Testing hierarchical pathway kinetics with residue data on dimethenamid and dimethenamid-P

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Introduction

The purpose of this document is to test demonstrate how nonlinear hierarchical models (NLHM) based on the parent degradation models SFO, FOMC, DFOP and HS, with parallel formation of two or more metabolites can be fitted with the mkin package.

It was assembled in the course of work package 1.2 of Project Number 173340 (Application of nonlinear hierarchical models to the kinetic evaluation of chemical degradation data) of the German Environment Agency carried out in 2022 and 2023.

The mkin package is used in version 1.2.2, which is currently under development. It contains the test data, and the functions used in the evaluations. The saemix package is used as a backend for fitting the NLHM, but is also loaded to make the convergence plot function available.

This document is processed with the knitr package, which also provides the kable function that is used to improve the display of tabular data in R markdown documents. For parallel processing, the parallel package is used.

```
library(mkin)
library(saemix)
library(parallel)
n_cores <- detectCores()
if (Sys.info()["sysname"] == "Windows") {
   cl <- makePSOCKcluster(n_cores)
} else {
   cl <- makeForkCluster(n_cores)
}</pre>
```

Data

The test data are available in the mkin package as an object of class mkindsg (mkin dataset group) under the identifier dimethenamid_2018. The following preprocessing steps are done in this document.

- The data available for the enantiomer dimethenamid-P (DMTAP) are renamed to have the same substance name as the data for the racemic mixture dimethenamid (DMTA). The reason for this is that no difference between their degradation behaviour was identified in the EU risk assessment.
- Unnecessary columns are discarded
- The observation times of each dataset are multiplied with the corresponding normalisation factor also available in the dataset, in order to make it possible to describe all datasets with a single set of parameters that are independent of temperature
- Finally, datasets observed in the same soil (Elliot 1 and Elliot 2) are combined, resulting in dimethenamid (DMTA) data from six soils.

The following commented R code performs this preprocessing.

```
# Apply a function to each of the seven datasets in the mkindsg object to create a list
dmta ds <- lapply(1:7, function(i) {</pre>
                                                                 # Get a dataset
  ds_i <- dimethenamid_2018$ds[[i]]$data
  ds_i[ds_i$name == "DMTAP", "name"] <- "DMTA"</pre>
                                                                 # Rename DMTAP to DMTA
  ds_i <- subset(ds_i, select = c("name", "time", "value")) # Select data</pre>
  ds i$time <- ds i$time * dimethenamid 2018$f time norm[i] # Normalise time
                                                                 # Return the dataset
  ds i
})
# Use dataset titles as names for the list elements
names(dmta_ds) <- sapply(dimethenamid_2018$ds, function(ds) ds$title)</pre>
# Combine data for Elliot soil to obtain a named list with six elements
dmta_ds[["Elliot"]] <- rbind(dmta_ds[["Elliot 1"]], dmta_ds[["Elliot 2"]]) #</pre>
dmta_ds[["Elliot 1"]] <- NULL</pre>
dmta_ds[["Elliot 2"]] <- NULL</pre>
```

The following tables show the 6 datasets.

```
for (ds_name in names(dmta_ds)) {
   print(
     kable(mkin_long_to_wide(dmta_ds[[ds_name]]),
        caption = paste("Dataset", ds_name),
        booktabs = TRUE, row.names = FALSE))
   cat("\n\\clearpage\n")
}
```

Table 1: Dataset Calke

time	DMTA	M23	M27	M31
0	95.8	NA	NA	NA
0	98.7	NA	NA	NA
14	60.5	4.1	1.5	2.0
30	39.1	5.3	2.4	2.1
59	15.2	6.0	3.2	2.2
120	4.8	4.3	3.8	1.8
120	4.6	4.1	3.7	2.1

