Homework 3

Radiative Transfer & Line Formation

Intro:

To better understand the processes involved in the light we receive from a star, astronomers spend a lot of time modeling stellar atmospheres. The models range from very simple one layer renderings to complex, multi-layer models. From what you have already seen in this course, there's a lot to consider when constructing an accurate model. For example, a good model needs to accurately represent the myriad of opacity contributions, line broadening mechanisms, and various excitation and ionization states of the atoms and molecules in the atmosphere. A good model also needs to include assumptions about the star, like mass, gravity, composition (metallicity), effective temperature, and microturbulence, to name a few. High resolution spectra are also required to reveal the intricate line structure to be modeled.

In this exercise you will study spectral line formation in simple one-and two-layer model atmospheres of the Sun. With the use of the provided Python script, you will vary the parameters of the “toy model” atmospheres that determine the emergent line profiles and describe what the various parameters reveal about the Sun. You will plot various profile fits (the fits to the absorption profiles of the spectral lines), after varying the parameters to get the best fit possible.

The goal of this exercise is to investigate how the emergent intensity of light through a medium is modified from its original form (the source function) by that medium. We'll be assuming a Plank (blackbody) function for the initial intensity, imposing a Voigt profile for the opacity, and passing our simple one- and two-layer model atmospheres. The part of the assignment that has you manipulate Voigt profiles will require you to do a little research on them and the significance of both the Gaussian & Lorentzian contributions to the profile to understand how best to manipulate the parameters and how to interpret your results.

At the end of this assignment you should be able to:
- Qualitatively state the relationship between the intensity of the radiation and the source function, both in LTE and for the sodium (Na) I D lines.
- Explain what goes into making the Voigt profile.
- Relate the shape of the damping profile to the extinction coefficient α and other parameters.
- State how the line opacity relate to the emergent profile for Na I D lines.
- Discuss the line shape under various conditions using the “Eddington-Barbier approximation”.
- Summarize your learning about absorption and emission lines in the Sun.

You will need the two files listed online:
- **line_functions.py,** a selection of useful python functions to help solve two simple model atmospheres. You can import this into you code like:
  - `import line_functions` or `from line_functions import *`
- **nai_d_data.txt**, a high resolution spectrum of the Sun, centered around the Na I D lines

Part 1:

As we have learned, a Voigt profile is a convolution of a Gaussian and a Lorentzian profile. To see how each influences the final Voigt profile, start by plotting a simple Gaussian and Lorentzian as shown here:
https://scipython.com/book/chapter-8-scipy/examples/the-voigt-profile/

Vary the parameters of each profile to see how the resulting Voigt profile changes as a result of your manipulations.

Question 1: How does increasing or decreasing α in the Gaussian profile change the resulting Voigt profile? Which part of the Voigt profile does it most affect (line core or wings)? Plot a few representative figures/curves and refer to them in your answer.

Question 2: How does increasing or decreasing γ in the Lorentzian profile change the resulting Voigt profile? Which part of the Voigt profile does it most affect (line core or wings)? Plot a few representative figures/curves and refer to them in your answer.

Question 3: Explain what a Voigt profile is, and how its maximum and shape are determined. Once you are comfortable with how the Gaussian and Lorentzian profile parameters affect the resulting Voigt profile, you are ready to move on to the next step.

Part 2:

Using the provided data file above, plot the the data in a well labeled figure indicating the two prominent Na I D (1 & 2) lines. They should be at:

- Na I D1 : 5895.924 $\mathring{A}$
- Na I D2 : 5889.950 $\mathring{A}$

Keep in mind that the units of wavelength are in Angstroms. Note these values are measured *in air, not vacuum* wavelengths, meaning they have been modified by the refraction of the atmosphere. The wavelengths in the spectrum data file are *vacuum* wavelengths so you'll want to convert them to air wavelengths to compare with the listed line centers above. The conversion is

$$\lambda_{air} = 0.99972683\,\lambda_{vac} + 0.0107 - \frac{196.25}{\lambda_{vac}}$$

where both wavelengths are expressed in Angstroms.

If you prefer to work in nanometers (nm), go ahead and convert the wavelengths in your program. We recommend that you double-check your units, especially as Angstroms and nanometers differ by a factor of 10.

Question 4: Plot the Na I D spectrum in air wavelengths. Identify D1 and D2. Can you identify any of the other lines in the high resolution spectrum? One is an iron (Fe) line and another is a titanium (Ti) line. Can you identify others? Hint: you'll likely need to do a little research.

