



Integrales Inmediatas

- 1) $\int (-3x^2 + 2x + 3) dx = -x^3 + x^2 + 3x + c$
- 2) $\int (x^3 - 2x^{-2} + 5x - 3x^2) dx = \frac{x^4}{4} + 2x^{-1} + \frac{5}{2}x^2 - x^3 + c$
- 3) $\int \frac{5x^3 + 3x^2 + 2x + 5}{x} dx = \frac{5}{3}x^3 + \frac{3}{2}x^2 + 2x + 5\ln|x| + c$
- 4) $\int \left(\sqrt[3]{x} + \frac{1}{x^2} - 2x^{-1} + \frac{1}{\sqrt{x}} \right) dx = \frac{3}{4}x^{\frac{4}{3}} - x^{-1} - 2\ln|x| + 2x^{\frac{1}{2}} + c$
- 5) $\int \frac{\sqrt{x}}{x} dx = 2x^{\frac{1}{2}} + c$
- 6) $\int \left(\frac{3x^3 - 2x^2}{\sqrt[3]{x^2}} \right) dx = \frac{9}{10}x^{\frac{10}{3}} - \frac{6}{7}x^{\frac{7}{3}} + c$
- 7) $\int \left(-\frac{3}{x} + \frac{1}{\cos^2(x)} \right) dx = -3\ln|x| + \operatorname{tg}(x) + c$
- 8) $\int \left(\frac{5}{1+x^2} + 3e^x \right) dx = 5\operatorname{arctg}(x) + 3e^x + c$
- 9) $\int \left(-e^x - \frac{1}{\operatorname{sen}^2(x)} + 4x - 2 \right) dx = -e^x + \cot g(x) + 2x^2 - 2x + c$
- 10) $\int \left(\frac{3}{\sqrt{1-x^2}} - 4 \right) dx = 3\operatorname{arcsen}(x) - 4x + c$

Integrales por sustitución

- 1) $\int (x^2 + 2)^4 x dx = \frac{(x^2 + 2)^5}{10} + c$
- 2) $\int (x^3 - 4)^2 x^2 dx = \frac{(x^3 - 4)^3}{9} + c$
- 3) $\int e^{3x^2} x dx = \frac{e^{3x^2}}{6} + c$
- 4) $\int \frac{x}{\sqrt{x^2 + 1}} dx = \sqrt{x^2 + 1} + c$
- 5) $\int \frac{\cos(x)}{\operatorname{sen}^3(x)} dx = \frac{\operatorname{sen}^{-2}(x)}{-2} + c$
- 6) $\int \frac{-3}{\sqrt{4+2x}} dx = -3(4+2x)^{\frac{1}{2}} + c$
- 7) $\int \cos^3(3x) \operatorname{sen}(3x) dx = -\frac{\cos^4(3x)}{12} + c$
- 8) $\int \frac{x^2}{x^3 + 2} dx = \frac{\ln|x^3 + 2|}{3} + c$
- 9) $\int \frac{\operatorname{sen}(x)}{1 - \cos(x)} dx = \ln|1 - \cos(x)| + c$
- 10) $\int \frac{\sqrt{\operatorname{tg}(x)}}{\cos^2(x)} dx = \frac{2}{3}(\operatorname{tg}(x))^{\frac{3}{2}} + c$
- 11) $\int \operatorname{tg}(x) dx = -\ln|\cos(x)| + c$
- 12) $\int \frac{\sqrt{2 + \ln(x)}}{x} dx = \frac{2}{3}(2 + \ln(x))^{\frac{3}{2}} + c$
- 13) $\int \sqrt{\cos(x)} \cdot \operatorname{sen}(x) dx = -\frac{2}{3}(\cos(x))^{\frac{3}{2}} + c$
- 14) $\int \frac{1}{x} \cdot \operatorname{sen}(\ln(x)) dx = -\cos(\ln(x)) + c$
- 15) $\int \frac{\cos(\sqrt{x})}{\sqrt{x}} dx = 2\operatorname{sen}(\sqrt{x}) + c$

