# CS440 (Intro to AI) - HW 3

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Note: For any Boolean variable $X$, the lowercase is used such that $P(x) = P(X = true)$ and $P(\neg x) = P(X = false)$.
If used in a sum, the lowercase is used such that $\sum_x P(x) = P(X = true) + P(X = false)$, following the textbook's example.

## 1 Problem 1

a) $P(a, b, c, d, e)$
$= P(a) * P(b) * P(c) * P(d|a, b) * P(e|b, c)$
$= 0.2 * 0.5 * 0.8 * 0.1 * 0.3$
$= 0.0024$

b) $P(\neg a, \neg b, \neg c, \neg d, \neg e)$
$= P(\neg a) * P(\neg b) * P(\neg c) * P(\neg d|\neg a, \neg b) * P(\neg e|\neg b, \neg c)$
$= 0.8 * 0.5 * 0.2 * 0.1 * 0.8$
$= 0.0064$

c) $P(\neg a|b, c, d, e) = P(\neg a, b, c, d, e)/P(b, c, d, e)$
$P(\neg a, b, c, d, e) = 0.8 * 0.5 * 0.8 * 0.6 * 0.3 = 0.0576$
$P(b, c, d, e) = P(a, b, c, d, e) + P(\neg a, b, c, d, e)$
$= 0.0576 + 0.0024 = 0.06$
$P(\neg a|b, c, d, e) = 0.0576/0.06$
$= 0.96$

## 2 Problem 2

Note: Each variable is referred to by their initial letter.

a) $P(B|j, m) = \alpha P(B) \sum_e P(e) \sum_a P(a|b, e)P(j|a)P(m|a)$

$$= \alpha P(B) \sum_e P(e) \left[ 0.9(0.7) \begin{pmatrix} 0.95 & 0.29 \\ 0.94 & 0.001 \end{pmatrix} + 0.05(0.01) \begin{pmatrix} 0.05 & 0.71 \\ 0.06 & 0.999 \end{pmatrix} \right]$$

$$= \alpha P(B) \sum_e P(e) \begin{pmatrix} 0.598525 & 0.183055 \\ 0.59223 & 0.0011295 \end{pmatrix}$$

$$= \alpha P(B) \left[ 0.002 \begin{pmatrix} 0.598525 \\ 0.183055 \end{pmatrix} + 0.998 \begin{pmatrix} 0.59223 \\ 0.0011295 \end{pmatrix} \right]$$

$$= \alpha \begin{pmatrix} 0.001 \\ 0.999 \end{pmatrix} \begin{pmatrix} 0.59224259 \\ 0.001493351 \end{pmatrix} = \alpha \begin{pmatrix} 0.00059224259 \\ 0.00149185765 \end{pmatrix}$$

$$= \langle \frac{0.00059224259}{0.00059224259 + 0.00149185765}, \frac{0.00149185765}{0.00059224259 + 0.00149185765} \rangle$$

$$P(Burglary|JohnCalls = true, MaryCalls = true) \approx \langle 0.284, 0.716 \rangle$$

b) Using enumeration, we would have to evaluate two binary trees (one for $x_1$ and one for $\neg x_i$), each with a depth of $n - 2$. As a result, the total work would be $O(2^n)$.

Using variable elimination, the factors never grow beyond two variables, as seen in the computation's first step:

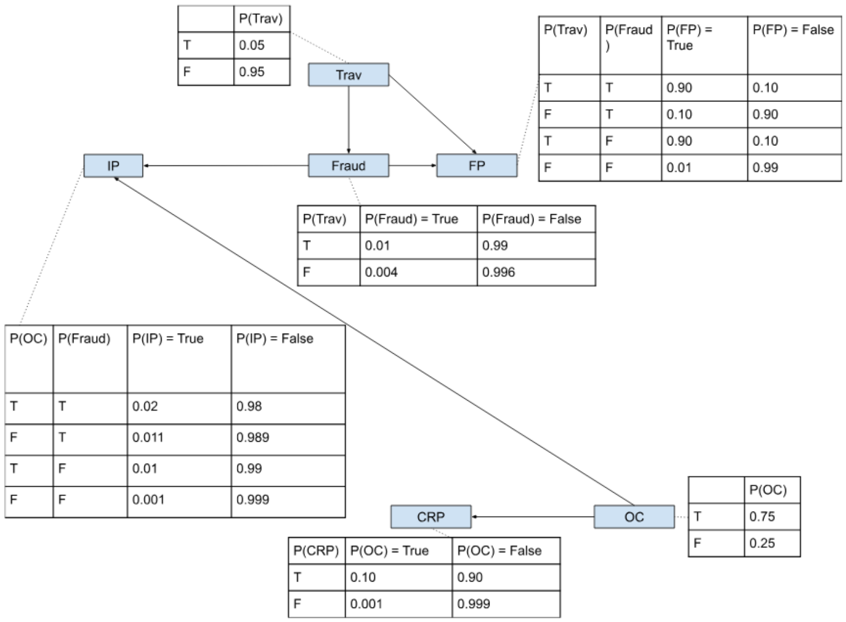
$$P(X_1|x_n) = \alpha P(X_1) \cdots \sum_{x_{n-2}} P(x_{n-2}|x_{n-3}) \sum_{x_{n-1}} P(x_{n-1}|x_{n-2}) P(x_n|x_{n-1})$$

