Equilibrium or Simple Rule at Wimbledon? An Empirical Study

Shih-Hsun Hsu, Chen-Ying Huang and Cheng-Tao Tang*

Nov. 2003

Abstract

We follow Walker and Wooders' (2001) empirical analysis to collect and study a broader data set in tennis, including male, female and junior matches. We find that there is mixed evidence in support of the minimax hypothesis. Granted, the plays in our data pass all the tests in Walker and Wooders (2001). However, we argue that not only the test on equal winning probabilities may lack power, but also the current serve choices may depend on past serve choices, the performance of past serve choices, or the time that the game has elapsed. We therefore examine the role that simple rules may play in determining the plays. For a significant number of top tennis players, some simple low-information rules outperform the minimax hypothesis. By comparing junior players with adult players, we find that the former tend to adopt simpler rules. The result of comparison between female and male players is inconclusive.

* Department of Economics, National Taiwan University.

1. Introduction

The theory of mixed strategy equilibrium does not fare particularly well in various experimental settings involving human subjects in the past few decades.[1] For instance, though the experiment by O'Neill (1987) is among the most celebrated designs that support the minimax hypothesis, his result was later challenged by Brown and Rosenthal (1990) because of the strong serial correlation in players' choices. Not until recently does the theory of minimax hypothesis regain its foothold. By analyzing field data in professional tennis matches in Grand Slam, Walker and Wooders (2001) propose a study that lends empirical support to the minimax hypothesis. They argue convincingly that, unlike subjects in labs, professional players have adequate experience to play games well and are highly motivated to win the games. Therefore, if equilibrium theory can ever predict behaviors, choices of the top players in tennis matches are the right place to run empirical tests on. Their result indicates that the win rates across strategies are not different and this is consistent with the equilibrium prediction. However, they fairly note that even top players tend to switch from one strategy to another too often, resulting in serial dependence. Palacios-Huerta (2003) goes a step further using data set from penalty kicks in professional soccer games where both the kicker and the goalie's choices are observable. He finds stronger evidence in favor of the equilibrium prediction than Walker and Wooders (2001). Not only the win rates across strategies are not different but also the serial independence of choices cannot be rejected. Chiappori, Levitt and Groseclose (2002) further deal with the heterogeneity of players and they cannot reject that soccer players are behaving optimally in penalty kicks. Comparing these results with those obtained in labs, they all demonstrate that players' experience and familiarity with the

[1] For example, Erev and Roth (1998) discuss twelve such experiments and their equilibrium predictions.

strategic situation may play an important role in determining whether the equilibrium theory fares well.

We follow this line of idea and examine the minimax hypothesis by collecting and studying a broader data set in tennis, including male, female and junior matches. Since a typical tennis match lasts long, one may wonder whether players indeed play their equilibrium strategies or "learn" throughout the game. We find that the support for the minimax hypothesis is mixed. According to Walker and Wooders (2001), if players are playing their equilibrium strategies, the winning probability for each of their pure strategies must be equal. Moreover, since players should maximize the chance of winning at each point, it is as if their strategies are generated as a binomial process which is independently and identically distributed (*i.i.d.* henceforth). Our analysis shows that, the null hypothesis of equal winning probability across strategies cannot be rejected. Neither can the null hypothesis of the serially independent choices be rejected. However, we find that the test on equal winning probability may lack power. Moreover, when we test the *i.i.d.* hypothesis in a slightly more general fashion, players' current serve choices may depend on past serve choices, the performance of past serve choices, or the time that the game has elapsed. These findings together suggest that there might be some links between current and past choices. Recently many studies have documented that learning models can better describe and predict experimental results than static Nash equilibrium, [2] we therefore propose several low-information learning rules [3] and formulate a regression framework. [4]

---

[2] See, for example, Erev and Roth (1998), Camerer and Ho (1999), and Feltovich (2000).

[3] See Mookherjee and Sopher (1994), Mitropoulos (2001), and Ho, Camerer and Wang (2002). Their studies examine some possible learning rules under low information. For instance, Ho, Camerer and Wang (2002) study learning to games where only the set of foregone payoffs from unchosen strategies are known.

[4] A related and interesting example can be found in the writing of the celebrated linguist Wittgenstein. Wittgenstein sometimes took tennis sport as an example when describing the learning process of languages and tried to make an analogy between tennis game and "language game." (Wittgenstein,

Comparing the predictions of different learning rules with those of the equilibrium theory on the basis of Akaike information and Schwarz criteria, we find that learning models perform better than the equilibrium theory for a significant number of top tennis players. However, for different players, different learning rules may best describe their behaviors. We further find that interestingly, junior players tend to adopt simpler learning rules than adult players do. Meanwhile, there is no conclusive evidence that male and female players adopt rules in a different way.

The structure of the paper is as follows. In section 2, we follow Walker and Wooders (2001) to model each point in a tennis match as a simple 2×2 normal form constant-sum game and briefly describe relevant aspects of Walker and Wooders' (2001) analysis. Section 3 describes our data set. In section 4, we first perform the tests proposed by Walker and Wooders (2001) on our data set. We then discuss the power of the test on equal winning probability and re-evaluate the result regarding the *i.i.d.* hypothesis. We test several learning rules and demonstrate the better performance of them in section 5. Section 6 contains some discussion and section 7 concludes.

2. Testing the Minimax Hypothesis

We follow Walker and Wooders (2001) to model each point in a tennis match between the server and the receiver as a simple 2×2 constant-sum normal form game. When the server serves, he can choose to serve to the left of the receiver (*L*) or to the right of the receiver (*R*). Simultaneously, when the server serves, the receiver is assumed to guess whether the serve will reach to his left (*L*) or right (*R*). Each player's payoffs are the corresponding probabilities that he will ultimately

---

"*Philosophical Investigations,*" ss. 68, 71.)

win the point, conditional on both players have made their left-or-right choices for that point. Let π_{sr} denote the server's probability of winning the point, where $s \in \{L, R\}$ denotes the server's choice and $r \in \{L, R\}$ denotes the receiver's choice. Since either the server or the receiver has to win the point, the receiver's probability of winning the point is $1-\pi_{sr}$. If the Mixed Strategy Condition[5] holds, the point game will have a unique Nash equilibrium in which both players use strictly mixed strategies. Following Walker and Wooders (2001), since the server and the serving court could matter, there will be four point games in every tennis match, depending on which player is serving for that point and whether the point is a deuce-court or an ad-court point. Figure 1 summarizes the basics of a point game.

According to Nash equilibrium, players should play their minimax strategies within a point game. This implies the following two testable predictions:

(1) The winning probability for each of the server's pure strategy should be the same. This prediction results because in a mixed strategy equilibrium, players should be indifferent with their left-or-right choices given their opponents are playing their best responses. Let q denote the receiver's probability of choosing L. From the server's perspective, equilibrium implies that $P_L^s = P_R^s$ where

$$P_L^s \equiv q \cdot \pi_{LL} + (1-q) \cdot \pi_{LR},$$

$$P_R^s \equiv q \cdot \pi_{RL} + (1-q) \cdot \pi_{RR}.$$

Note that P_L^s is the server's expected probability of winning a point by serving to the left of the receiver while P_R^s is that by serving to the right of the receiver.

[5] It is reasonable to assume that the server's probability of winning a point is lower when the receiver chooses the same strategy. That is to say,

$$\pi_{LL} < \pi_{RL} \quad \text{and} \quad \pi_{RR} < \pi_{LR},$$

$$\pi_{LL} < \pi_{LR} \quad \text{and} \quad \pi_{RR} < \pi_{RL}.$$

The above inequalities are called the Mixed Strategy Condition. This reflects the idea that if the receiver guesses correctly about the server's right-or-left choice, he will be better prepared and thus more likely to win that point.

(2) The server's left-or-right choices in a given point game must be serially independent because he should play the same Nash equilibrium strategy for every point in a point game.[6] This implies that the serve choices will be random draws from a binomial process which is *i.i.d.* across all serves in a given point game.

By (1) and (2), we can therefore formulate and test these two fundamental hypotheses implied by von Neumann's Minimax Theorem.

3. Data

Our data set comprises three major groups (male, female and junior) and is colleted from videotapes or directly recorded on the spot. We have 10 matches in male tennis, 9 in female and 8 in junior. Since each match has 4 point games (depending on the server and the serving court), we therefore have 40 point games in male tennis, 36 in female and 32 in junior. The data covers the top-level players in Grand Slam finals (both male and female) over the past two decades. Therefore, it is fair to say that they are all highly motivated. In junior group, the matches include final, quarter final and second round in both Grand Slam and tournaments because it is hard to get data for junior players.[7] According to tennis rules, male tennis matches in Grand Slam can last at most five sets. Female and junior matches can last at most three sets.

The first three tables summarize the data. Each row of the tables corresponds to a point game. We first index each point game. For each of them, we state the following information in order: the match and its year, the server, the serving court, the number of times that the server chooses L, the number of times that the server chooses R, the total number of serves, the number of times that the server chooses

[6] See Walker and Wooders (1999).

[7] Few games are missing at the beginning of three junior matches. They are 2000 Wimbledon (Salerni vs. Perediynis), 2003 Australian Open (Scherer vs. Cvetkovic) and 2003 Australian Open (Tsonga vs. Feeney). However, this does not affect the continuity of our data.

L and wins, the number of times that the server chooses R and wins, the fraction of times that the server wins if he chooses L, the fraction of times that the server wins if he chooses R, Pearson statistic and its p-value (the latter two will be explained in the next section).[8] The winner of each match is indicated in boldface.

4. Testing the Equilibrium and a Reappraisal

4.1 Test of Equal Winning Probability

We first run the test of equal winning probability. Following Walker and Wooders (2001), we conduct both Pearson's chi-square goodness-of-fit and the Kolmogorov-Smirnov (KS henceforth) test. We index each point game by i. For each point game i, let p_j^i denote the probability that the server will win the point when he uses strategy $j \in \{L, R\}$. Let n_j^i denote the number of times that the server chooses j. Let N_{jS}^i denote the number of times for which the server wins when he chooses j. Let N_{jF}^i denote the number of times for which the server loses when he chooses j. For each point game i, under the null hypothesis, $p_L^i = p_R^i = p^i$. The maximum likelihood estimate of p^i is $\frac{N_{LS}^i + N_{RS}^i}{n_L^i + n_R^i}$. The Pearson statistic for point game i is

$$Q^i = \sum_{j \in \{L,R\}} \left[\frac{\left(N_{jS}^i - n_j^i p^i\right)^2}{n_j^i p^i} + \frac{\left(N_{jF}^i - n_j^i\left(1 - p^i\right)\right)^2}{n_j^i\left(1 - p^i\right)} \right].$$

If we substitute its maximum likelihood estimate for p^i, the Pearson statistic is distributed asymptotically as chi-square with 1 degree of freedom if the null hypothesis is true.

The results in Tables 1-3 show that the null hypothesis is not rejected for most

---

[8] We consider only the "first" serve direction and whether the server ultimately wins that point in every point game.

point games in each group. In other words, under the conventional 5% or 10% significance levels, the equal winning probability in each group can hardly be rejected, although the number of rejections in male group (2 for 5% and 6 for 10%) is slightly higher than that in female (0 for 5% and 1 for 10%) or junior groups (1 for 5% and 3 for 10%). We then turn to the joint test to examine whether the data in each group is consistent with the equilibrium theory. The statistic for the Pearson joint test is the sum of the individual test statistic Q^i, *i.e.* $\sum_{i=1}^{N_k} Q^i$, where N_k denotes the number of point games in group *k* and $k \in \{male, female, junior\}$. Under the joint null hypothesis, this statistic is distributed as chi-square with N_k degrees of freedom. Note that this joint test allows the parameters p_L^i and p_R^i to vary across different point games within a group. The corresponding p-values are 0.067 for male players, 0.716 for female, 0.551 for junior. Under the 10% significance level, the hypothesis of equal winning probability can be rejected for male players. However, if we drop the match which generates the extreme value (*i.e.* Becker serving to Lendl in the ad court in 1989 Wimbledon), the p-value of the rest 36 point games will rise to 0.285 where the null hypothesis cannot be rejected under any conventional significance level. In general, the equal-winning-probability hypothesis fares well for female and junior players.[9]

Since Pearson joint test would be problematic in detecting how the data is generated,[10] we therefore turn to compare the observed distribution with the predicted one by the KS test. As Walker and Wooders (2001) suggest, the p-values associated with the realized Q^i values should be N_k draws from the uniform

[9] One may be concerned by the results for the female and junior players because there are fewer observations for these two groups. That is, fewer observations may cause the problem that it is unlikely to reject the minimax hypothesis when it is false. One alternative way to resolve this is to merge ad-court and deuce-court data into one for a given server. The results are as follows. The p-values of Pearson statistic are 0.349 and 0.465 for female and junior players respectively. The KS statistic introduced later yields similar results that the equal winning probability hypothesis cannot be rejected.

[10] See p. 121 of Gibbons, J. and Chakraborti, S. (1992).

distribution $U[0,1]$ under the joint null hypothesis for group *k*. We present a visual comparison by drawing the cumulative distribution function (CDF henceforth) of the p-values associated with the realized Q^i values for each group and that of a uniform distribution in Figure 2. As a result, the KS statistics are 0.778, 0.578, and 0.641 for male, female and junior players respectively, which are all far from the critical value at 10% level.[11] Thus, we cannot reject the null hypothesis that jointly, players in each group are behaving according to equilibrium.

Curiously, the results of junior players contradict the original conjecture we had before running the empirical tests that junior players might not play minimax as well as adult players due to limited rationality. Nevertheless, the results of both the Pearson joint test and the KS test so far indicate the validity of the equilibrium prediction for equal winning probability. These are consistent with Walker and Wooders (2001).

4.2 Test of Serial Independence

We next examine the serial independence of players' serve choices. For each point game *i*, let $s^i = \{s_1^i, s_2^i, ..., s_{n_L^i + n_R^i}^i\}$, where $s_n^i \in \{L, R\}$ is the direction of the n_{th} serve, and n_L^i, n_R^i are the number of serves to the left and that to the right respectively. Therefore, there are $n_L^i + n_R^i$ serves in total in point game *i*. In words, s^i is the list of direction of serves in the order observed. The examination of serial independence we conduct is runs test, where a run is the maximal string of identical serve directions. Denote the number of runs in s^i as r^i. Under the null hypothesis of serial independence, the probability of having r^i runs in a sequence with n_L^i serves to the left and n_R^i serves to the right is denoted as $f(r^i; n_L^i, n_R^i)$. Let $F(r^i; n_L^i, n_R^i)$ be the probability of having r^i or fewer runs.

