

QAPLIB-Problem Instances and Solutions

R.E. BURKARD, E. ÇELA, S.E. KARISCH and F. RENDL

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Compressed Data and Solutions

Data: [qapdata.tar.gz](#) (453187 KB). Solutions: [qapsoln.tar.gz](#) (9836 KB). ("gunzip qapxxxx.tar.gz" and "tar xf qapxxxx.tar". This should result in 136 instances and 111 solutions.)

Description

The instances are listed in alphabetical order by the names of their authors. We shortly characterize the examples by indicating their generation. All the instances in the current version are pure quadratic.

If not stated otherwise the examples are symmetric.

The format of the problem data is

n A B

where n is the size of the instance, and A and

B are either

flow or distance matrix.

This corresponds to a QAP of the form

$$\min_{p \in \Pi} \sum_{i,j \in [n]} a_{ij} p(i) p(j) + \sum_{i,j \in [n]} b_{ij} p(i) p(j)$$

where p is a permutation.

We quote the filename under which it is stored in the library and report the size of the problem. Then the objective function value of the best known feasible solution is given. In parentheses we indicate whether this solution is optimal or derived by a heuristic. The heuristics that are used are

- ant systems: (ANT) [Stuetzle:97].

- genetic hybrids: (GEN) [FlFe:94],(GEN-2) [OsRu:96],(GEN-3) [TaGa:97], and (GEN-4) [Mise:04].
- a greedy randomized adaptive search procedure: (GRASP) [LiPaRe:94],
- scatter search: (ScS) [CuMaMiTa:97],
- simulated annealing: (SIM-1) [BuRe:84],(SIM-2)[ThBo:94] and(SIM-3) [Mise:03], and
- simulated jumping: (SIMJ) [Amin:98].
- tabu search:parallel adaptive tabu search (PA-TS)[TaHaGe:97],reactive tabu search (Re-TS) [BaTe:94],robust tabu search (Ro-TS) [Taillard:91,Taillard:94],strict tabu search (S-TS) [Skorin:90], (TS-1) [Iriyama:97], (TS-2) [Mise:05] and (ITS) [Mise:08]

If available we give a link to a solution for the instances. The format

of these files is

```
n      sol

p
```

where n gives the size of the instance, sol is the

objective function value and p a corresponding permutation,

i.e.

```

      ---  ---
      \    \

sol =  /    /    a    b    .

      ---  ---  ij    p(i),p(j)

      i    j
```

For optimal solutions we enclose the optimal permutation while for nonoptimal solutions lower bounds are given. We also give explicit reference who solved hard instances of size $n \geq 15$ first. The lower bounds given in the tables are

- the Gilmore-Lawler bound: (GLB)[Gilmore:62, Lawler:63],
- the elimination bound: (ELI) [HaReWo:92],
- an interior point based linear programming bound: (IPLP) [ReRaDr:94]
- a triangle decomposition bound: (TDB) [KaRe:95a],
- a semidefinite programming bound: (SDP)[ZhKaReWo:96],
- a bound based on a dual procedure: (DP)[HaGr:95],
- a bound based on a cutting plane approach: (CUT)[Kaibel:97],
- a dual framework based bound: (DFB)[KaCeClEs:98],
- a lift-and-project relaxation bound: (L&P)[http://www.optimization-online.org/DB_HTML/2004/06/890.html],
- a level-2 RLT bound: (RLT2)[HHJGSR:01],
- a semidefinite programming bound: (SDP1)[DKSo:07], and
- a semidefinite relaxation-matrix splitting bound: (SDRMS)[http://www.optimization-online.org/DB_HTML/2009/02/2220.html],

When lower bounds are included we also give the relative gap between best feasible solution and best known lower bound in percent, i.e.

$$gap = (solution - bound) / (solution) * 100 \%$$

R.E. Burkard and J. Offermann [BuOf:77]

The data of the first matrix correspond to the typing-time of an average stenotypist, while the second matrix describes the frequency of pairs of letters in different languages taken over 100,000 pairs for examples a-f and over 187,778 pairs for examples g-h. (Note that the solutions are not scaled for a flow matrix of 100,000 pairs anymore.)

One also distinguishes between two types of typewriter keyboards.

