The genesis of geostatistics in gold and diamond industries

Danie Krige[1] and Wynand Kleingeld[2]

[1] Florida Hill, South Africa
[2] De Beers MRM, UK

1 Introduction

Geostatistics has had a phenomenal half a century of development and achievements, and to which George Matheron has made invaluable contributions throughout. The genesis of geostatistics is clearly linked to South Africa and more specifically to gold mining in the Witwatersrand basin, which started towards the end of the 19^{th}century. In the later stages of the development of geostatistics the problems inherent in the valuation of diamond deposits presented a new field for geostatistical contributions.

2 The Influence of Gold and the Origin of Geostatistics

For economic reasons this gold mining was and still is conducted on a selective block basis and calls for intensive and regular sampling of underground exposures of the ore bodies. This resulted in the accumulation over many years of massive data sets conducive to statistical analysis and the study of frequency distribution models. In the pre-geostatistics period ore reserve blocks were valued on the arithmetic averages of samples from the block peripheries; as these blocks were being mined the advancing stope faces inside the blocks were also sampled regularly to yield extensive follow-up block values. Comparisons of these follow-up grades with the original block estimates provide an obvious opportunity for statistical analyses such as frequency distribution studies and classical correlations. However, this opportunity remained dormant until the 1940's.

At that time, extensive exploration of virgin properties in the new South African gold fields by deep drilling was also taking place. Grade estimates for these new mines had to be based on limited sets of drill hole grades with no proper basis for estimating the effects of selective mining to economic cut-off grades. Fortunately, the ideal venue for access to all this data from numerous existing mines and from the new gold fields was provided by the records in the Government Mining Engineer's Department where the first author was privileged to be employed in the late 1940's. He was introduced to the statistical approach for the processing of this data by the earlier initial work by Sichel [27], Ross [26] and De Wijs [3, 4]. This led to a first set of publications ([8, 9]) which in turn introduced, inter alia, Allais [1] and Matheron [15] to the subject.

This paper is, thus confined to ore valuations in the mining field.

2.1 Frequency Distribution Models

The skew nature of the gold grade frequency distributions was first observed by Watermeyer [33] and later studied by Truscott [32]. But these studies were done without the knowledge of the lognormal model and were unsuccessful.

Real progress was absent until the 1940's when Sichel [27] suggested the use of **the lognormal model**. He was a classical statistician and in the mining field he concentrated his efforts on frequency distribution models. He developed the T-estimator with its appropriate confidence limits [28]. This estimator is more efficient than the arithmetic mean, but is strictly valid only for a random set of data which follows the lognormal model exactly. Departures from the 2-parameter lognormal model, as observed in practice, were largely overcome with the introduction in 1960 of the 3-parameter lognormal model [10] which requires an additive constant before taking logarithms. However, there were still cases which could not be covered properly by the 3-parameter lognormal, and led to the introduction of the more flexible Compound Lognormal Distribution [30], originally developed by Sichel for diamond distributions. This model is very flexible and caters specifically for a tail of high values which is much longer than that for earlier models. This development is covered in the second part of this paper.

2.2 The genesis and early development of geostatistics

Geostatistical concepts originated in the late 1940's when Ross applied the lognormal model to a variety of actual gold grade data [26], de Wijs showed how the differences between individual grades depended on their distances apart [3, 4] and particularly when the basic concept of gold ore grades as a variable with a spatial structure was introduced in 1951/2 [8, 9]. The objective of this latter work was to develop more efficient grade estimates for new mines and for ore blocks on existing mines.

The first paper [8] was aimed at finding an explanation for the experience on all the gold mines for many decades, of ore reserve estimates during subsequent mining consistently showing a significant under-valuation in the lower grade categories and the reverse for estimates in the higher grade categories. Classical statistical correlation and regression analyses proved this to be an unavoidable result of block estimates subject to error and to conditional biases [8]. In the proper perspective it was essential to no longer view the peripheral data used for individual block estimates and the ore blocks themselves in isolation. It was essential to see **the peripheral data as part of an extensive spread of data (the data population)** in stopes and development ends in the relevant mine section; also to accept the grade of the ore block concerned as part of a collection of block grades (both intact and already mined out), i.e. as a member of **a population of oreblock grades**.