[11] The critical values of the KS statistic at 5% and 10% level are 1.328 and 1.194, respectively.

The null hypothesis of serial independence in point game i is rejected at 5% significance level if either $F(r^i; n_L^i, n_R^i) < 0.025$ or $1 - F(r^i; n_L^i, n_R^i) < 0.025$, where the first inequality corresponds to the case that the probability of having r^i or fewer runs is less than 2.5% (too few runs) and the second corresponds to the case that the probability of having r^i or more runs is less than 2.5% (too many runs). Walker and Wooders (2001) find that players' choices are not serially independent which leads to the conclusion that even the best tennis players tend to switch from one direction to another too often. In comparison with their finding, our result is quite different. Only fewer point games in our data can the null hypothesis be rejected. We report the results in Tables 4-6.

For each point game i, in addition to the same basic information as reported in Tables 1-3, we report the number of runs, the CDF of having r^i-1 or fewer runs, and the CDF of having r^i or fewer runs. At 5% significance level, there are 2 rejections for male players because of too many runs, 1 for female because of too many runs, and 1 for junior because of too few runs. At 10% significance level, there are 4 rejections for male players because of too many runs, 1 for female because of too many runs, and 4 for junior (2 for too many runs and 2 for too few runs). As for joint test, the KS statistics of the joint null hypothesis that the serves are serially independent within a certain group are 0.867 for male, 0.831 for female and 0.597 for junior, of which the p-values are all far from the rejection region under the conventional significance level[12]. Figure 3 offers a visual comparison of the empirical CDF and the predicted CDF under the null hypothesis.[13] In brief, we cannot reject the null hypothesis that jointly, choices in

---

[12] For the critical value of the KS statistic, please refer to footnote 11.

[13] Following Walker & Wooders (2001), the joint KS statisitc is constructed by picking a random draw d^i from the uniform distribution $U[F(r^i - 1; n_L^i, n_R^i),\ F(r^i; n_L^i, n_R^i)]$ in each point game i. Under the null hypothesis of serial independence in point game i, the statistic d^i is distributed as $U[0,1]$. The average value of KS statistic we report here is obtained by performing ten thousand trials with such random draws for each point game.

each group are serially independent.

4.3 The Power of Tests

So far we have basically applied the tests in Walker and Wooders (2001) to our data and generally confirmed the validity of the minimax hypothesis in these tests. We now reexamine these test results with a critical eye. We first turn to the issue regarding the power of the tests. We concentrate on the Pearson joint test of equal winning probability as Walker and Wooders (2001) did. Without loss of generality, we take the male data as an example to see whether the test has the power to detect deviations from the minimax play by running Monte Carlo simulations.

Consider first a 2×2 point game model where $\pi_{LL} = 0.53$, $\pi_{LR} = 0.88$, $\pi_{RL} = 0.79$, $\pi_{RR} = 0.33$ (see Figure 4).[14] If players follow their minimax strategies, this model predicts that servers will serve to the receivers' left side with probability 0.568. Note that in our male tennis data, 56.8% of all serves are indeed to the left of the receivers. The probability that the servers will win a point (*i.e.*, the game's value) is 0.643. Note again that in our male tennis data servers indeed win 64.3% of all points. Let θ be the proportion that receivers choose to play *L*, then under the null hypothesis one can calculate that $\theta = 0.68$. The Pearson statistic $\sum_{i=1}^{N_{male}} Q^i$ is distributed as chi-square with 40 degrees of freedom. To depict the power function, we randomly generate ten thousand times for 40 point games at any fixed value of θ, evaluate the servers' probability of winning across *L* and *R* by the payoff matrix, and compute the frequency where the Pearson joint test rejects the null hypothesis under 5% significance level (*i.e.*, an estimate of the power of the test under θ). This process is performed for many values of θ.

On the basis of the above payoff matrix, the Pearson joint test has good power against alternative hypothesis when the true value of θ differs from 0.68. This is

[14] The payoff matrix satisfies the Mixed Strategy Condition so that there exists a unique mixed strategy equilibrium in this example.

consistent with Walker and Wooders (2001).

However, to sort things out, we consider another 2×2 point game with the following parameters: $\pi_{LL} = 0.62$, $\pi_{LR} = 0.67$, $\pi_{RL} = 0.66$, $\pi_{RR} = 0.59$ (see Figure 5). Based on this payoff matrix, the minimax theory also implies that servers serve to the left with probability 0.568 and win a point with probability 0.643. The null hypothesis for receivers' choosing *L* is also 0.68. Nevertheless, a quick glance at the power function shows that it does poorly in detecting deviations from the minimax play. Since the payoff matrix of any point games is not observable, we are led to question that non-minimax behaviors may also lead to acceptance of equal winning probability of server's left-or-right choices.

To make the point clearer, note that as econometricians, we only observe the value of the game and the proportion of servers' serves to the left. The payoff matrix is not directly observable. When applying Monte Carlo simulations, given the observable value of the game and the proportion of servers' serves to the left, we actually have latitude in choosing two additional parameters (θ, π_{LL}) due to limited information in the payoff matrix.[15] If we focus only on the parameter θ but ignore the fact that π_{LL} can also vary, the power function thus depicted can be misleading. In other words, the surface that relates equal winning probabilities to the minimax play could be quite flat around the equilibrium strategy for some payoff matrix. In statistical terminology, tests based on equal winning probability for point games may lack power when some certain plausible alternative hypothesis is considered.[16] Similar remarks can be made about tests on female and junior groups. We conjecture that this lack of power might help explain why

[15] Let *p*, *V* denote the probability of servers' choosing *L* and the game's value respectively. Given these observable variables—*p* and *V*, we can thus derive π_{LR}, π_{RL} and π_{RR} given any pair (θ, π_{LL}) as follows: $\pi_{LR} = \frac{V - \pi_{LL} \cdot \theta}{1-\theta}$, $\pi_{RL} = \frac{V - \pi_{LL} \cdot p}{1-p}$, $\pi_{RR} = \frac{V}{1-\theta} - \frac{\theta \cdot (V - \pi_{LL} \cdot p)}{(1-\theta)\cdot(1-p)}$.

[16] This lack of power will become severe when the test of equal winning probabilities is performed individually on each point game.

junior players' choices are quite consistent with the equilibrium prediction, a result different from our conjecture.

4.4 Re-evaluation of the I.I.D Hypothesis

Although our results based on the runs tests are in favor of serial independence and thus different from those in Walker and Wooders (2001), yet we argue that the question of serial independence should be further addressed. Explicitly, the *i.i.d.* hypothesis requires not only that the current serve choice should be independent of past serve choices, but also that the current serve choice should have nothing to do with other past plays, such as the successful past choices. In other words, passing the runs test is necessary but not sufficient to pass the *i.i.d.* hypothesis. For this reason, we further examine whether past serve choices, the performances of past serve choices and other possible variables might play a role in determining the current serve choice.

To address this, we follow Brown and Rosenthal (1990) and propose a probit equation (Equation No.1) for each point game in which the dependent variable is a dummy for the server's choice of *R*. Denote this dummy by *D*. Then *D* takes the value of 1 if the server chooses *R* and 0 if the server chooses *L*. The independent variables we try are the following: the first and second lagged dummies for server's choice of *R*, the first and second lagged dummies for the server's successful choice of *R* (we denote this by *RW*, short for "right and win," which takes the value of 1 if the server chooses *R* and wins and 0 otherwise), the first and second lagged dummies for the server's successful choice of *L* (we denote this by *LW*, short for "left and win," which takes the value of 1 if the server chooses *L* and wins and 0 otherwise),[17] and a time trend denoted by *T* which measures the

[17] We allow successful past choices of *R* and *L* to affect the current serve choice differently. This allows servers to care about a particular side and whether serving to that side wins. For instance, a server may care more about the performance when he serves to the receiver's "weaker" side.

number of serves that has occurred till now (this is included to allow for the situation that players may raise their possibility of playing a certain strategy as time goes by).[18] The results are shown in the first part of Table 7.

There are six tests in Equation No. 1. In test #1, the null hypothesis that all the explanatory variables are jointly insignificant is rejected for five point games at 0.05 level (twelve at 0.1 level). Other tests help identify the source of the failure in the joint null hypothesis. Test #2 considers whether servers' own past choices affect their current play. Test #3 tackles whether successful past plays have an impact on the current choice. Tests #4 and #5 respectively measure the influence of past successful choices of a specific serve direction (*RW* or *LW*) on the current play. Test #6 deals with the case where players raise the probability of serving to a certain side as time goes by. If we consider all six tests reported in Table 7, twenty-two point games are inconsistent with the minimax theory at 10% significance level (twelve at 5% level). In brief, about 20% of all point games do not seem to satisfy the *i.i.d.* hypothesis at 10% level and roughly 10% at 5% level. Therefore, it is reasonable to question the earlier result which supports serial independence.

Recall that the runs test performed in section 4.2 measures whether serves are randomly generated in an ordered list of direction of serves. The idea behind the runs test corresponds well to that behind test #2 in Equation No. 1. This is because test #2 assesses whether the current serve choices depend on the first and second lagged serve choices. If serves are serially independent, there should be no systematic relationship between the current serve choice and its lags. This is generally confirmed here, as there are only 2 rejections at 5% significance level

[18] We have tried to include additional lags (the third or fourth lags) in the probit equation. However, we find that these additional lags are insignificant in explaining the current serve choice. Therefore, we only report the result with two lags.

and 3 at 10% in test #2. Moreover, recall that the runs test fairs quite well in our analysis. These together suggest that we may drop the past serve choices from the regression. We therefore formulate another probit equation (equation No.2) by dropping the first and second lags of past serve choices. This can help increase the degree of freedom and may be especially important for the analysis of the junior data because the data are shorter for junior group. The second part of Table 7 summarizes this result. In comparison with equation No.1, the number of rejection rises up in each test, especially for junior group. There are twenty-seven point games (25% of all) inconsistent with the minimax play at 10% significance level for all the five tests we consider.

To this point, combining with the problem that Pearson test may lack power, we tentatively conclude that there might be some alternative hypothesis that can better explain the data than the minimax hypothesis. Since results in Table 7 suggest that past plays seem to affect current ones, we therefore turn to a more elaborate analysis in which we propose various rules to model this influence and apply them to our data. We aim at characterizing the effect of various past plays on the current serve choice by different rules. Since various learning models in the literature provide good frameworks under which past behaviors affect current ones, we next turn to a brief discussion about how to apply the concept of learning models to our data where payoff information is very limited. The discussion gives us a guideline to propose the rules that we will consider. Therefore, some of the rules we propose in the following have the flavor of learning embedded. Moreover, by proposing several rules and discussing the cognitive ability involved in these various rules, we will also be able to select the best rule in fitting players' serve choices and categorize players' "level of cognitive ability." This will ultimately lead us to compare whether for different group of players, the best rule may differ in a certain way.

5. Various Rules in Characterizing Players' Choices

5.1 Learning Models and Rules

Learning models can vary in the way that they focus on different psychological effects. To a certain degree, they may generate different results in predicting people's behavior. Despite the variety of learning models, they can be classified into two broad branches: reinforcement-based models and belief-based models.[19] Reinforcement-based models assume that players' strategies are reinforced by the corresponding payoffs they earn. The higher the payoff is, the more strongly their choices are reinforced later. Therefore, players care about the payoffs from different strategies and it is the relative payoffs that are sufficient in guiding them to make choices. They generally do not explicitly form beliefs about what other people will do once the information about their payoffs is realized. Belief-learning models, on the other hand, require that players hold beliefs by observing the previous plays of other players. Given these beliefs, they then choose best responses to maximize their expected payoffs. Despite of the difference between them, some authors argue that reinforcement-based and belief-based models might be different only on the surface. The nature of the two learning models, however, can be quite similar.[20] In short, both learning models provide considerable insights in explaining players' behaviors in strategic situations.

It would seem straightforward to apply either model to our data. However,

[19] Several papers deal with one kind of model only. Cheung and Friedman (1997) estimate a fictitious play model on individual-level data while Roth and Erev (1995) posit a reinforcement model in several games. The studies of belief or reinforcement learning have their own explanatory power. Some studies compare the model of reinforcement with that of fictitious play, including Ho and Weigelt (1996), and Battalio, Samuelson and Van Huyck (1997). Overall, comparisons appear to favor reinforcement in constant-sum games and belief learning in coordination games (see Camerer and Ho (1999)).

[20] See Camerer and Ho (1999) and Hopkins (2002).

several obstacles stand in the way of such a foolhardy application. When applying belief-based models, not only every entry in the payoff matrix of a server has to be known in order to calculate his best response, but also the receiver's choices have to be observed to us as econometricians so that we can estimate how the server develops his belief. However, the payoff matrix in a tennis match is not directly observable. Moreover, unlike goalies' choices in penalty kicks, receivers' guesses about servers' choices can hardly be told at all. Overall, the information available to us as econometricians is quite low and partial. If we were to apply reinforcement-based models to the data, these problems still exist but may be less severe. Note that the spirit of reinforcement-based models is that when a strategy does well, it will be adopted more often later. Following this line, we can model players' behaviors (with the essence of reinforcement learning) as follows. Whenever the server wins a point, this choice of serve direction will be reinforced. This corresponds well to the idea behind reinforcement learning when information is partial or low.[21]

We now propose several rules following the discussion above. They can be divided into two general classes, based on whether the concept of reinforcement learning is involved.

(1) Rules that reinforcement learning is not involved: We consider two rules in this class. The first rule is that the servers' choice of direction may be affected by the number of serves they have made, a proxy for the time that the game has elapsed. For this rule, we evaluate the impact of T, the time trend, on the dummy D for server's choice of R. Both variables, T and D, are already defined in section 4. The second rule we consider is that players may simply keep track of past serve choices and these past choices may affect their current play. For this simple rule,

[21] A similar idea has been used to describe subjects' behaviors in environments with partial information by Ho, Camerer and Wang (2002).

we will see how the variable D is affected by its own lags. This spirit of this rule resembles that underlying the runs test in section 4.2.

(2) Rules that reinforcement learning is involved: We consider three rules in this class. It is possible that players care more about the performance of a particular serve direction.[22] To capture this, we evaluate how the variable D is influenced by lags of RW, recalling that RW takes the value of 1 when players serve to the right and win that point and 0 otherwise as defined in section 4. This is the third rule we consider. Similarly, we also evaluate how D can be affected by lags of LW, where LW takes the value of 1 when players serve to the left and win that point and 0 otherwise as defined in section 4. This is the fourth rule. We propose one more rule. If players are even more sophisticated, they may be *motivated* to play R more often either because they win a point by serving to the right or they lose that point by serving to the left.[23] We denote this by WD, standing for winning difference. WD takes the value of 1 when a player either serves to the right and wins that point, or he serves to the left and loses that point. It takes the value of 0 otherwise. This is the fifth rule.