The instances are asymmetric.

name	n	feas. solution	bound	gap
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* Bur26a 26 5426670 (OPT) (26 15 11 7 4 12 13 2 6 18 1 5 9 21 8
14 3 20 19 25 17 10 16 24 23 22)

* Bur26b 26 3817852 (OPT) (17 11 26 7 4 14 6 22 23 18 5 9 1 21
8 12 3 19 20 15 10 25 24 16 13 2)

* Bur26c 26 5426795 (OPT) (12 3 2 13 16 25 11 15 10 9 18 19 8
20 4 21 1 5 14 24 22 6 23 7 26 17)

* Bur26d 26 3821225 (OPT) (3 22 11 2 16 26 8 15 21 9 19 12 18
20 23 25 14 5 1 6 13 24 4 7 17 10)

* Bur26e 26 5386879 (OPT) (14 4 13 7 16 25 26 17 1 15 12 20 18
19 3 8 21 9 5 24 6 10 22 2 23 11)

* Bur26f 26 3782044 (OPT) (7 2 13 17 16 26 23 1 10 15 19 20 18
12 14 25 21 5 9 3 6 24 22 4 11 8)

* Bur26g 26 10117172 (OPT) (22 11 2 23 13 25 24 8 1 21 20 4 7 18
12 15 9 19 5 26 16 6 14 3 17 10)

* Bur26h 26 7098658 (OPT) (22 16 3 12 6 24 17 1 8 21 20 4 7 18
14 15 9 5 19 2 11 13 23 26 25 10)

N. Christofides and E. Benavent [ChBe:89]

One matrix is the adjacency matrix of a weighted tree the other that

of a complete graph.

name	n	solution	permutation

<u>Chr12a</u>	12	<u>9552</u> (OPT)	(7,5,12,2,1,3,9,11,10,6,8,4)
<u>Chr12b</u>	12	<u>9742</u> (OPT)	(5,7,1,10,11,3,4,2,9,6,12,8)
<u>Chr12c</u>	12	<u>11156</u> (OPT)	(7,5,1,3,10,4,8,6,9,11,2,12)
<u>Chr15a</u>	15	<u>9896</u> (OPT)	(5,10,8,13,12,11,14,2,4,6,7,15,3,1,9)
<u>Chr15b</u>	15	<u>7990</u> (OPT)	(4,13,15,1,9,2,5,12,6,14,7,3,10,11,8)
<u>Chr15c</u>	15	<u>9504</u> (OPT)	(13,2,5,7,8,1,14,6,4,3,15,9,12,11,10)
<u>Chr18a</u>	18	<u>11098</u> (OPT)	(3,13,6,4,18,12,10,5,1,11,8,7,17,14,9,16,15,2)
<u>Chr18b</u>	18	<u>1534</u> (OPT)	(1,2,4,3,5,6,8,9,7,12,10,11,13,14,16,15,17,18)
<u>Chr20a</u>	20	<u>2192</u> (OPT)	(3,20,7,18,9,12,19,4,10,11,1,6,15,8,2,5,14,16,13,17)

Chr20b 20 2298 (OPT)
(20,3,9,7,1,12,16,6,8,14,10,4,5,13,17,2,18,11,19,15)

Chr20c 20 14142 (OPT)
(12,6,9,2,10,11,3,4,15,18,7,13,16,5,14,17,19,1,8,20)

Chr22a 22 6156 (OPT)
(15,2,21,8,16,1,7,18,14,13,5,17,6,11,3,4,20,19,9,22,10,12)

Chr22b 22 6194 (OPT)
(10,19,3,1,20,2,6,4,7,8,17,12,11,15,21,13,9,5,22,14,18,16)

Chr25a 25 3796 (OPT)
(25,12,5,3,18,4,16,8,20,10,14,6,15,23,24,19,13,1,21,11,17,2,22,7,9)

A.N. Elshafei [Elshafei:77]

The data describe the distances of 19 different facilities of a hospital and the flow of patients between those locations. The optimal solution was first found by [Mautor:92].

name	n	solution	permutation
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<u>Els19</u>	19	<u>17212548</u>	(OPT)
(9,10,7,18,14,19,13,17,6,11,4,5,12,8,15,16,1,2,3)			

