3) $f(x) = e^x \quad a = 0$

Nos estan pidiendo que calculemos el menor n tal que $|R_{n,0}(x)| < \frac{1}{10}$, calculamos $R_{n,0}(x)$

$$R_{n,a}(x) = \frac{f^{(n+1)}(t)}{(n+1)!}(x-a)^{(n+1)}$$

Sabemos que $f^{(n)}(x) = e^x$.

Tambien sabemos que el valor de x que se nos da cumple que $0 \leq x \leq 1$, osea $\underbrace{0 \leq \frac{1}{e^2}}_{} < 1$. (*)

Por lo que $x > a$, entonces nuestro valor $t \in (a, x)$.

$$R_{n,0}(x) = \frac{e^t}{(n+1)!}\left(\frac{1}{e^2} - 0\right)^{(n+1)} \overset{(*)}{\leq} \boxed{\frac{e^1}{(n+1)!}\left(\frac{1}{e^2}\right)^{(n+1)}}$$

Ahora demosle valores a n

$$n = 0 \Rightarrow \frac{e}{(0+1)!}\left(\frac{1}{e^2}\right)^{(0+1)} = \frac{e}{e^2} = \frac{1}{e} > \frac{1}{10} \quad \times \text{ no es menor}$$

$$n = 1 \Rightarrow \frac{e}{(1+1)!}\left(\frac{1}{e^2}\right)^{(1+1)} = \frac{e}{2!} \cdot \frac{1}{e^4} = \frac{e}{2e^4} = \frac{1}{2e^3} < \frac{1}{10} \quad \checkmark$$

El menor n tal que $R_{n,0}\left(\frac{1}{e^2}\right) < \frac{1}{10}$ es $n = 1$

Ramos Julian
Hoja 4

HOJA N°	
FECHA	

i) $F(x,y,z) = x^2 + y^2 + 2z^2 \quad P_0 = (1, 1, \frac{1}{\sqrt{2}})$

Ec. del plano tangente a la superficie de nivel que pasa por P_0

$\langle (x,y,z) - P_0, \nabla f(P_0) \rangle = 0$

Calculo $\nabla f(P_0)$:

$f_x(x,y,z) = 2x \quad f_y(x,y,z) = 2y \quad f_z(x,y,z) = -4z$

$f_x(1,1,\frac{1}{\sqrt{2}}) = 2.1 \quad f_y(1,1,\frac{1}{\sqrt{2}}) = 2.1 \quad f_z(x,y,z) = \frac{-4}{\sqrt{2}}$

$\nabla F(1,1,\frac{1}{\sqrt{2}}) = (2, 2, -\frac{4}{\sqrt{2}})$

$\langle (x,y,z) - (1,1,\frac{1}{\sqrt{2}}), (2,2,-\frac{4}{\sqrt{2}}) \rangle = 0$

$\langle ((x-1),(y-1),(z-\frac{1}{\sqrt{2}})), (2,2,-\frac{4}{\sqrt{2}}) \rangle = 0$

$$\boxed{2(x-1) + 2(y-1) - \frac{4}{\sqrt{2}}(z - \frac{1}{\sqrt{2}}) = 0}$$