Table 2: Dataset Borstel

time	DMTA	M23	M27	M31
0.000000	100.5	NA	NA	NA
0.000000	99.6	NA	NA	NA
1.941295	91.9	0.4	NA	NA
1.941295	91.3	0.5	0.3	0.1
6.794534	81.8	1.2	0.8	1.0
6.794534	82.1	1.3	0.9	0.9
13.589067	69.1	2.8	1.4	2.0
13.589067	68.0	2.0	1.4	2.5
27.178135	51.4	2.9	2.7	4.3
27.178135	51.4	4.9	2.6	3.2
56.297565	27.6	12.2	4.4	4.3
56.297565	26.8	12.2	4.7	4.8
86.387643	15.7	12.2	5.4	5.0
86.387643	15.3	12.0	5.2	5.1
115.507073	7.9	10.4	5.4	4.3
115.507073	8.1	11.6	5.4	4.4

Table 3: Dataset Flaach

time	DMTA	M23	M27	M31
0.0000000	96.5	NA	NA	NA
0.0000000	96.8	NA	NA	NA
0.0000000	97.0	NA	NA	NA
0.6233856	82.9	0.7	1.1	0.3
0.6233856	86.7	0.7	1.1	0.3
0.6233856	87.4	0.2	0.3	0.1
1.8701567	72.8	2.2	2.6	0.7
1.8701567	69.9	1.8	2.4	0.6
1.8701567	71.9	1.6	2.3	0.7
4.3636989	51.4	4.1	5.0	1.3
4.3636989	52.9	4.2	5.9	1.2
4.3636989	48.6	4.2	4.8	1.4
8.7273979	28.5	7.5	8.5	2.4
8.7273979	27.3	7.1	8.5	2.1
8.7273979	27.5	7.5	8.3	2.3
13.0910968	14.8	8.4	9.3	3.3
13.0910968	13.4	6.8	8.7	2.4
13.0910968	14.4	8.0	9.1	2.6
17.4547957	7.7	7.2	8.6	4.0
17.4547957	7.3	7.2	8.5	3.6
17.4547957	8.1	6.9	8.9	3.3
26.1821936	2.0	4.9	8.1	2.1
26.1821936	1.5	4.3	7.7	1.7
26.1821936	1.9	4.5	7.4	1.8
34.9095915	1.3	3.8	5.9	1.6
34.9095915	1.0	3.1	6.0	1.6
34.9095915	1.1	3.1	5.9	1.4
43.6369893	0.9	2.7	5.6	1.8
43.6369893	0.7	2.3	5.2	1.5
43.6369893	0.7	2.1	5.6	1.3
52.3643872	0.6	1.6	4.3	1.2
52.3643872	0.4	1.1	3.7	0.9
52.3643872	0.5	1.3	3.9	1.1
74.8062674	0.4	0.4	2.5	0.5
74.8062674	0.3	0.4	2.4	0.5
74.8062674	0.3	0.3	2.2	0.3

Table 4: Dataset BBA 2.2

time	DMTA	M23	M27	M31
0.0000000	98.09	NA	NA	NA
0.0000000	98.77	NA	NA	NA
0.7678922	93.52	0.36	0.42	0.36
0.7678922	92.03	0.40	0.47	0.33
2.3036765	88.39	1.03	0.71	0.55
2.3036765	87.18	1.07	0.82	0.64
5.3752452	69.38	3.60	2.19	1.94
5.3752452	71.06	3.66	2.28	1.62
10.7504904	45.21	6.97	5.45	4.22
10.7504904	46.81	7.22	5.19	4.37
16.1257355	30.54	8.65	8.81	6.31
16.1257355	30.07	8.38	7.93	6.85
21.5009807	21.60	9.10	10.25	7.05
21.5009807	20.41	8.63	10.77	6.84
32.2514711	9.10	7.63	10.89	6.53
32.2514711	9.70	8.01	10.85	7.11
43.0019614	6.58	6.40	10.41	6.06
43.0019614	6.31	6.35	10.35	6.05
53.7524518	3.47	5.35	9.92	5.50
53.7524518	3.52	5.06	9.42	5.07
64.5029421	3.40	5.14	9.15	4.94
64.5029421	3.67	5.91	9.25	4.39
91.3791680	1.62	3.35	7.14	3.64
91.3791680	1.62	2.87	7.13	3.55