Integrales por Partes

- 1) $\int \ln(x) x^2 dx = \frac{1}{3} \ln(x) \cdot x^3 - \frac{1}{9} x^3 + c$
- 2) $\int x \cdot \operatorname{sen}(x) dx = -x \cdot \cos(x) + \operatorname{sen}(x) + c$
- 3) $\int x e^x dx = e^x x - e^x + c$
- 4) $\int \ln(x) dx = \ln(x) \cdot x - x + c$
- 5) $\int \ln(x) \sqrt{x} dx = \frac{2}{3} \ln(x) \cdot x^{\frac{3}{2}} - \frac{4}{9} x^{\frac{3}{2}} + c$
- 6) $\int 2x \cos(3x) dx = \frac{2}{3} x \cdot \operatorname{sen}(3x) + \frac{2}{9} \cos(3x) + c$
- 7) $\int \frac{x}{e^x} dx = -e^{-x}(x+1) + c$
- 8) $\int x^2 e^{3x} dx = e^{3x} \left(\frac{x^2}{3} - \frac{2}{9}x + \frac{2}{27} \right) + c$
- 9) $\int (x+1)^2 e^x dx = (x+1)^2 e^x - 2(x+1)e^x + 2e^x + c$
- 10) $\int \ln(x^2 + 1) dx = \ln(x^2 + 1)x - \frac{1}{2} \ln(x^2 + 1) + c$
- 11) $\int \operatorname{arctg}(x) dx = \operatorname{arctg}(x) \cdot x - \frac{1}{2} \ln|x^2 + 1| + c$
- 12) $\int \operatorname{arcsen}(x) dx = \operatorname{arcsen}(x) \cdot x + (1-x^2)^{\frac{1}{2}} + c$
- 13) $\int \arccos(x) dx = \arccos(x) \cdot x - (1-x^2)^{\frac{1}{2}} + c$
- 14) $\int e^x \operatorname{sen}(x) dx = \frac{1}{2} e^x (\operatorname{sen}(x) - \cos(x)) + c$
- 15) $\int e^x \cos(x) dx = \frac{1}{2} e^x (\cos(x) + \operatorname{sen}(x)) + c$
- 16) $\int \operatorname{sen}(\ln x) dx = \frac{1}{2} x (\operatorname{sen}(\ln x) - \cos(\ln x)) + c$



Integrales de funciones racionales

Raíces Simples

$$1) \int \frac{3x+1}{x^2-x-2} dx = \frac{7}{3} \ln|x-2| + \frac{2}{3} \ln|x+1| + c$$

$$2) \int \frac{-x+4}{x^2+x-6} dx = \frac{2}{5} \ln|x-2| - \frac{7}{5} \ln|x+3| + c$$

$$3) \int \frac{dx}{x^2-4} = \frac{1}{4} \ln|x-2| - \frac{1}{4} \ln|x+2| + c$$

$$4) \int \frac{dx}{4-x^2} = -\frac{1}{4} \ln|x-2| + \frac{1}{4} \ln|x+2| + c$$

$$5) \int \frac{2x^2-3}{(x-1)(x+2)(x-2)} dx = \frac{1}{3} \ln|x-1| + \frac{5}{12} \ln|x+2| + \frac{5}{4} \ln|x-2| + c$$

$$6) \int \frac{x^3+2x+1}{x^2+x-6} dx = \frac{1}{2} x^2 - x + \frac{13}{5} \ln|x-2| + \frac{32}{5} \ln|x+3| + c$$

