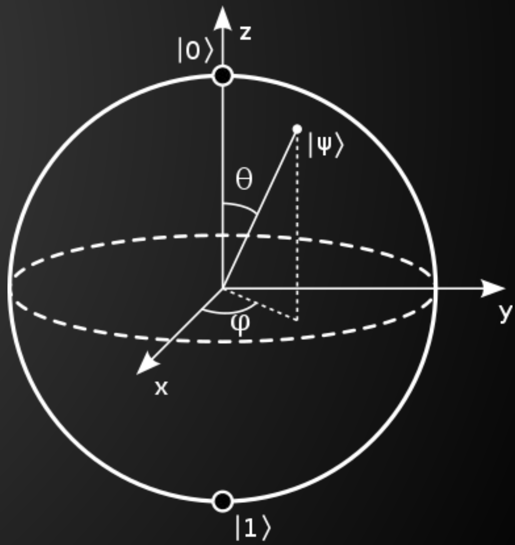
UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
School of Professional & Executive Development

Quantum Credit Risk Analysis

Presented by:
Arjun Puppala, Jose Ramon Aleman, Marti Ciurana

Tutor:
Giulio Gasbarri



Introduction

The credit risk analysis determines a borrower's ability to meet their debt obligations, by quantifying the risk of loss that the lender is exposed to.

Using quantum computers, we can get faster and more accurate solutions. Banks and asset managers optimize portfolios based on computationally intense models that process large sets of variables, and for this reason, they will be able to more effectively analyze large or unstructured data sets. The ability to speedily identify an optimal risk-adjusted portfolio is likely to create significant competitive advantage.

Problem

Description, Definition, and Parameters

Problem Description

We can describe the problem as the simple default risk that a lender takes on, in the chance that a borrower will be unable to make the required payments on their debt obligation. Measuring this probability is called the default risk. For example:

- A consumer may fail to make a payment due on a mortgage loan, credit card, line of credit, or other loan.
- A company is unable to repay asset-secured fixed or floating charge debt.
- A business or consumer does not pay a trade invoice when due.

Problem Definition

- We analyze the credit risk of a portfolio of k assets. The default probability of every asset follows a Gaussian Conditional Independence model

$$p_k(z) = F\left(\frac{F^{-1}(p_k^0) - \sqrt{\rho_k}z}{\sqrt{1-\rho_k}}\right)$$

- We are interested in analyzing risk measures of the total loss

$$L = \sum_{k=1}^{K} \lambda_k X_k(Z)$$

Problem Definition

- Value at Risk (VaR) is a measure of the risk of loss for investments. It estimates how much a set of investments might lose

$$\mathrm{VaR}_\alpha(L) = \inf\{x \mid \mathbb{P}[L <= x] \geq 1 - \alpha\}$$

- Conditional VaR (CVaR), also known as the expected shortfall, is a risk assessment measure that quantifies the amount of tail risk an investment portfolio has.

$$\mathrm{CVaR}_\alpha(L) = \mathbb{E}[L \mid L \geq \mathrm{VaR}_\alpha(L)]$$

Problem Parameters

1. Number of qubits used to represent Z, denoted by n_z

2. Truncation value for Z, denoted by, $z_{\max}$ i.e., Z is assumed to take equidistant values in $\{-z_{\max}, \dots, +z_{\max}\}$

3. Base default probabilities for each asset : $p_0^k \in (0, 1), k = 1, \dots, K$

4. Sensitivities of the default probabilities with respect to Z, denoted by $\rho_k \in [0, 1)$

5. Loss given default for asset k, denoted by λ_k

6. Confidence level for VaR / CVaR $\alpha \in [0, 1]$

Methods

MC, QAE, Grover's Algorithm, QFT, IQAE

Methods

To compare the classical and quantum approach, we use the computation of risk measures of a simple two-asset (2) portfolio. The methods used were:

- Classical Method: Monte Carlo (MC)
- Quantum Method: Iterative Quantum Amplitude Estimation (IQAE)

Value at risk (VaR) is a statistic that quantifies the extent of possible financial losses within a firm, portfolio, or position over a specific time frame.

The CVar quantifies the average expected loss.

We use Qiskit to compute the quantum method.

Classical Method: Monte Carlo (MC)

This method relies on repeated random sampling to obtain numerical results, using randomness to solve problems that might be deterministic in principle.

MC recalculates results over and over, using a different set of random numbers between min and max values. This process can be repeated thousands of times for a more accurate result.

