RT – An illustrated version

The equation for each of the 4 basic objects required in the RT subject is really simple when these objects are placed at the center of the frame of reference:

$X^2 + Y^2 + Z^2 = radius^2$ for a sphere centered on (0,0,0).

If your sphere is located somewhere else, the equation is more complex:

$(X - center\_X)^2 + (Y - center\_Y)^2 + (Z - center\_Z)^2 = radius^2$

As soon as you manipulate this equation to obtain the radius intersection, it becomes even more complex.

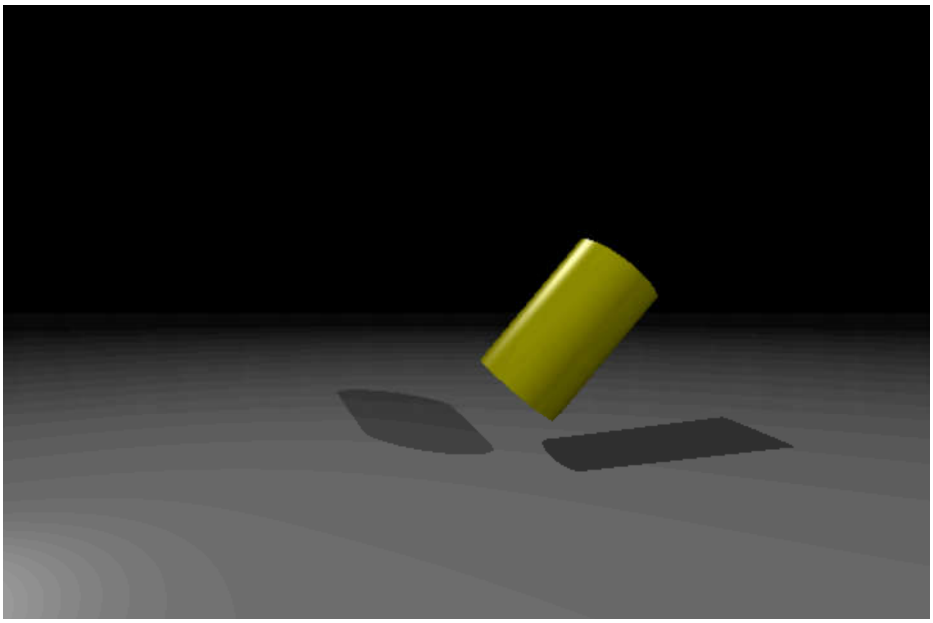
Even though no tutorials (or at least none I was able to find) talks about this, there is a way to simplify the maths, which is often the part that people hates, you're free to use it or not: it is basically the change of frame of reference using rotations and translations.

In summary:

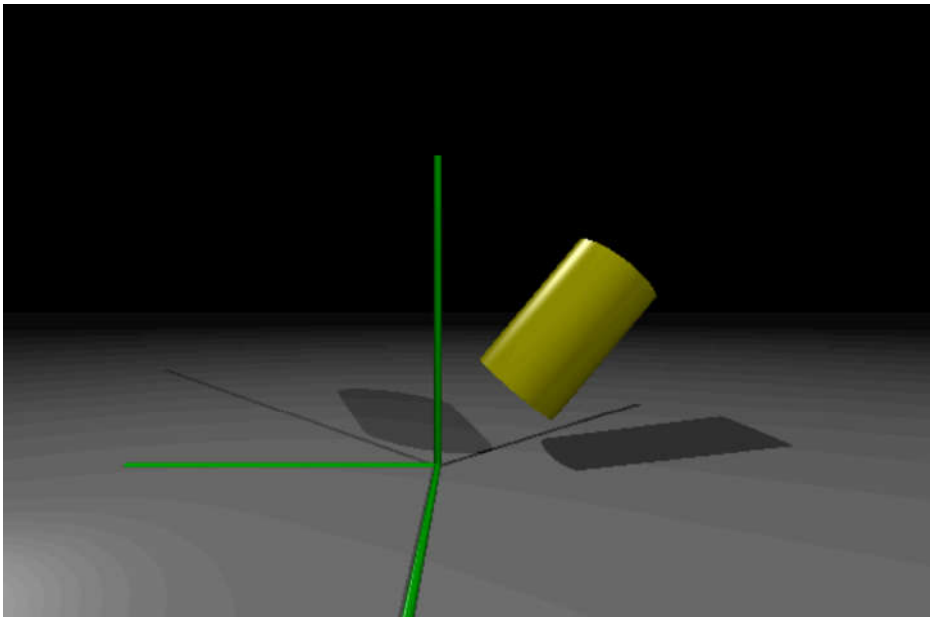
- Transform coordinates of the vantage point and of the system's principal ray to a system where the tested object is positioned at the center and on a specific axis (for the cone or the cylinder, you can use Z axis for example). I'll use afterwards the terms "real position" and "simple position".
- The object's equation becomes then easy to manipulate. Calculate the distance to intersection and coordinates of intersection point.
- Change the system back to its original state. The vantage point distance stays the same, coordinates changes. The change back is a necessity for interactions with other elements of the scene, mainly the lights.

Illustration

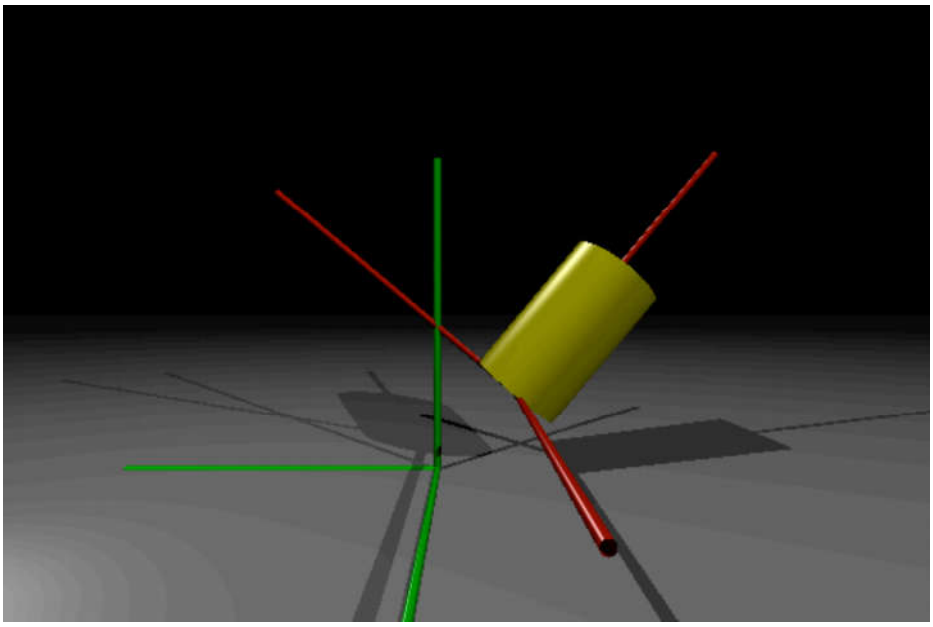
A cylinder (sliced for visibility reasons) in a random position in space. It's is real position. Its equation is complex to manipulate and to solve.