In this way, **the spatial concept was introduced as well as the concept of support** in moving from individual sample grades (point supports), to block grades. A mathematical model was first set up of the lognormal distribution of actual block values in a mine section. The errors in assigning the limited peripheral grades to the blocks were super imposed on the actual grades to yield the corresponding distribution of block estimates. On correlating these two sets of block values on a classical statistical basis, the averages of the actual block values relative to the corresponding block estimates in grade categories could be observed, i.e. the curvilinear regression of actuals on estimates. This was a theoretical follow-up exercise to simulate the results actually observed in practice. It provided the statistical explanation of the natural phenomena of the unavoidable under- and over-valuation features as mentioned above, i.e. the **inherent conditional biases**. The use of the lognormal model also covered **the curvilinear nature of the regression trend** as observed in practice.

The very fact that a correlation exists between the block estimates and the internal actual grades emphasises the presence of a spatial structure. With the explanation of these conditional biases, the initial application in practice was to apply the trend, or regression, of actuals (or follow-ups) versus estimates –as observed from the mine records or modelled geostatistically– to the orthodox block estimates so as to eliminate these biases. As the regressed estimates were, in effect, weighted averages of the peripheral estimates and the global mean grade of the mine section, it was **the first application of what became known as kriging. It can be labelled Simple Elementary Kriging**, being based on the spatial correlation between the peripheral values and the actual grades of the ore inside the ore blocks, and giving proper weight to the data outside the block periphery via the mean.

During the 1950's several large gold mines introduced regression techniques for their ore reserve estimates on a routine basis. It is instructive to observe that on the gold mines the improvement in the standard of block valuations due to the elimination of conditional biases accounts for some 70% of the total level of improvement achievable today with the most sophisticated geostatistical techniques. It is for this reason, that so much stress is placed on the elimination of conditional bias (so called "conditional unbiasedness").

In the second paper [9], **the spatial structure of the grade data** from 91 drill holes in the main sector of the new Orange Free State gold field was defined by the log-variance/log-area relationship (see Fig. 1). This demonstrated the so-called **Krige formula** (point variance within a large area minus the average point variance within ore blocks = the variance of block values within the large area) as well as the so-called **permanence of the lognormal model** for different support sizes. On this basis the lognormal model for the expected distribution of ore block grades was modelled with a global mean grade as estimated from the 91 drill hole grades. It led successfully to meaningful tonnage and grade estimates for a range of cut-off grades, i.e. the **first version of the now well known tonnage-grade curve**. Without a proper block distribution model the orthodox approach would have based the tonnage-grade estimates directly on the individual drill hole grades with seriously misleading results globally and for individual mines.

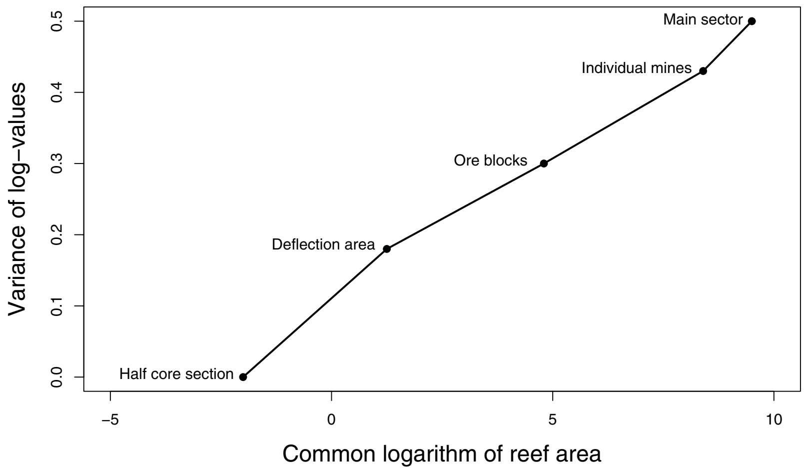

Fig. 1. Dispersion variance versus domain size: example of gold data from the Orange Free State. The horizontal axis represents the area of the domain in logarithmic scale, from 10^{-5} to 10^{10} ft^2