B. Eschermann and H.J. Wunderlich [EsWu:90]

These examples stem from an application in computer science, from the testing of self-testable sequential circuits. The amount of additional hardware for the testing should be minimized. The optimal solutions are due to [ClPe:94] ($n=16$) and [BrClMaPe:96] ($n=32$).

name	n	feas.sol.	permutation/bound	gap

<u>Esc16a</u>	16	<u>68</u> (OPT)	(2,14,10,16,5,3,7,8,4,6,12,11,15,13,9,1)	
<u>Esc16b</u>	16	<u>292</u> (OPT)	(6,3,7,5,13,1,15,2,4,11,9,14,10,12,8,16)	
<u>Esc16c</u>	16	<u>160</u> (OPT)	(11,14,10,16,12,8,9,3,13,6,5,7,15,2,1,4)	
<u>Esc16d</u>	16	<u>16</u> (OPT)	(14,2,12,5,6,16,8,10,3,9,13,7,11,15,4,1)	
<u>Esc16e</u>	16	<u>28</u> (OPT)	(16,7,8,15,9,12,14,10,11,2,6,5,13,4,3,1)	
<u>Esc16f</u>	16	<u>0</u> (OPT)	(1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16)	
<u>Esc16g</u>	16	<u>26</u> (OPT)	(8,11,9,12,15,16,14,10,7,6,2,5,13,4,3,1)	
<u>Esc16h</u>	16	<u>996</u> (OPT)	(13,9,10,15,3,11,4,16,12,7,8,5,6,2,1,14)	

Esc16i 16 14 (OPT) (13, 9, 11, 3, 7, 5, 6, 2, 1, 15, 4, 14, 12, 10, 8, 16)

Esc16j 16 8 (OPT) (8, 3, 16, 14, 2, 12, 10, 6, 9, 5, 13, 11, 4, 7, 15, 1)

* Esc32a 32 130 (OPT)
(11, 3, 7, 23, 19, 27, 15, 14, 20, 17, 28, 9, 12, 4, 8, 2, 26, 24, 32, 13, 22, 25, 6, 18, 29, 10, 30, 21, 1, 5, 16, 31)

* Esc32b 32 168 (OPT)
(15, 31, 7, 8, 23, 24, 16, 32, 14, 10, 30, 26, 5, 6, 13, 9, 2, 1, 21, 22, 29, 25, 18, 17, 12, 27, 20, 11, 3, 19, 28, 4)

* Esc32c 32 642 (OPT)
(15, 12, 27, 13, 22, 8, 24, 23, 20, 19, 4, 2, 1, 7, 6, 3, 5, 18, 17, 21, 14, 29, 16, 32, 26, 11, 31, 30, 28, 10, 25, 9)

* Esc32d 32 200 (OPT)
(18, 29, 10, 2, 25, 32, 22, 20, 24, 17, 30, 9, 1, 26, 31, 21, 19, 23, 27, 16, 13, 6, 3, 11, 15, 7, 8, 5, 14, 4, 12, 28)

Esc32e 32 2 (OPT)
(1, 2, 5, 6, 8, 16, 13, 19, 9, 32, 7, 22, 24, 20, 4, 12, 3,

17, 29, 21, 11, 25, 27, 18, 30, 31, 23, 28, 14, 15, 26, 10)

Esc32g 32 6 (OPT)
(14, 15, 16, 12, 11, 26, 30, 10, 25, 8, 29, 22, 31, 28,

13, 1, 19, 9, 17, 32, 24, 18, 4, 2, 20, 5, 21, 3, 7, 6, 23, 27)

* Esc32h 32 438 (OPT)
(1,19,29,22,12,4,30,25,9,7,27,11,21,6,5,13,14,

31,10,28,8,3,23,26,17,2,32,15,24,18,20,16)

* Esc64a 64 116 (OPT)
(1,2,9,50,3,61,4,62,5,54,64,6,7,52,56,8,55,10,63,18,11,51,12,13,14,15,2
0,43,16,41,17,47,23,19,24,21,53,22,28,25,26,27,29,60,30,59,31,32,33,34,
35,36,37,38,39,40,42,44,45,46,48,49,57,58)