The error goes down like the reciprocal of the square root of the number of points. The estimate has nothing to do with the dimension, in that sense, MC integration is indeed independent of dimension.

Quantum Amplitude Estimation (QAE)

This canonical version uses quantum phase estimation along with a set of m additional evaluation qubits to find an estimate $\tilde{a}$, that is restricted to the grid

$$\tilde{a} \in \{\sin^2(\pi y/2^m) : y = 0, \ldots, 2^{m-1}\}$$

More evaluation qubits produce a finer sampling grid, therefore the accuracy of the algorithm increases with m

Using a maximum likelihood post processing, this grid constraint can be avoided.

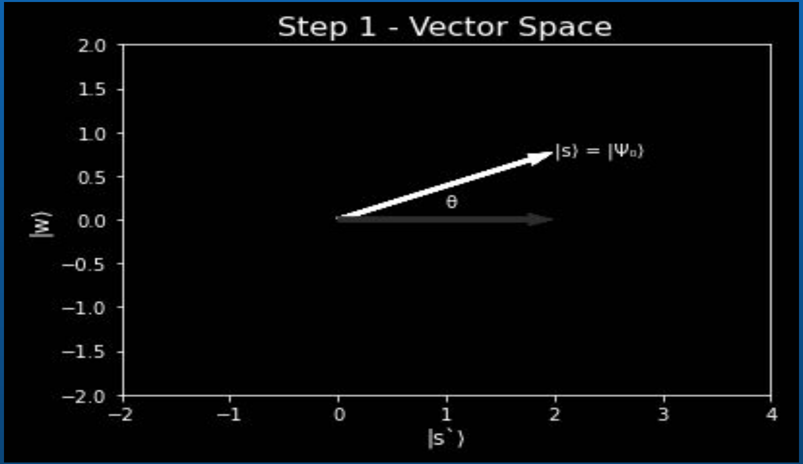
Grover’s Algorithm

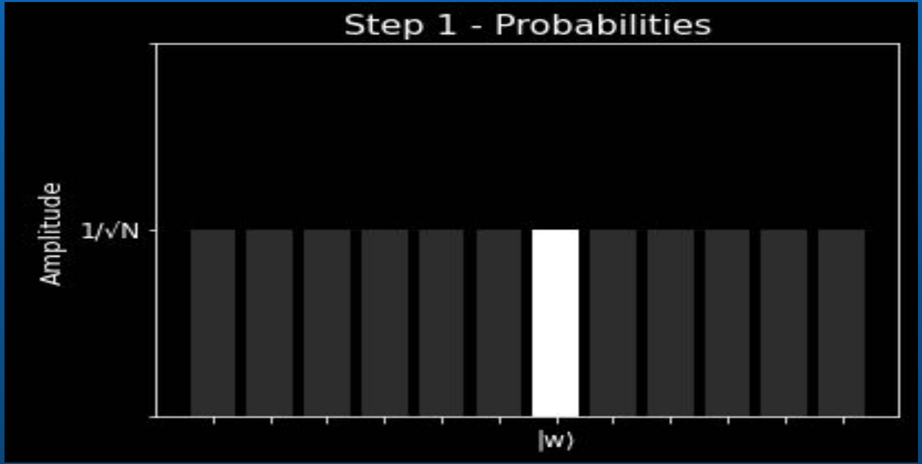
- This algorithm can speed up an unstructured search problem quadratically.

- Before looking at the list of items, we have no idea where the marked item is. Therefore, any guess of its location is as good as any other, which can be expressed in terms of a uniform superposition:

