to another humanity element. We also give several expressions to analytically describe the dynamics of love. Finally, we introduce potential applications of our formulations. The most significant application is to indicate someone's fabulous love through Dirac's delta function.

1 Introduction

Love is a emotional foundation of humanity cultures. This emotion or feeling is transmitted unidirectionally in general cases, and can usually be intensified at the situation when the love transmission becomes bidirectional. Many mathematicians and psychologists tried to describe the love as an interaction between human units, however since love has an extremely complicated mechanism, a fully mathematical description of love and its dynamics is impossible, at least not before we fully understanding turbulence, entropy and emergence.

For the same reason of complicity of love, we will not try to define a well-defined formulation for our model of love, but a simplified version instead. This simplification also coincides with the characteristics of love, such as uncertainty and pathological. It just like love itself: *no impeccable deduction is required but only unconditional empathy, is sufficient to solve the problem.* We also going to give some elegant formulae to show your love to another.

2 The Human Set

Before discussing any mathematical representations of love, we have to start with defining the objectives for which the love plays as an interaction between them. Here, on the Planet Earth, these objectives are called 'Humans'.

Definition 2.1. The *set of all Humanity*, or the *Human set*, is denoted and defined by:

$$\mathbb{H} := \{h \mid h \text{ is a human}\}.$$

For any $h \in \mathbb{H}$, we call h as a *human element*, or a *human* for short. More specifically, h should be an alive human at a certain time. It's not difficult to realise that $\mathbb{H}$ is not perfectly well-defined, since we haven't define what is a human. But for simplicity and for readers to understand this paper, we will not give a rigorous definition of a human.

There could be some philosophical problems in the non-well definition of a human and then $\mathbb{H}$. For example, what is the essential of a human, the body or the consciousness? Or the soul? If we can upload the full consciousness of a $h \in \mathbb{H}$ to the internet, after the old body of h is destroyed, is it still an element of $\mathbb{H}$? Many problems will appear. So to avoid these ultimate philosophical problems, we will consider only the common sense, so $h \in \mathbb{H}$ is just a common human, you (if you are not an artificial intelligence) and those around you (hopefully they are also actually belongs to $\mathbb{H}$).

There are also some interesting facts about $\mathbb{H}$. For instance, $|\mathbb{H}|$ is total populations of all humanity. And any subset $H \subset \mathbb{H}$ is a group of human. Physically, the human set $\mathbb{H}$ and its subsets are

*please send any feedback or suggestions to js2722@cam.ac.uk.

actually time-dependent, in an apparently way. The elements of $\mathbb{H}$ are always changing under the flowing of their proper time, since the day they exist. Thus, we can formulate one sad fact:

$$\lim_{t\to\pm\infty} \mathbb{H}(t) = \varnothing,$$

where t is the classical time and measured in its natural way, regardless of any relativistic and quantum effects.

2.1 Subset and social circle

In order to describe the measurement of love from one human unit to another, it is better to shrink our domain of human. We must start by describing a prerequisite condition of love, 'knowing'. Even it is possible for a human element $h \in \mathbb{H}$ to love another non-human element $\eta \notin \mathbb{H}$, but h knowing η must still be true before loving.

Definition 2.2. Let $x, y \in \mathbb{H}$. We say that x *knows* y (denote $x \star y$) if x can organise information of its memories to uniquely describe y. The form of information can be any languages or visualisation that are recognisable by some $z \in \mathbb{H}$ with $z \neq x$.

Note that $\star$ is clearly not a symmetric relation, that is, $x \star y$ does not necessarily imply $y \star x$. Also note that this definition is not very exact, since the memory of a $x \in \mathbb{H}$ is not clearly defined. Again, using common sense. Then we can further define the effective human set of any human element.

Definition 2.3. The *effective human subset (EHS) with respect to* x is defined as:

$$K(x) := \{h \in \mathbb{H} \mid x \star h\} \subset \mathbb{H},$$

and similarly, the *special EHS with respect to* x:

$$K^*(x) := \{h \in K(x) \mid h \star x\} \subset K(x).$$

It is obviously that $K^*(x)$ represents the social circle set of x. And any emotion of an $x \in \mathbb{H}$ can only be applied on $y \in K(x)$.

3 The Time-independent Love Function Time Evolution

We now have our domain of love to apply on, let's begin formulating love by considering a simple case, where time evolution is not included. This is the steady state function of love. Although everyone knows that the love is not steady in general.

3.1 The General Form

Definition 3.1. The *general form of the time-independent love function* is given by:

$$\heartsuit : \mathbb{H} \times \mathbb{H} \to [0, \infty) \qquad \text{defined by} \qquad \heartsuit(x, y) = \text{how much does } x \text{ love } y.$$

Obviously, $\heartsuit(x, y)$ is dimensionless, and its value is dependent on which basis of unit we use, and it has no standard unit. For example, we can use a factor unit, measure $\heartsuit(x, y)$ in units of $\heartsuit(z, y)$ for some $z \in \mathbb{H}$. We can also use $\heartsuit(x, x)$ as a unit, since it must exist for where $\heartsuit(x, y)$ is defined. Note that the value of $\heartsuit(x, y)$ is non-negative for all $x, y \in \mathbb{H}$, this means that we don't consider any form of anti-love. In some culture, hatred may be regarded as the opposite side of love, however, we ignore these and consider love independently, as non-negative values. Similar to $\star$, $\heartsuit$ is generally non-symmetry, means $\heartsuit(x, y) \neq \heartsuit(y, x)$. This is another sad fact and convention about love.

Definition 3.2. The *total love* of someone $x \in \mathbb{H}$ is defined by:

$$T(x) = \sum_{i=1}^{|\mathbb{H}|} \heartsuit(x, h_i).$$

Note that this total love gives the sum of love of x among all humanity, includes x itself.