In each point game i, a stochastic decision formula that allows for a catch-all intercept is studied. Let $X_{it}(-s)$ denote the sth lag of variable $X \in \{D, RW, LW, WD\}$ at the tth serve in point game i. For instance, $D_{i5}(-3)$ is the 3rd lag of the direction of serve at the 5th serve in point game i. Equivalently, $D_{i5}(-3)=1$ if the direction of serve at the second serve in point game i is to the right and 0 otherwise. Let $G[\cdot]$ indicates the probit CDF. For each rule (except for the 1st rule), serving to the right will be chosen with probability

$$Pr(D_{it}=1|\alpha_i, \beta_{i1}, ..., \beta_{im}) = G[\alpha_i + \textstyle\sum_{s=1}^{m} \beta_{is} X_{it}(-s)],$$

22 For instance, a player may care about the performance when serving to the receiver’s “weaker” side.
23 In Mookherjee and Sopher (1994), a strategy is motivated (or vindicated) either when it goes well or the other strategy does badly.

where X corresponds to the explanatory variable underlying that particular rule, $m \leq t$ reflects how long a player's memory is, α_i is a catch-all intercept, and β_{is} measures how importantly $X_{it}(-s)$ affects the current choice of serve direction D_{it}. Note that α_i describes a player's idiosyncratic tendency for serving to the right.

5.2 Estimation Procedure

To sort out which type of rules can better predict players' choices, for each rule, we trace the past observations of its underlying explanatory variable for as long as needed. Take the second rule, where players' current serve direction may be affected by their past serve directions, for example. If m=2, then we conduct the following regression

$$Pr(D_{it} = 1 | \alpha_i, \beta_{i1}, \beta_{i2}) = G[\alpha_i + \beta_{i1} D_{it}(-1) + \beta_{i2} D_{it}(-2)].$$

The regression says that the current serve direction may be affected by the serve direction one serve ago and that two serves ago. The regressions for rules 3 to 5 can be constructed in a similar fashion. There is an exception though. For the first rule, since it is meant to capture how the current serve direction depends on the number of serves happened so far (represented by the variable T), we simply regress the current direction choice on T using the probit function, or

$$Pr(D_{it} = 1 | \alpha_i, \beta_i) = G[\alpha_i + \beta_i T].$$

To make a comparison to the equilibrium prediction, we also run a regression where the only regressor is the catch-all intercept (α_i). This accounts for the equilibrium prediction where the probability of current direction choice is independent of any past choices.

5.3 Minimax or Simple Rules?

For the regressions of each point game we run, we compute the value of the log likelihood function of the predicted play for each point game. The concept of this criterion is derived by summing over all serves the predicted probability of each serve direction if indeed this serve direction is chosen. Formally, denote $\hat{p}_t(R)$ the predicted probability that R will be chosen for the tth serve by the rule in concern. Denote R the set of serves in our sample that R is actually chosen. Denote L the set of serves in our sample that L is actually chosen. Then

$$ln(\text{likelihood}) = \sum_{t \in R} ln(\hat{p}_t(R)) + \sum_{t \in L} ln(1 - \hat{p}_t(R)).$$

Note that the predicted probability will not decrease when more explanatory parameters are introduced. We therefore use Akaike information criterion (AIC henceforth) and Schwarz criterion (SC henceforth) to select the best fitting rule. Both criteria are based on the value of the log likelihood function but they impose some penalties depending on the number of the parameters and that of observations. The AIC is $-2LL/N + 2k/N$ and the SC is $-2LL/N + k(\log N)/N$ where k is the number of estimated parameters, LL is the value of log likelihood function with k estimated parameters, and N is the number of observations. Both criteria are to provide a measure that strikes a balance between the measure of goodness-of-fit and the parsimonious specification of the model. SC asks more penalties relative to AIC.

For each rule in rules 2 to 5, since m, the length of the memory, should be determined endogenously from the data, we first search the optimal number of m that minimizes each criterion respectively by increasing the value of m until the AIC or SC increases steadily. In our data, we find that for most players, the optimal number of m is predominantly less than 3, reflecting that they have rather short memories. We then compare the AIC and SC values of the five rules (where the optimal m is used for rules 2 to 5) with those of equilibrium (where only the constant term α_i is used as the regressor) in each point game. In Tables 8-10, in regard to the five rules, we report only the rule that fits the data best in the last column and its AIC and SC values in the second to last column. We call this the best rule. We also report the AIC and SC values for the equilibrium in the third to last column. In summary, according to AIC, the best rule fits better than the equilibrium prediction in a significant number of point games. With thirty-one out of forty point games for male players, twenty-two out of thirty-six for female players, and twenty-seven out of thirty-two for junior players, the best rules fit better than the equilibrium prediction. In total, in 74% of the 108 point games, the best rules perform better than the equilibrium prediction. Since SC penalizes more heavily on models with more regressors than AIC and in the regression for testing the equilibrium prediction, the only regressor is the constant intercept term α_i, it is no wonder that the proportion where the best rules outperform the equilibrium prediction will be reduced if we use SC instead. We find that by applying SC, in 38% of the point games, the best rules outperform the equilibrium prediction. In particular, in nine out of forty for male players, fifteen out of thirty-six for female players, and seventeen out of thirty-two for junior players, best rules fit the data better than the equilibrium prediction. We take this as strong evidence against the minimax hypothesis.

Moreover, even if the equilibrium prediction outperforms the best rule in a point game, it is still possible that the set of coefficients β_i s' for the best rule is jointly significantly different from zero, i.e. past choices do affect the current serve direction. To highlight this, we test, for the best rule in each point game, whether the set of coefficients β_i s' is jointly different from zero. Table 11 summarizes this result under both 0.05 and 0.1 levels applying either information criterion. It suggests that the statistical significance of the best rules in explaining players' behavior is the *rule* rather than an exception. There are 61% of point games where the corresponding β_i s' are significant at 0.1 level under AIC, 56% under SC. This lends additional support to the idea that players are indeed affected by the past plays and therefore describing their choices by the various rules we consider is appropriate.

5.4 Junior vs. Adult Players

Do players adopt different rules? The five rules we consider impose quite different cognitive ability on players. To begin with, for the two rules where reinforcement is not involved (the first two rules), players only need to keep track of either the number of serves that has occurred or their own past serve choices. For the three rules where reinforcement is involved (rules 3-5), they not only need to keep track of their past serve choices but also the performance or even the performance difference of past choices. This clearly requires more cognitive ability. Lastly, for players to play according to their equilibrium strategies, they certainly have to be super rational in the sense that all the rationality assumptions required by the minimax hypothesis must be met.[24] This brings in an interesting question: Are junior players, in any sense, more or less "rational" than adult players?

We make an attempt to answer this question as follows. As discussed above, we divide all the rules and the equilibrium prediction into three classes, depending on the degree of cognitive ability or rationality involved. We lump the two rules where reinforcement is not involved into the "Low" class, presumably because the cognitive ability thus required is limited. The three other rules where reinforcement learning is involved is categorized as the "Medium" class since better cognitive ability is necessary in accordance with these rules. Finally, the equilibrium prediction is assigned to the "High" class.

[24] Note that rules 3-5 have the flavor of reinforcement learning embedded. If a player's behavior is best characterized by the equilibrium, we can question on how he knows to play the equilibrium strategy. One possible answer is that we can think of an equilibrium as one where all the possible learning has been completed before the game starts. In other words, players have completed their reinforcement learning to play the equilibrium or they have learned to play the equilibrium.

Based on Tables 8-10, we can assign each point game into either the “Low,” “Medium” or “High” class by comparing the AIC or SC values for all the regressions that we run. For instance, the best rule, the 3rd rule, outperforms the equilibrium prediction under AIC for the first point game in male (Borg serving to McEnroe in the ad court in the 1980 Wimbledon) and thus this point game is assigned to class “Medium” under AIC since the 3rd rule is categorized into this class. Combining the point games in male and female groups, we make bar charts to compare junior players’ choices with adult players’ in Figure 6. Under AIC, we find the proportion of players using “Medium” rules is highest for both junior and adult players. More importantly, the distributions of rationality for adult players first order stochastic dominate those for junior players under both AIC and SC. For instance, by AIC, the proportion of junior players classified into “Low” (rules 1 and 2) is 0.313, higher than that of adult players, which is 0.276. The proportion of junior players classified into “Medium” (rules 3, 4, and 5) is 0.531, higher than that of adult players, which is 0.421. A similar result holds under SC. We then turn to the comparison between the distributions of cognitive ability or rationality of male and female players. Here the evidence is mixed. Under AIC, there is no first order stochastic dominance relationship between the distributions of female and male players. However, the distribution of male players’ choices does first order stochastic dominate that of female players under SC. Note how this is in sharp contrast to the Pearson test of equal winning probabilities where the number of rejections arises mostly for male players.

6. Discussion

6.1 Payoff Change within Point Games?

Our study is based on the assumption that there is a fixed payoff matrix for every point game. In particular, the payoff matrix does not change with time in each point game. We now turn to verify this assumption more carefully.

The idea is as follows. A priori, we do not know when the payoff matrix will change in a point game. Since there are several sets in a point game, we naturally focus on whether there is any payoff change from set to set instead. Recall that for each point, we only observe the server's choice of direction and whether he wins that point. If the payoff does not change, the observed choices should not change from set to set either. Following Chiappori, Levitt and Groseclose (2002),[25] we regress separately the server's choice of direction and whether he wins the point on the constant and a collection of dummies characterizing which set the point is in. The null hypothesis is that the coefficients of the set dummies are jointly zero, reflecting no effect of the sets on the observed variables. At the 0.1 significance level, regarding whether the server wins the point, in seven out of 100 point games are the coefficients of the set dummies significant. For the server's choice of direction, there are eleven out of 100 point games in which the null hypothesis is rejected.[26] Admittedly, we may be led to question that the sets do not affect the server's choice of direction since there are eleven rejections whereas by chance, one would expect only ten. We therefore turn to conduct a joint likelihood ratio test where the null hypothesis is that the set dummies do not affect the server's choice of direction in the 100 point games. The p-value is 0.184, which is not significant at any usual significance level.[27] These results are thus consistent with

[25] Chiappori, Levitt and Groseclose (2002) consider whether goalies are homogeneous. They basically regress four outcome variables (whether the kick is successful, whether the kicker shoots right, whether the kicker shoots in the middle, and whether the goalie jumps right) separately on goalie-fixed effects. The null hypothesis is therefore the goalie-fixed effects are jointly insignificant from zero.

[26] The total number of point games where we run these regressions is 100, because two junior matches are too short to do so.

[27] The null hypothesis implies that the statistic $-2(\sum_{i=1}^{N} L_i - \sum_{i=1}^{N} l_i)$ has the asymptotic $\chi^2(d)$ distribution, where N denotes the total number of point games, L_i is the maximum log likelihood of the regression without the set dummies, while l_i is the maximum log likelihood of the regression with the set dummies, and $d =$ the number of the estimated parameters that are added while considering set variables.

the view that there is no statistically significant payoff change, at least from set to set. We further note that if we adopt a more cautious view to drop the point games where the coefficients of the set dummies are significant at 10% level in either regression, among the remaining point games, the relation that the rules characterizing the adult players first order stochastic dominate those of the junior players still holds.[28]

6.2 Learning Minimax?

Our results so far generally point to the direction that the minimax hypothesis should be more carefully considered. In doing so, we adopt the concept of reinforcement learning and propose several simple rules to account for the serial dependence in the data. However, we hasten to add that our results should not be read as a counterevidence of the minimax hypothesis. We interpret our finding as an indication that if the minimax hypothesis holds, it holds in a constrained way. For instance, it is still possible that the minimax hypothesis describes players' behaviors well in the later stage of a game when they have more experiences. In other words, players may "learn" to play minimax. To address this possibility, we take male tennis matches, which have longer observations, as an example and repeat some of our earlier analyses. As a first and preliminary attempt, we divide the data in each point game into the first half and the second half of the game, and perform the Pearson test of equal winning probability and the runs test on them separately. If the null hypotheses fare better in the second half of the game than in the first half, then it may be suggestive of learning to play minimax.

We find that the p-values of the Pearson joint statistic are 0.582 for the first half and 0.312 for the second half. The p-values of the KS statistic (of the joint

[28] Under AIC, the proportions of Low, Median and High in adult players are 0.27, 0.397 and 0.333, while those of junior players are 0.35, 0.45 and 0.2, respectively. Under SC, the proportions of Low, Median and High in adult players are 0.127, 0.143 and 0.73, while those of junior players are 0.15, 0.4 and 0.45, respectively.

null hypothesis that the probability of winning is equal across strategies) are 0.735 and 0.709 for the first half and the second half respectively. As for the runs test, the p-values of the KS statistic (of the joint null hypothesis that the serves are serially independent) are 0.046 for the first half and 0.35 for the second half. In summary, there seems to be less serial dependence in the second half of the game according to the results of the runs test. We do not want to make too much out of this preliminary finding as dividing the data into two halves is quite arbitrary. Yet this does hint on the possibility of learning to play minimax.

7. Concluding Remarks

While the theory of mixed strategy equilibrium has not been entirely consistent with the flurry of the empirical evidence from experimental settings involving human subjects in the past few decades, the empirical finding in Walker and Wooders (2001) undoubtedly makes a mark in testing the minimax hypothesis. However, our finding, from studying a broader natural data set in tennis, suggests that the minimax hypothesis should be further questioned. Not only should one examine the robustness of equal winning probability in a more careful way, but also the phenomenon of serial dependence needs to be re-addressed.

Since we do find evidence regarding impacts of past plays on current choices, we therefore turn to model this dependence by discussing how learning models can apply to our data and proposing several rules. When estimating various rules to account for players' choices, we find that for a significant number of point games, the best rules outperform the equilibrium prediction. More interestingly, we demonstrate that junior players tend to adopt simpler rules than adult players do. Overall, we see our work as a primitive attempt in applying rules (or learning models) to field data. More elaborate empirical analyses seem imminent to improve our understanding of choices under strategic situations.

References

Brown, J. and Rosenthal, R. (1990). "Testing the Minimax Hypothesis: A Reexamination of O'Neill's Experiment," *Econometrica* **5**, 1065-1081

Battalio, R.; Samuelson, L. and Van Huyck, J. (2003). "Risk Dominance, Payoff Dominance and Probabilistic Choice Learning," Wisconsin Madison—Social Systems *Working Paper No. 2*.