These developments were published in 1951/1952 and aroused world-wide interest in the subject now known as geostatistics, particularly in French circles. Matheron and Duval translated these two papers and re-published them in 1955 together with two personal contributions by them ([15, 5]). This was followed by a paper on exploration prospects in the Sahara by Allais [1]. Matheron [15] in particular covered the more theoretical background underlying the two basic South African papers and the models involved in all this work. He showed, for example, that **the permanence of the lognormal model** can only logically apply where a spatial structure is present and that the positive correlation between the log variances and the mean grades of lognormal distributions – the so-called **proportional effect** – is an inherent feature of the lognormal model. This contribution by Matheron was accompanied and/or followed in the 1950's and 1960's by numerous other notes and publications in French and the introduction of **the theory of regionalised variables**. Matheron's first geostatistical papers in English were in the 1960's ([16, 17]), followed by an English monograph in 1971 ([18]) which covered the theory in detail.

Modelling of the spatial structure is basic to any geostatistical approach. The original approach in South Africa [9] was followed by extensive correlations of pairs of grades for different lags and the results were modelled by **correlograms and covariograms** [11, 12] and used in multiple regression techniques to arrive at the relative weights to be applied to the data available for a block valuation. This was already introduced on a routine basis on some of the large gold mines for ore reserve estimates in the early 1960's and was called weighted moving average estimates until at Matheron's insistence the term **kriging** prevailed. In the mean time Matheron covered a continuous series of further developments of geostatistical models based on the now generally applied **variogram** for defining spatial structures.

The critical need for identifying likely changes in the characteristics of the spatial structure between sections within the ore body, such as grade continuity levels, anisotropy directions, etc. was also already stressed during the 1960's. This basic tenet of geostatistics has been and is still widely met in practice via the linkage of these characteristics with changes in **geological and/or mineralogical parameters** which can more readily be modelled.

2.3 The main basic tenets of geostatistics

Virtually all the fundamental concepts and tenets of geostatistics were established in these early years and are still applicable today.

1. The use of **appropriate parametric distribution models** when practical for confidence limits of estimates of the mean grade. Various non-parametric approaches have been developed, but face the common problem that the pattern of the observed point distribution accepted as the model can be misleading for the upper tail of the distribution unless a very large data base is available.
2. **Spatial structures** generally present in ore bodies and with characteristics associated with geological and mineralogical features.
3. The concepts of **support sizes and types, the proportionally effect** and models for estimating the **SMU (Selected Mining Unit) block distribution** parameters directly or indirectly from the point value distribution.
4. **Kriging** for block estimates. Many types of kriging have been developed but all allow to eliminate - or at least reduce - the conditional bias.
5. If block valuations are done before the actual selective mining stage - when the final data becomes available - the estimates will be **smoothed** and have to be post-processed. Meaningful post-processing techniques were set up after the early stages of the development of geostatistics, as well

as the so-called non-linear geostatistics (see the paper by Rivoirard in the present volume, page 17).

2.4 Conditional unbiasedness

Early developments in South Africa generally retained strong links with classical statistics, particularly through the preference on the gold mines for simple kriging. This is essentially a classical multiple regression technique based on the mean grade for the ore body or local part thereof and on the corresponding spatial structure to provide the necessary covariance's for the solution of the matrix equations involved. However, where block valuations are based on exploratory data to be supplemented at the final mining stage by additional data more closely spaced, the earlier block estimates will be smoothed compared to the final estimates and have to be post-processed before declaring ore reserves and doing mine planning and feasibility studies.

Arising from this problem and the fact that the mean grade as used in simple kriging itself changes within an ore body, a general preference developped for ordinary kriging, which relies only on the data as accessed for the kriging of each block. There is no objection, in principle, to this approach provided the data accessed is adequate to effectively provide a close grade level for the local area encompassing the block. This will ensure conditional unbiasedness but will not overcome the smoothing problem. Also the effort time and cost involved in kriging each block on a relatively large data base required to meet this objective (say 50 instead of only 5, 10 or even 20 values), led to the widespread use of a limited search routine, and lately also to simulation as an alternative. These practices can reduce or eliminate smoothing, but unfortunately, re-introduce conditional biases as prevailed in the pre-geostatistical period and cannot be post-processed unless the conditional biases are first removed. Although such estimates could provide acceptable global tonnage-grade figures for the whole ore body, they could still be seriously conditionally biased for sections of the ore body as will be mined sequentially over short time periods [13] and thus be unacceptable.