3.2 The Special Form

The above general form is not concise enough to be involved in many beautiful formulae. It is way too general, but for human's emotion, we prefer to focus on certain persons. Moreover, it is clearly that for any $h \in \mathbb{H}$, $\heartsuit(h, x) = 0$ for any $x \notin K(x)$. So we want to focus on the actual reachable social connections for any certain human. Thus we introduce the normalised special form.

Definition 3.3. For each $x \in \mathbb{H}$, the *special form of the time-independent love function* is given by:

$$\heartsuit_x : K(x) \to [0, 1] \qquad \text{defined by} \qquad \heartsuit_x(y) = \frac{\heartsuit(x, y)|_{K(x)}}{T(x)}.$$

This expression is especially usable when x is a certain person. Now $\heartsuit_x(y)$ gives the percentage of how much x loves y, or equivalently it gives the importance of y to x in comparison to any other $z \in \mathbb{H}$. Hence, it is likely to have, for some $x, y, z \in \mathbb{H}$, $\heartsuit(x, y) > \heartsuit(z, y)$ but $\heartsuit_x(y) < \heartsuit_z(y)$. Also notice the normalisation of this special form:

$$\sum_{i=1}^{|K(x)|} \heartsuit_x(h_i) = 1 \qquad \longrightarrow \qquad \int_{K(x)} \heartsuit_x(h)\, dh = 1.$$

4 Time Evolution of the Love Function

As we stated in Section 2, $\mathbb{H}$ evolves in time. This natural law is also applied to our love function. Hence, we can propose a kind of love dynamics. It is straightforward to add a time parameter to the general form of the love function.

Definition 4.1. The *general form of the love function* is given by:

$$\heartsuit(x, y, t) = \text{how much does } x \text{ love } y \text{ at time } t.$$

which is extended from the steady state. Again the time t is classical, and is measured and meaningful only for where both x and y exist. However, if we consider all x, y, t as variables, the function will be over complicated, as it measures the love of all human on all human through all times. For simplicity, we will ony consider for some specific $a, b \in \mathbb{H}$. In the case, note that $a(t), b(t)$ will also change with time. Hence the equation is given by the chain rule:

$$\frac{d\heartsuit_a^b(t)}{dt} := \frac{d\heartsuit(a, b, t)}{dt} = \frac{\partial\heartsuit}{\partial a}\frac{da}{dt} + \frac{\partial\heartsuit}{\partial b}\frac{db}{dt} + \frac{\partial\heartsuit}{\partial t}.$$

This tells us, no matter in a relationship or just in other kinds of love, the change of love is dependent on the change of people. Even the host x is not changing with time, the love will still change if the objective is changing. Inevitable results.

5 Applications

The aim of this paper and above preparations are for this section. We are going to introduce several applications to show somebody's love. By using above formulations of love function, one can transmit information in an extremely concentrated way. Let's make the following convention in the context of this section:

Denotation. $m \in \mathbb{H}$ is *'me'* or the subjective and $u \in \mathbb{H}$ is *'you'* or the objective.

5.1 How to show your adoration to someone

5.1.1 Monotone increasing love

One apparent way of showing love is simply stating its dynamic. Using the following inequality:

$$\frac{d\heartsuit_m^u(t)}{dt} \geq 0,$$

which means *'my love on you is a monotone increasing function, it's never fall.'*

5.1.2 Limit to love

Alternatively, we could use limit to show our love:

$$\lim_{h \to u} \heartsuit_m(h) = 1.$$

Since the range of $\heartsuit_m(h)$ is to be less or equal to 1 by construction, equaling to 1 means this is all love of m. Hence, this limit directly implies that, *'the one will have all of my love can only be you.'* However, this expression also contains a trick. Everyone who have learnt calculus formally, knows that the limit exists does not indicate that the function at u must exists. Hence it is still possible that $\heartsuit_m(u) < 1$, or even does not exist. *m just love the one who feels like u.* Oh, a sad story.

5.1.3 The hyper-love

Now we come to our fantastic mathematical expression. It is named as the **Sun's Hyper-love Formula**:

$$\heartsuit_m(h) = \delta(h - u), \quad \forall h \in \mathbb{H},$$

where δ is known as the *Dirac delta function*, which can be represented as:

$$\delta(x - y) = \begin{cases} 0, & \text{if } x \neq y; \\ \infty, & \text{if } x = y. \end{cases}$$

At first glance, seems this formula breaks the co-domain definition of the specific love function, as it is more than 1. However this is just a mathematical trick, which is more marvelous, the delta function still satisfies the normalisation:

$$\int_{\mathbb{R}} \delta(x - y)\, dx = 1 \qquad \Longrightarrow \qquad \int_{\mathbb{H}} \delta(h - u)\, dh = 1,$$

so that the total amount of m's love is still 100%, does not overflow the co-domain. You can imagine this function as a single rectangle, placed on h, with width of ε and length $1/\varepsilon$. Hence for all $\varepsilon > 0$, its area (the integration) is 1. But as $\varepsilon \to 0$, its length goes to infinity.

Hence the Sun's formula means, *'even my flush body is restricted by the natural laws, I still found a way, for my soul to love you up to the infinity and to the eternity.'*

5.2 Further theories

Proposition 5.1. *The Sun's Preference Theorem of Love*:

$$\forall x \in \mathbb{H}, \exists s \in \mathbb{H}, \forall y \in \mathbb{H} \setminus \{x\}, \quad \text{s.t. } \heartsuit_s(x) > \heartsuit_s(y).$$

The meaning of this theorem is, *'for everyone in the world, there will always exist someone, whose favorite is you, who will give you all of their preferences.'*