Camerer, C. and Ho, T-H (1999). "Experience-Weighted Attraction Learning in Normal Form Games," *Econometrica* **67**, 827-874

Cheung, Y-W and Friedman, D. (1997). "Individual Learning in Normal Form Games: Some Laboratory Results," *Games and Economic Behavior* **19**, 46-47

Chiappori, P-A; Levitt, S. and Groseclose, T. (2002). "Testing Mixed-Strategy Equilibria When Players Are Heterogeneous: The Case of Penalty Kicks in Soccer," *American Economic Review* **92**, 1138-1151

Erev, I. and Roth, A. E. (1998). "Predicting How People Play Games: Reinforcement Learning in Experimental Games with Unique, Mixed Strategy Equilibria," *American Economic Review* **88**, 848-881

Feltovich, N. (2000). "Reinforcement-Based vs. Belief-Based Learning Models in Experimental Asymmetric-Information Games," *Econometrica* **68**, 605-641

Gibbons, J. and Chakraborti, S. (1992). *Nonparametric Statistical Inference*, New York: Marcel Dekker.

Ho, T-H and Weigelt, K. (1996). "Task Complexity, Equilibrium Selection, and Learning: An Experimental Study," *Management Science* **42**, 659-679

Ho, T-H; Camerer C. and Wang, X. (2002). "Individual Differences in EWA Learning with Partial Payoff Information," *mimeo*.

Hopkins, E. (2002). "Two Competing Models of How People Learn in Games," *Econometrica* **70**, 2141-2166

Mitropoulos, A. (2001). “Learning under Minimal Information: An Experiment on Mutual Fate Control,” *Journal of Economic Psychology* **22**, 523-557

Mookherjee, D. and Sopher, B. (1994). “Learning Behavior in an Experimental Matching Pennies Game,” *Games and Economic Behavior* **7**, 62-91

O’Neill, B. (1987). “Nonmetric Test of the Minimax Theory of Two-Person Zerosum Games,” *Proceedings of National Academy of Sciences, U.S.A.* 84, 2106-2109

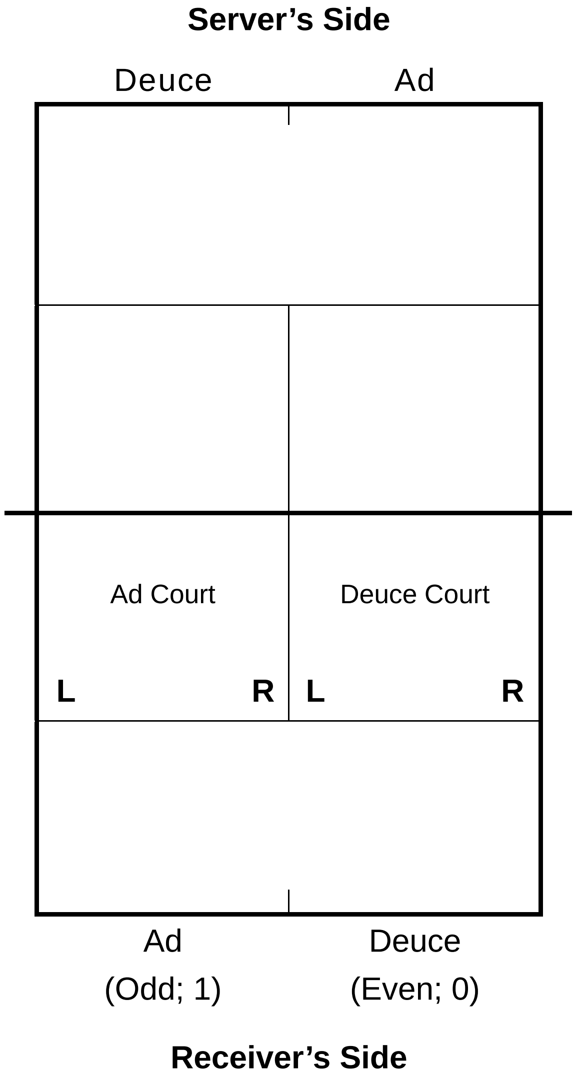
Palacios-Huerta, I. (2003). “Professionals Play Minimax,” *Review of Economic Studies* **70**, 395-415

Roth, A. E. and Erev, I. (1995). “Learning in Extensive-Form Games: Experimental Data and Simple Dynamic Models in the Intermediate Term,” *Games and Economic Behavior* **8**, 16-212

Walker, M. and Wooders, J. (*2000*). “Binary Markov Games,” *University of Arizona Working Paper No.00-12*.

Walker, M. and Wooders, J. (2001). “Minimax Play at Wimbledon,” *American Economic Review* **91**, 1521-1538

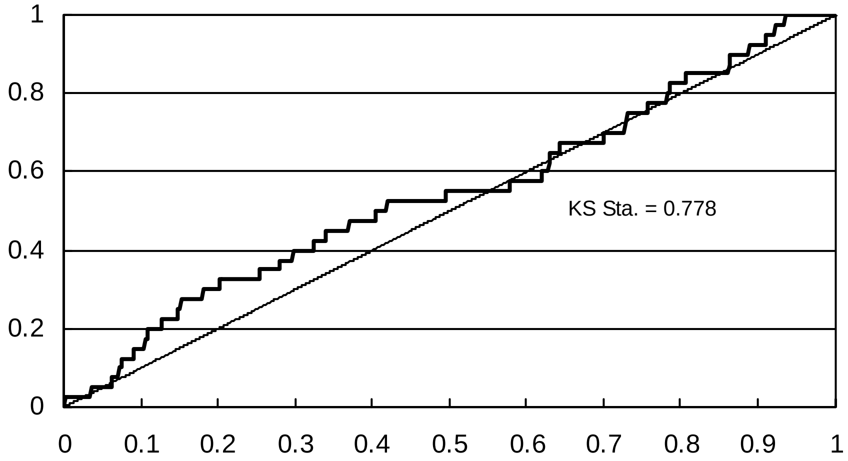
Figure 1
The Tennis Court and A Typical Game



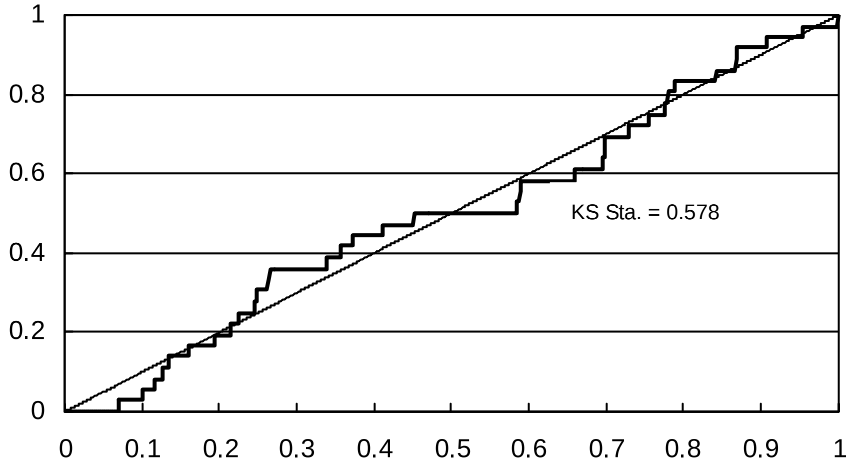
Receiver

		L	R
Server	**L**	π_{LL}	π_{LR}
	R	π_{RL}	π_{RR}

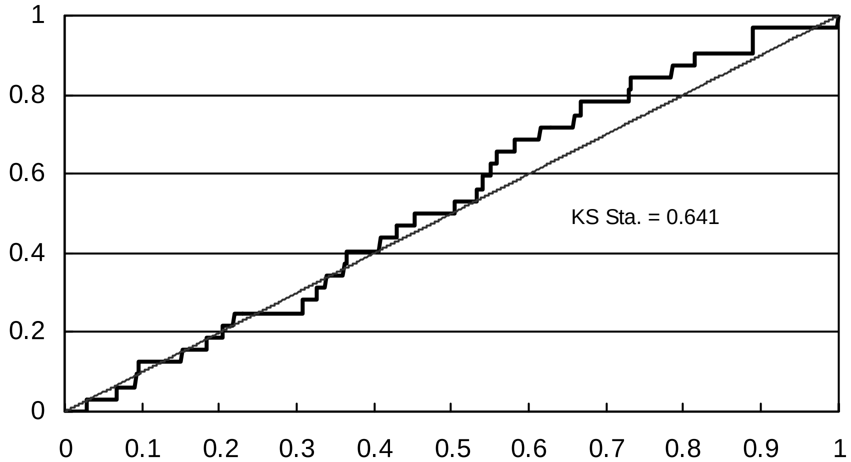
Figure 2
KS Test for Equal Winning Probabilities



Male

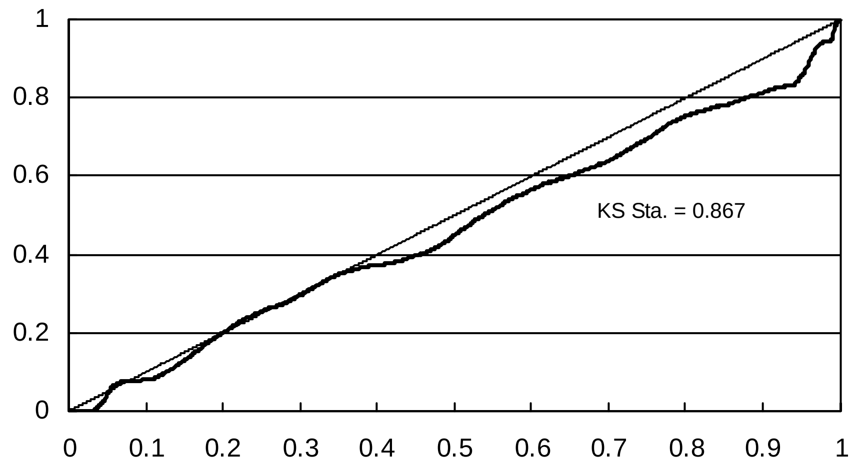


Female

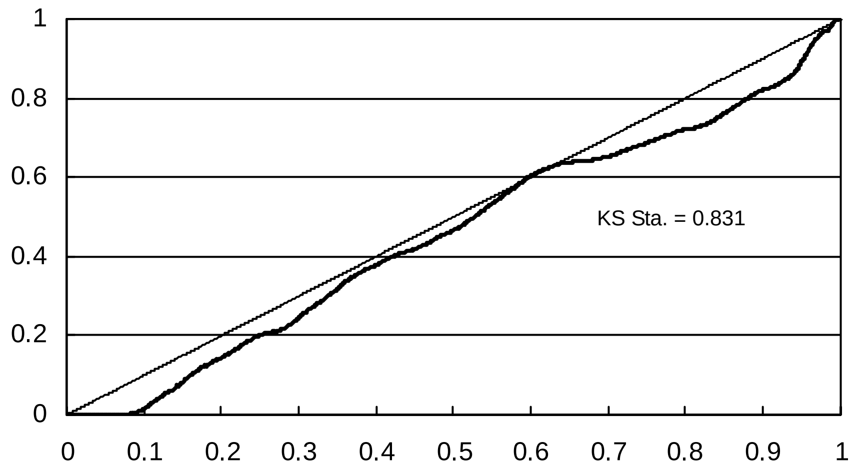


Junior

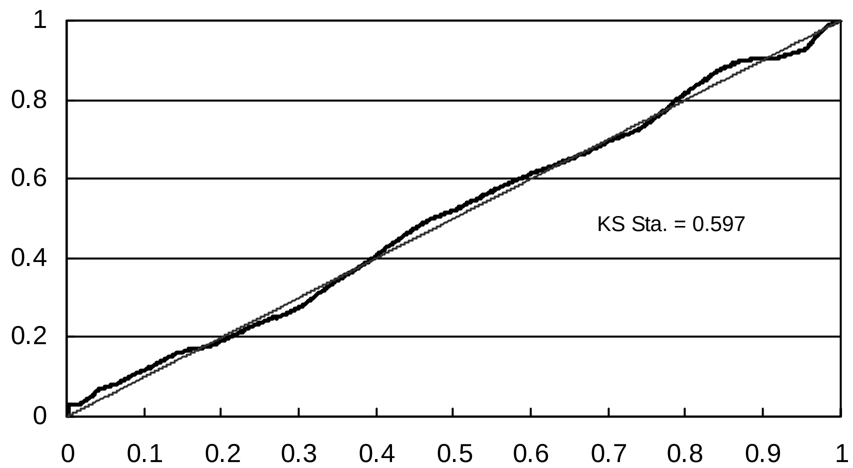
Figure 3
KS Runs Tests for Serial Independence



Male



Female



Junior

Figure 4
The Power Function in Example 1

Receiver

		L	R	Server's Minimax
Server	**L**	0.53	0.88	0.568
	R	0.79	0.33	0.432
	Receiver's Minimax	0.68	0.32	

Game's Value = 0.643

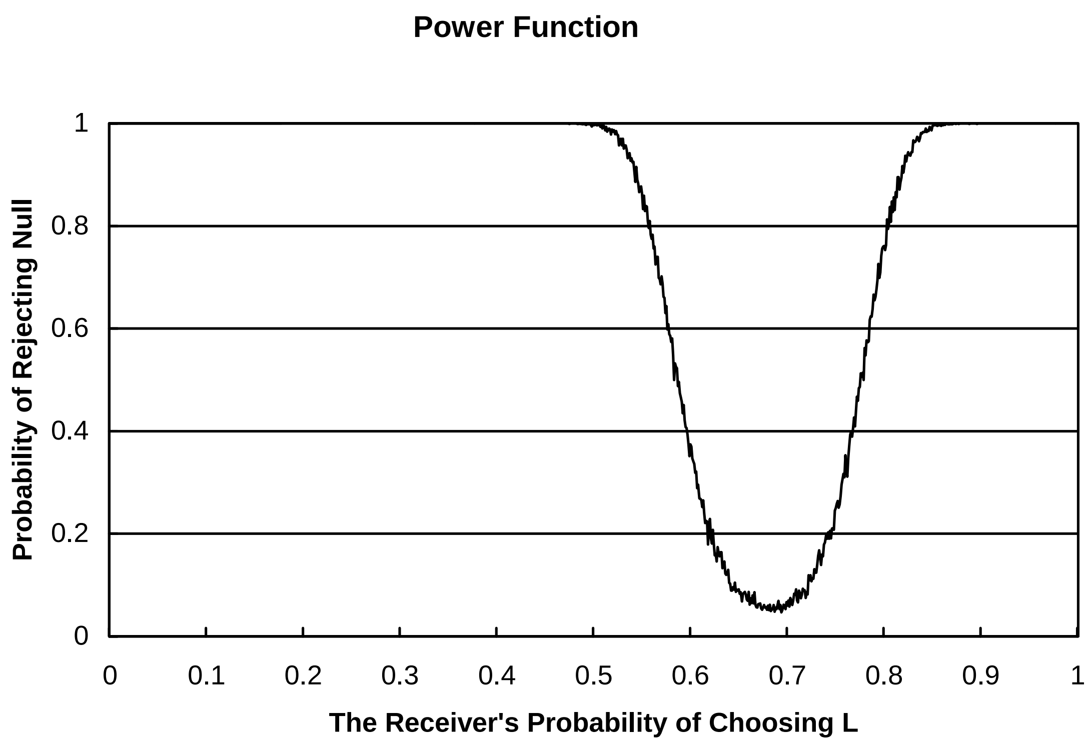


Figure 5
The Power Function in Example 2

Receiver

		L	R	Server's Minimax
Server	**L**	0.62	0.67	0.568
	R	0.66	0.59	0.432
	Receiver's Minimax	0.68	0.32	