3 The influence of the specificity of the diamond mining industry on the development of geostatistics

3.1 Introduction

Diamonds are a unique commodity in the realms of mining and its associated disciplines. The particulate nature of the diamond affects the processes of exploration, evaluation, mining and metallurgy and especially the way in which diamonds are valued and sold.

The evaluation of alluvial diamond deposits specifically drew the attention of some of the greatest minds in the field of geostatistics and this would eventually give rise to novel ways of dealing with sampling and estimation techniques in the case of discrete particle deposits.

The diagram shown in Fig. 2 illustrates the complex nature of diamond deposits compared with other mineral commodities. The average grade for diamond deposits is generally very low and a high degree of geological discontinuity exists, particularly in the case of marine placer deposits where selective mining is absolutely essential.

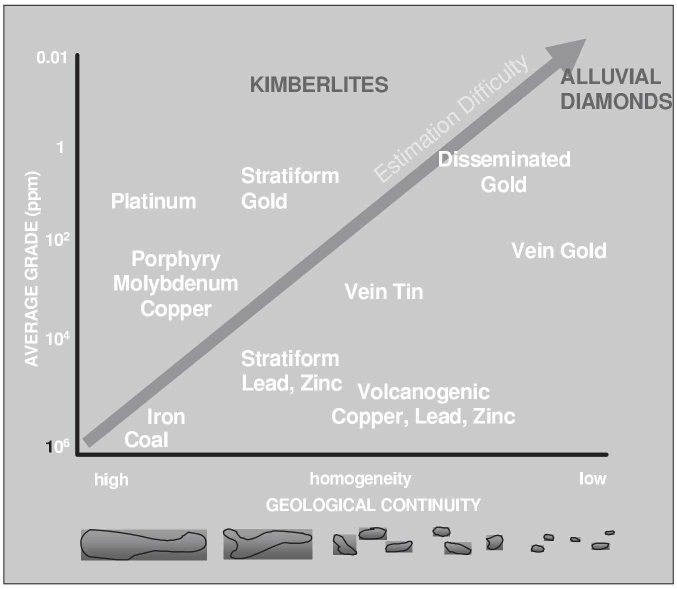

Fig. 2. Plotting the average grade versus the mineralization continuity of various minerals

Groundbreaking work by Sichel during the early seventies [29] gave rise to a statistical approach on how to evaluate these deposits, and models such as the compound Poisson for diamond density distributions [20, 21] and the Compound Lognormal for diamond size distributions were developed. The adaptation of these models had significant impact later on the estimation of other minerals [31].

In the 10 years from 1980 to 1990 a substantial research effort [24] was directed towards understanding the following issues;

1. The complex nature of the geology that gave rise to the discrete particle mineral deposit.
2. The problems associated with the sampling of deposits of this nature and the fact that the sampling could produce non representative results since the sample size is smaller than the trap sites in which the particles occur.

3. The statistical models required to cater for the extremely skewed sampling data, with the emphasis on smoothing the shape of the curve, and to increase sample representavity.
4. Methods to obtain local reserve estimates, including confidence limits, which require a local distribution density function.
5. The need to produce bivariate representation of the probability mass functions for the non-linear kriging procedures used.

The outcome of the research culminated in a geostatistical approach to the development of the ideas of cluster sampling, discrete isofactorial models that could be used in the Disjunctive Kriging estimation process to develop estimates and the Cox Process for discrete particle simulation.

In essence it could thus be stated that the research undertaken on the evaluation of alluvial diamond deposits gave rise to discrete geostatistics.

The research would not have been possible without the substantial input from people such as Matheron, Sichel and others such as Lantuéjoul and Lajaunie.

3.2 Geology

In the research, ancient beach deposits were considered where mineralization is largely confined to basal gravel horizons. Those are located in one or more marine abrasion platforms, usually cut into schists and phyllites.