Table 5: Dataset BBA 2.3

$_{ m time}$	DMTA	M23	M27	M31
0.0000000	99.33	NA	NA	NA
0.0000000	97.44	NA	NA	NA
0.6733938	93.73	0.18	0.50	0.47
0.6733938	93.77	0.18	0.83	0.34
2.0201814	87.84	0.52	1.25	1.00
2.0201814	89.82	0.43	1.09	0.89
4.7137565	71.61	1.19	3.28	3.58
4.7137565	71.42	1.11	3.24	3.41
9.4275131	45.60	2.26	7.17	8.74
9.4275131	45.42	1.99	7.91	8.28
14.1412696	31.12	2.81	10.15	9.67
14.1412696	31.68	2.83	9.55	8.95
18.8550262	23.20	3.39	12.09	10.34
18.8550262	24.13	3.56	11.89	10.00
28.2825393	9.43	3.49	13.32	7.89
28.2825393	9.82	3.28	12.05	8.13
37.7100523	7.08	2.80	10.04	5.06
37.7100523	8.64	2.97	10.78	5.54
47.1375654	4.41	2.42	9.32	3.79
47.1375654	4.78	2.51	9.62	4.11
56.5650785	4.92	2.22	8.00	3.11
56.5650785	5.08	1.95	8.45	2.98
80.1338612	2.13	1.28	5.71	1.78
80.1338612	2.23	0.99	3.33	1.55

Table 6: Dataset Elliot

time	DMTA	M23	M27	M31
0.000000	97.5	NA	NA	NA
0.000000	100.7	NA	NA	NA
1.228478	86.4	NA	NA	NA
1.228478	88.5	NA	NA	1.5
3.685435	69.8	2.8	2.3	5.0
3.685435	77.1	1.7	2.1	2.4
8.599349	59.0	4.3	4.0	4.3
8.599349	54.2	5.8	3.4	5.0
17.198697	31.3	8.2	6.6	8.0
17.198697	33.5	5.2	6.9	7.7
25.798046	19.6	5.1	8.2	7.8
25.798046	20.9	6.1	8.8	6.5
34.397395	13.3	6.0	9.7	8.0
34.397395	15.8	6.0	8.8	7.4
51.596092	6.7	5.0	8.3	6.9
51.596092	8.7	4.2	9.2	9.0
68.794789	8.8	3.9	9.3	5.5
68.794789	8.7	2.9	8.5	6.1
103.192184	6.0	1.9	8.6	6.1
103.192184	4.4	1.5	6.0	4.0
146.188928	3.3	2.0	5.6	3.1
146.188928	2.8	2.3	4.5	2.9
223.583066	1.4	1.2	4.1	1.8
223.583066	1.8	1.9	3.9	2.6
0.000000	93.4	NA	NA	NA
0.000000	103.2	NA	NA	NA
1.228478	89.2	NA	NA	1.3
1.228478	86.6	NA	NA	NA
3.685435	78.2	2.6	1.0	3.1
3.685435	78.1	2.4	2.6	2.3
8.599349	55.6	5.5	4.5	3.4
8.599349	53.0	5.6	4.6	4.3
17.198697	33.7	7.3	7.6	7.8
17.198697	33.2	6.5	6.7	8.7
25.798046	20.9	5.8	8.7	7.7
25.798046	19.9	7.7	7.6	6.5
34.397395	18.2	7.8	8.0	6.3
34.397395	12.7	7.3	8.6	8.7
51.596092	7.8	7.0	7.4	5.7
51.596092	9.0	6.3	7.2	4.2
68.794789	11.4	4.3	10.3	3.2
68.794789	9.0	3.8	9.4	4.2
103.192184	3.9	2.6	6.5	3.8
103.192184	4.4	2.8	6.9	4.0
146.188928	2.6	1.6	4.6	4.5
146.188928	3.4	1.1	4.5	4.5
223.583066	2.0	1.4	4.3	3.8
223.583066	1.7	1.3	4.2	2.3

