

Lab2

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Probability distributions in R and Stan

Task 1 (8 points)

Study the previous code provided on probability distribution functions in R (see the file on BbLearn). Your task is to choose a continuous probability distribution other than the Gaussian distribution and complete the following (2 points each). You can use the Stan functions to help.

1. Create a custom function to calculate the relative probability density of a given observation of a continuous random variate.

```
#Uniform Distribution
pdf_func = function(x, lower, upper){
  if((x >= lower) & (x <= upper)){
    result = 1/(upper-lower);
  }
  else{
    result = 0;
  }
  return(result)
}

pdf_func(x=0, lower=-10, upper=10)

## [1] 0.05
```

2. Compare your function to R's built-in density function (i.e., show that they give the same output).

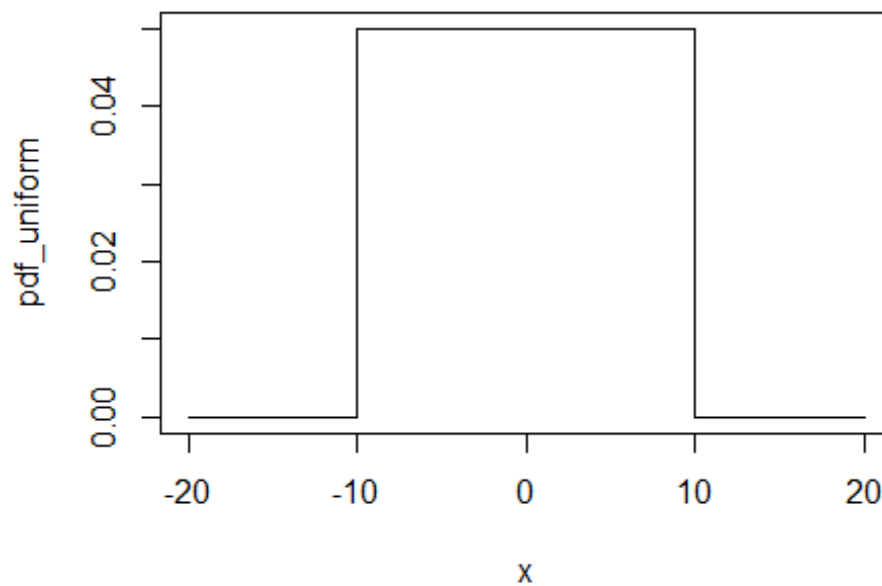
```
#Uniform Distribution

dunif(0, min = -10, max = 10)

## [1] 0.05
```

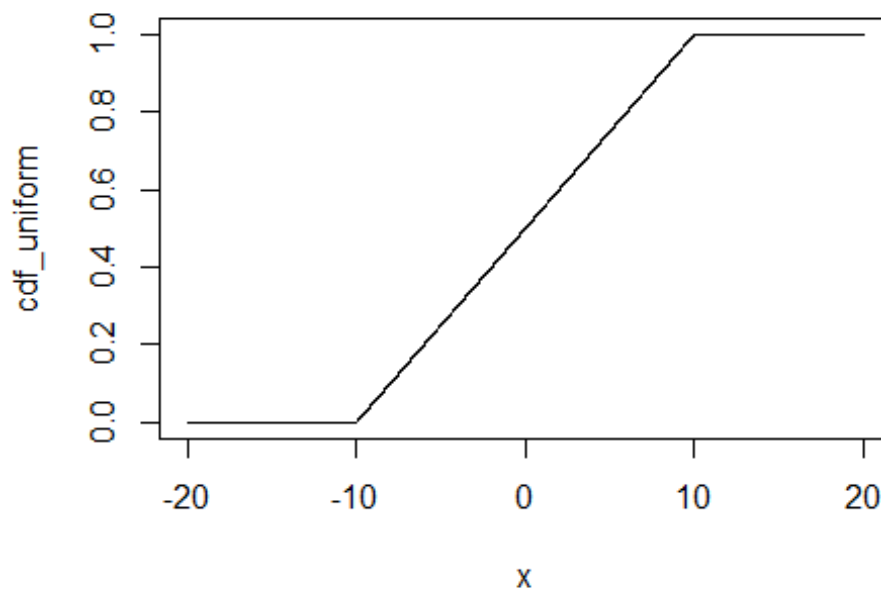
3. Plot the probability density function $P(y|\theta)$, where θ is a vector of user-specified parameters that define the probability distribution of interest.

```
#Uniform Distribution  
x = seq(-20,20,length.out = 10000)  
  
pdf_uniform = dunif(x,-10,10)  
  
plot(pdf_uniform ~ x,type = "l")
```



4. Plot the cumulative density function $F(y|\theta)$, as above.

```
#Uniform Distribution  
x = seq(-20,20,length.out = 10000)  
  
cdf_uniform = punif(x,-10,10)  
  
plot(cdf_uniform ~ x,type = "l")
```



Task 2 (12 points)

Your task is to now choose a discrete probability distribution, and use the guided code above to complete the following tasks. Again, see the Stan functions documentation for assistance.

```
# Load the rstan package
library(rstan)

# Set some useful options
options(mc.cores = parallel::detectCores())
rstan_options(auto_write = TRUE)
```