The shist bedrock is extensively gullied by wave action assuming a characteristic pattern well developped. The gullies are controlled by the slope and the structure of the bedrock and by the presence of boulders and gravel. The pot-holes and gullies act as particle trap sites and can contain high concentrations of the mineral.

Though some trap sites can contain high concentrations they could be surrounded by sites that have low concentration or even be barren. This high degree of variability is related to the complex interaction of geological controls during deposition. The chance of finding a particle in this type of deposit is related to the chance of sampling a trap site and the distribution of particles in the trap site.

The distribution of particles is different for each beach, corresponding to the different marine transgressions and is related to the length of stillstand of the sea which influenced the degree of abrasion of the marine platform and the degree of reworking that took place. It is also influenced by the amount of mineral bearing gravel that was available during the transgression period.

Thus the distribution of particles is directly related to the presence of particles in the gravel, the quantity and quality of the trap sites and to the degree of reworking of the gravel.

In a certain area a characteristic gully pattern is normally formed with parallel gullies at relatively constant distances apart. The trap sites in these gullies also show characteristic patterns with the typical size of a trap site 5m along and 3m across a gully, but variable from area to area.

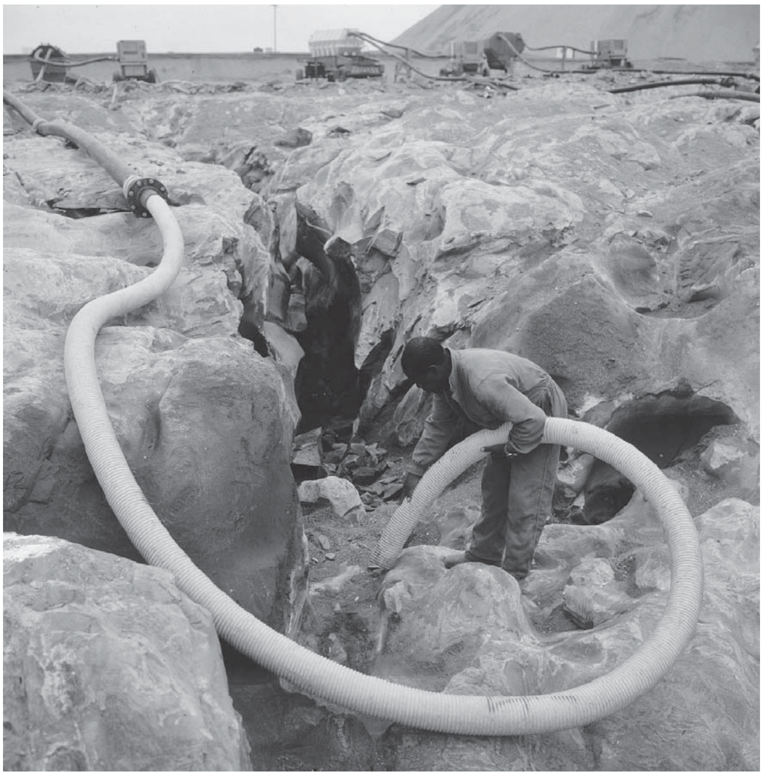

Fig. 3. Mining an alluvial deposit. The diamonds are trapped in the pot-holes and the gullies of the bedrock

3.3 Sampling

From the outset the sampling of these types of deposits posed significant problems, even at elevated sample support sizes (5 square meters) up to 90% of the samples did not contain any particle. In contrast, there are rare occurrences where several hundred particles were recovered in one sample unit located over a natural trap site such as a deep pot-hole.

Based on the geological model it became obvious that normal sampling theory as applied to homogeneous mineralization was not applicable in the case of marine placers where mineralization was concentrated in trap sites [29].

Other examples occur in vein and alluvial deposits of diamonds and gold. In such deposits, two different factors account for the grade variability at two different scales, firstly the spatial distribution of the trap sites, and secondly the dispersion of the mineralization within the trap sites.

The existence of several scales of variability makes sampling a very complex operation. As a matter of fact, a set of samples of a given size may not account for all the scales of variability. Using many small samples, traps are well detected, but their mineralization contents are poorly assessed. Using a limited number of larger samples, the quality of the traps is better known, but it becomes more difficult to assess the distribution of the traps sites.

