STA 570 - Exam 2

1.) a.) $n = \left(\frac{1.96 \times 36}{6}\right)^2 = 138.3$ so use $n = 139$.

b.) 80 ± 6 or $(74, 86 \text{ hrs})$.

2.) Paired data results in $df = n-1 = 9$ so Researcher A performed the paired t-test.

3.) a.) $\alpha = 1 - pt(1.9, 99) \approx 0.03$

b.) Smaller. A larger t-crit equates to a smaller α-value.

c.) $\beta = P(\text{type II error}) = P(\text{fail to reject } H_0 \text{ when } H_0 \text{ is false})$

If in reality, $\mu_a = 2.5$, the type II error probability will be larger than the corresponding value for $\mu_a = 3.0$ since $\mu_a = 2.5$ is closer to $\mu_0 = 2.0$. Graphing the cases involving $\mu_0 = 2.0$ compared to μ_a may be helpful.

4.) a.) A: Bootstrap distribution

B: Permulation distribution

b.) Using graph A, include about $200 \times 0.95 = 210$ dots of the center dots yielding an approximate 95% for $\mu_{oval} - \mu_{standard}$ of $(0, 2)$.

c.) Using graph A, $\bar{x}_{oval} - \bar{x}_{standard} = 1$

d.) p-value $\approx 2 \times \frac{4}{200} = 0.04$

5.) c. and d.

6.) a.) $H_0: \mu = 170$

$H_a: \mu > 170$

b.) $t = \frac{\bar{x} - 170}{s/\sqrt{n}} = \frac{182 - 170}{8/\sqrt{5}} = \frac{12}{3.578} \approx 3.35$. Under H_0, $t \sim$ t-distribution with $df = n-1 = 4$

c.) p-value $= P(t_4 > 3.35) = 1 - pt(3.35, 4) = 0.014$

d.) A random sample of coffee temperatures from a normally distributed population of coffee temperatures.

e.) p-value $\leq \alpha$ so reject H_0. Sample evidence supports the claim that the true mean coffee temperature exceeds the industry standard.

f.) $qt(0.975, 4) = 2.776$ so $\bar{x} \pm 2.776 \cdot \frac{8}{\sqrt{5}}$ is 182 ± 9.93 or $(172.07, 191.93°F$

7.) a.) H_0: Data are normally distributed

H_a: Data are not normally distributed

b.) $H_0: \sigma_1^2 = \sigma_2^2$

$H_a: \sigma_1^2 \neq \sigma_2^2$

8.) a.) $H_0: \sigma_1^2 = \sigma_2^2$

$H_a: \sigma_1^2 \neq \sigma_2^2$

b.) $f = \frac{s_1^2}{s_2^2} = \frac{3^2}{2^2} = \frac{9}{4} = 2.25$. Under H_0, $f \sim F_{29,39}$

c.) p-value $= 2P(F_{29,39} > 2.25) = 2(1 - pf(2.25, 29, 39)) = 0.0186$

d.) Two independent random samples, the first being 30 randomly selected wheel weights under the new process and the second being 40 randomly selected wheel weights under the old process. Additionally, the population of wheel weights under the new process is assumed to be normally distribute and the population of wheel weights under the old process is also normally distributed.