$$|s\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle$$

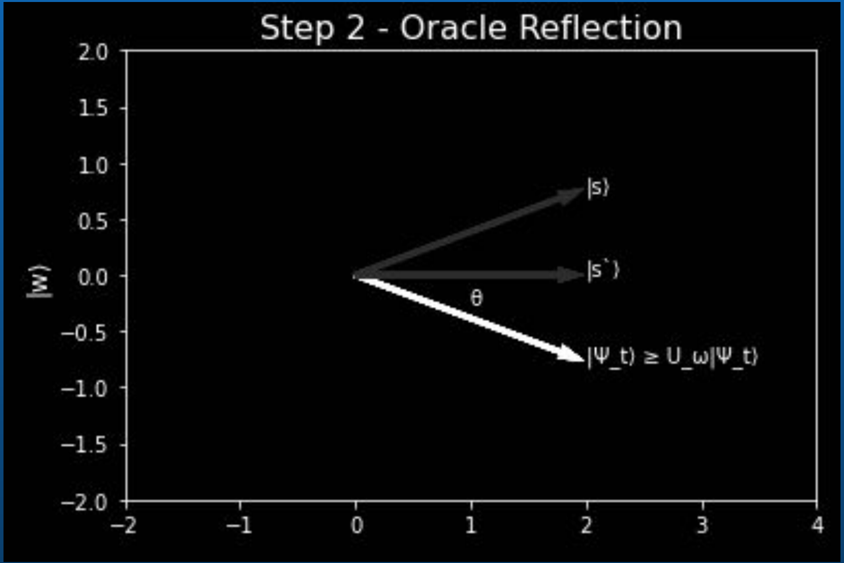
Step 1 - Initialize System

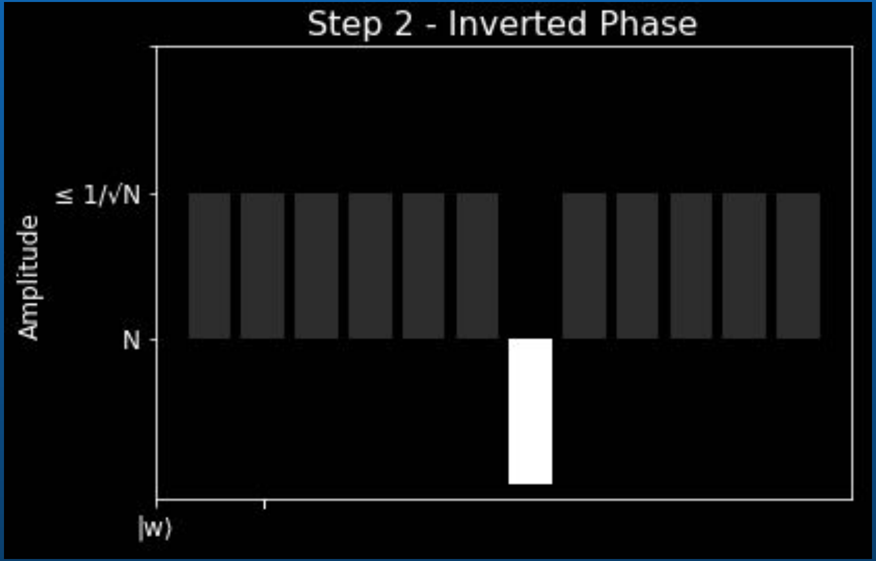




$$|s\rangle = \sin\theta|w\rangle + \cos\theta|s'\rangle \longrightarrow \theta = \arcsin\langle s|w\rangle = \arcsin\frac{1}{\sqrt{N}}$$

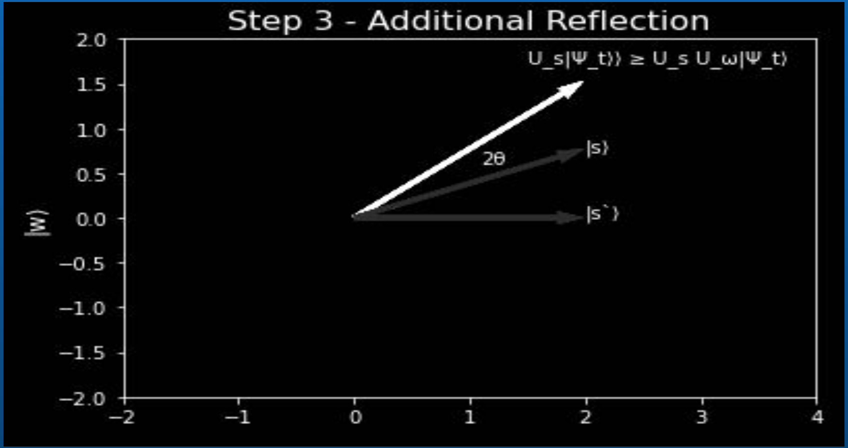
Step 2 - Oracle Reflection and Inverted Phase