* Esc128 128 64 (OPT) (1,2,3,4,117,5,6,7,8,9,10,11,12,73,13,89,

14,15,16,17,18,19,20,21,22,23,24,53,25,26,102,

27,104,28,118,120,29,30,31,80,32,111,112,34,35,

36,97,98,38,39,100,40,99,41,42,43,44,45,46,47,

48,33,49,37,51,121,52,54,122,55,123,56,124,57,

125,58,126,59,127,128,81,60,61,62,63,64,113,

105,66,67,68,69,70,65,71,72,74,75,76,77,78,79,

82, 83, 84, 85, 86, 87, 88, 90, 50, 91, 114, 92, 93, 94, 95,

96, 101, 103, 106, 107, 108, 109, 110, 115, 116, 119)

S.W. Hadley, F. Rendl and H. Wolkowicz [[HaReWo:92](#)]

The first matrix represents Manhattan distances of a connected cellular complex in the plane while the entries in the flow matrix are drawn uniformly from the interval $[1, n]$. The proof of optimality of the solution for $n=16$ is due to [[HaGrHa:96](#)], for $n=18$ and $n=20$ due to [[BrClMaPe:96](#)]

name	n	solution	permutation

Had12	12	1652 (OPT)	(3, 10, 11, 2, 12, 5, 6, 7, 8, 1, 4, 9)
Had14	14	2724 (OPT)	(8, 13, 10, 5, 12, 11, 2, 14, 3, 6, 7, 1, 9, 4)
Had16	16	3720 (OPT)	(9, 4, 16, 1, 7, 8, 6, 14, 15, 11, 12, 10, 5, 3, 2, 13)
Had18	18	5358 (OPT)	(8, 15, 16, 6, 7, 18, 14, 11, 1, 10, 12, 5, 3, 13, 2, 17, 9, 4)

```
Had20 20 6922 (OPT)
(8,15,16,14,19,6,7,17,1,12,10,11,5,20,2,3,4,9,18,13)
```

J. Krarup and P.M. Pruzan [KrPr:78]

The instances contain real world data and were used to plan the
Klinikum Regensburg in Germany.

name	n	feas. solution	permutation/bound	gap

* <u>Kra30a</u>	30	88900 (OPT)		
(23,10,28,29,21,7,13,24,20,8,9,19,25,27,15,				
4,22,12,6,5,16,11,3,2,17,1,30,26,18,14)				
* <u>Kra30b</u>	30	91420 (OPT)		
(23,26,19,25,20,22,11,8,9,14,27,30,12,6,28,				
24,21,18,1,7,10,29,13,5,2,17,3,15,4,16)				
* <u>Kra32</u>	32	88700 (OPT)		
(31,23,18,21,22,19,10,11,15,9,30,29,14,12,17,26,				
27,28,1,7,6,25,5,3,8,24,32,13,2,20,4,16)				

Y. Li and P.M. Pardalos [LiPa:92]

These instances come from problem generators described in [LiPa:92].

The generators provide asymmetric instances with known optimal solutions.

name	n	solution

<u>Lipa20a</u>	20	<u>3683</u> (OPT)
<u>Lipa20b</u>	20	<u>27076</u> (OPT)
<u>Lipa30a</u>	30	<u>13178</u> (OPT)
<u>Lipa30b</u>	30	<u>151426</u> (OPT)
<u>Lipa40a</u>	40	<u>31538</u> (OPT)
<u>Lipa40b</u>	40	<u>476581</u> (OPT)
<u>Lipa50a</u>	50	<u>62093</u> (OPT)
<u>Lipa50b</u>	50	<u>1210244</u> (OPT)
<u>Lipa60a</u>	60	<u>107218</u> (OPT)

Lipa60b 60 2520135 (OPT)

Lipa70a 70 169755 (OPT)

Lipa70b 70 4603200 (OPT)

Lipa80a 80 253195 (OPT)

Lipa80b 80 7763962 (OPT)

Lipa90a 90 360630 (OPT)

Lipa90b 90 12490441 (OPT)

C.E. Nugent, T.E. Vollmann and J. Ruml [NuVoRu:68]

The following problem instances are probably the most used. The distance matrix contains Manhattan distances of rectangular grids. The instances of size $n = \{14, 16, 17, 18, 21, 22, 24, 25\}$ were constructed out of the larger instances by deleting certain rows and columns, see Clausen and Perregaard [ClPe:94]. The optimal solutions are also due to [ClPe:94]. For Nug21 and Nug22 optimality was proved by [BrClMaPe:96], for Nug24 by [CEKPT:96].