$$= \alpha P(X_1) \cdots \sum_{x_{n-2}} P(x_{n-2}|x_{n-3}) \sum_{x_{n-1}} f_{X_{n-1}}(x_{n-1}, x_{n-2}) f_{X_n}(x_{n-1})$$

$$= \alpha P(X_1) \cdots \sum_{x_{n-2}} P(x_{n-2}|x_{n-3}) f_{\overline{X_{n-1}} X_n}(x_{n-2})$$

Since the work done in the first step is a constant independent of $n$, the total work using variable elimination would be $O(n)$.

# 3 Problem 3




| | P(Trav) |
|---|---|
| T | 0.05 |
| F | 0.95 |

| P(Trav) | P(Fraud) | P(FP) = True | P(FP) = False |
|---|---|---|---|
| T | T | 0.90 | 0.10 |
| F | T | 0.10 | 0.90 |
| T | F | 0.90 | 0.10 |
| F | F | 0.01 | 0.99 |

| P(Trav) | P(Fraud) = True | P(Fraud) = False |
|---|---|---|
| T | 0.01 | 0.99 |
| F | 0.004 | 0.996 |

| P(OC) | P(Fraud) | P(IP) = True | P(IP) = False |
|---|---|---|---|
| T | T | 0.02 | 0.98 |
| F | T | 0.011 | 0.989 |
| T | F | 0.01 | 0.99 |
| F | F | 0.001 | 0.999 |

| | P(OC) |
|---|---|
| T | 0.75 |
| F | 0.25 |

| P(CRP) | P(OC) = True | P(OC) = False |
|---|---|---|
| T | 0.10 | 0.90 |
| F | 0.001 | 0.999 |

a)

b) $P(fraud) = P(fraud|trav)P(trav) + P(fraud|\neg trav)P(\neg trav)$
$= (0.01 * 0.05) + (0.004 * 0.95) = 0.0043$

$P(fraud|fp, \neg ip, crp) = P(fp, \neg ip, crp|fraud)P(fraud)/P(fp, \neg ip, crp)$

- $P(fp, \neg ip, crp|fraud)$
  $= \sum_{trav} \sum_{oc} P(fp|trav, fraud)P(\neg ip|oc, fraud)P(crp|oc)P(trav)P(oc)$
  - (Trav = True) and (OC = True):
    $P(fp|trav, fraud) = 0.9$
    $P(\neg ip|oc, fraud) = 0.98$
    $P(crp|oc) = 0.1$
    $P(trav) = 0.05$
    $P(oc) = 0.75$
    $0.9 * 0.98 * 0.1 * 0.05 * 0.75 = 0.0033075$
  - (Trav = True) and (OC = False):
    $P(fp|trav, fraud) = 0.9$
    $P(\neg ip|\neg oc, fraud) = 0.989$
    $P(crp|\neg oc) = 0.001$
    $P(trav) = 0.05$
    $P(\neg oc) = 0.25$
    $0.9 * 0.989 * 0.001 * 0.05 * 0.25 = 0.00001112625$
  - (Trav = False) and (OC = True):
    $P(fp|\neg trav, fraud) = 0.1$

$P(\neg ip|oc, fraud) = 0.98$
$P(crp|oc) = 0.1$
$P(\neg trav) = 0.95$
$P(oc) = 0.75$
$0.1 * 0.98 * 0.1 * 0.95 * 0.75 = 0.0069825$

- (Trav = False) and (OC = False):
  $P(fp|\neg trav, fraud) = 0.1$
  $P(\neg ip|\neg oc, fraud) = 0.989$
  $P(crp|\neg oc) = 0.001$
  $P(\neg trav) = 0.95$
  $P(\neg oc) = 0.25$
  $0.1 * 0.989 * 0.001 * 0.95 * 0.25 = 0.00002348875$