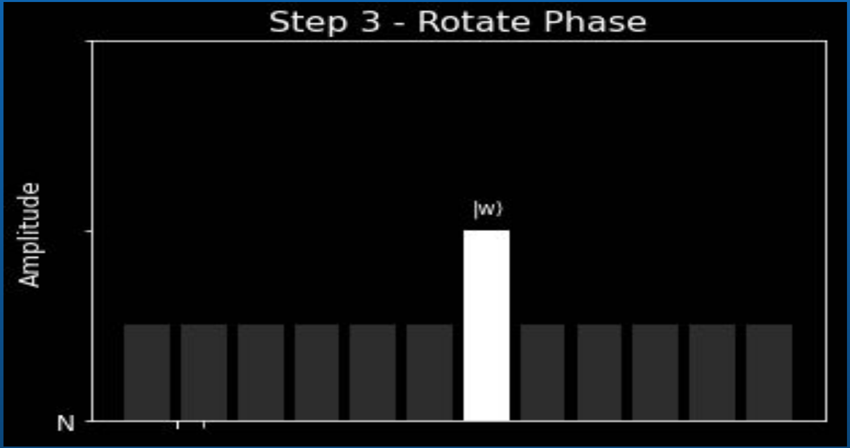




$$|s\rangle : U_s = 2|s\rangle\langle s| - 1$$

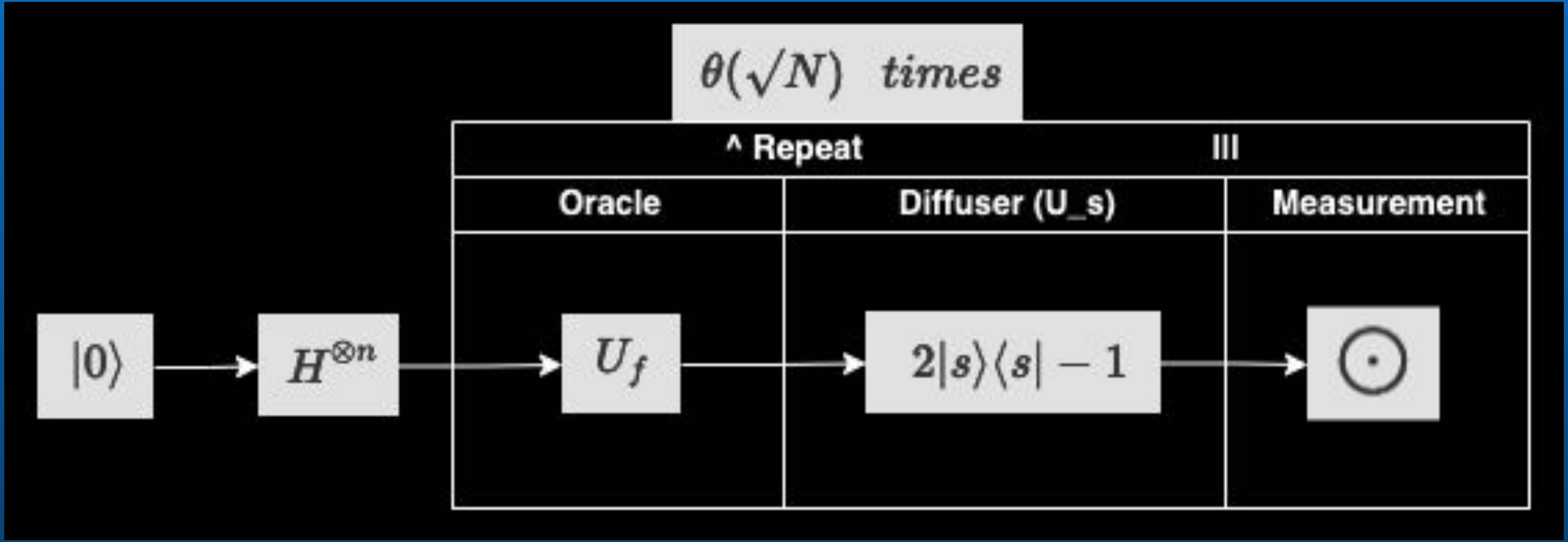
Step 3 - Additional Reflection and Rotate Phase





