6.1

$LDL =$

$$\begin{pmatrix} L_{00} & 0 & 0 \\ \lambda_{10} e_L^T & 1 & 0 \\ 0 & \lambda_{21} e_F & L_{22} \end{pmatrix} \begin{pmatrix} D_{00} & 0 & 0 \\ 0 & \delta_1 & 0 \\ 0 & 0 & D_{22} \end{pmatrix} \begin{pmatrix} L_{00} & 0 & 0 \\ \lambda_{10} e_L^T & 1 & 0 \\ 0 & \lambda_{21} e_F & L_{22} \end{pmatrix}^T$$

$=$

$$LD = \begin{pmatrix} L_{00} D_{00} & 0 & 0 \\ \lambda_{10} e_L^T D_{00} & \delta_1 & 0 \\ 0 & \lambda_{21} e_F & L_{22} D_{22} \end{pmatrix}$$

$LD \times L^T$

$$\begin{pmatrix} L_{00} & 0 & 0 \\ \lambda_{10} e_L^T & 1 & 0 \\ 0 & \lambda_{21} e_F & L_{22} \end{pmatrix}^T = \begin{pmatrix} L_{00}^T & \lambda_{10} e_L & 0 \\ 0 & 1 & \lambda_{21} e_F^T \\ 0 & 0 & L_{22}^T \end{pmatrix} = L^T$$

$$LDL^T = \begin{pmatrix} L_{00} D_{00} L_{00}^T & L_{00} D_{00} \lambda_{10} e_L & 0 \\ L_{00} D_{00} \lambda_{10} e_L & \lambda_{10} e_L^T D_{00} \lambda_{10} e_L + \delta_1 & \delta_1 \lambda_{21} e_F^T \\ 0 & \lambda_{21} e_f & \lambda_{21} e_f \lambda_{21} e_f^T L_{22} D_{22} L_{22}^T \end{pmatrix}$$

$$A = U \Sigma U^T$$

$$\left(\begin{array}{c|c|c} U_{00} & U_{01} e_L & 0 \\ \hline 0 & 1 & U_{12} e_f^T \\ \hline 0 & 0 & U_{22} \end{array}\right) \left(\begin{array}{c|c|c} \Sigma_{00} & 0 & 0 \\ \hline 0 & \Sigma_1 & 0 \\ \hline 0 & 0 & \Sigma_{22} \end{array}\right) \left(\begin{array}{c|c|c} U_{00} & U_{01} e_L & 0 \\ \hline 0 & 1 & U_{12} e_f^T \\ \hline 0 & 0 & U_{22} \end{array}\right)^T$$

$$\left(\begin{array}{c|c|c} U_{00} \Sigma_{00} & U_{01} e_L \Sigma_1 & 0 \\ \hline 0 & \Sigma_1 & U_{12} e_f^T \Sigma_{22} \\ \hline 0 & 0 & U_{22} \Sigma_{22} \end{array}\right) \left(\begin{array}{c|c|c} U_{00}^T & 0 & 0 \\ \hline U_{01} e_L^T & 1 & 0 \\ \hline 0 & U_{12} e_f & U_{22}^T \end{array}\right)$$

$$\left(\begin{array}{c|c|c} U_{00} \Sigma_{00} U_{00}^T + U_{01} e_L \Sigma_1 U_{01} e_L^T & U_{01} e_L \Sigma_1 & 0 \\ \hline \Sigma_1 U_{01} e_L^T & \Sigma_1 + U_{12} e_f^T \Sigma_{22} U_{12} e_f & U_{12} e_f^T \Sigma_{22} U_{22} \\ \hline 0 & U_{22} \Sigma_{22} U_{12} e_f & U_{22} \Sigma_{22} U_{22}^T \end{array}\right)$$

Given

$$\left(\begin{array}{c|c|c} A_{00} & \alpha_{10}e_L & 0 \\ \hline \alpha_{10}e_L^T & \alpha_{11} & \alpha_{21}e_F^T \\ \hline 0 & \alpha_{21}e_f & A_{22} \end{array}\right) = \left(\begin{array}{c|c|c} L_{00} & 0 & 0 \\ \hline \lambda_{10}e_L^T & 1 & U_{12}e_F^T \\ \hline 0 & 0 & U_{22} \end{array}\right)\left(\begin{array}{c|c|c} D_{00} & 0 & 0 \\ \hline 0 & \phi_1 & 0 \\ \hline 0 & 0 & \varepsilon_{22} \end{array}\right)\left(\begin{array}{c|c|c} L_{00} & 0 & 0 \\ \hline \lambda_{10}e_L^T & 1 & U_{12}e_F^T \\ \hline 0 & 0 & U_{22} \end{array}\right)^T$$

LD

$$\Rightarrow \left(\begin{array}{c|c|c} L_{00}D_{00} & 0 & 0 \\ \hline \lambda_{10}e_L^T D_{00} & \phi_1 & U_{12}e_f^T\varepsilon_{22} \\ \hline 0 & 0 & U_{22}\varepsilon_{22} \end{array}\right)\left(\begin{array}{c|c|c} L_{00}^T & \lambda_{10}e_L & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & U_{12}e_f & U_{22}^T \end{array}\right)$$

$$\left(\begin{array}{c|c|c} L_{00}D_{00}L_{00}^T & L_{00}D_{00}\lambda_{10}e_L & 0 \\ \hline \lambda_{10}e_L^T D_{00}L_{00}^T & \lambda_{10}e_L^T D_{00}\lambda_{10}e_L + \phi_1 + U_{12}e_F^T\varepsilon_{22}U_{12}e_f & U_{12}e_F^T\varepsilon_{22}U_{22}^T \\ \hline 0 & U_{22}\varepsilon_{22}U_{12}e_f & U_{22}\varepsilon_{22}U_{22}^T \end{array}\right) =$$

$$\left(\begin{array}{c|c|c} A_{00} & \alpha_{10}e_L & 0 \\ \hline \alpha_{10}e_L^T & \alpha_{11} & \alpha_{21}e_F^T \\ \hline 0 & \alpha_{21} & A_{22} \end{array}\right)$$

$$\lambda_{10eL}^T D_{00} \lambda_{10eL} + \delta_1 = \varepsilon_1 + U_{12eF}^T \xi_{22} U_{12eF} = \lambda_{10e}^T D_{00} \lambda_{10eL} + \phi_1 + U_{12eF}^T \xi_{22} U_{12eF}$$

$$\phi = \delta_1 + \varepsilon_1 - \alpha_{11}$$

$$\phi = \lambda_{10eL}^T D_{00} \lambda_{10eL} + \delta_1 + \varepsilon_1 + U_{12}^{eT} \xi_{22} U_{12}^{eF} - \lambda_{10e}^T D_{00} \lambda_{10eL} + \phi_1 + U_{12eF}^T \xi_{22} U_{12eF} =$$

$$\phi + \alpha_{11} = \delta_1 + \varepsilon_1$$

$$\delta_1 + \varepsilon_1 = \lambda_{10eL}^T D_{00} \lambda_{10eL} + \delta_1 + \varepsilon_1 U_{12eF}^T \xi_{22} U_{12eF}$$

$$\phi = \delta_1 + \varepsilon_1 - \alpha_{11}$$

- ignoring the cost of the computed factorizations the additional cost for the twisted factorization would be $O(1)$ since it only involves solving a linear equation that solves for α_{11} based on the equation above seeing how the rest of the values are known.

- if we compute l of them in $O(1)$ then the number the cost would be dependent on the number of twisted factorizations which depend on the number of non zero entries so $O(n)$ where n would be the number of factorizations.