The methodology involved in sampling such highly dispersed type ore bodies was addressed in [7]. The paper presents several results on the sampling of highly dispersed type ore bodies and highlights the two major problems encountered when sampling under such conditions.

It also addresses the problem of defining a representative sample support size and making it operatory by resorting to a cluster sampling approach.

During the research it became evident that limited experience exists in the field of in situ sampling, especially when stratification is present.

3.4 Estimation

In his book "Estimer et choisir" [19], Matheron discusses certain fundamental differences between the approaches adopted in statistics and geostatistics. Fundamentally statistics is involved in the estimation of parameters for a chosen probability model, whereas geostatistics is involved with the estimation of a spatial average for a natural phenomenon.

However, in the case of discrete particle deposits where the bases for estimation (variogram and histogram) are not well defined due to the non representative nature of sampling, the necessity of using statistical models in estimation was evident [6]. Matheron highlights that the model is not the deposit and notes that substantial research is needed to explain the variation of model parameters with the geology of a deposit. Such work was carried out by Oosterveld *et al.* [25].

The need for introducing a statistical model for local reserve estimation was clearly indicated and research was done to provide a method to estimate the number of particles expected in an in situ reserve block of specific support size.

The introduction of geostatistics contributed to the understanding and quantification of the risk associated with grade estimation. Uncertainty was defined in terms of confidence limits derived from a modelled probability distribution for the grades of the mining blocks.

The skewness of the sample grades gave rise to skew distributions for the block estimates and their error distributions. This led to research in the field of non-linear kriging, more specifically disjunctive kriging under appropriate discrete isofactorial models [22]. A suitable statistical model which represents a discrete type of particle density distribution had to take into account the distributional characteristics of the trap sites as well as the particles contained within the trap sites. The most important problem in mining geostatistics, i.e. that of change of support, was also addressed. The inference of the parameters is a challenging problem and practical aspects of implementing discrete isofactorial methodology are presented in [14].

The high degree of geological discontinuity also led to research towards a connectivity index in mining. This problem occurs when mining at high cut-off grades where only a fraction of the selective units is above cut-off and where the blocks are split into disjoint patches that cannot be accessed economically during mining [2].

4 Conclusion

The immensity of the South African deposits amongst others that of gold and diamonds has produced the background to the development of geostatistics. The evaluation of deposits drew the attention of the most prominent geostatisticians of our time. Fortunately, we had people such as Matheron and Sichel to assist in the phenomenal development that took place in the last 50 years.

References

1. Allais, M.: Method of appraising economic prospects of mining exploration over large territories; Algerian Sahara Case Study. Man. Sci., **3-4**, 285–347 (1957)
2. Allard, D., Armstrong, M., Kleingeld, W.J.: The need of a connectivity index in mining geostatistics. In: Dimitrakopoulos, R. (ed) Geostatistics for the next century. Kluwer, Dordrecht (1994)
3. De Wijs, H.J.: Statistics of ore distribution. Part 1. J. of the Roy. Netherl. Geol. and Min. Soc., **13-11**, 365–375 (1951)
4. De Wijs, H.J.: Statistics of ore distribution. Part 2. J. of the Roy. Netherl. Geol. and Min. Soc., **15-1**, 12–24 (1953)
5. Duval, R.: A propos de l'échantillonnage des gisements. Annales des Mines, **12**, 76–79 (1955)
6. Kleingeld, W.J.: La géostatistique pour des variables discrètes. PhD Thesis, Ecole des Mines de Paris (1987)
7. Kleingeld, W.J., Lantuéjoul, C.: Sampling of orebodies with a highly dispersed mineralization. In: Soares, A. (ed) Geostatistics Tróia'92. Kluwer, Dordrecht (1993)
8. Krige, D.G.: A statistical approach to some basic mine valuation problems on the Witwatersrand. J. Chem. Metall. Min. Soc.South Afr., **52**, 119–139 (1951)
9. Krige, D.G.: A statistical analysis of some of the borehole values in the Orange Free State goldfield. J. Chem. Metall. Min. Soc.South Afr., **53-3**, 47–64 (1952)
10. Krige, D.G.: On the departure of ore value distributions from the lognormal model in South African gold mines. J. South Afr.Inst. Min. Metal, **61**, 231–244 (1960)
11. Krige, D.G.: Statistical applications in mine valuation. J. Inst. Mine Survey South Afr., **12-2**, 45–84 (1962)
12. Krige, D.G.: Statistical applications in mine valuation. J. Inst. Mine Survey South Afr., **12-3**, 95–136 (1962)