Game's Value = 0.643

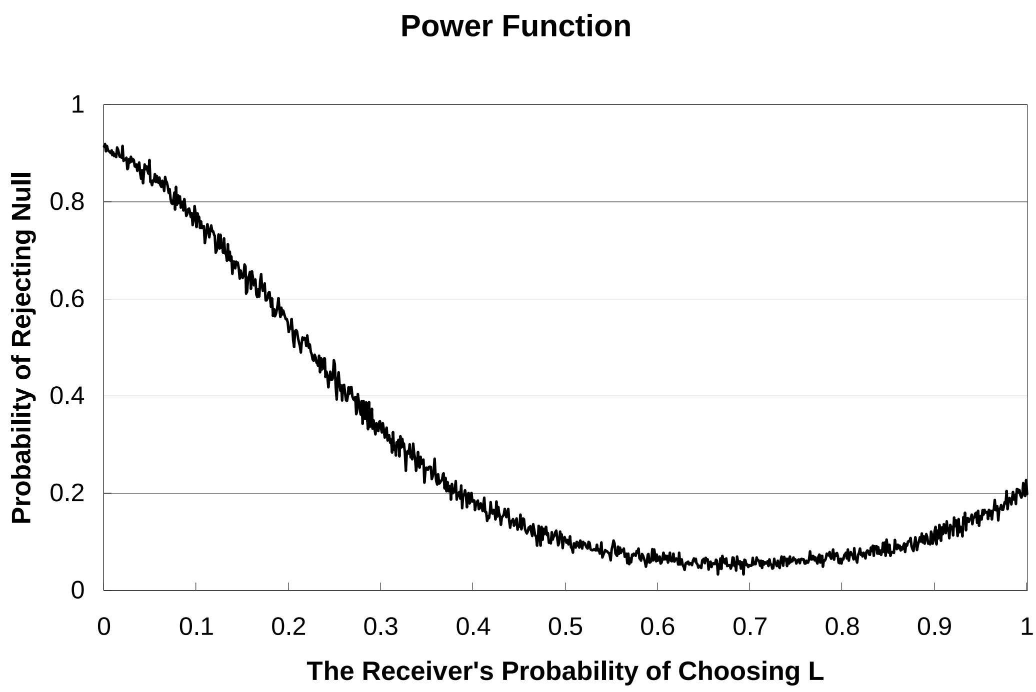


Figure 6

The Distribution of Classes of Cognitive Ability for Junior and Adult Players

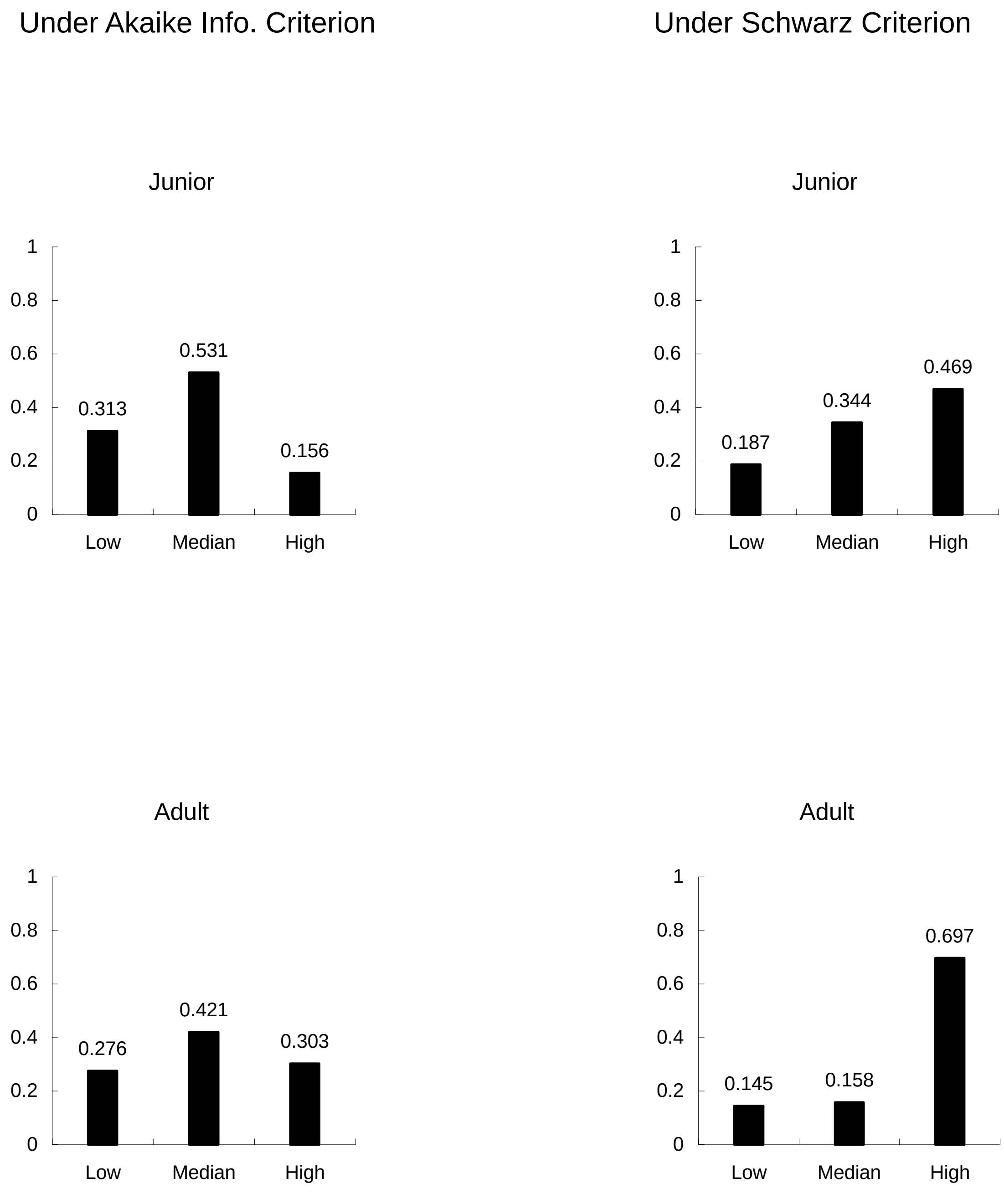


Figure 7

The Distribution of Classes of Cognitive Ability for Male and Female Players

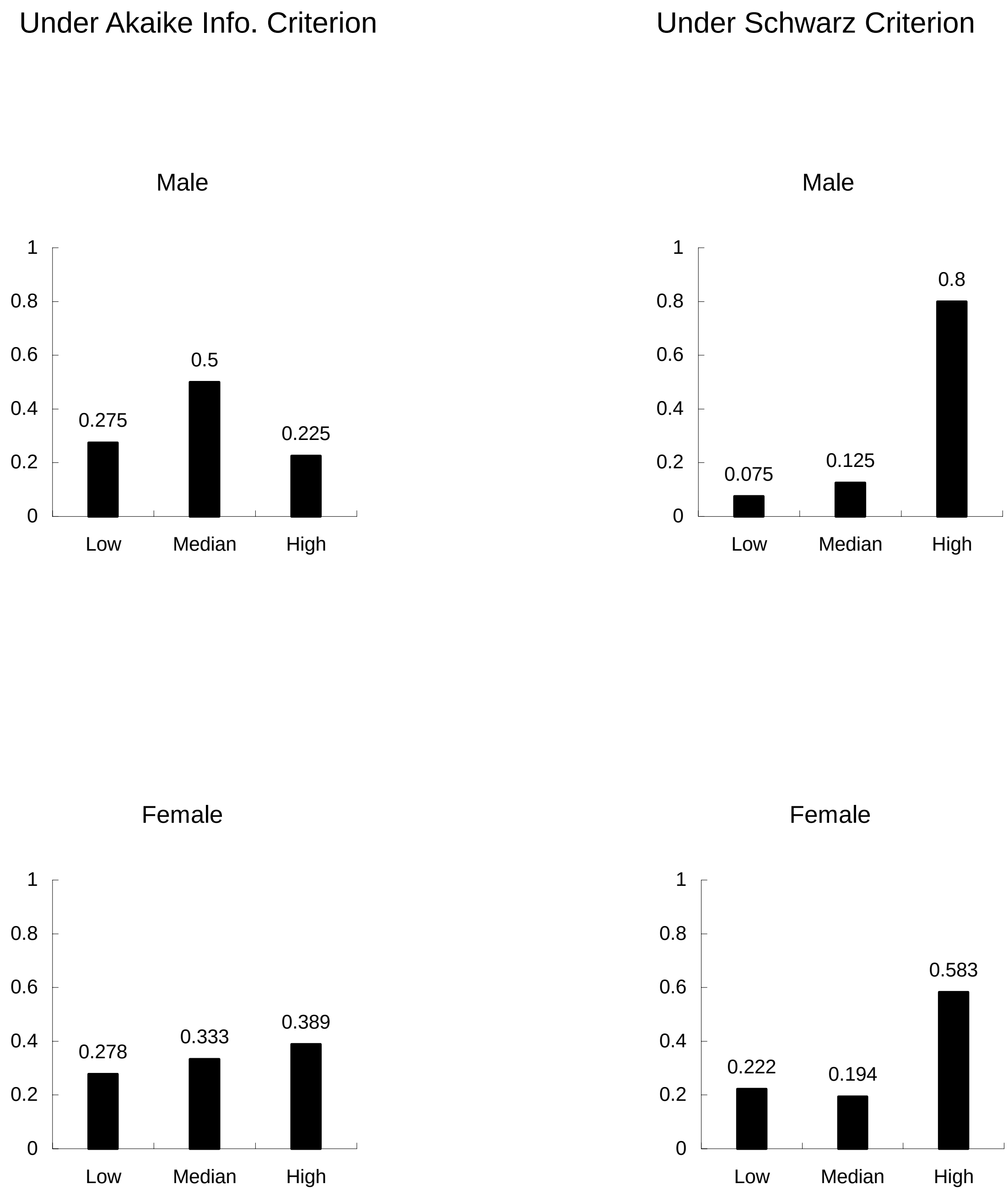


Table 1
Test of Equal Winning Probabilities in Male Tennis

Index	Male Play Match	Server	Court	Serve Direction L	R	Total	Points Won L	R	Win Rate L	R	Pearson Sta.	p-Value	
1	80 WIMBLEDON	Borg	Ad	15	74	89	10	51	0.667	0.689	0.029	0.864	
2	80 WIMBLEDON	Borg	Deuce	32	57	89	26	36	0.813	0.632	3.174	0.075	**
3	80 WIMBLEDON	McEnroe	Ad	45	30	75	29	19	0.644	0.633	0.010	0.922	
4	80 WIMBLEDON	McEnroe	Deuce	47	38	85	31	29	0.660	0.763	1.086	0.297	
5	80 U.S. OPEN	McEnroe	Ad	42	36	78	23	27	0.548	0.750	3.450	0.063	**
6	80 U.S. OPEN	McEnroe	Deuce	60	28	88	38	16	0.633	0.571	0.309	0.579	
7	80 U.S. OPEN	Borg	Ad	23	53	76	15	33	0.652	0.623	0.060	0.806	
8	80 U.S. OPEN	Borg	Deuce	27	53	80	18	27	0.667	0.509	1.797	0.180	
9	85 ROLAND GARROS	Wilander	Ad	36	12	48	23	8	0.639	0.667	0.030	0.862	
10	85 ROLAND GARROS	Wilander	Deuce	18	36	54	10	18	0.556	0.500	0.148	0.700	
11	85 ROLAND GARROS	Lendl	Ad	39	9	48	20	6	0.513	0.667	0.697	0.404	
12	85 ROLAND GARROS	Lendl	Deuce	30	19	49	13	9	0.433	0.474	0.077	0.782	
13	89 WIMBLEDON	Becker	Ad	32	31	63	29	16	0.906	0.516	11.743	0.001	*
14	89 WIMBLEDON	Becker	Deuce	41	25	66	29	15	0.707	0.600	0.805	0.370	
15	89 WIMBLEDON	Lendl	Ad	50	35	85	27	23	0.540	0.657	1.166	0.280	
16	89 WIMBLEDON	Lendl	Deuce	60	31	91	39	19	0.650	0.613	0.122	0.727	
17	89 U.S. OPEN	Becker	Ad	48	10	58	31	4	0.646	0.400	2.090	0.148	
18	89 U.S. OPEN	Becker	Deuce	44	27	71	27	15	0.614	0.556	0.234	0.629	
19	89 U.S. OPEN	Lendl	Ad	34	22	56	19	16	0.559	0.727	1.617	0.203	
20	89 U.S. OPEN	Lendl	Deuce	33	26	59	18	21	0.545	0.808	4.463	0.035	*
21	92 AUSTRALIAN OPEN	Courier	Ad	34	18	52	22	12	0.647	0.667	0.020	0.888	
22	92 AUSTRALIAN OPEN	Courier	Deuce	30	20	50	20	12	0.667	0.600	0.231	0.630	
23	92 AUSTRALIAN OPEN	Edberg	Ad	40	6	46	24	3	0.600	0.500	0.215	0.643	
24	92 AUSTRALIAN OPEN	Edberg	Deuce	26	16	42	17	8	0.654	0.500	0.973	0.324	
25	95 ROLAND GARROS	Muster	Ad	27	8	35	21	4	0.778	0.500	2.333	0.127	
26	95 ROLAND GARROS	Muster	Deuce	30	8	38	23	5	0.767	0.625	0.654	0.419	
27	95 ROLAND GARROS	Chang	Ad	38	2	40	22	0	0.579	0.000	2.573	0.109	
28	95 ROLAND GARROS	Chang	Deuce	21	24	45	16	12	0.762	0.500	3.268	0.071	**
29	95 U.S. OPEN	Sampras	Ad	19	38	57	11	29	0.579	0.763	2.054	0.152	
30	95 U.S. OPEN	Sampras	Deuce	30	28	58	20	24	0.667	0.857	2.870	0.090	**
31	95 U.S. OPEN	Agassi	Ad	41	13	54	31	11	0.756	0.846	0.463	0.496	
32	95 U.S. OPEN	Agassi	Deuce	35	25	60	21	14	0.600	0.560	0.096	0.757	
33	00 AUSTRALIAN OPEN	Agassi	Ad	30	25	55	19	12	0.633	0.480	1.304	0.254	
34	00 AUSTRALIAN OPEN	Agassi	Deuce	32	28	60	23	21	0.719	0.750	0.075	0.785	
35	00 AUSTRALIAN OPEN	Kafelnikov	Ad	28	24	52	15	14	0.536	0.583	0.119	0.730	
36	00 AUSTRALIAN OPEN	Kafelnikov	Deuce	31	27	58	21	15	0.677	0.556	0.910	0.340	
37	01 WIMBLEDON	Ivanisevic	Ad	48	26	74	30	21	0.625	0.808	2.628	0.105	
38	01 WIMBLEDON	Ivanisevic	Deuce	60	23	83	41	17	0.683	0.739	0.246	0.620	
39	01 WIMBLEDON	Rafter	Ad	27	32	59	20	24	0.741	0.750	0.007	0.935	
40	01 WIMBLEDON	Rafter	Deuce	31	33	64	22	23	0.710	0.697	0.012	0.911	
	Joint Test			1414	1076	2490	914	689	0.646	0.640	54.157	0.067	