$P(fp, \neg ip, crp|fraud) = 0.0033075 + 0.00001112625 + 0.0069825 + 0.00002348875 = 0.010324615$

- $P(fp, \neg ip, crp|\neg fraud)$
  $= \sum_{trav} \sum_{oc} P(fp|trav, \neg fraud)P(\neg ip|OC, \neg fraud)P(crp|oc)P(trav)P(oc)$
  - (Trav = True) and (OC = True):
    $P(fp|trav, \neg fraud) = 0.9$
    $P(\neg ip|oc, \neg fraud) = 0.99$
    $P(crp|oc) = 0.1$
    $P(trav) = 0.05$
    $P(oc) = 0.75$
    $0.9 * 0.99 * 0.1 * 0.05 * 0.75 = 0.00334125$
  - (Trav = True) and (OC = False):
    $P(fp|trav, \neg fraud) = 0.9$
    $P(\neg ip|\neg oc, \neg fraud) = 0.999$
    $P(crp|\neg oc) = 0.001$
    $P(trav) = 0.05$
    $P(\neg oc) = 0.25$
    $0.9 * 0.999 * 0.001 * 0.05 * 0.25 = 0.00001123875$
  - (Trav = False) and (OC = True):
    $P(fp|\neg trav, \neg fraud) = 0.01$
    $P(\neg ip|oc, \neg fraud) = 0.99$
    $P(crp|oc) = 0.1$
    $P(\neg trav) = 0.95$
    $P(oc) = 0.75$
    $0.01 * 0.99 * 0.1 * 0.95 * 0.75 = 0.000705375$
  - (Trav = False) and (OC = False):
    $P(fp|\neg trav, \neg fraud) = 0.01$
    $P(\neg ip|\neg oc, \neg fraud) = 0.999$
    $P(crp|\neg oc) = 0.001$
    $P(\neg trav) = 0.95$
    $P(\neg oc) = 0.25$
    $0.01 * 0.999 * 0.001 * 0.95 * 0.25 = 0.000002372625$

$P(fp, \neg ip, crp|\neg fraud) = 0.00334125 + 0.00001123875 + 0.000705375 + 0.000002372625 = 0.004060236375$

$P(fp, \neg ip, crp) = (0.010324615 * 0.0043) + (0.004060236375 * 0.9957) = 0.004087173203$

$P(fraud|fp, \neg ip, crp) = P(fp, \neg ip, crp|fraud)P(fraud)/P(fp, \neg ip, crp) = (0.010324615 * 0.0043)/0.004087173203 = 0.01086223712$

Prior probability of fraud: $P(fraud) = 0.0043$
Probability of fraud with evidence: $P(fraud|fp, \neg ip, crp) \approx 0.0109$

# 4 Problem 4

1. $P(X_3|hot_1, cold_2, cold_3) = \alpha \sum_{x_1} \sum_{x_2} P(X_3, hot_1, cold_2, cold_3, x_1, x_2)$
$= \alpha \sum_{x_1} \sum_{x_2} P(X_3|x_2)P(hot_1|x_1)P(cold_2|x_2)P(cold_3|X_3)P(x_1)P(x_2|x_1)$
$= \alpha \sum_{x_1} P(x_1) \sum_{x_2} P(X_3|x_2)P(hot_1|x_1)P(cold_2|x_2)P(cold_3|X_3)P(x_2|x_1)$
$= \alpha \sum_{x_2} P(X_3|x_2)P(hot_1|X_1 = A)P(cold_2|x_2)P(cold_3|X_3)P(x_2|x_1 = A) + 0$
$= \alpha(0.8[P(X_3|X_2 = B)P(hot_1|X_1 = A)P(cold_2|X_2 = B)P(cold_3|X_3)] + 0.2[P(X_3|X_2 = A)P(hot_1|X_1 = A)P(cold_2|X_2 = A)P(cold_3|X_3)]$
$= \alpha(0.8 * P(X_3|X_2 = B) * 1 * P(cold|X_3)) + 0$
$= \alpha(0.8 \begin{pmatrix} 0 \\ 0.2 \\ 0.8 \\ 0 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{pmatrix})$
$= \alpha\langle 0, 0.16, 0.64, 0, 0, 0\rangle = \langle 0, 0.2, 0.8, 0, 0, 0\rangle$

$$P(X_3 = x_3|hot_1, cold_2, cold_3) = \begin{cases} 0.2 & x_3 = B \\ 0.8 & x_3 = C \\ 0 & \text{otherwise} \end{cases}$$