The instances of size $n=27$ and $n=28$ were constructed out

of the instance of

size $n=30$ by deleting the three or two last facilities, respectively,

and were solved by Anstreicher, Brixius, Goux, and Linderoth.

Aslo Nug 30 was solved by these authors.

The solution was found by applying a

branch and bound algorithm, see Anstreicher and Brixius

[AnBr2:00].

The involved bound was based on convex quadratic programming, see

Anstreicher and Brixius

[AnBr1:00].

name	n	feas.sol.	permutation/bound	gap

<u>Nug12</u>	12	<u>578</u> (OPT)	(12,7,9,3,4,8,11,1,5,6,10,2)	
<u>Nug14</u>	14	<u>1014</u> (OPT)	(9,8,13,2,1,11,7,14,3,4,12,5,6,10)	
<u>Nug15</u>	15	<u>1150</u> (OPT)	(1,2,13,8,9,4,3,14,7,11,10,15,6,5,12)	
<u>Nug16a</u>	16	<u>1610</u> (OPT)	(9,14,2,15,16,3,10,12,8,11,6,5,7,1,4,13)	
<u>Nug16b</u>	16	<u>1240</u> (OPT)	(16,12,13,8,4,2,9,11,15,10,7,3,14,6,1,5)	
<u>Nug17</u>	17	<u>1732</u> (OPT)	(16,15,2,14,9,11,8,12,10,3,4,1,7,6,13,17,5)	

Nug18 18 1930 (OPT)
 (10,3,14,2,18,6,7,12,15,4,5,1,11,8,17,13,9,16)

Nug20 20 2570 (OPT)
 (18,14,10,3,9,4,2,12,11,16,19,15,20,8,13,17,5,7,1,6)

Nug21 21 2438 (OPT)
 (4,21,3,9,13,2,5,14,18,11,16,10,6,15,20,19,8,7,1,12,17)

Nug22 22 3596 (OPT)
 (2,21,9,10,7,3,1,19,8,20,17,5,13,6,12,16,11,22,18,14,15)

Nug24 24 3488 (OPT)
 (17,8,11,23,4,20,15,19,22,18,3,14,1,10,7,9,16,21,24,12,6,13,5,2)

Nug25 25 3744 (OPT)
 (5,11,20,15,22,2,25,8,9,1,18,16,3,6,19,24,21,14,7,10,17,12,4,23,13)

* Nug27 27 5234 (OPT)
 (23,18,3,1,27,17,5,12,7,15,4,26,8,19,20,2,24,21,14,10,9,13,22,25,6,16,11)

* Nug28 28 5166 (OPT)
 (18,21,9,1,28,20,11,3,13,12,10,19,14,22,15,2,25,16,4,23,7,17,24,26,5,27,8,6)

* Nug30 30 6124 (OPT) (5 12 6 13 2 21 26 24 10 9 29 28 17 1 8 7
 19 25 23 22 11 16 30 4 15 18 27 3 14 20)

C. Roucairol [Roucairol:87]

The entries of the matrices are chosen from the interval [1,100].

```
name      n      feas.sol.      permutation
-----
-----

Rou12    12    235528 (OPT)      (6,5,11,9,2,8,3,1,12,7,4,10)

Rou15    15    354210 (OPT)      (12,6,8,13,5,3,15,2,7,1,9,10,4,14,11)

Rou20    20    725522 (OPT)
(1,19,2,14,10,16,11,20,9,5,7,4,8,18,15,3,12,17,13,6)
```

M. Scriabin and R.C. Vergin [ScVe:75]

The distances of these problems are rectangular. The optimal solution for the instance of size $n=20$ was found by [Mautor:92].

```
name      n      solution      permutation
-----
-----

Scr12    12    31410 (OPT)      (8,6,3,2,10,1,5,9,4,7,12,11)

Scr15    15    51140 (OPT)      (15,7,11,8,1,4,3,2,12,6,13,5,14,10,9)
```

Scr20 20 110030 (OPT)
(20,7,12,6,4,8,3,2,14,11,18,9,19,15,16,17,13,5,10,1)

J. Skorin-Kapov [Skorin:90]

The distances of these problems are rectangular and the entries in flow matrices are pseudorandom numbers.

