Week 14 - Social Network Graphs 2

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Exercise 10.5.1 (section 10.5.5)

If, in Fig. 10.20 you start the walk from Picture 2, what will be the similarity to Picture 2 of the other two pictures? Which do you expect to be more similar to Picture 2?

Answer

Confirm code works with getting same answer from book's example:

```
n_iters <- 100;
M <- matrix(c(0, 0, 0, 4/15, 2/5,
              0, 0, 0, 4/15, 0,
              0, 0, 0, 4/15, 2/5,
              2/5, 4/5, 2/5, 0, 0,
              2/5, 0, 2/5, 0, 0), ncol=5, byrow = TRUE);
beta <- 0.8;
en <- c(1,0,0,0,0);

Mnew <- M + matrix(rep((1-beta)*en, 5), ncol=5);

# Iterate 100 times
vn <- en;
for (i in 1:n_iters) {
  vn <- Mnew %*% vn;
}

# Print final results, which matcth the book
vn
```

```
##            [,1]
## [1,] 0.34461028
## [2,] 0.06633499
## [3,] 0.14461028
## [4,] 0.24875622
## [5,] 0.19568823
```

Lets do the same for N = 1, which is picture 2:

```
n_iters <- 100;
M <- matrix(c(0, 0, 0, 4/15, 2/5,
              0, 0, 0, 4/15, 0,
              0, 0, 0, 4/15, 2/5,
              2/5, 4/5, 2/5, 0, 0,
              2/5, 0, 2/5, 0, 0), ncol=5, byrow = TRUE);
beta <- 0.8;
```

```r
en <- c(0,1,0,0,0);

Mnew <- M + matrix(rep((1-beta)*en, 5), ncol=5);

# Iterate 100 times
vn <- en;
for (i in 1:n_iters) {
  vn <- Mnew %*% vn;
}

# Print final results, which matcth the book
vn
```

```
##           [,1]
## [1,] 0.1326700
## [2,] 0.2902156
## [3,] 0.1326700
## [4,] 0.3383085
## [5,] 0.1061360
```

Notice, that the similarity to Picture 1 and Picture 3 is the same; since its symmetrical we should not expect anything different.

Use R for 10.7.1 (section 10.7.8)

Exercise 10.7.1 : For the graph of Fig. 10.9, which we repeat here as Fig. 10.22:

(a) If the graph is represented as a directed graph, how many arcs are there?
(b) What are the neighborhood profiles for nodes A and B?
(c) What is the diameter of the graph?
(d) How many pairs are in the transitive closure? Hint : Do not forget that there are paths of length greater than zero from a node to itself in this graph.
(e) If we compute the transitive closure by recursive doubling, how many rounds are needed?

Answer

(a) There are 12 edges in the graph. To convert it from undirected to directed, we would double that amount (for each edge x-y, create two edges x -> y and y -> x). Therefore, **24 edges in the directed graph**.

(b)

$$N(A,1) = \{A, B, C\}$$
$$N(A,2) = \{A, B, C, H, D\}$$
$$N(A,3) = \{A, B, C, H, D, I, G, E, F\}$$

So the neighborhood profile for A is: **3, 5, 9**

$$N(B,1) = \{A, B, C, H\}$$
$$N(B,2) = \{A, B, C, H, D, I, G\}$$
$$N(B,3) = \{A, B, C, H, D, I, G, E, F\}$$

So the neighborhood profile for A is: **4, 7, 9**

(c) The diameter of a directed graph is the smallest integer d such that for every two nodes u and v there is a path of length d or less from u to v. In this case, the **diameter is 3**, since every node is at most 3 steps from another. Can be derived visually or by noticing that 3 is the largest path in part (b) and we know this graph is symetrical with respect to these nodes to every other possible node, so there can be no longer paths.

(d)

(e) From the textbook: “By a recursive-doubling technique, we can double the length of paths we know about in a single round. Thus, on a graph of diameter d, we need only $log_2(d)$ rounds, rather than d rounds”. Since the diameter is 3, we need only $log2(3)$ rounds, which is **2 rounds** when the deciam is rounded up. This saves us only one round, which is not spectacular.