2. $P(X_2|hot_1, cold_2, cold_3) = \alpha \sum_{x_1} \sum_{x_3} P(X_2, hot_1, cold_2, cold_3, x_1, x_3)$
$= \alpha \sum_{x_1} P(x_1) \sum_{x_3} P(X_2|x_1)P(hot_1|x_1)P(cold_2|X_2)P(cold_3|x_3)P(x_3|X_2)$
$= \alpha \sum_{x_3} P(X_2|X_1 = A)P(hot_1|X_1 = A)P(cold_2|X_2)P(cold_3|x_3)P(x_3|X_2) + 0$
$= \alpha \begin{pmatrix} 0.2 \\ 0.8 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \times 1 \times \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} \sum_{x_3} P(cold_3|x_3)P(x_3|X_2)$

$$= \alpha \begin{pmatrix} 0 \\ 0.8 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \times (0 + \begin{pmatrix} 0 \\ 0.2 \\ 0.8 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0.2 \\ 0.8 \\ 0 \\ 0 \end{pmatrix} + 0 + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.2 \\ 0.8 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix})$$

$= \alpha\langle 0, 0.16, 0, 0, 0, 0\rangle = \langle 0, 1, 0, 0, 0, 0\rangle$

$$P(X_2 = x_2|hot_1, cold_2, cold_3) = \begin{cases} 1 & x_2 = B \\ 0 & \text{otherwise} \end{cases}$$

3. $P(E_4|hot_1, cold_2, cold_3) = \alpha \sum_{x_1} \sum_{x_2} \sum_{x_3} \sum_{x_4} P(E_4, hot_1, cold_2, cold_3, x_1, x_2, x_3, x_4)$
   $= \alpha \sum_{x_1} P(x_1) \sum_{x_2} \sum_{x_3} \sum_{x_4} [P(E_4|x_4)P(hot_1|x_1)P(cold_2|x_2)$
   $P(cold_3|x_3)P(x_2|x_1)P(x_3|x_2)P(x_4|x_3)]$
   $= \alpha \sum_{x_2} \sum_{x_3} \sum_{x_4} [P(E_4|x_4) * 1 * P(cold_2|x_2)P(cold_3|x_3)P(x_2|X_1 = A)P(x_3|x_2)P(x_4|x_3)]$
   $+0$
   $= \alpha(0.8[\sum_{x_3} \sum_{x_4} P(E_4|x_4)P(cold_2|X_2 = B)P(cold_3|x_3)P(x_3|X_2 = B)P(x_4|x_3)]$
   $+0.2[\sum_{x_3} \sum_{x_4} P(E_4|x_4)P(cold_2|X_2 = A)P(cold_3|x_3)P(x_3|X_2 = A)P(x_4|x_3)])$
   $= 0.8\alpha \sum_{x_3} \sum_{x_4} P(E_4|x_4) * 1 * P(cold_3|x_3)P(x_3|X_2 = B)P(x_4|x_3) + 0$
   $= \alpha \sum_{x_3} P(cold_3|x_3)P(x_3|X_2 = B) * ([P(E_4|x_4 = A)P(X_4 = A|x_3)]$
   $+[P(E_4|x_4 = B)P(X_4 = B|x_3)] + [P(E_4|x_4 = C)P(X_4 = C|x_3)]$
   $+[P(E_4|x_4 = D)P(X_4 = D|x_3)] + [P(E_4|x_4 = E)P(X_4 = E|x_3)]$
   $+[P(E_4|x_4 = F)P(X_4 = F|x_3)])$
   $= \alpha(0 + 0.2 \left[0 + \begin{pmatrix} 0 \\ 1 \end{pmatrix} 0.2 + \begin{pmatrix} 0 \\ 1 \end{pmatrix} 0.8 + 0 + 0 + 0\right] + 0.8 \left[0 + 0 + \begin{pmatrix} 0 \\ 1 \end{pmatrix} 0.2 + \begin{pmatrix} 1 \\ 0 \end{pmatrix} 0.8\right]$
   $+0 + 0 + 0)$
   $= \alpha\langle 0.64, 0.36\rangle = \langle 0.64, 0.36\rangle$