name	n	feas.sol.	bound	gap

<u>Sko42</u>	42	<u>15812</u> (Ro-TS)	14934 (TDB)	5.56 %
<u>Sko49</u>	49	<u>23386</u> (Ro-TS)	22004 (TDB)	5.91 %
<u>Sko56</u>	56	<u>34458</u> (Ro-TS)	32610 (TDB)	5.37 %
<u>Sko64</u>	64	<u>48498</u> (Ro-TS)	45736 (TDB)	5.70 %
<u>Sko72</u>	72	<u>66256</u> (Ro-TS)	62691 (TDB)	5.38 %
<u>Sko81</u>	81	<u>90998</u> (GEN)	86072 (TDB)	5.41 %
* <u>Sko90</u>	90	<u>115534</u> (Ro-TS)	109030 (SDRMS-SUM)	5.63 %
* <u>Sko100a</u>	100	<u>152002</u> (GEN)	143846 (SDRMS-SUM)	5.37 %

* _	<u>Sko100b</u>	100	<u>153890</u> (GEN)	145522 (SDRMS-SUM)	5.44 %
* _	<u>Sko100c</u>	100	<u>147862</u> (GEN)	139881 (SDRMS-SUM)	5.54 %
* _	<u>Sko100d</u>	100	<u>149576</u> (GEN)	141289 (SDRMS-SUM)	5.54 %
* _	<u>Sko100e</u>	100	<u>149150</u> (GEN)	140893 (SDRMS-SUM)	5.54 %
* _	<u>Sko100f</u>	100	<u>149036</u> (GEN)	140691 (SDRMS-SUM)	5.60 %

L. Steinberg [Steinberg:61]

The three instances model the backboard wiring problem. The distances in the first one are Manhattan, in the second squared Euclidean, and in the third one Euclidean distances (multiplied by 1000).

	name	n	feas.sol.	permutation/bound	gap

* _	<u>Ste36a</u>	36	<u>9526</u> (OPT)	(35, 5, 6, 12, 11, 27, 26, 25, 24, 9, 4, 1, 13, 20, 14, 23, 21, 22, 2, 8, 10, 7, 28, 19, 32, 34, 33, 17, 18, 3, 15, 16, 29, 30, 31, 36)	
* _	<u>Ste36b</u>	36	<u>15852</u> (OPT)	(35, 31, 30, 29, 28, 1, 15, 9, 16, 33, 34, 32, 19, 20, 7, 10, 18, 17, 26, 25, 23, 14, 12, 13, 4, 8, 2, 24, 22, 21, 27, 11, 6, 5, 3, 36)	
* _	<u>Ste36c</u>	36	<u>8239110</u> (OPT)	(3, 19, 29, 21, 30, 31, 13, 20, 2, 12, 32, 23, 22, 24, 4, 1, 10, 11, 15, 14, 26, 27, 25, 36, 35, 34, 33, 5, 6, 7, 8, 16, 18, 17, 28, 9)	

E.D. Taillard

[Taillard:91,Taillard:94]

The instances Tai_{xxa} are uniformly generated and were proposed in

[Taillard:91]. The other problems were introduced in

[Taillard:94]. Problems Tai_{xxb} are asymmetric and randomly

generated. Instances Tai_{xxc} occur in the generation of grey patterns. The optimality of

the solutions for Tai_{17a} and Tai_{20a} was proved by [BrClMaPe:96],

while the method of [HaGrHa:96] proved optimality of Tai_{20b} and the Tai_{25a}.

Giovannetti [Giovannetti:97] showed the optimality of Tai_{25b}.

Drezner [Drez:06] proved the optimality of the Tai_{64c}. Drezner's method is branch and bound. His bound exploits the special structure of the problem.

name	n	feas.sol.	permutation/bound	gap

<u>Tai12a</u>	12	<u>224416</u> (OPT)	(8,1,6,2,11,10,3,5,9,7,12,4)	
<u>Tai12b</u>	12	<u>39464925</u> (OPT)	(9,4,6,3,11,7,12,2,8,10,1,5)	
<u>Tai15a</u>	15	<u>388214</u> (OPT)	(5,10,4,13,2,9,1,11,12,14,7,15,3,8,6)	
<u>Tai15b</u>	15	<u>51765268</u> (OPT)	(1,9,4,6,8,15,7,11,3,5,2,14,13,12,10)	
<u>Tai17a</u>	17	<u>491812</u> (OPT)	(12,2,6,7,4,8,14,5,11,3,16,13,17,9,1,10,15)	