Raíces Múltiples

$$1) \int \frac{x+1}{x^2+4x+4} dx = \frac{1}{x+2} + \ln|x+2| + c$$

$$2) \int \frac{x-1}{x(x+2)^2} dx = -\frac{\ln|x|}{4} - \frac{3}{2(x+2)} + \frac{\ln|x+2|}{4} + c$$

$$3) \int \frac{dx}{x(x+1)^2} = \ln|x| + \frac{1}{x+1} - \ln|x+1| + c$$

$$4) \int \frac{x^3-2}{x^3-2x^2+x} dx = x - 2 \ln|x| + \frac{1}{x-1} + 4 \ln|x-1| + c$$

$$5) \int \frac{x^2+2x+2}{x^3(x+1)} dx = \frac{-1}{x^2} + \ln|x| - \ln|x+1| + c$$

$$6) \int \frac{x^2+6x+3}{(x-3)^2(x+1)^2} dx = \frac{-15}{8(x-3)} - \frac{3}{16} \ln|x-3| + \frac{1}{8(x+1)} + \frac{3}{16} \ln|x+1| + c$$

Raíces complejas

$$1) \int \frac{dx}{x^2+2x+5} = \frac{1}{2} \arctg\left(\frac{x+1}{2}\right) + c$$

$$2) \int \frac{3}{x^2-4x+25} dx = \frac{\sqrt{21}}{7} \arctg\left(\frac{x-2}{\sqrt{21}}\right) + c$$

$$3) \int \frac{x+2}{x^2-2x+5} dx = \frac{1}{2} \left[\ln|x^2-2x+5| + 3 \arctg\left(\frac{x-1}{2}\right) \right] + c$$

$$4) \int \frac{2x-1}{x^2+x+1} dx = \ln|x^2+x+1| - \frac{4}{3} \sqrt{3} \arctg\left(\frac{2x+1}{\sqrt{3}}\right) + c$$

$$5) \int \frac{x}{(x+1)(x^2+4)} dx = \frac{1}{5} \left[-\ln|x+1| - \frac{1}{2} \ln|x^2+4| + 2 \arctg\left(\frac{x}{2}\right) \right] + c$$

$$7) \int \frac{(x-2) dx}{(x^2+4x+8)(x+2)} = -\ln|x+2| + \frac{1}{2} \ln|x^2+4x+8| + \frac{1}{2} \arctg\left(\frac{x+2}{2}\right) + c$$

Integrales de funciones racionales de seno y coseno

$$\boxed{\begin{matrix} \operatorname{sen}(x) = \frac{2u}{1+u^2} & \cos(x) = \frac{1-u^2}{1+u^2} & dx = \frac{2}{1+u^2} du & u = \operatorname{tg}\left(\frac{x}{2}\right) \end{matrix}}$$

$$1) \int \frac{dx}{1+\operatorname{sen}(x)+\cos(x)} = \ln\left| \operatorname{tg}\left(\frac{x}{2}\right) + 1 \right| + c$$

$$2) \int \frac{\cos(x)}{1+\cos(x)} dx = -\operatorname{tg}\left(\frac{x}{2}\right) + x + c$$

$$3) \int \frac{dx}{3+\cos(x)+2\operatorname{sen}(x)} = \arctg\left(\operatorname{tg}\left(\frac{x}{2}\right) + 1\right) + c$$

$$4) \int \frac{dx}{3+5\cos(x)} = \frac{1}{4} \ln\left| \operatorname{tg}\left(\frac{x}{2}\right) + 2 \right| - \frac{1}{4} \ln\left| \operatorname{tg}\left(\frac{x}{2}\right) - 2 \right| + c$$