   $P(hot_4|hot_1, cold_2, cold_3) = 0.64$

4. $P(X_4|hot_1, cold_2, cold_3) = \sum_{x_1} \sum_{x_2} \sum_{x_3} P(X_4, hot_1, cold_2, cold_3, x_1, x_2, x_3)$
   $= \alpha \sum_{x_1} P(x_1)P(hot_1|x_1) \sum_{x_2} P(cold_2|x_2)P(x_2|x_1) \sum_{x_3} P(X_4|x_3)P(cold_3|x_3)P(x_3|x_2)$
   $= \alpha \sum_{x_2} P(cold_2|x_2)P(x_2|X_1 = A) \sum_{x_3} P(X_4|x_3)P(cold_3|x_3)P(x_3|x_2) + 0$
   $= 0.8\alpha \sum_{x_3} P(X_4|x_3)P(cold_3|x_3)P(x_3|X_2 = B) + 0$
   $$= \alpha(0 + \begin{pmatrix} 0 \\ 0.2 \\ 0.8 \\ 0 \\ 0 \\ 0 \end{pmatrix} (1)(0.2) + \begin{pmatrix} 0 \\ 0 \\ 0.2 \\ 0.8 \\ 0 \\ 0 \end{pmatrix} (1)(0.8) + 0 + 0 + 0)$$

$= \alpha\langle 0, 0.04, 0.32, 0.64, 0, 0\rangle = \langle 0, 0.04, 0.32, 0.64, 0, 0\rangle$

$$P(X_4 = x_4|hot_1, cold_2, cold_3) = \begin{cases} 0.04 & x_4 = B \\ 0.32 & x_4 = C \\ 0.64 & x_4 = D \\ 0 & \text{otherwise} \end{cases}$$

5. $P(hot_4, hot_5, cold_6|hot_1, cold_2, cold_3)$
$= P(hot_4|hot_1, cold_2, cold_3) * P(hot_5|hot_1, cold_2, cold_3) * P(cold_6|hot_1, cold_2, cold_3)$

   $P(hot_4|hot_1, cold_2, cold_3) = 0.64$ (as found in part 3)

   $P(E_5|hot_1, cold_2, cold_3)$
$= \alpha \sum_{x_1} P(hot_1|x_1)P(x_1) \sum_{x_2} P(cold_2|x_2)P(x_2|x_1) \sum_{x_3} P(cold_3|x_3)P(x_3|x_2)$
$\sum_{x_4} P(E_4|x_4)P(x_4|x_3) \sum_{x_5} P(E_5|x_5)P(x_5|x_4)$
$= \alpha \sum_{x_2} P(cold_2|x_2)P(x_2|X_1 = A) \sum_{x_3} P(cold_3|x_3)P(x_3|x_2)$
$\sum_{x_4} P(E_4|x_4)P(x_4|x_3) \sum_{x_5} P(E_5|x_5)P(x_5|x_4) + 0$
$= 0.8\alpha \sum_{x_3} P(cold_3|x_3)P(x_3|X_2 = B) \sum_{x_4} P(E_4|x_4)P(x_4|x_3)$
$\sum_{x_5} P(E_5|x_5)P(x_5|x_4) + 0$
$= \alpha \sum_{x_3} P(cold_3|x_3)P(x_3|X_2 = B) \sum_{x_4} P(E_4|x_4)P(x_4|x_3)$
$*([P(E_5|x_5 = A)P(X_5 = A|x_4)] + [P(E_5|x_5 = B)P(X_5 = B|x_4)]$
$+[P(E_5|x_5 = C)P(X_5 = C|x_4)] + [P(E_5|x_5 = D)P(X_5 = D|x_4)]$
$+[P(E_5|x_5 = E)P(X_5 = E|x_4)] + [P(E_5|x_5 = F)P(X_5 = F|x_4)])$
$= \alpha \sum_{x_3} P(cold_3|x_3)P(x_3|X_2 = B) * (P(E_4|X_4 = A)P(X_4 = A|x_3) *$
$[\begin{pmatrix}1\\0\end{pmatrix}(0.2) + \begin{pmatrix}0\\1\end{pmatrix}(0.8) + 0 + 0 + 0 + 0] + P(E_4|X_4 = B)P(X_4 = B|x_3) *$
$[0 + \begin{pmatrix}0\\1\end{pmatrix}(0.2) + \begin{pmatrix}0\\1\end{pmatrix}(0.8) + 0 + 0 + 0] + P(E_4|X_4 = C)P(X_4 = C|x_3) *$
$[0 + 0 + \begin{pmatrix}0\\1\end{pmatrix}(0.2) + \begin{pmatrix}1\\0\end{pmatrix}(0.8) + 0 + 0] + P(E_4|X_4 = D)P(X_4 = D|x_3) *$
$[0 + 0 + 0 + \begin{pmatrix}1\\0\end{pmatrix}(0.2) + \begin{pmatrix}0\\1\end{pmatrix}(0.8) + 0] + P(E_4|X_4 = E)P(X_4 = E|x_3) *$
$[0 + 0 + 0 + 0 + \begin{pmatrix}0\\1\end{pmatrix}(0.2) + \begin{pmatrix}0\\1\end{pmatrix}(0.8)] + P(E_4|X_4 = F)P(X_4 = F|x_3) *$
$[0 + 0 + 0 + 0 + 0 + \begin{pmatrix}0\\1\end{pmatrix}])$
$= \alpha \sum_{x_3} P(cold_3|x_3)P(x_3|X_2 = B) * (\begin{pmatrix}1\\0\end{pmatrix} P(X_4 = A|x_3) \begin{pmatrix}0.2\\0.8\end{pmatrix}$
$+ \begin{pmatrix}0\\1\end{pmatrix} P(X_4 = B|x_3) \begin{pmatrix}0\\1\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix} P(X_4 = C|x_3) \begin{pmatrix}0.8\\0.2\end{pmatrix}$
$+ \begin{pmatrix}1\\0\end{pmatrix} P(X_4 = D|x_3) \begin{pmatrix}0.2\\0.8\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix} P(X_4 = E|x_3) \begin{pmatrix}0\\1\end{pmatrix}$
$+ \begin{pmatrix}0\\1\end{pmatrix} P(X_4 = F|x_3) \begin{pmatrix}0\\1\end{pmatrix})$