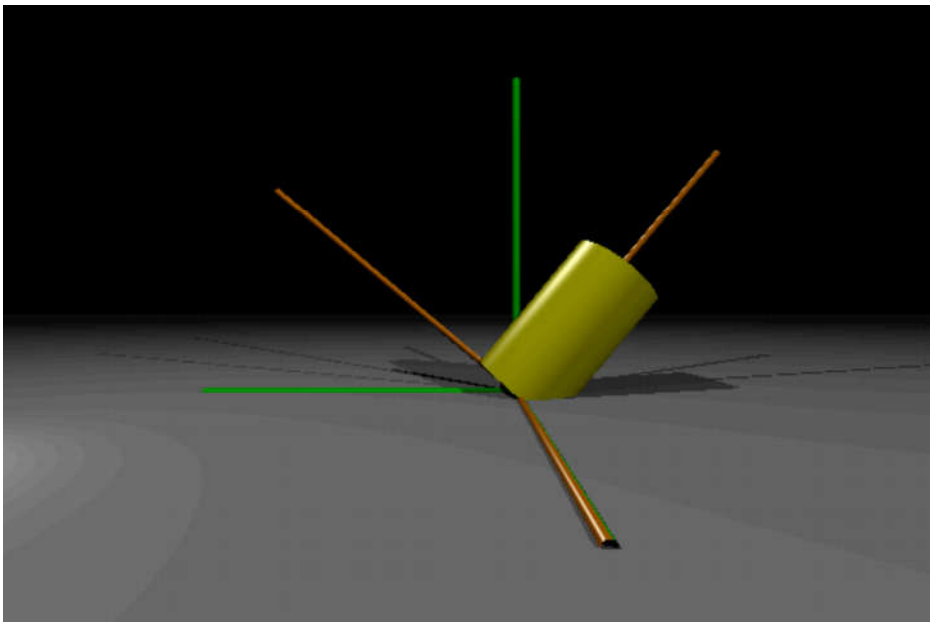
In green the main system



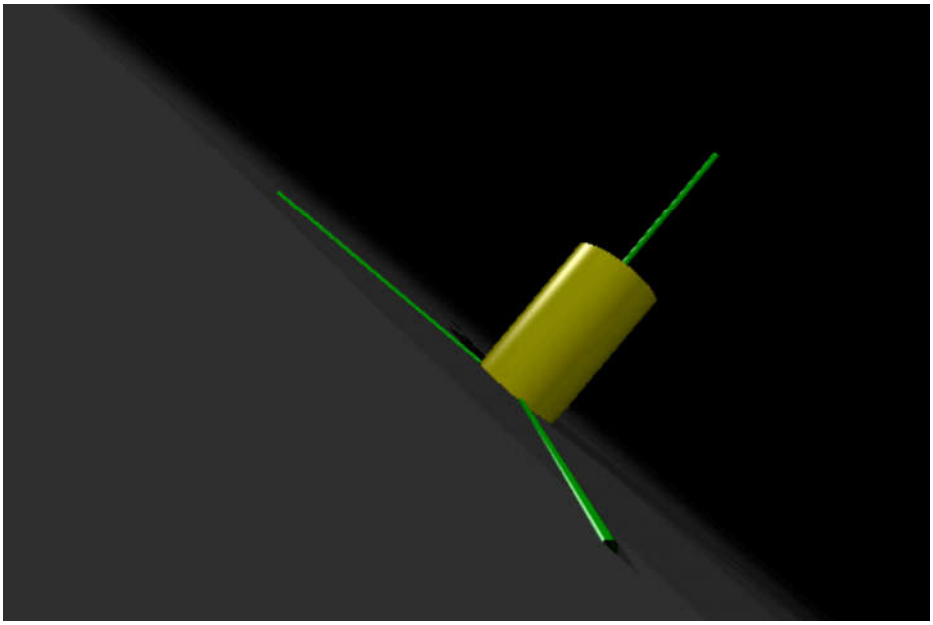
The cylinder's equation would be simpler to manipulate in the red system. The first possible solution is to ask ourselves what is the eye's and the vision vector's coordinates (the one that goes through a defined pixel) in the red system, the cylinder's formula being already known.



Another solution consists of keeping the eye and the cylinder together so that the intersection stays the same, and move the system to bring it to the green system starting with a translation to center both systems on the same origin…



…and then rotations to make everything identical



Both approaches require the same calculations, a translation and 3 rotations. To your matrix !!