$$|\psi_t\rangle = (U_s U_f)^t |s\rangle$$

Grover’s Algorithm Diagram



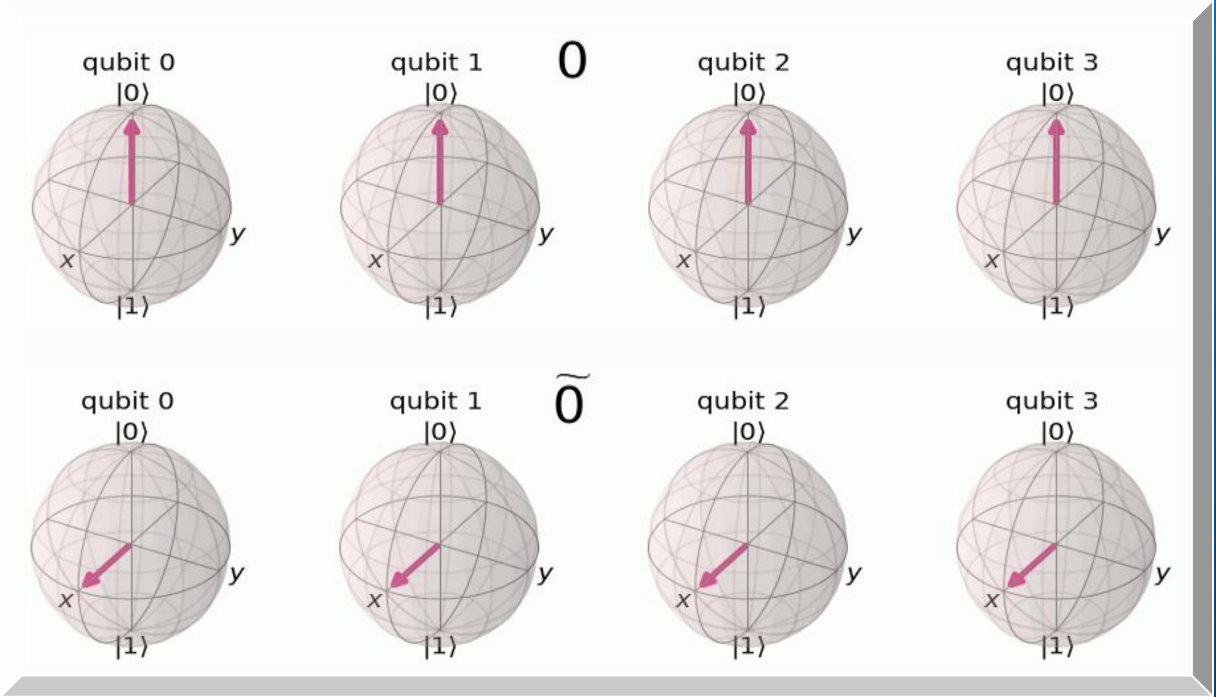
Quantum Fourier Transform (QFT)

This is the quantum equivalent implementation of the discrete Fourier Transform over the amplitudes of a wavefunction. The discrete Fourier transform acts on a vector $(x0, \ldots, x_{N-1})$ and maps it to the vector $(y0, \ldots, y_{N-1})$ according to the formula:

$$y_k = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} x_j \omega_N^{jk} \longrightarrow \begin{aligned} |X\rangle &= \textstyle\sum_{j=0}^{N-1} x_j |j\rangle \\ |Y\rangle &= \textstyle\sum_{k=0}^{N-1} y_k |k\rangle \end{aligned}$$

$$\omega_N^{jk} = e^{2\pi i \frac{jk}{N}}$$

QFT - Counting in Fourier basis



The number we want to store dictates the angle at which each qubit is rotated around the Z-axis

Iterative QAE (IQAE)

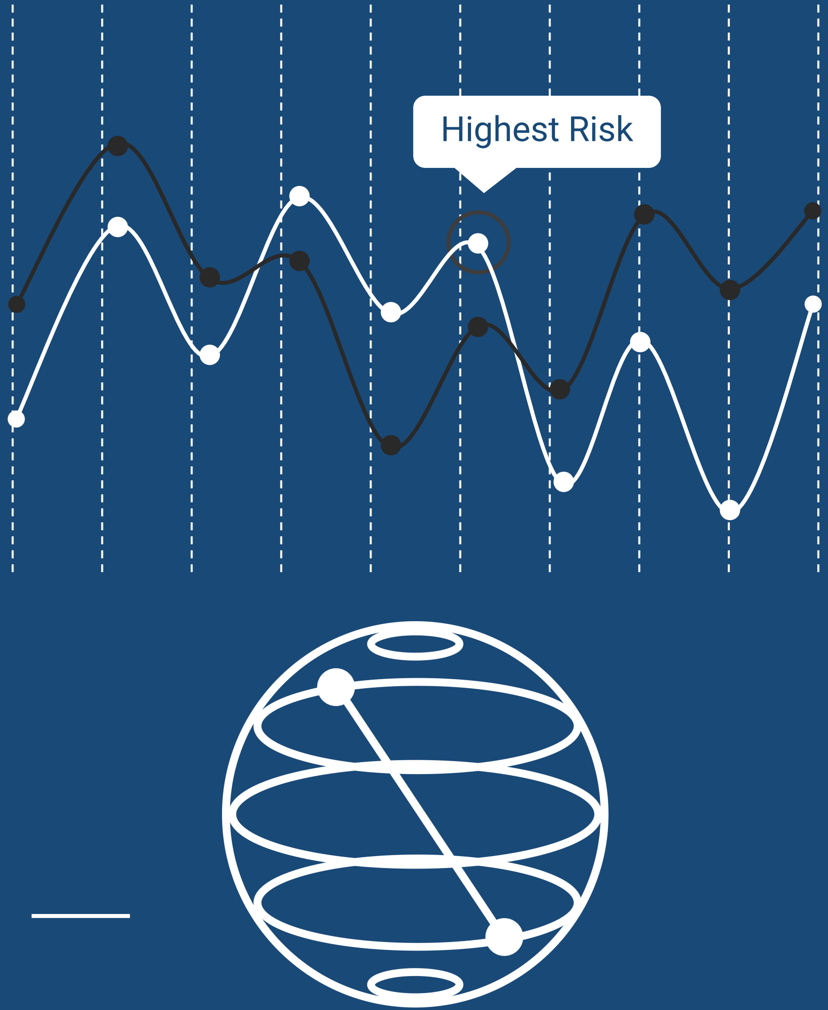
This variant of QAE is only based on Grover's algorithm, for this reason, it differs from the original as it does not rely on the Quantum Phase Estimation (QPE).

