“How to Obtain the Lorentz Space Contraction Formula for a Moving Rod from Knowledge of the Positions of its Ends at Different Times” Simplified.

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In a recent paper [1] the length contraction for a rod in uniform motion is derived performing two measurements at arbitrary times. It is considered that “Provided that the velocity of the rod is known, this derivation does not require the simultaneous measurement of the two ends. Lorentz transformations are involved, a fact that obscures the physics behind the problem. We present a variant of a similar problem proposed by Kard [2] in which the algebraic structure of the factor $\gamma(V)$ that relates the length of a rod in its rest frame (proper length) and the length of the same rod measured by observers relative to whom it moves with constant speed V in the direction which joins its ends. The proposed strategy is sketched in Figure 1. Consider two rods of proper length $L_{0,1}$ and $L_{0,2}$ located along the x axis. The left ends of the two rods coincide at the origin of time with the origin O. We consider first the situation when the first rod is in a state of rest, the second rod moving with velocity V. The length of the moving rod measured by observers relative to whom $L_{0,1}$ is in a state of rest is $\gamma(V)L_{0,2}$ where $\gamma(v)$ is an unknown dimensionless function of V but not of $L_{0,2}$. At the origin of time, a source of light S located at the origin O emits a light signal in the positive direction of the x axis. Suppose that the right end of the moving rod and the light signal arrive simultaneously at the right end of the stationary rod (Figure 2). We take into account the fact that we can add or subtract only length measured by observers of the same inertial reference frame. The geometry of Figure 2 suggests considering that

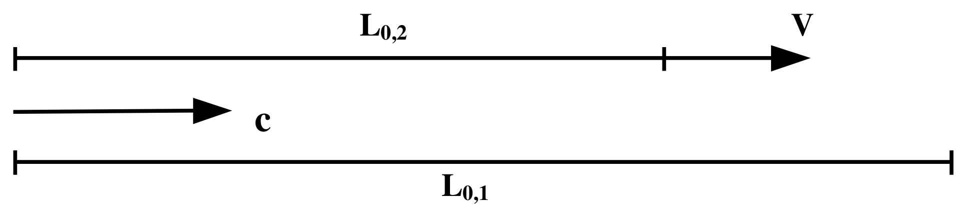

Figure 1. Initial location of rods $L_{0,1}$ and $L_{0,2}$

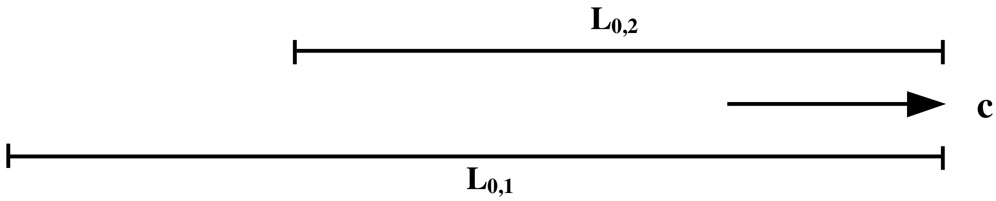

Figure 2. Final location of the involved rods

$$L_{0,1} = \gamma(V) L_{0,2} + \lambda \tag{1}$$

where $\lambda = L_{0,1} - L_{0,2}$ represents the difference between the proper lengths of the two rods. The simultaneous arrival of the right end of the moving rod and of the light signal at the right end of the stationary rod imposes the condition

$$\frac{L_{0,1}}{c} = \frac{\lambda}{V}. \tag{2}$$

Detecting the same situation from the point of view of observers relative to whom the rod of proper length $L_{0,2}$ is in a state of rest we have

$$\gamma(V) L_{0,1} = \lambda' + L_{0,2} \tag{3}$$

and

$$\frac{L_{0,2}}{c} = \frac{\lambda'}{V} \tag{4}$$

Eliminating λ and λ' we obtain

$$\gamma(V) L_{0,1} = \left(1 + \frac{V}{c}\right) L_{0,2} \tag{5}$$

$$\gamma(V) L_{0,2} = \left(1 - \frac{V}{c}\right) L_{0,1}. \tag{6}$$

Multiplying (5) and (6) side by side the result is

$$\gamma(V) = \sqrt{1 - \frac{V^2}{c^2}} \tag{7}$$

and so the relationship between the proper length of a rod L_0 and the length of the same rod L measured by observers relative to whom it moves with constant speed V in the direction which joins its ends is

$$L = L_0 \sqrt{1 - \frac{V^2}{c^2}}. \tag{8}$$

Consider a rod of proper length L_0, at rest in I' inertial reference frame. At its left end we find a source of light S', and a mirror M' at its right end. Reference frame I'

is in the standard configuration with reference frame I relative to which it moves with constant speed V in the positive direction of the overlapped axes x,x'. At the origin of time (t'=0) S' emits a light signal toward the mirror which returns after reflection to O' at a time t' and so

$$2L_0 = c(t'-0) = c\Delta t' \qquad \textbf{(9)}$$

$$L_0 = \frac{c}{2}\Delta t' . \qquad \textbf{(10)}$$

Detecting the same experiment from I we have in the case when the light signal is outgoing

$$c\Delta t_1 = L + V\Delta t_1 \qquad \textbf{(11)}$$

whereas in the case of the reflected signal we have

$$c\Delta t_2 = L - V\Delta t_2 \qquad \textbf{(12)}$$

where L represents the measured length of the rod by observers of I. From (11) and (12) we obtain that the total travel time is

$$\Delta t = \Delta t_1 + \Delta t_2 = \frac{2L}{1-\frac{V^2}{c^2}} . \qquad \textbf{(13)}$$

But

$$\Delta t = \sqrt{\frac{\Delta t'}{1-\frac{V^2}{c^2}}} \qquad \textbf{(14)}$$

accounting for the time dilation effect, we obtain taking into account (10) we recover

$$L = L_0\sqrt{1-\frac{V^2}{c^2}} . \qquad \textbf{(15)}$$

If in [1] equations (8) and (15) are derived imposing a non-simultaneous detection of the space-time coordinates of the ends of the rod, involving their Lorentz transformations we have presented above two derivations which are associated with the non-simultaneous detection but without using the Lorentz transformations a fact that increases the transparence and the accessibility of the approaches presented above.

References

[1] M. Fernandez Guasti and C. Zagoya, "How to obtain the Lorentz space contraction formula for a moving rod from knowledge of the positions of its ends at different times," Eur.J.Phys. 30, 253-258 (2009)

[2] Leo Karlov, " Paul Kard and Lorentz-free special relativity," Phys.Educ. 24, 165-168 (1989)