Digesto de Funciones de Listas y Propiedades

Definiciones

1. Largo de una lista:

$$
\begin{array}{|l}
\# : [A] \to Num \\
\hline
\#[\,] \doteq 0 \\
\#(x \triangleright xs) \doteq \#xs + 1
\end{array}
$$

2. Indexar:

$$
\begin{array}{|l}
.\ : [A] \to Num \to A \\
\hline
(x \triangleright xs)\,!\,0 \doteq x \\
(x \triangleright xs)\,!\,(n+1) \doteq xs\,!\,n
\end{array}
$$

3. Tirar:

$$
\begin{array}{|l}
\downarrow : [A] \to Num \to [A] \\
\hline
[\,] \downarrow n \doteq [\,] \\
(x \triangleright xs) \downarrow 0 \doteq x \triangleright xs \\
(x \triangleright xs) \downarrow (n+1) \doteq xs \downarrow n
\end{array}
$$

4. Tomar:

$$
\begin{array}{|l}
\uparrow : [A] \to Num \to [A] \\
\hline
[\,] \uparrow n \doteq [\,] \\
(x \triangleright xs) \uparrow 0 \doteq [\,] \\
(x \triangleright xs) \uparrow (n+1) \doteq x \triangleright (xs \uparrow n)
\end{array}
$$

5. Concatenar:

$$
\begin{array}{|l}
+\!\!+\ : [A] \to [A] \to [A] \\
\hline
[\,] +\!\!+ ys \doteq ys \\
(x \triangleright xs) +\!\!+ ys \doteq x \triangleright (xs +\!\!+ ys)
\end{array}
$$

6. Head (cabeza):

$$
\begin{array}{|l}
hd : [A] \to A \\
\hline
hd.(x \triangleright xs) \doteq x
\end{array}
$$

7. Tail (cola):

$$
\begin{array}{|l}
tl : [A] \to [A] \\
\hline
tl.(x \triangleright xs) \doteq xs
\end{array}
$$

Propiedades

1. Constructores de lista ($[\,]$, $\triangleright$):

$$x \triangleright xs \neq [\,]$$
$$(x \triangleright xs) = (y \triangleright ys) \quad \equiv \quad x = y \wedge xs = ys$$

2. Concatenación:

$$
\begin{aligned}
(xs \mathbin{+\!\!+} ys) = [\,] \quad &\equiv \quad xs = [\,] \wedge ys = [\,] \\
(xs \mathbin{+\!\!+} ys) \mathbin{+\!\!+} zs \quad &= \quad xs \mathbin{+\!\!+} (ys \mathbin{+\!\!+} zs) \\
(xs \mathbin{+\!\!+} ys)\,!\,i \quad &= \quad (\ i < \#xs \to xs\,!\,i \\
&\qquad\quad \square\ i \geq \#xs \to ys\,!\,(i - \#xs) \\
&\qquad\quad)
\end{aligned}
$$

3. Longitud:

$$
\begin{aligned}
\#(xs \mathbin{+\!\!+} ys) \quad &= \quad \#xs + \#ys \\
\#(xs{\uparrow}n) \quad &= \quad n \,\text{mín}\, \#xs \\
\#(xs{\downarrow}n) \quad &= \quad (\#xs - n) \,\text{máx}\, 0
\end{aligned}
$$