Integrales de funciones trigonométricas

Potencias impares de seno y coseno

$$\boxed{\cos^2(x) + \operatorname{sen}^2(x) = 1 \Rightarrow \cos^2(x) = 1 - \operatorname{sen}^2(x) \quad \wedge \quad \operatorname{sen}^2(x) = 1 - \cos^2(x)}$$

$$1) \int \operatorname{sen}^3(x) dx = -\cos(x) + \frac{\cos^3(x)}{3} + c$$

$$2) \int \cos^3(2x) dx = \frac{1}{2} \operatorname{sen}(2x) - \frac{1}{6} \operatorname{sen}^3(2x) + c$$

$$3) \int \operatorname{sen}^3\left(\frac{x}{2}\right) dx = -2 \cos\left(\frac{x}{2}\right) + \frac{2}{3} \cos^3\left(\frac{x}{2}\right) + c$$

$$4) \int \operatorname{sen}^5(-x) dx = \cos(-x) - \frac{2}{3} \cos^3(-x) + \frac{1}{5} \cos^5(-x) + c$$



Potencias pares de seno y coseno

$$\boxed{\sin^2(x) = \frac{1 - \cos(2x)}{2} \quad \cos^2(x) = \frac{1 + \cos(2x)}{2}}$$

$$1) \int \sin^2(x) dx = \frac{1}{2} \left(x - \frac{\sin(2x)}{2} \right) + c$$

$$2) \int \sin^2(3x) dx = \frac{1}{2} \left(x - \frac{\sin(6x)}{6} \right) + c$$

$$3) \int \cos^2(2x) dx = \frac{1}{2} \left(x + \frac{\sin(4x)}{4} \right) + c$$

$$4) \int \sin^4(-x) dx = \frac{1}{4} \left(\frac{3}{2} x - \frac{\sin(-4x)}{8} + \sin(-2x) \right) + c$$

Producto de potencias pares e impares de seno y coseno

$$1) \int \sin^2(x) \cos^2(x) dx = \frac{1}{4} \left[\frac{1}{2} x - \frac{1}{8} \sin(4x) \right] + c$$

$$2) \int \sin^2(2x) \cos^2(2x) dx = \frac{1}{4} \left[\frac{1}{2} x - \frac{1}{16} \sin(8x) \right] + c$$

$$3) \int \sin^2(x) \cos^4(x) dx = \frac{1}{16} \left[x - \frac{1}{4} \sin(4x) + \frac{1}{3} \sin^3(2x) \right] + c$$

$$4) \int \frac{\sin^3(x)}{\sqrt{\cos(x)}} dx = -2 \cos^{\frac{1}{2}}(x) + \frac{2}{5} \cos^{\frac{5}{2}}(x) + c$$

$$5) \int \cos^3(x) \sqrt[3]{\sin(x)} dx = \frac{3}{4} \sin^{\frac{4}{3}}(x) - \frac{3}{10} \sin^{\frac{10}{3}}(x) + c$$

$$6) \int \sin^4(x) \cos^2(x) dx = \frac{1}{8} \left[\frac{1}{2} x - \frac{1}{8} \sin(4x) + \frac{1}{6} \sin^3(2x) \right] + c$$

Integración de funciones Irracionales

$$1) \int \frac{dx}{\sqrt{x+1}} = 2 \left[\sqrt{x+1} - \ln|\sqrt{x+1}| \right] + c$$

$$2) \int \frac{x}{\sqrt[3]{x+1}} dx = \frac{3}{5} \sqrt[3]{(x+1)^5} - \frac{3}{2} \sqrt[3]{(x+1)^2} + c$$

$$3) \int \frac{dx}{(2-x)\sqrt{1-x}} = -2 \operatorname{arctg}(\sqrt{1-x}) + c$$

$$4) \int \frac{\sqrt{x}}{x+2} dx = 2 \left[\sqrt{x} - \sqrt{2} \operatorname{arctg} \left(\sqrt{\frac{x}{2}} \right) \right] + c$$

$$5) \int \frac{dx}{\sqrt{2x-1} - \sqrt[4]{2x-1}} = \sqrt{2x-1} + 2\sqrt[4]{2x-1} + 2 \ln|\sqrt[4]{2x-1} - 1| + c$$

$$6) \int \frac{dx}{\sqrt{x} + \sqrt[3]{x}} = 2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6 \ln(\sqrt[6]{x} + 1) + c$$

$$7) \int \frac{dx}{\sqrt[3]{x+1} + \sqrt{x+1}} = 2\sqrt{x+1} - 3\sqrt[3]{x+1} + 6\sqrt[6]{x+1} - 6 \ln|\sqrt[6]{x+1} + 1| + c$$

$$8) \int \frac{dx}{(x-2)\sqrt{x+2}} = \frac{1}{2} \ln \left| \frac{\sqrt{x+2} - 2}{\sqrt{x+2} + 2} \right| + c$$