Separate evaluations

As a first step to obtain suitable starting parameters for the NLHM fits, we do separate fits of several variants of the pathway model used previously (Ranke et al. 2021), varying the kinetic model for the parent compound. Because the SFORB model often provides faster convergence than the DFOP model, and can sometimes be fitted where the DFOP model results in errors, it is included in the set of parent models tested here.

```
if (!dir.exists("dmta_dlls")) dir.create("dmta_dlls")
m_sfo_path_1 <- mkinmod(</pre>
  DMTA = mkinsub("SFO", c("M23", "M27", "M31")),
 M23 = mkinsub("SFO"),
 M27 = mkinsub("SFO"),
 M31 = mkinsub("SFO", "M27", sink = FALSE),
  name = "m_sfo_path", dll_dir = "dmta_dlls",
  unload = TRUE, overwrite = TRUE,
  quiet = TRUE
)
m_fomc_path_1 <- mkinmod(</pre>
  DMTA = mkinsub("FOMC", c("M23", "M27", "M31")),
 M23 = mkinsub("SFO"),
 M27 = mkinsub("SFO"),
 M31 = mkinsub("SFO", "M27", sink = FALSE),
  name = "m_fomc_path", dll_dir = "dmta_dlls",
  unload = TRUE, overwrite = TRUE,
  quiet = TRUE
)
m_dfop_path_1 <- mkinmod(</pre>
  DMTA = mkinsub("DFOP", c("M23", "M27", "M31")),
 M23 = mkinsub("SFO"),
  M27 = mkinsub("SFO"),
 M31 = mkinsub("SFO", "M27", sink = FALSE),
  name = "m_dfop_path", dll_dir = "dmta_dlls",
  unload = TRUE, overwrite = TRUE,
  quiet = TRUE
m_sforb_path_1 <- mkinmod(</pre>
 DMTA = mkinsub("SFORB", c("M23", "M27", "M31")),
  M23 = mkinsub("SFO"),
 M27 = mkinsub("SFO"),
 M31 = mkinsub("SFO", "M27", sink = FALSE),
  name = "m_sforb_path", dll_dir = "dmta_dlls",
  unload = TRUE, overwrite = TRUE,
  quiet = TRUE
m_hs_path_1 <- mkinmod(</pre>
 DMTA = mkinsub("HS", c("M23", "M27", "M31")),
 M23 = mkinsub("SFO"),
 M27 = mkinsub("SFO"),
 M31 = mkinsub("SFO", "M27", sink = FALSE),
  name = "m_hs_path", dll_dir = "dmta_dlls",
  unload = TRUE, overwrite = TRUE,
  quiet = TRUE
deg_mods_1 <- list(</pre>
 sfo_path_1 = m_sfo_path_1,
 fomc_path_1 = m_fomc_path_1,
 dfop_path_1 = m_dfop_path_1,
```

```
sforb_path_1 = m_sforb_path_1,
hs_path_1 = m_hs_path_1)

sep_1_const <- mmkin(
   deg_mods_1,
   dmta_ds,
   error_model = "const",
   quiet = TRUE)

status(sep_1_const) |> kable()
```

	Calke	Borstel	Flaach	BBA 2.2	BBA 2.3	Elliot
sfo_path_1	OK	OK	OK	OK	OK	OK
$fomc_path_1$	OK	OK	OK	OK	OK	OK
$dfop_path_1$	OK	OK	\mathbf{C}	OK	OK	OK
$sforb_path_1$	OK	OK	\mathbf{C}	OK	OK	OK
hs_path_1	\mathbf{C}	\mathbf{C}	\mathbf{C}	\mathbf{C}	\mathbf{C}	С