It works like this:

- IQAE iteratively applies carefully selected Grover iterations to find an estimate for the target amplitude.
- The output of the algorithm is an estimate for the amplitude a, that with at least probability 1 - alpha has an error of epsilon.
- The number of A operator calls scales linearly in 1/epsilon (up to a logarithmic factor)

Results

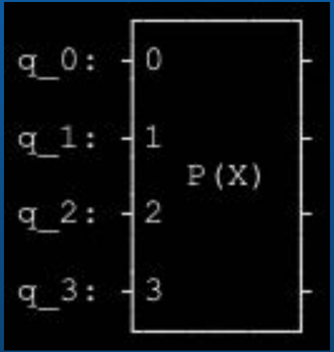
UM, EL, VaR, CVaR



Uncertainty Model (UM)

We construct a circuit that loads the UM, by creating a quantum state in a register of n_z qubits that represents Z following a standard normal distribution.

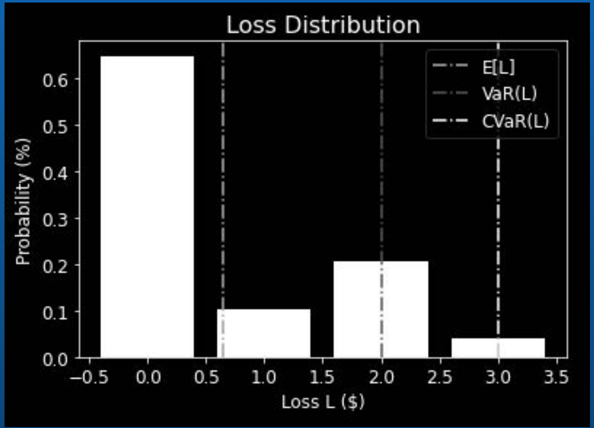
- The resulting quantum state can be written as:

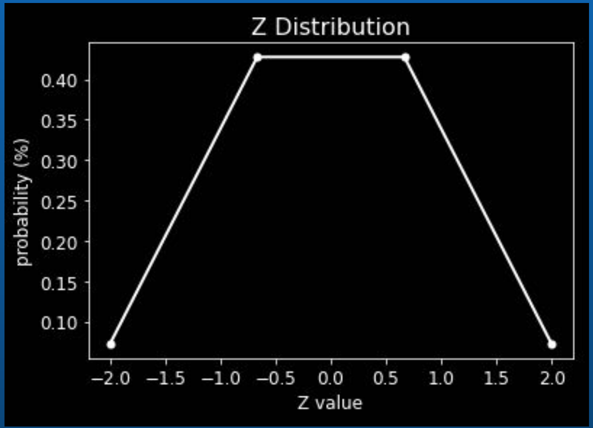


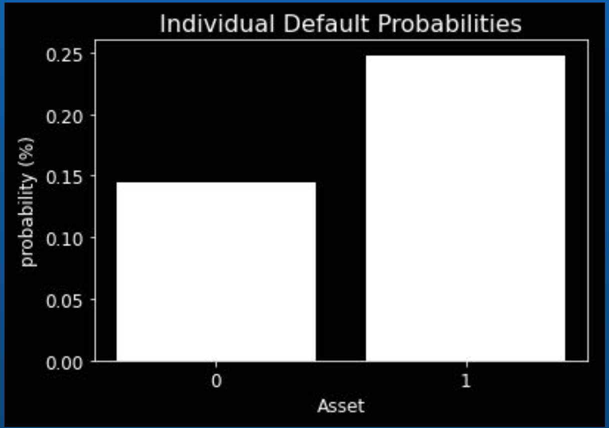
$$|\Psi\rangle = \sum_{i=0}^{2^{n_z}-1} \sqrt{p_z^i}|z_i\rangle \bigotimes_{k=1}^{K} \left(\sqrt{1-p_k(z_i)}|0\rangle + \sqrt{p_k(z_i)}|1\rangle\right)$$

```
P[L] <= VaR[L]]:                  0.9591
Expected Loss E[L]:               0.6409
Value at Risk VaR[L]:             2.0000
Conditional Value at Risk CVaR[L]: 3.0000
```

Loss Distribution, Z Distribution, Individual Probs

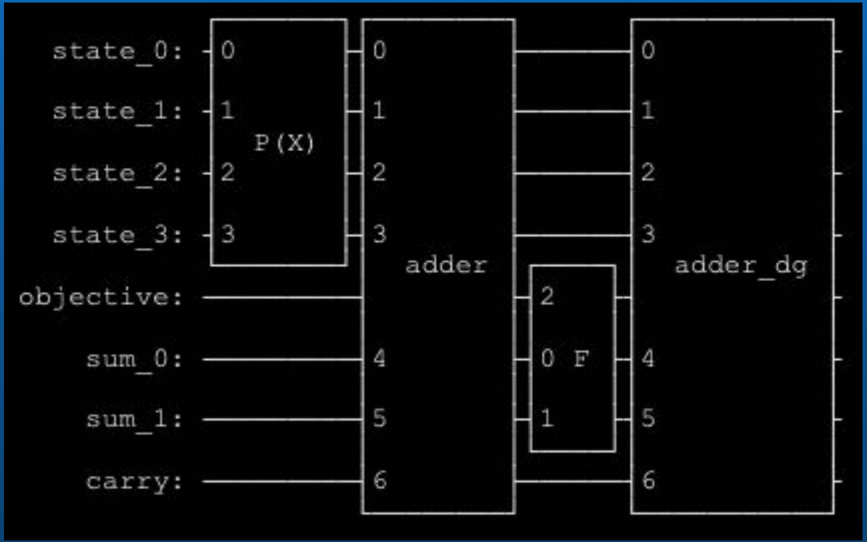






Expected Loss (EL)

To estimate the expected loss, we first apply a weighted sum operator to sum up individual losses to total loss:



Number of qubits to represent the result:

$$n_s = \lfloor \log_2(\lambda_1 + \ldots + \lambda_K) \rfloor + 1$$

```
Exact Expected Loss:    0.6409
Exact Operator Value:   0.3906
Mapped Operator value:  0.6639
```

IQAE

```
Exact value:            0.6409
Estimated value:        0.6263
Confidence interval:    [0.5577, 0.6950]
```

Value at Risk (VaR)

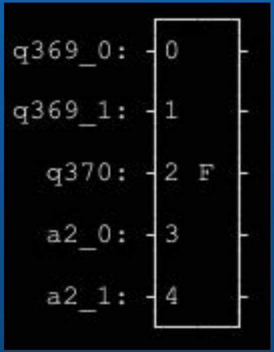
We use a bisection search and IQAE to efficiently evaluate the Cumulative Distribution Function (CDF) to estimate the value at risk.

```
...............................................................
 Start bisection search for target value: 0.950
...............................................................
low_level     low_value     level     value     high_level     high_value
...............................................................
-1            0.000         1         0.752     3              1.000
 1            0.752         2         0.959     3              1.000
...............................................................
 Finished bisection search
...............................................................



Estimated Value at Risk:   2
Exact Value at Risk:       2
Estimated Probability:     0.959
Exact Probability:         0.959
```

Conditional Value at Risk (CVaR)

The expected value of the loss conditional to it being larger than or equal to the VaR. we evaluate a piecewise linear objective function, dependent on the total loss, that is given by:

$$f(L) = \begin{cases} 0 & \text{if} \quad L \leq VaR \\ L & \text{if} \quad L > VaR \end{cases}$$




```
Estimated CVaR: 3.2834
Exact CVaR:     3.0000
```