* denotes rejection of equal winning probability at 5% level.

** denotes rejection of equal winning probability at 10% level.

Table 2
Test of Equal Winning Probabilities in Female Tennis

	Female Play			Serve Direction			Points Won		Win Rate				
Index	Match	Server	Court	L	R	Total	L	R	L	R	Pearson Sta.	p-Value	
1	85 AUSTRALIAN OPEN	Navratilova	Ad	21	20	41	17	13	0.810	0.650	1.328	0.249	
2	85 AUSTRALIAN OPEN	Navratilova	Deuce	22	14	36	12	4	0.545	0.286	2.338	0.126	
3	85 AUSTRALIAN OPEN	Evert	Ad	18	16	34	9	8	0.500	0.500	0.000	1.000	
4	85 AUSTRALIAN OPEN	Evert	Deuce	5	32	37	2	17	0.400	0.531	0.298	0.585	
5	87 WIMBLEDON	Navratilova	Ad	21	7	28	17	6	0.810	0.857	0.081	0.776	
6	87 WIMBLEDON	Navratilova	Deuce	29	6	35	17	3	0.586	0.500	0.151	0.698	
7	87 WIMBLEDON	Graf	Ad	13	16	29	7	11	0.538	0.688	0.677	0.411	
8	87 WIMBLEDON	Graf	Deuce	11	20	31	8	13	0.727	0.650	0.194	0.660	
9	87 U.S. OPEN	Navratilova	Ad	25	12	37	14	9	0.560	0.750	1.244	0.265	
10	87 U.S. OPEN	Navratilova	Deuce	24	10	34	16	9	0.667	0.900	1.975	0.160	
11	87 U.S. OPEN	Graf	Ad	13	11	24	8	6	0.615	0.545	0.120	0.729	
12	87 U.S. OPEN	Graf	Deuce	12	14	26	6	10	0.500	0.714	1.254	0.263	
13	92 ROLAND GARROS	Seles	Ad	34	15	49	23	6	0.676	0.400	3.293	0.070	**
14	92 ROLAND GARROS	Seles	Deuce	29	22	51	16	13	0.552	0.591	0.078	0.780	
15	92 ROLAND GARROS	Graf	Ad	33	27	60	14	16	0.424	0.593	1.684	0.194	
16	92 ROLAND GARROS	Graf	Deuce	36	27	63	17	17	0.472	0.630	1.539	0.215	
17	92 U.S. OPEN	Seles	Ad	13	13	26	7	6	0.538	0.462	0.154	0.695	
18	92 U.S. OPEN	Seles	Deuce	18	9	27	13	7	0.722	0.778	0.096	0.756	
19	92 U.S. OPEN	Sanchez	Ad	9	25	34	2	10	0.222	0.400	0.916	0.339	
20	92 U.S. OPEN	Sanchez	Deuce	21	12	33	12	8	0.571	0.667	0.290	0.590	
21	97 WIMBLEDON	Hingis	Ad	26	14	40	14	8	0.538	0.571	0.040	0.842	
22	97 WIMBLEDON	Hingis	Deuce	15	29	44	8	16	0.533	0.552	0.013	0.908	
23	97 WIMBLEDON	Novotna	Ad	14	20	34	8	12	0.571	0.600	0.028	0.868	
24	97 WIMBLEDON	Novotna	Deuce	29	14	43	13	10	0.448	0.714	2.686	0.101	
25	99 ROLAND GARROS	Graf	Ad	22	21	43	13	10	0.591	0.476	0.568	0.451	
26	99 ROLAND GARROS	Graf	Deuce	23	20	43	14	12	0.609	0.600	0.003	0.954	
27	99 ROLAND GARROS	Hingis	Ad	36	9	45	14	6	0.389	0.667	2.250	0.134	
28	99 ROLAND GARROS	Hingis	Deuce	32	18	50	17	10	0.531	0.556	0.027	0.869	
29	00 U.S. OPEN	V. Williams	Ad	11	21	32	5	14	0.455	0.667	1.347	0.246	
30	00 U.S. OPEN	V. Williams	Deuce	17	20	37	10	13	0.588	0.650	0.149	0.699	
31	00 U.S. OPEN	Davenport	Ad	14	14	28	8	11	0.571	0.786	1.474	0.225	
32	00 U.S. OPEN	Davenport	Deuce	10	21	31	4	12	0.400	0.571	0.797	0.372	
33	02 AUSTRALIAN OPEN	Capriati	Ad	13	29	42	6	16	0.462	0.552	0.293	0.588	
34	02 AUSTRALIAN OPEN	Capriati	Deuce	20	22	42	11	13	0.550	0.591	0.072	0.789	
35	02 AUSTRALIAN OPEN	Hingis	Ad	33	16	49	17	6	0.515	0.375	0.850	0.357	
36	02 AUSTRALIAN OPEN	Hingis	Deuce	26	23	49	16	9	0.615	0.391	2.452	0.117	
	Joint Test			748	639	1387	415	370	0.555	0.579	30.758	0.716	

* denotes rejection of equal winning probability at 5% level.

** denotes rejection of equal winning probability at 10% level.

Table 3
Test of Equal Winning Probabilities in Junior Tennis

Index	Junior Play Match	Server	Court	Serve Direction L	R	Total	Points Won L	R	Win Rate L	R	Pearson Sta.	p-Value	
1	96 AVVENIRE TOURNAMENT	Middleton	Ad	17	10	27	11	5	0.647	0.500	0.564	0.453	
2	96 AVVENIRE TOURNAMENT	Middleton	Deuce	22	11	33	17	7	0.773	0.636	0.688	0.407	
3	96 AVVENIRE TOURNAMENT	Kalvaria	Ad	21	8	29	12	4	0.571	0.500	0.120	0.730	
4	96 AVVENIRE TOURNAMENT	Kalvaria	Deuce	11	15	26	8	7	0.727	0.467	1.766	0.184	
5	00 WIMBLEDON	Salerni	Ad	8	8	16	3	4	0.375	0.500	0.254	0.614	
6	00 WIMBLEDON	Salerni	Deuce	8	9	17	3	7	0.375	0.778	2.837	0.092	**
7	00 WIMBLEDON	Perediynis	Ad	6	8	14	2	4	0.333	0.500	0.389	0.533	
8	00 WIMBLEDON	Perediynis	Deuce	10	6	16	3	1	0.300	0.167	0.356	0.551	
9	02 AVVENIRE TOURNAMENT	Gonzalez	Ad	13	11	24	5	7	0.385	0.636	1.510	0.219	
10	02 AVVENIRE TOURNAMENT	Gonzalez	Deuce	9	13	22	6	4	0.667	0.308	2.764	0.096	
11	02 AVVENIRE TOURNAMENT	Sanchez	Ad	15	6	21	7	3	0.467	0.500	0.019	0.890	
12	02 AVVENIRE TOURNAMENT	Sanchez	Deuce	13	9	22	4	2	0.308	0.222	0.196	0.658	
13	03 AUSTRALIAN OPEN (Qrt)	Baqhdatis	Ad	9	7	16	9	4	1.000	0.571	4.747	0.029	*
14	03 AUSTRALIAN OPEN (Qrt)	Baqhdatis	Deuce	10	12	22	9	9	0.900	0.750	0.825	0.364	
15	03 AUSTRALIAN OPEN (Qrt)	Evans	Ad	12	8	20	5	5	0.417	0.625	0.833	0.361	
16	03 AUSTRALIAN OPEN (Qrt)	Evans	Deuce	18	6	24	12	2	0.667	0.333	2.057	0.151	
17	03 AUSTRALIAN OPEN (2nd)	Bauer	Ad	19	12	31	11	6	0.579	0.500	0.185	0.667	
18	03 AUSTRALIAN OPEN (2nd)	Bauer	Deuce	6	27	33	5	17	0.833	0.630	0.917	0.338	
19	03 AUSTRALIAN OPEN (2nd)	Kerber	Ad	28	12	40	13	3	0.464	0.250	1.607	0.205	
20	03 AUSTRALIAN OPEN (2nd)	Kerber	Deuce	21	20	41	12	11	0.571	0.550	0.019	0.890	
21	03 AUSTRALIAN OPEN (2nd)	Dellacqua	Ad	18	7	25	15	5	0.833	0.714	0.446	0.504	
22	03 AUSTRALIAN OPEN (2nd)	Dellacqua	Deuce	21	6	27	15	3	0.714	0.500	0.964	0.326	
23	03 AUSTRALIAN OPEN (2nd)	Kim	Ad	6	28	34	4	17	0.667	0.607	0.074	0.785	
24	03 AUSTRALIAN OPEN (2nd)	Kim	Deuce	13	21	34	8	10	0.615	0.476	0.624	0.429	
25	03 AUSTRALIAN OPEN (2nd)	Scherer	Ad	11	7	18	7	5	0.636	0.714	0.117	0.732	
26	03 AUSTRALIAN OPEN (2nd)	Scherer	Deuce	11	9	20	6	6	0.545	0.667	0.303	0.582	
27	03 AUSTRALIAN OPEN (2nd)	Cvetkovic	Ad	6	6	12	4	3	0.667	0.500	0.343	0.558	
28	03 AUSTRALIAN OPEN (2nd)	Cvetkovic	Deuce	6	7	13	5	4	0.833	0.571	1.040	0.308	
29	03 AUSTRALIAN OPEN (2nd)	Tsonga	Ad	11	5	16	10	4	0.909	0.800	0.374	0.541	
30	03 AUSTRALIAN OPEN (2nd)	Tsonga	Deuce	8	10	18	6	7	0.750	0.700	0.055	0.814	
31	03 AUSTRALIAN OPEN (2nd)	Feeney	Ad	14	6	20	7	3	0.500	0.500	0.000	1.000	
32	03 AUSTRALIAN OPEN (2nd)	Feeney	Deuce	12	8	20	4	6	0.333	0.750	3.333	0.068	**
	Joint Test			413	338	751	248	185	0.600	0.547	30.327	0.551	

* denotes rejection of equal winning probability at 5% level.
** denotes rejection of equal winning probability at 10% level.

Table 4
Runs Test in Male Tennis

	Male Play			Serve Direction			Runs			
Index	Match	Server	Court	L	R	Total	r_i	F(r_i -1)	F(r_i)	
1	80 WIMBLEDON	Borg	Ad	14	70	84	27	0.766	0.927	
2	80 WIMBLEDON	Borg	Deuce	31	51	82	40	0.495	0.584	
3	80 WIMBLEDON	McEnroe	Ad	42	29	71	42	0.939	0.963	
4	80 WIMBLEDON	McEnroe	Deuce	44	34	78	47	0.951	0.971	**
5	80 U.S. OPEN	McEnroe	Ad	40	35	75	38	0.422	0.516	
6	80 U.S. OPEN	McEnroe	Deuce	57	28	85	32	0.043	0.067	
7	80 U.S. OPEN	Borg	Ad	22	51	73	32	0.475	0.570	
8	80 U.S. OPEN	Borg	Deuce	26	46	72	31	0.168	0.243	
9	85 ROLAND GARROS	Wilander	Ad	36	12	48	18	0.285	0.396	
10	85 ROLAND GARROS	Wilander	Deuce	18	36	54	26	0.563	0.669	
11	85 ROLAND GARROS	Lendl	Ad	39	9	48	19	0.903	1.000	
12	85 ROLAND GARROS	Lendl	Deuce	30	19	49	27	0.748	0.839	
13	89 WIMBLEDON	Becker	Ad	32	31	63	32	0.400	0.502	
14	89 WIMBLEDON	Becker	Deuce	41	25	66	28	0.116	0.173	
15	89 WIMBLEDON	Lendl	Ad	50	35	85	45	0.698	0.773	
16	89 WIMBLEDON	Lendl	Deuce	60	31	91	44	0.650	0.725	
17	89 U.S. OPEN	Becker	Ad	48	10	58	15	0.076	0.184	
18	89 U.S. OPEN	Becker	Deuce	44	27	71	37	0.693	0.781	
19	89 U.S. OPEN	Lendl	Ad	34	22	56	29	0.585	0.693	
20	89 U.S. OPEN	Lendl	Deuce	33	26	59	28	0.245	0.337	
21	92 AUSTRALIAN OPEN	Courier	Ad	34	18	52	32	0.988	0.994	*
22	92 AUSTRALIAN OPEN	Courier	Deuce	30	20	50	29	0.850	0.912	
23	92 AUSTRALIAN OPEN	Edberg	Ad	40	6	46	11	0.213	0.529	
24	92 AUSTRALIAN OPEN	Edberg	Deuce	26	16	42	22	0.135	0.253	
25	95 ROLAND GARROS	Muster	Ad	27	8	35	15	0.672	0.878	
26	95 ROLAND GARROS	Muster	Deuce	30	8	38	14	0.479	0.615	
27	95 ROLAND GARROS	Chang	Ad	38	2	40	5	0.146	1.000	
28	95 ROLAND GARROS	Chang	Deuce	21	24	45	29	0.940	0.968	
29	95 U.S. OPEN	Sampras	Ad	19	38	57	27	0.510	0.639	
30	95 U.S. OPEN	Sampras	Deuce	30	28	58	28	0.256	0.350	
31	95 U.S. OPEN	Agassi	Ad	41	13	54	27	0.989	1.000	*
32	95 U.S. OPEN	Agassi	Deuce	35	25	60	26	0.106	0.163	
33	00 AUSTRALIAN OPEN	Agassi	Ad	30	25	55	22	0.031	0.056	
34	00 AUSTRALIAN OPEN	Agassi	Deuce	32	28	60	38	0.959	0.978	**
35	00 AUSTRALIAN OPEN	Kafelnikov	Ad	28	24	52	24	0.172	0.255	
36	00 AUSTRALIAN OPEN	Kafelnikov	Deuce	31	27	58	30	0.461	0.568	
37	01 WIMBLEDON	Ivanisevic	Ad	48	26	74	33	0.279	0.377	
38	01 WIMBLEDON	Ivanisevic	Deuce	60	23	83	28	0.035	0.057	
39	01 WIMBLEDON	Rafter	Ad	27	32	59	33	0.721	0.802	
40	01 WIMBLEDON	Rafter	Deuce	31	32	63	29	0.156	0.223	

* denotes rejection of serial independence at 5% level.