$$= \alpha(0 + 0.2[0 + 0.2 \begin{pmatrix}0\\1\end{pmatrix} + 0.8 \begin{pmatrix}0\\0.2\end{pmatrix} + 0 + 0 + 0]$$
$$+0.8[0 + 0 + 0.2 \begin{pmatrix}0\\0.2\end{pmatrix} + 0.8 \begin{pmatrix}0.2\\0\end{pmatrix} + 0 + 0] + 0 + 0 + 0)$$
$$= \alpha(0.2 \begin{pmatrix}0\\0.36\end{pmatrix} + 0.8 \begin{pmatrix}0.16\\0.04\end{pmatrix}) = \alpha\langle 0.128, 0.104\rangle \approx \langle 0.551724138, 0.448275862\rangle$$

$P(hot_5|hot_1, cold_2, cold_3) \approx 0.551724138$

$P(E_6|hot_1, cold_2, cold_3)$
$= \alpha \sum_{x_1} P(hot_1|x_1)P(x_1) \sum_{x_2} P(cold_2|x_2)P(x_2|x_1) \sum_{x_3} P(cold_3|x_3)P(x_3|x_2)$
$\sum_{x_4} P(E_4|x_4)P(x_4|x_3) \sum_{x_5} P(E_5|x_5)P(x_5|x_4) \sum_{x_6} P(E_6|x_6)P(x_6|x_5)$
$= \alpha \sum_{x_2} P(cold_2|x_2)P(x_2|X_1 = A) \sum_{x_3} P(cold_3|x_3)P(x_3|x_2)$
$\sum_{x_4} P(E_4|x_4)P(x_4|x_3) \sum_{x_5} P(E_5|x_5)P(x_5|x_4) \sum_{x_6} P(E_6|x_6)P(x_6|x_5) + 0$
$= 0.8\alpha \sum_{x_3} P(cold_3|x_3)P(x_3|X_2 = B) \sum_{x_4} P(E_4|x_4)P(x_4|x_3)$
$\sum_{x_5} P(E_5|x_5)P(x_5|x_4) \sum_{x_6} P(E_6|x_6)P(x_6|x_5) + 0$
$= \alpha \sum_{x_3} P(cold_3|x_3)P(x_3|X_2 = B) \sum_{x_4} P(E_4|x_4)P(x_4|x_3) \sum_{x_5} P(E_5|x_5)P(x_5|x_4)$
$*([P(E_6|x_6 = A)P(X_6 = A|x_5)] + [P(E_6|x_6 = B)P(X_6 = B|x_5)]$
$+[P(E_6|x_6 = C)P(X_6 = C|x_5)] + [P(E_6|x_6 = D)P(X_6 = D|x_5)]$
$+[P(E_6|x_6 = E)P(X_6 = E|x_5)] + [P(E_6|x_6 = F)P(X_6 = F|x_5)])$
$= \alpha \sum_{x_3} P(cold_3|x_3)P(x_3|X_2 = B) \sum_{x_4} P(E_4|x_4)P(x_4|x_3)$
$$*(\begin{pmatrix}1\\0\end{pmatrix} P(X_5 = A|x_4)[\begin{pmatrix}1\\0\end{pmatrix}(0.2) + \begin{pmatrix}0\\1\end{pmatrix}(0.8) + 0 + 0 + 0 + 0]$$
$$+ \begin{pmatrix}0\\1\end{pmatrix} P(X_5 = B|x_4)[0 + \begin{pmatrix}0\\1\end{pmatrix}(0.2) + \begin{pmatrix}0\\1\end{pmatrix}(0.8) + 0 + 0 + 0]$$
$$+ \begin{pmatrix}0\\1\end{pmatrix} P(X_5 = C|x_4)[0 + 0 + \begin{pmatrix}0\\1\end{pmatrix}(0.2) + \begin{pmatrix}1\\0\end{pmatrix}(0.8) + 0 + 0]$$
$$+ \begin{pmatrix}1\\0\end{pmatrix} P(X_5 = D|x_4)[0 + 0 + 0 + \begin{pmatrix}1\\0\end{pmatrix}(0.2) + \begin{pmatrix}0\\1\end{pmatrix}(0.8) + 0]$$
$$+ \begin{pmatrix}0\\1\end{pmatrix} P(X_5 = E|x_4)[0 + 0 + 0 + 0 + \begin{pmatrix}0\\1\end{pmatrix}(0.2) + \begin{pmatrix}0\\1\end{pmatrix}(0.8)]$$