1. Create a pmf_sim.stan file that generates the PMF of your chosen distribution and that draws random values from the distribution (5 points).

```
# Define data inputs

## Number of random draws:
n_draws = 1000
## Number of points used for PMF:
n_seq = 101
## Sequence of observed y, for PMF:
y_pmf = seq(0, 100, length.out = n_seq)
```

```

# Store the required data in a list
sim_data = list(n_draws = n_draws,
                n_seq = n_seq,
                y_pmf = y_pmf)

#-----
#-----
# Run the simulation:
sim_fit =
  stan(file="pmf_sim.stan",
        data=sim_data,
        iter=1,
        chains=1,
        algorithm="Fixed_param"
  )

##
## SAMPLING FOR MODEL 'pmf_sim' NOW (CHAIN 1).
## Chain 1: Iteration: 1 / 1 [100%] (Sampling)
## Chain 1:
## Chain 1: Elapsed Time: 0 seconds (Warm-up)
## Chain 1:           0 seconds (Sampling)
## Chain 1:           0 seconds (Total)
## Chain 1:

# The function extract() will create a list of output
# Note that "lp__" is irrelevant here, but will become
# very important later in the course.

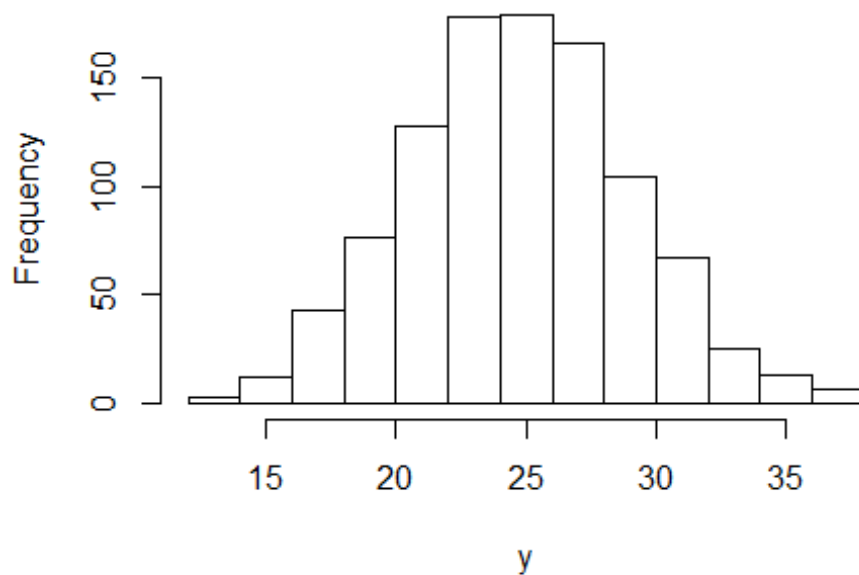
sim_out = extract(sim_fit)
str(sim_out)

## List of 3
## $ log_lik      : num [1, 1:101] -28.8 -25.3 -22.5 -20.1 -18 ...
## ..- attr(*, "dimnames")=List of 2
## .. ..$ iterations: NULL
## .. ..$           : NULL
## $ rand_binomial: num [1, 1:1000] 22 24 37 25 18 25 21 31 23 27 ...
## ..- attr(*, "dimnames")=List of 2
## .. ..$ iterations: NULL
## .. ..$           : NULL
## $ lp__         : num [1(1d)] 0
## ..- attr(*, "dimnames")=List of 1
## .. ..$ iterations: NULL

```

2. Plot a histogram of the random draws (2 points).

```
# Plot the random draws:
hist(sim_out$rand_binomial[, 1:n_draws], main = "", xlab = "y")
```

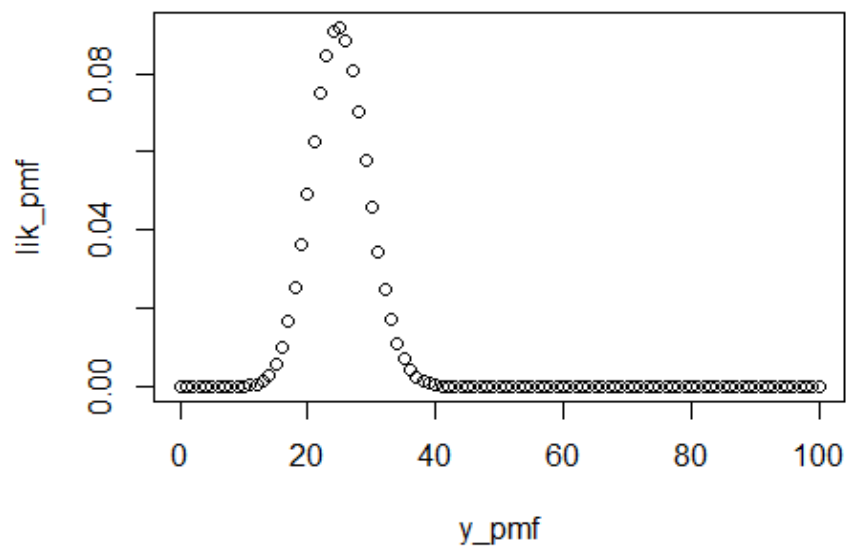


```
mean(sim_out$rand_binomial[, 1:n_draws])
## [1] 25.237
var(sim_out$rand_binomial[, 1:n_draws])
## [1] 18.27711
```

3. Plot the PMF using bars or points (3 points).

```
# Plot the PMF
## First get back to usual scale:
lik_pmf = exp(sim_out$log_lik[, 1:n_seq])

## Now plot:
plot(lik_pmf ~ y_pmf, type = "p",)
```



4. Compare the PMF generated in Stan to the PMF generated by R's built-in functions (2 points).

```
# Plot the PMF
lik_pmf_R = dbinom(y_pmf, size=100, prob = 0.25)
plot(lik_pmf_R ~ y_pmf, type = "p")
```