Integral Definida $\boxed{\int_a^b f(x) dx = F(x) \Big|_a^b = F(b) - F(a)}$

$$\int_0^1 \ln(x^2 + 1) dx$$

$$\int_{-1}^1 \frac{x^2}{x^2 - 9} dx$$

$$\int_1^e \frac{1}{(1 + \ln(x))x} dx$$

$$\int_0^1 \frac{1}{(1 + e^x)e^{-x}} dx$$

$$\int_0^\pi \frac{\sin(x)}{1 + \cos^2(x)} dx$$

$$\int_0^{3\pi/2} \frac{\cos(x)}{1 + \sin^2(x)} dx$$

$$\int_1^e \sqrt[3]{x} \ln(x) dx$$

$$\int_0^4 \frac{x}{\sqrt{x} + 2} dx$$

$$\int_0^{1/2} (3x+1)e^{2x} dx$$

$$\int_0^1 \frac{(x+1)}{x^2 + 1} dx$$

$$\int_0^1 \ln(x+1) dx$$

$$\int_{\pi/4}^{\pi/2} \frac{\cos(x)}{\sin^2(x)} dx$$

$$\int_1^9 \frac{1 + \sqrt{x}}{x^2} dx$$

$$\int_{\pi/4}^{3\pi/4} -\cos 2x dx$$

$$\int_0^{\pi/4} \sin^2(x) dx$$

$$\int_2^3 \frac{3+x}{x^2 - 2x + 1} dx$$


Área Debajo de una función $\int_a^b f(x) dx$

1) $y = x^2 - 2x$ entre $x = 1$ $x = 3$ Rta: 2

3) $y = x^2 + 2x - 3$ entre $x = 1$ $x = 2$ Rta: $\frac{7}{3}$

5) $y = x^3 + 3x^2$ entre $x = 0$ $x = 2$ Rta: 12

2) $y = -x^2 + 4$ entre $x = -2$ $x = 2$ Rta: $\frac{32}{3}$

4) $y = x^3 - 2x$ entre $x = -2$ $x = 0$ Rta: 2

6) $y = \sin(x)$ entre $x = \pi$ $x = 2\pi$ Rta: 2

Área entre funciones $\int_a^b [f(x) - g(x)] dx$

1) $y = x$ $y = x^2$ Rta: $\frac{1}{6}$

3) $y = x - 2$ $y = 2x - x^2$ Rta: $\frac{9}{2}$

5) $y = x^2 + 1$ $y = -x^2 + 3$ Rta: $\frac{8}{3}$

7) area $y = x^3$ y su derivada

9) Área entre $y^2 = 2x$; $x^2 = 2y$

11) Área entre $y = x^4$ $y = x^2$

13) Area $y = x^2 + 3$; $y = x$; $x = -1$; $x = 1$

2) $y = x - 2$ $y = x^2 - 4$ Rta: $\frac{9}{2}$

4) $y = x^2 - 4x + 1$ $y = -x + 5$ Rta: $\frac{125}{6}$

6) $y = 5x^2$ $y = x^2 + 1$ Rta: $\frac{2}{3}$

8) Área entre $y = x$ $y = x^2$; $y = 4$

10) Área entre $y = 4x^2$; $y = x^2 + 3$

12) Área entre $y = x^2 + 1$; $y = -x^2 + 3$

14) Area entre $y = 2x$; $y = x^2 - 4x$

Einstein