$$+ \begin{pmatrix}0\\1\end{pmatrix} P(X_5 = F)[0 + 0 + 0 + 0 + 0 + \begin{pmatrix}0\\1\end{pmatrix}])$$
$= \alpha \sum_{x_3} P(cold_3|x_3)P(x_3|X_2 = B) \sum_{x_4} P(E_4|x_4)P(x_4|x_3)$
$$*(\begin{pmatrix}0.2\\0\end{pmatrix} P(X_5 = A|x_4) + \begin{pmatrix}0\\1\end{pmatrix} P(X_5 = B|x_4) + \begin{pmatrix}0\\0.2\end{pmatrix} P(X_5 = C|x_4)$$
$$+ \begin{pmatrix}0.2\\0\end{pmatrix} P(X_5 = D|x_4) + \begin{pmatrix}0\\1\end{pmatrix} P(X_5 = E|x_4) + \begin{pmatrix}0\\1\end{pmatrix} P(X_5 = F|x_4))$$
$= \alpha \sum_{x_3} P(cold_3|x_3)P(x_3|X_2 = B)$
$$*(\begin{pmatrix}1\\0\end{pmatrix} P(X_4 = A|x_3)[\begin{pmatrix}0.04\\0\end{pmatrix} + \begin{pmatrix}0\\0.8\end{pmatrix} + 0 + 0 + 0 + 0]$$
$$+ \begin{pmatrix}0\\1\end{pmatrix} P(X_4 = B|x_3)[0 + \begin{pmatrix}0\\0.2\end{pmatrix} + \begin{pmatrix}0\\0.16\end{pmatrix} + 0 + 0 + 0 + 0]$$
$$+ \begin{pmatrix}0\\1\end{pmatrix} P(X_4 = C|x_3)[0 + 0 + \begin{pmatrix}0\\0.04\end{pmatrix} + \begin{pmatrix}0.16\\0\end{pmatrix} + 0 + 0]$$

$+\begin{pmatrix}1\\0\end{pmatrix} P(X_4 = D|x_3)[0+0+0+\begin{pmatrix}0.04\\0\end{pmatrix}+\begin{pmatrix}0\\0.8\end{pmatrix}+0]$
$+\begin{pmatrix}0\\1\end{pmatrix} P(X_4 = E|x_3)[0+0+0+0+\begin{pmatrix}0\\0.2\end{pmatrix}+\begin{pmatrix}0\\0.8\end{pmatrix}]$
$+\begin{pmatrix}0\\1\end{pmatrix} P(X_4 = F|x_3)[0+0+0+0+0+\begin{pmatrix}0\\1\end{pmatrix}])$
$= \alpha \sum_{x_3} P(cold_3|x_3)P(x_3|X_2 = B)$
$*(\begin{pmatrix}0.04\\0\end{pmatrix} P(X_4 = A|x_3) + \begin{pmatrix}0\\0.36\end{pmatrix} P(X_4 = B|x_3) + \begin{pmatrix}0\\0.04\end{pmatrix} P(X_4 = C|x_3)$
$+\begin{pmatrix}0.04\\0\end{pmatrix} P(X_4 = D|x_3) + \begin{pmatrix}0\\1\end{pmatrix} P(X_4 = E|x_3) + \begin{pmatrix}0\\1\end{pmatrix} P(X_4 = F|x_3))$
$= \alpha(0 + 0.2[0 + \begin{pmatrix}0\\0.36\end{pmatrix}(0.2) + \begin{pmatrix}0\\0.04\end{pmatrix}(0.8) + 0 + 0 + 0]$
$+0.8[0 + 0 + \begin{pmatrix}0\\0.04\end{pmatrix}(0.2) + \begin{pmatrix}0.04\\0\end{pmatrix}(0.8) + 0 + 0] + 0 + 0 + 0)$
$= \alpha(0.2\begin{pmatrix}0\\0.104\end{pmatrix} + 0.8\begin{pmatrix}0.032\\0.008\end{pmatrix}) = \alpha\langle 0.0256, 0.0272\rangle$
$\approx \langle 0.484848485, 0.515151515\rangle$