** denotes rejection of serial independence at 10% level.

Table 5
Runs Test in Female Tennis

Index	Female Play Match	Server	Court	Serve Direction L	R	Total	Runs r_i	$F(r_i - 1)$	$F(r_i)$	
1	85 AUSTRALIAN OPEN	Navratilova	Ad	17	19	36	21	0.702	0.806	
2	85 AUSTRALIAN OPEN	Navratilova	Deuce	19	14	33	20	0.805	0.890	
3	85 AUSTRALIAN OPEN	Evert	Ad	14	15	29	22	0.946	0.975	
4	85 AUSTRALIAN OPEN	Evert	Deuce	4	30	34	9	0.466	0.610	
5	87 WIMBLEDON	Navratilova	Ad	21	7	28	11	0.281	0.502	
6	87 WIMBLEDON	Navratilova	Deuce	29	6	35	11	0.331	0.647	
7	87 WIMBLEDON	Graf	Ad	13	16	29	17	0.671	0.793	
8	87 WIMBLEDON	Graf	Deuce	11	20	31	19	0.905	0.963	
9	87 U.S. OPEN	Navratilova	Ad	25	12	37	15	0.148	0.259	
10	87 U.S. OPEN	Navratilova	Deuce	24	10	34	13	0.133	0.252	
11	87 U.S. OPEN	Graf	Ad	13	11	24	12	0.273	0.433	
12	87 U.S. OPEN	Graf	Deuce	12	14	26	14	0.430	0.594	
13	92 ROLAND GARROS	Seles	Ad	34	15	49	25	0.810	0.903	
14	92 ROLAND GARROS	Seles	Deuce	29	22	51	22	0.096	0.155	
15	92 ROLAND GARROS	Graf	Ad	33	27	60	29	0.282	0.376	
16	92 ROLAND GARROS	Graf	Deuce	36	27	63	30	0.270	0.362	
17	92 U.S. OPEN	Seles	Ad	13	13	26	18	0.919	0.966	
18	92 U.S. OPEN	Seles	Deuce	18	9	27	17	0.939	0.984	
19	92 U.S. OPEN	Sanchez	Ad	9	25	34	17	0.828	0.947	
20	92 U.S. OPEN	Sanchez	Deuce	21	12	33	14	0.145	0.246	
21	97 WIMBLEDON	Hingis	Ad	26	14	40	21	0.668	0.794	
22	97 WIMBLEDON	Hingis	Deuce	15	29	44	21	0.454	0.598	
23	97 WIMBLEDON	Novotna	Ad	11	18	29	19	0.939	0.978	
24	97 WIMBLEDON	Novotna	Deuce	28	13	41	19	0.449	0.609	
25	99 ROLAND GARROS	Graf	Ad	22	21	43	30	0.985	0.994	*
26	99 ROLAND GARROS	Graf	Deuce	23	20	43	26	0.831	0.899	
27	99 ROLAND GARROS	Hingis	Ad	36	9	45	16	0.515	0.636	
28	99 ROLAND GARROS	Hingis	Deuce	32	18	50	22	0.217	0.312	
29	00 U.S. OPEN	V. Williams	Ad	11	21	32	16	0.510	0.654	
30	00 U.S. OPEN	V. Williams	Deuce	17	20	37	16	0.096	0.168	
31	00 U.S. OPEN	Davenport	Ad	14	14	28	14	0.280	0.427	
32	00 U.S. OPEN	Davenport	Deuce	10	21	31	14	0.331	0.478	
33	02 AUSTRALIAN OPEN	Capriati	Ad	11	25	36	14	0.138	0.232	
34	02 AUSTRALIAN OPEN	Capriati	Deuce	14	20	34	18	0.503	0.642	
35	02 AUSTRALIAN OPEN	Hingis	Ad	31	16	47	21	0.293	0.422	
36	02 AUSTRALIAN OPEN	Hingis	Deuce	26	23	49	21	0.078	0.128	

* denotes rejection of serial independence at 5% level.

** denotes rejection of serial independence at 10% level.

Table 6
Runs Test in Junior Tennis

	Junior Play			Serve Direction			Runs			
Index	Match	Server	Court	L	R	Total	r_i	F(r_i -1)	F(r_i)	
1	96 AVVENIRE TOURNAMENT	Middleton	Ad	17	10	27	12	0.189	0.320	
2	96 AVVENIRE TOURNAMENT	Middleton	Deuce	22	11	33	18	0.771	0.866	
3	96 AVVENIRE TOURNAMENT	Kalvaria	Ad	21	8	29	10	0.077	0.156	
4	96 AVVENIRE TOURNAMENT	Kalvaria	Deuce	11	15	26	9	0.016	0.042	**
5	00 WIMBLEDON	Salerni	Ad	8	8	16	10	0.595	0.786	
6	00 WIMBLEDON	Salerni	Deuce	8	9	17	10	0.500	0.702	
7	00 WIMBLEDON	Perediynis	Ad	6	8	14	5	0.028	0.086	
8	00 WIMBLEDON	Perediynis	Deuce	10	6	16	9	0.497	0.706	
9	02 AVVENIRE TOURNAMENT	Gonzalez	Ad	13	11	24	12	0.273	0.433	
10	02 AVVENIRE TOURNAMENT	Gonzalez	Deuce	9	13	22	11	0.305	0.472	
11	02 AVVENIRE TOURNAMENT	Sanchez	Ad	15	6	21	11	0.668	0.871	
12	02 AVVENIRE TOURNAMENT	Sanchez	Deuce	13	9	22	11	0.305	0.472	
13	03 AUSTRALIAN OPEN (Qrt)	Baqhdatis	Ad	9	7	16	7	0.108	0.231	
14	03 AUSTRALIAN OPEN (Qrt)	Baqhdatis	Deuce	10	12	22	12	0.425	0.605	
15	03 AUSTRALIAN OPEN (Qrt)	Evans	Ad	12	8	20	14	0.920	0.971	
16	03 AUSTRALIAN OPEN (Qrt)	Evans	Deuce	18	6	24	10	0.392	0.569	
17	03 AUSTRALIAN OPEN (2nd)	Bauer	Ad	19	12	31	15	0.319	0.466	
18	03 AUSTRALIAN OPEN (2nd)	Bauer	Deuce	6	27	33	12	0.673	0.792	
19	03 AUSTRALIAN OPEN (2nd)	Kerber	Ad	28	12	40	20	0.747	0.840	
20	03 AUSTRALIAN OPEN (2nd)	Kerber	Deuce	21	20	41	19	0.173	0.264	
21	03 AUSTRALIAN OPEN (2nd)	Dellacqua	Ad	18	7	25	13	0.741	0.908	
22	03 AUSTRALIAN OPEN (2nd)	Dellacqua	Deuce	21	6	27	8	0.062	0.139	
23	03 AUSTRALIAN OPEN (2nd)	Kim	Ad	6	28	34	10	0.216	0.347	
24	03 AUSTRALIAN OPEN (2nd)	Kim	Deuce	13	21	34	16	0.282	0.415	
25	03 AUSTRALIAN OPEN (2nd)	Scherer	Ad	11	7	18	9	0.296	0.484	
26	03 AUSTRALIAN OPEN (2nd)	Scherer	Deuce	11	9	20	15	0.955	0.985	**
27	03 AUSTRALIAN OPEN (2nd)	Cvetkovic	Ad	6	6	12	7	0.392	0.608	
28	03 AUSTRALIAN OPEN (2nd)	Cvetkovic	Deuce	6	7	13	9	0.733	0.879	
29	03 AUSTRALIAN OPEN (2nd)	Tsonga	Ad	11	5	16	9	0.626	0.846	
30	03 AUSTRALIAN OPEN (2nd)	Tsonga	Deuce	8	10	18	4	0.000	0.003	*
31	03 AUSTRALIAN OPEN (2nd)	Feeney	Ad	14	6	20	13	0.956	1.000	**
32	03 AUSTRALIAN OPEN (2nd)	Feeney	Deuce	12	8	20	11	0.480	0.663	

* denotes rejection of serial independence at 5% level.

** denotes rejection of serial independence at 10% level.

Table 7
Results of Significance Tests from Probit Equations for Serve Choices

Estimating Equation No. 1

$$D = G[a_0 + a_1 lag(D) + a_2 lag2(D) + b_1 lag(RW) + b_2 lag2(RW) + c_1 lag(LW) + c_2 lag2(LW) + d_0 T]$$

Null Hypothesis		Point Games Where the Null Hypothesis is Rejected at the 0.05, 0.1 Levels	
		0.05 Level	0.1 Level
1. $a_1, a_2, b_1, b_2, c_1, c_2, d_0 = 0$	**Male**	6, 17, 31	6, 15, 17, 21, 22, 31, 34
	Female	13, 32	3, 13, 16, 20, 32
	Junior		
2. $a_1, a_2 = 0$	**Male**	22, 34	22, 34
	Female		3
	Junior		
3. $b_1, b_2, c_1, c_2 = 0$	**Male**	6,15	6, 15
	Female		
	Junior		
4. $b_1, b_2 = 0$	**Male**		12, 34
	Female	16	16
	Junior		
5. $c_1, c_2 = 0$	**Male**	6, 15, 30	6, 15, 30
	Female		
	Junior		
6. $d_0 = 0$	**Male**	17, 20, 31	7, 10, 11, 17, 20, 31
	Female	13, 16	3, 13, 16, 20, 30, 36
	Junior	3	3, 24

Estimating Equation No. 2

$$D = G[a_0 + a_1 lag(RW) + a_2 lag2(RW) + b_1 lag(LW) + b_2 lag2(LW) + d_0 T]$$

Null Hypothesis		Point Games Where the Null Hypothesis is Rejected at the 0.05, 0.1 Levels	
		0.05 Level	0.1 Level
1. $a_1, a_2, b_1, b_2, d_0 = 0$	**Male**	6, 13, 15, 17, 31	6, 13, 15, 17, 30, 31
	Female	8, 13, 16, 18, 20, 32	8, 13, 16, 18, 20, 32
	Junior	8, 14, 15, 30	8, 14, 15, 18, 26, 30
2. $a_1, a_2, b_1, b_2 = 0$	**Male**	15	13, 15
	Female		
	Junior		
3. $a_1, a_2 = 0$	**Male**	13	13
	Female	12, 16	8, 12, 16, 19
	Junior		
4. $b_1, b_2 = 0$	**Male**	15	12, 15
	Female		17, 33
	Junior		
5. $d_0 = 0$	**Male**	17, 20, 31	7, 17, 20, 31
	Female	8, 13, 16	8, 13, 16, 20
	Junior	3	3, 26, 32

Table 8
Model Selection under AIC, SC Criterion in Male Tennis

Index	Male Play Match	Server	Court	Criterion	Equilibrium Value	Best Rule Value	Rule
1	80 WIMBLEDON	Borg	Ad	AIC	0.9296	0.9070	3
				SC	0.9576	0.9696	3
2	80 WIMBLEDON	Borg	Deuce	AIC	1.3288	1.3417	4
				SC	1.3568	1.4072	1
3	80 WIMBLEDON	McEnroe	Ad	AIC	1.3727	1.3243	3
				SC	1.4036	1.3865	3
4	80 WIMBLEDON	McEnroe	Deuce	AIC	1.3986	1.3810	4
				SC	1.4273	1.4389	4
5	80 U.S. OPEN	McEnroe	Ad	AIC	1.4060	1.4164	2
				SC	1.4362	1.4887	1
6	80 U.S. OPEN	McEnroe	Deuce	AIC	1.2737	1.0636	3
				SC	1.3019	1.1493	3
7	80 U.S. OPEN	Borg	Ad	AIC	1.2525	1.2457	5
				SC	1.2831	1.3075	5
8	80 U.S. OPEN	Borg	Deuce	AIC	1.3037	1.3033	3
				SC	1.3335	1.3633	3
9	85 ROLAND GARROS	Wilander	Ad	AIC	1.1308	1.0062	3
				SC	1.1701	1.1504	1
10	85 ROLAND GARROS	Wilander	Deuce	AIC	1.3101	1.2794	1
				SC	1.3469	1.3530	1
11	85 ROLAND GARROS	Lendl	Ad	AIC	1.0068	0.9703	2
				SC	1.0458	1.0490	2
12	85 ROLAND GARROS	Lendl	Deuce	AIC	1.3763	1.3633	3
				SC	1.4149	1.4727	3
13	89 WIMBLEDON	Becker	Ad	AIC	1.4178	1.3751	4
				SC	1.4518	1.4739	1
14	89 WIMBLEDON	Becker	Deuce	AIC	1.3572	1.3546	4
				SC	1.3904	1.4215	4
15	89 WIMBLEDON	Lendl	Ad	AIC	1.3785	1.3320	3
				SC	1.4073	1.4168	3
16	89 WIMBLEDON	Lendl	Deuce	AIC	1.3049	1.3114	4
				SC	1.3325	1.3669	4
17	89 U.S. OPEN	Becker	Ad	AIC	0.9539	0.6960	1
				SC	0.9894	0.7671	1
18	89 U.S. OPEN	Becker	Deuce	AIC	1.3566	1.3481	4
				SC	1.3884	1.4123	4
19	89 U.S. OPEN	Lendl	Ad	AIC	1.3757	1.3980	1
				SC	1.4119	1.4703	1
20	89 U.S. OPEN	Lendl	Deuce	AIC	1.4061	1.3725	1
				SC	1.4413	1.4429	1
21	92 AUSTRALIAN OPEN	Courier	Ad	AIC	1.3285	1.1431	4
				SC	1.3661	1.2336	4
22	92 AUSTRALIAN OPEN	Courier	Deuce	AIC	1.3860	1.3115	2
				SC	1.4243	1.4284	2
23	92 AUSTRALIAN OPEN	Edberg	Ad	AIC	0.8179	0.8008	3
				SC	0.8577	0.8900	1
24	92 AUSTRALIAN OPEN	Edberg	Deuce	AIC	1.3767	1.4094	1
				SC	1.4180	1.4922	1