13. Krige, D.G., Assibey-Bonsu, W.: Valuation of recoverable resources by kriging, direct conditioning and simulation. In: Jiang, Y. (ed) Proceedings of the 29th symposium on application of computers in the mineral industries. Balkema, Rotterdam (2001)
14. Lajaunie, C., Lantuéjoul, C.: Setting up a general methodology for discrete isofactorial models. In: Armstrong, M. (ed) Geostatistics. Kluwer, Dordrecht (1989)
15. Matheron, G.: Application des méthodes statistiques à l'évaluation des gisements. Annales des Mines, **12**, 50–75 (1955)
16. Matheron, G.: Principles of geostatistics. Econ. Geol., **58**, 1246-1266 (1963)
17. Matheron, G.: Kriging, or polynomial interpolation procedures? Can. Min. Met. Bull., **11**, 240–244 (1967)
18. Matheron, G.: The theory of regionalized variables and its applications. School of Mines, Paris (1971)
19. Matheron, G.: Estimer et choisir. School of Mines, Paris (1978)
20. Matheron, G.: Quatre familles discrètes. Technical Report N-703. Centre de Géostatistique, Ecole des Mines de Paris (1981)
21. Matheron, G.: Deux autres familles, non moins discrètes mais plus nombreuses. Technical Report N-717. Centre de Géostatistique, Ecole des Mines de Paris (1981)
22. Matheron, G.: Une méthodologie générale pour les modèles isofactoriels discrets. Sciences de la Terre, **21**, 1–64 (1984)
23. Matheron, G.: Two classes of isofactorial models. In Armstrong, M. (ed) Geostatistics. Kluwer, Dordrecht (1989)
24. Matheron, G., Kleingeld, W.J.: The evolution of geostatistics. Proceedings of the 20th APCOM international symposium. SAIMM, Johannesburg (1987)
25. Oosterveld, M.M., Campbell, D.C., Hazel, K.R.: Geology related to statistical evaluation parameters for a diamondiferous beach deposit. South Afr. Inst. Min. Met., 129–136 (1987)
26. Ross, F.W.J.: The development and some practical applications of statistical value distribution theory for Witwatersrand auriferous deposits. MSc thesis, Univ. of the Witwatersrand (1950)
27. Sichel, H.S.: An experimental and theoretical investigation of bias error in mine sampling with special reference to narrow gold reefs. Trans. Inst. Min. Metall. Lond., **56**, 403–473 (1947)
28. Sichel, H.S.: New methods in the statistical evaluation of mine sampling data. Bull. Inst. Min. Metall. Lond., 261–288 (1952)
29. Sichel, H.S.: Statistical valuation of diamondiferous deposits. In: Salamon, M.G.D., Lancaster, F.H. (eds) Proceedings of the 11th APCOM international symposium. J. South Afr. Inst. Min. Met., Johannesburg (1973)
30. Sichel, H.S., Kleingeld, W.J., Assibey-Bonsu, W.: A comparative study of the frequency distribution models for use in ore valuation. J. South Afr. Inst. Min. Met., **92**, 235–243 (1992)
31. Sichel, H.S., Dohm, C.E., Kleingeld, W.J., New generalised model of observed ore value distributions. Trans. Inst. Min. Metall., **104**, 79–124 (1995)
32. Trustcott, S.J.: The computation of the probable value of ore reserves from assay results. Trans. Inst. Min. Metall. Lond., **39**, 482–496 (1929)
33. Watermeyer, G.A.: Application of the theory of probability in the determination of ore reserves. J. Chem. Metall. Min. Soc. South Afr., **19**, 97–107 (1919)