$P(cold_6|hot_1, hot_2, hot_3) \approx 0.515151515$

$P(hot_4, hot_5, cold_6|hot_1, cold_2, cold_3) = 0.64 * 0.551724138 * 0.515151515$
$\approx 0.1819$

# 5 Problem 5

1. $V^\pi(s) = R(s, \pi(s)) + \gamma \sum_{s'} T(s, \pi(s), s')V^\pi(s')$

2. Initial Policy: $\pi(A) = 1, \pi(B) = 1$
   Initialize: $V_0^\pi(A) = 0, V_0^\pi(B) = 0$

   Iteration 1
   State A: $V_1^\pi(A) = R(A, 1) + \sum_{s'} T(A, 1, s')V_0^\pi(s')$
   $= R(A, 1) + T(A, 1, A)V_0^\pi(A) + T(A, 1, B)V_0^\pi(B)$
   $= 0 + (1 * 0) + (0 * 0) = 0$
   State B: $V_1^\pi(B) = R(B, 1) + \sum_{s'} T(B, 1, s')V_0^\pi(s')$
   $= R(B, 1) + T(B, 1, A)V_0^\pi(A) + T(B, 1, B)V_0^\pi(B)$
   $= 5 + (0 * 0) + (1 * 0) = 5$

   Iteration 2
   State A: $V_2^\pi(A) = R(A, 1) + \sum_{s'} T(A, 1, s')V_1^\pi(s')$
   $= R(A, 1) + T(A, 1, A)V_1^\pi(A) + T(A, 1, B)V_1^\pi(B)$
   $= 0 + (1 * 0) + (0 * 5) = 0$
   State B: $V_2^\pi(B) = R(B, 1) + \sum_{s'} T(B, 1, s')V_1^\pi(s')$
   $= R(B, 1) + T(B, 1, A)V_1^\pi(A) + T(B, 1, B)V_1^\pi(B)$

$= 5 + (0 * 0) + (1 * 5) = 10$

$V_2^\pi(A) = 0,\ V_2^\pi(B) = 10$

3. $\pi_{\text{new}}(s) = \text{argmax}_a(R(s,a) + \gamma \sum_{s'} T(s,a,s')V_2^\pi(s'))$
   State A
   Action 1: $Q(A,1) = R(A,1) + \gamma \sum_{s'} T(A,1,s')V_2^\pi(s')$
   $= R(A,1) + T(A,1,A)V_2^\pi(A) + T(A,1,B)V_2^\pi(B)$
   $= 0 + (1 * 0) + (0 * 10) = 0$
   Action 2: $Q(A,2) = R(A,2) + \gamma \sum_{s'} T(A,2,s')V_2^\pi(s')$
   $= R(A,2) + T(A,2,A)V_2^\pi(A) + T(A,2,B)V_2^\pi(B)$
   $= -1 + (0.5 * 0) + (0.5 * 10) = 4$

   $\pi_{\text{new}}(A) = \text{argmax}_a(0,4) = 2$

   State B
   Action 1: $Q(B,1) = R(B,1) + \gamma \sum_{s'} T(B,1,s')V_2^\pi(s')$
   $= R(B,1) + T(B,1,A)V_2^\pi(A) + T(B,1,B)V_2^\pi(B)$
   $= 5 + (0 * 0) + (1 * 10) = 15$
   Action 2: $Q(B,2) = R(B,2) + \gamma \sum_{s'} T(B,2,s')V_2^\pi(s')$
   $= R(B,2) + T(B,2,A)V_2^\pi(A) + T(B,2,B)V_2^\pi(B)$
   $= 0 + (0 * 0) + (1 * 10) = 10$

   $\pi_{\text{new}}(B) = \text{argmax}_a(15,10) = 1$

   $\pi_{\text{new}}(A) = 2,\ \pi_{\text{new}}(B) = 1$