25	95 ROLAND GARROS	Muster	Ad	AIC	1.1322	1.1102	4
				SC	1.1767	1.2095	1
26	95 ROLAND GARROS	Muster	Deuce	AIC	1.0819	1.0237	2
				SC	1.1250	1.1953	1
27	95 ROLAND GARROS	Chang	Ad	AIC	0.4470	0.4128	1
				SC	0.4893	0.4973	1
28	95 ROLAND GARROS	Chang	Deuce	AIC	1.4263	1.3653	2
				SC	1.4664	1.4793	2
29	95 U.S. OPEN	Sampras	Ad	AIC	1.3081	1.2941	4
				SC	1.3440	1.3664	4
30	95 U.S. OPEN	Sampras	Deuce	AIC	1.4196	1.3282	3
				SC	1.4551	1.4367	3
31	95 U.S. OPEN	Agassi	Ad	AIC	1.1409	0.9551	3
				SC	1.1777	1.1016	2
32	95 U.S. OPEN	Agassi	Deuce	AIC	1.3917	1.3711	1
				SC	1.4266	1.4409	1
33	00 AUSTRALIAN OPEN	Agassi	Ad	AIC	1.4144	1.3273	2
				SC	1.4509	1.4314	4
34	00 AUSTRALIAN OPEN	Agassi	Deuce	AIC	1.4152	1.3442	4
				SC	1.4501	1.4535	2
35	00 AUSTRALIAN OPEN	Kafelnikov	Ad	AIC	1.4188	1.4292	3
				SC	1.4564	1.5050	3
36	00 AUSTRALIAN OPEN	Kafelnikov	Deuce	AIC	1.4160	1.4357	4
				SC	1.4515	1.5074	4
37	01 WIMBLEDON	Ivanisevic	Ad	AIC	1.3236	1.3504	1
				SC	1.3547	1.4127	1
38	01 WIMBLEDON	Ivanisevic	Deuce	AIC	1.2045	1.1831	2
				SC	1.2336	1.2418	2
39	01 WIMBLEDON	Rafter	Ad	AIC	1.4130	1.4312	2
				SC	1.4482	1.5022	2
40	01 WIMBLEDON	Rafter	Deuce	AIC	1.4166	1.3139	4
				SC	1.4503	1.4884	4

Notes: In each point game, the model (equilibrium or the best rule) that fits better is underlined for each criterion. The last column indicates the best rule. Recall that rules 1-5 correspond to T, D, RW, LW, and WD, respectively.

Table 9
Model Selection under AIC, SC Criterion in Female Tennis

Index	Female Play Match	Server	Court	Criterion	Equilibrium Value	Best Rule Value	Rule
1	85 AUSTRALIAN OPEN	Navratilova	Ad	AIC	1.4345	1.4460	2
				SC	1.4763	1.5305	2
2	85 AUSTRALIAN OPEN	Navratilova	Deuce	AIC	1.3921	1.3957	2
				SC	1.4360	1.5215	5
3	85 AUSTRALIAN OPEN	Evert	Ad	AIC	1.4417	1.3813	2
				SC	1.4865	1.4831	2
4	85 AUSTRALIAN OPEN	Evert	Deuce	AIC	0.8461	0.8604	3
				SC	0.8897	0.9790	1
5	87 WIMBLEDON	Navratilova	Ad	AIC	1.1961	0.9952	2
				SC	1.2437	1.1403	2
6	87 WIMBLEDON	Navratilova	Deuce	AIC	0.9734	1.0136	4
				SC	1.0179	1.1034	4
7	87 WIMBLEDON	Graf	Ad	AIC	1.4445	1.4699	1
				SC	1.4917	1.5642	1
8	87 WIMBLEDON	Graf	Deuce	AIC	1.3653	1.3123	4
				SC	1.4116	1.4057	4
9	87 U.S. OPEN	Navratilova	Ad	AIC	1.3142	1.3421	3
				SC	1.3578	1.4301	3
10	87 U.S. OPEN	Navratilova	Deuce	AIC	1.2704	1.1471	4
				SC	1.3153	1.2988	4
11	87 U.S. OPEN	Graf	Ad	AIC	1.4627	1.4893	1
				SC	1.5118	1.5874	1
12	87 U.S. OPEN	Graf	Deuce	AIC	1.4573	1.3128	2
				SC	1.5057	1.4601	2
13	92 ROLAND GARROS	Seles	Ad	AIC	1.2727	1.1800	1
				SC	1.3114	1.2572	1
14	92 ROLAND GARROS	Seles	Deuce	AIC	1.4066	1.3928	1
				SC	1.4445	1.4686	1
15	92 ROLAND GARROS	Graf	Ad	AIC	1.4096	1.4341	3
				SC	1.4445	1.5045	3
16	92 ROLAND GARROS	Graf	Deuce	AIC	1.3976	1.3680	1
				SC	1.4316	1.4361	1
17	92 U.S. OPEN	Seles	Ad	AIC	1.4632	1.4130	2
				SC	1.5116	1.5105	2
18	92 U.S. OPEN	Seles	Deuce	AIC	1.3471	1.1083	4
				SC	1.3951	1.2546	4
19	92 U.S. OPEN	Sanchez	Ad	AIC	1.2147	1.0239	5
				SC	1.2596	1.1146	5
20	92 U.S. OPEN	Sanchez	Deuce	AIC	1.3716	0.9599	3
				SC	1.4169	1.1956	3
21	97 WIMBLEDON	Hingis	Ad	AIC	1.3449	1.3425	5
				SC	1.3871	1.4278	5
22	97 WIMBLEDON	Hingis	Deuce	AIC	1.3287	1.3288	3
				SC	1.3693	1.4107	3
23	97 WIMBLEDON	Novotna	Ad	AIC	1.4138	1.3458	2
				SC	1.4587	1.4365	2
24	97 WIMBLEDON	Novotna	Deuce	AIC	1.3085	1.3020	5
				SC	1.3495	1.4177	2

25	99 ROLAND GARROS	Graf	Ad	AIC	1.4323	1.3300	2
				SC	1.4732	1.4128	2
26	99 ROLAND GARROS	Graf	Deuce	AIC	1.4279	1.3616	1
				SC	1.4689	1.4435	1
27	99 ROLAND GARROS	Hingis	Ad	AIC	1.0452	1.0897	1
				SC	1.0854	1.1700	1
28	99 ROLAND GARROS	Hingis	Deuce	AIC	1.3468	1.3402	3
				SC	1.3851	1.4402	3
29	00 U.S. OPEN	V. Williams	Ad	AIC	1.3495	1.3232	3
				SC	1.3953	1.4157	3
30	00 U.S. OPEN	V. Williams	Deuce	AIC	1.4338	1.4042	1
				SC	1.4773	1.4913	1
31	00 U.S. OPEN	Davenport	Ad	AIC	1.4577	1.4945	4
				SC	1.5053	1.5905	4
32	00 U.S. OPEN	Davenport	Deuce	AIC	1.3221	1.0400	4
				SC	1.3684	1.2022	4
33	02 AUSTRALIAN OPEN	Capriati	Ad	AIC	1.2851	1.2016	3
				SC	1.3264	1.2852	3
34	02 AUSTRALIAN OPEN	Capriati	Deuce	AIC	1.4316	1.4522	5
				SC	1.4730	1.5470	1
35	02 AUSTRALIAN OPEN	Hingis	Ad	AIC	1.3042	1.3072	3
				SC	1.3428	1.3879	3
36	02 AUSTRALIAN OPEN	Hingis	Deuce	AIC	1.4234	1.4281	3
				SC	1.4620	1.5059	1

Notes: In each point game, the model (equilibrium or the best rule) that fits better is underlined for each criterion. The last column indicates the best rule. Recall that rules 1-5 correspond to T, D, RW, LW, and WD, respectively.

Table 10
Model Selection under AIC, SC Criterion in Junior Tennis

Index	Junior Play Match	Server	Court	Criterion	Equilibrium Value	Best Rule Value	Rule
1	96 AVVENIRE TOURNAMENT	Middleton	Ad	AIC	1.3924	1.2036	4
				SC	1.4404	1.4110	4
2	96 AVVENIRE TOURNAMENT	Middleton	Deuce	AIC	1.3336	1.3232	2
				SC	1.3790	1.4148	2
3	96 AVVENIRE TOURNAMENT	Kalvaria	Ad	AIC	1.2470	1.1356	1
				SC	1.2941	1.2299	1
4	96 AVVENIRE TOURNAMENT	Kalvaria	Deuce	AIC	1.4395	1.2626	4
				SC	1.4878	1.4601	4
5	00 WIMBLEDON	Salerni	Ad	AIC	1.5113	1.4730	4
				SC	1.5596	1.6468	4
6	00 WIMBLEDON	Salerni	Deuce	AIC	1.5005	1.4813	1
				SC	1.5495	1.5793	1
7	00 WIMBLEDON	Perediynis	Ad	AIC	1.5087	1.4342	3
				SC	1.5543	1.5789	3
8	00 WIMBLEDON	Perediynis	Deuce	AIC	1.4481	1.4269	3
				SC	1.4964	1.5855	1
9	02 AVVENIRE TOURNAMENT	Gonzalez	Ad	AIC	1.4627	1.3416	5
				SC	1.5118	1.5020	5
10	02 AVVENIRE TOURNAMENT	Gonzalez	Deuce	AIC	1.4440	1.5057	4
				SC	1.4936	1.6052	4
11	02 AVVENIRE TOURNAMENT	Sanchez	Ad	AIC	1.2918	1.2642	1
				SC	1.3415	1.3637	1
12	02 AVVENIRE TOURNAMENT	Sanchez	Deuce	AIC	1.4440	1.4116	4
				SC	1.4936	1.5216	4
13	03 AUSTRALIAN OPEN (Qrt)	Baqhdatis	Ad	AIC	1.4956	1.2469	4
				SC	1.5439	1.3839	4
14	03 AUSTRALIAN OPEN (Qrt)	Baqhdatis	Deuce	AIC	1.4689	1.2209	2
				SC	1.5185	1.3703	2
15	03 AUSTRALIAN OPEN (Qrt)	Evans	Ad	AIC	1.3949	1.0581	2
				SC	1.4447	1.2064	2
16	03 AUSTRALIAN OPEN (Qrt)	Evans	Deuce	AIC	1.2080	1.0518	5
				SC	1.2571	1.1505	5
17	03 AUSTRALIAN OPEN (2nd)	Bauer	Ad	AIC	1.4247	1.2691	4
				SC	1.4709	1.4594	4
18	03 AUSTRALIAN OPEN (2nd)	Bauer	Deuce	AIC	1.0089	0.9336	3
				SC	1.0542	1.0252	3
19	03 AUSTRALIAN OPEN (2nd)	Kerber	Ad	AIC	1.2717	1.2389	4
				SC	1.3140	1.3242	4
20	03 AUSTRALIAN OPEN (2nd)	Kerber	Deuce	AIC	1.4297	1.4351	1
				SC	1.4715	1.5187	1
21	03 AUSTRALIAN OPEN (2nd)	Dellacqua	Ad	AIC	1.2659	1.0476	2
				SC	1.3147	1.2403	4
22	03 AUSTRALIAN OPEN (2nd)	Dellacqua	Deuce	AIC	1.2186	0.9932	2
				SC	1.2666	1.1395	2
23	03 AUSTRALIAN OPEN (2nd)	Kim	Ad	AIC	0.9908	0.7874	4
				SC	1.0357	1.0209	4
24	03 AUSTRALIAN OPEN (2nd)	Kim	Deuce	AIC	1.4138	1.3948	1
				SC	1.4587	1.4846	1

25	03 AUSTRALIAN OPEN (2nd)	Scherer	Ad	AIC	1.4476	<u>1.2381</u>	5
				SC	1.4971	<u>1.3851</u>	5
26	03 AUSTRALIAN OPEN (2nd)	Scherer	Deuce	AIC	1.4763	<u>1.2868</u>	2
				SC	1.5261	<u>1.3862</u>	2
27	03 AUSTRALIAN OPEN (2nd)	Cvetkovic	Ad	AIC	1.5530	<u>1.5340</u>	5
				SC	<u>1.5934</u>	1.6064	5
28	03 AUSTRALIAN OPEN (2nd)	Cvetkovic	Deuce	AIC	1.5342	<u>1.4580</u>	4
				SC	1.5777	<u>1.5388</u>	4
29	03 AUSTRALIAN OPEN (2nd)	Tsonga	Ad	AIC	<u>1.3672</u>	1.4515	3
				SC	<u>1.4155</u>	1.5459	3
30	03 AUSTRALIAN OPEN (2nd)	Tsonga	Deuce	AIC	1.4850	<u>1.0917</u>	3
				SC	1.5345	<u>1.2598</u>	2
31	03 AUSTRALIAN OPEN (2nd)	Feeney	Ad	AIC	1.3217	<u>1.1311</u>	2
				SC	1.3715	<u>1.2544</u>	2
32	03 AUSTRALIAN OPEN (2nd)	Feeney	Deuce	AIC	<u>1.4460</u>	1.4512	1
				SC	<u>1.4958</u>	1.5507	1

Notes: In each point game, the model (equilibrium or the best rule) that fits better is underlined for each criterion. The last column indicates the best rule. Recall that rules 1-5 correspond to T, D, RW, LW, and WD, respectively.