Tai20a 20 703482 (OPT)
(10,9,12,20,19,3,14,6,17,11,5,7,15,16,18,2,4,8,13,1)

* Tai20b 20 122455319 (OPT)
(8,16,14,17,4,11,3,19,7,9,1,15,6,13,10,2,5,20,18,12)

* Tai25a 25 1167256 (OPT)
(9,4,6,11,5,1,15,10,14,3,17,12,19,18,23,8,21,2,22,7,16,20,24,25,13)

* Tai25b 25 344355646 (OPT)
(4,15,10,9,13,5,25,19,7,3,17,6,18,20,16,2,22,23,8,11,21,24,14,12,1)

Tai30a 30 1818146 (Ro-TS) 1706855 (L&P) 6.12
%

* Tai30b 30 637117113 (OPT) (4 8 11 15 17 20 21 5 14 30 2 13
6 29 10 26 27 24 28 22 12 9 7 23 19 18 25 16 1 3)

Tai35a 35 2422002 (Ro-TS) 2216627 (L&P) 8,48
%

* Tai35b 35 283315445 (Ro-TS) 242172800 (SDRMS-SUM) 14.52
%

Tai40a 40 3139370 (Ro-TS) 2843274 (L&P) 9.43
%

* Tai40b 40 637250948 (Ro-TS) 564428353 (SDRMS-SUM) 11.43
%

* %	<u>Tai50a</u>	50	<u>4938796</u>	(ITS)	4390920	(L&P)	11.09
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* %	<u>Tai50b</u>	50	<u>458821517</u>	(Ro-TS)	395543467	(SDRMS-SUM)	13.79
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* 22.59 %	<u>Tai60a</u>	60	<u>7205962</u>	(TS-2)	5578356	(SDRMS)	
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* %	<u>Tai60b</u>	60	<u>608215054</u>	(Ro-TS)	542376603	(SDRMS-SUM)	10.82
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	<u>Tai64c</u>	64	<u>1855928</u>	(BandB)			
	(1,3,5,15,17,20,30,35,40,45,49,51,55,2,4,6,7,8,9,10,11,12,13,14,16,18,19,21,22,23,24,25,26,27,28,29,31,32,33,34,36,37,38,39,41,42,43,44,46,47,48,50,52,53,54,56,57,58,59,60,61,62,63,64)						

* %	<u>Tai80a</u>	80	<u>13499184</u>	(ITS)	10501941	(DP)	22.20
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* %	<u>Tai80b</u>	80	<u>818415043</u>	(Ro-TS)	717907288	(SDRMS-SUM)	12.28
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* 24.86 %	<u>Tai100a</u>	100	<u>21052466</u>	(ITS)	15844731	(SDRMS)	
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* %	<u>Tai100b</u>	100	<u>1185996137</u>	(Ro-TS)	1058131796	(SDRMS-SUM)	10.78
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* %	<u>Tai150b</u>	150	<u>498896643</u>	(GEN-3)	441786736	(SDRMS-SUM)	11.45
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* %	<u>Tai256c</u>	256	<u>44759294</u>	(ANT)	43849646	(SDRMS)	2.03
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U.W. Thonemann and A. Bölte [ThBo:94]

The distances of these instances are rectangular.

name	n	feas.sol.	bound	gap

* <u>Tho30</u>	30	149936 (OPT)		
(8,6,20,17,19,12,29,15,1,2,30,11,13,28,23,				
27,16,22,10,21,25,24,26,18,3,14,7,5,9,4)				
<u>Tho40</u>	40	<u>240516</u> (SIM-2)	224414 (L&P)	6.69 %
* <u>Tho150</u>	150	<u>8133398</u> (SIM-3)	7620628 (TDB)	6.30 %

M.R. Wilhelm and T.L. Ward [WiWa:87]

The distances of these problems are rectangular.

name	n	feas.sol.	bound	gap

<u>Wil50</u>	50	<u>48816</u> (SIM-2)	47098 (TDB)	3.52 %

* W11100 100 273038 (GEN) 264442 (SDRMS-SUM) 3.15 %



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