IQAE

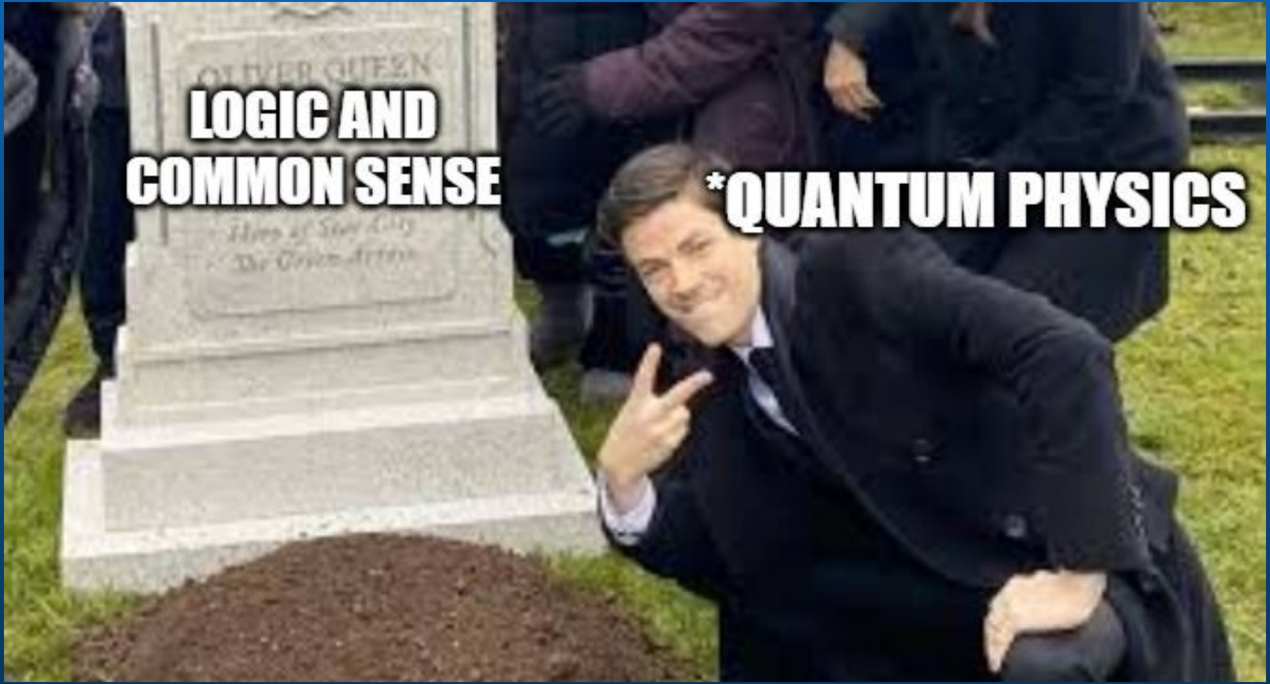
```
Exact CVaR:     3.0000
Estimated CVaR: 3.2491
```

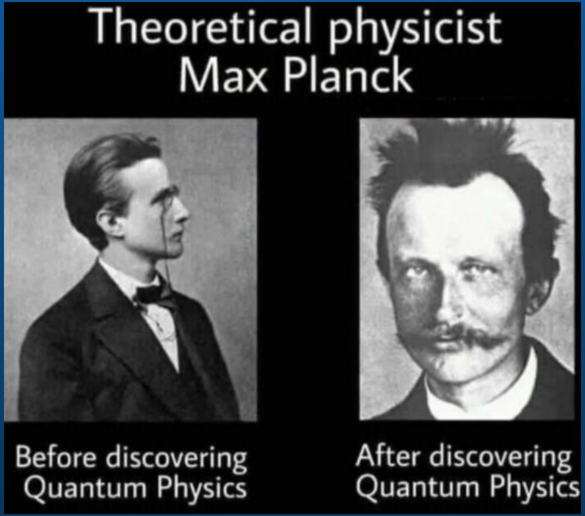
Conclusions

Quantum computers (QC) are crippled by errors in the form of noise, however, correction schemes exist but consume a large number of qubits, relatively to qubits for actual computation. When the number of qubits required increases, the computing will not viable.

The application of this quantum algorithm (or quantum computing in general) is not feasible due to the noise and the quantity of qubits required.

Thank you for your time!





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