You'll want to focus on just the Na I D lines, so make two figures, one for each feature, and restrict your plot axes to "zoom" in on each feature. We suggest plotting them separately and restricting your x-axes to roughly 5888-5892 for the D2 line and 5894-5898 for the D1 line, but feel free to pick whatever works for you. Look carefully at the features for each line.

Question 5: What do you observe about each line feature? List some general observations. Hint: are they both at the same depth? Do they both appear to be symmetric? etc…

Part 3:

Now, compute a model solar Na I D1 & D2 line using the *one-layer model atmosphere* in the python script we provided. Assume LTE holds for the continuum processes at these wavelengths (being dominated by H- bound-free transitions), the assumption of LTE line formation implies that you can simply set

$$S_\lambda = B_\lambda(T)$$

The one-layer atmosphere function in the script then solves:

$$I_\lambda = I_0\, e^{-\tau_\lambda} + B_\lambda\,(1 - e^{-\tau_\lambda})$$

where the optical depth is given by the Voigt profile. Since we are not exploring the other properties of the medium in our toy model atmosphere (e.g. composition, density, thickness…) it is OK to just assume the same τ_λ throughout the atmosphere.

Adjust the parameters of the Voigt profile (α and γ) and the conditions of the one-layer atmosphere (T and I_0) to achieve the best fit possible. You are free to just tune these parameters by hand, and not use any kind of “solver”. The intensities in the provided spectrum are *relative* to the continuum (hence the baseline level of intensity = 1). To make things easier, we suggest you keep the “surface” temperature fixed at 5770 K. This is the temperature of the medium we see radiance from (i.e. the “surface”). You should normalize your resulting spectrum by this Planck function as well. Importantly: use the same parameters to model both the Na I D1 and D2 lines.

Question 6: Qualitatively, how good is your fit? Discuss this in terms of how well your model fits both the wings and the core of the data for *both* the Na I D1 and D2 lines.

Question 7: Did your model fit the data better for for one line than the other? Why/why not? Do you feel like either/both fits were very good? Why or why not?

Question 8: Which parameters did you have to adjust most/least? What does this tell you about the conditions where the Na I D lines form in the atmosphere of the Sun?

For the above questions, plot a well labeled figure showing your best fit (show both the data points and the fit in your figure). Once you are comfortable with how the one-layer atmosphere works, you are ready to move onto the next step.

Part 4:

Now you will compute a model solar Na I D1 & D2 line using the *two-layer model atmosphere* in the python script we provided. Assume LTE holds for the continuum processes at these wavelengths as you did for the one-layer model.

Adjust the parameters of the Voigt profiles (one profile per layer) and two-layer atmosphere to achieve the best fit possible. Keep in mind that you are essentially solving the one-layer atmosphere twice, which means you have a few additional "knobs" to adjust as the light travels from one layer to the next. Be sure to scale the normalization of each of your Voigt profiles by a factor of 0.5. This ensures that the each layer of your atmosphere contributes equally to the overall emergent intensity.

Question 9: How good is your fit? Discuss this in terms of how well your model fits both the wings and the core of the data for both the Na I D1 & D2 lines.

Question 10: Did your model fit the data better for for one line (1 or 2) than the other? Why/why not? Do you feel like either/both fits were very good? Why or why not?

Question 1`: Which parameters did you have to adjust most/least? What does this tell you about the conditions where Na I D lines form in the atmosphere of the Sun?

Question 11: Which model worked better? In other words, which atmosphere (one-layer or two-layer) model did a better job or render a more realistic representation of the Na I D lines? Explain which model atmosphere (one- or two-layer) best modeled the spectrum and why you think it did a better job.

For the above questions, plot a well labeled figure showing your best fit to the data (show both the data points and the fit in your figure).

Question 13: How could your model be improved? What else could/should your model atmosphere include to render an even more accurate/realistic fit to the data? You may need to do a little research on model atmospheres to see what more advanced models include and why.

When you are happy with your figures and responses to the questions above:
Question 14: Write a brief summary (~200 words or 1-2 well-written paragraphs) summarizing your results and what you have learned from this atmosphere modeling exercise.

Turn in your write-up, including the labeled plots, as a PDF. Remember to include an attribution for any group work! Also turn in your code or Jupyter Notebook used to solve the assignment. Note: we'd like to be able to run your code to check that it actually works, so be sure (if using Jupyter notebooks) to check that it runs "top down"!

Use the Dropbox upload link. **DUE: Feb 8, 11PM PST**