All separate pathway fits with SFO or FOMC for the parent and constant variance converged (status OK). Most fits with DFOP or SFORB for the parent converged as well. The fits with HS for the parent did not converge with default settings.

```
sep_1_tc <- update(sep_1_const, error_model = "tc")
status(sep_1_tc) |> kable()
```

	Calke	Borstel	Flaach	BBA 2.2	BBA 2.3	Elliot
sfo_path_1	OK	OK	OK	OK	OK	OK
$fomc_path_1$	OK	OK	\mathbf{C}	OK	OK	\mathbf{C}
$dfop_path_1$	OK	\mathbf{C}	OK	OK	OK	OK
$sforb_path_1$	OK	\mathbf{C}	OK	OK	OK	OK
hs_path_1	С	С	С	С	С	OK

With the two-component error model, the set of fits with convergence problems is slightly different, with convergence problems appearing for different data sets when applying the DFOP and SFORB model and some additional convergence problems when using the FOMC model for the parent.

Hierarchichal model fits

The following code fits two sets of the corresponding hierarchical models to the data, one assuming constant variance, and one assuming two-component error.

```
saem_1 <- mhmkin(list(sep_1_const, sep_1_tc))</pre>
```

The run time for these fits was around two hours on five year old hardware. After a recent hardware upgrade these fits complete in less than twenty minutes.

status(saem_1) |> kable()

	const	tc
sfo_path_1	OK	OK
$fomc_path_1$	OK	OK
$dfop_path_1$	OK	OK
$sforb_path_1$	OK	OK
hs_path_1	OK	OK

According to the status function, all fits terminated successfully.

```
anova(saem_1) |> kable(digits = 1)
```

Warning in FUN(X[[i]], ...): Could not obtain log likelihood with 'is' method for sforb_path_1 const

	npar	AIC	BIC	Lik
sfo_path_1 const	17	2291.8	2288.3	-1128.9
$sfo_path_1 tc$	18	2276.3	2272.5	-1120.1
$fomc_path_1 const$	19	2099.0	2095.0	-1030.5
$fomc_path_1 tc$	20	1939.6	1935.5	-949.8
$dfop_path_1 const$	21	2038.8	2034.4	-998.4
$hs_path_1 const$	21	2024.2	2019.8	-991.1
$dfop_path_1 tc$	22	1879.8	1875.2	-917.9
$sforb_path_1 tc$	22	1832.9	1828.3	-894.4
$hs_path_1 tc$	22	1831.4	1826.8	-893.7

When the goodness-of-fit of the models is compared, a warning is obtained, indicating that the likelihood of the pathway fit with SFORB for the parent compound and constant variance could not be calculated with importance sampling (method 'is'). As this is the default method on which all AIC and BIC comparisons are based, this variant is not included in the model comparison table. Comparing the goodness-of-fit of the remaining models, HS model model with two-component error provides the best fit. However, for batch experiments performed with constant conditions such as the experiments evaluated here, there is no reason to assume a discontinuity, so the SFORB model is preferable from a mechanistic viewpoint. In addition, the information criteria AIC and BIC are very similar for HS and SFORB. Therefore, the SFORB model is selected here for further refinements.

Parameter identifiability based on the Fisher Information Matrix

Using the illparms function, ill-defined statistical model parameters such as standard deviations of the degradation parameters in the population and error model parameters can be found.

illparms(saem_1) |> kable()

	const	tc
sfo_path_1		$sd(DMTA_0)$
$fomc_path_1$		$sd(DMTA_0)$
$dfop_path_1$		
$sforb_path_1$		$sd(log_k_DMTA_bound_free)$
hs_path_1		$sd(log_tb)$

When using constant variance, no ill-defined variance parameters are identified with the illparms function in any of the degradation models. When using the two-component error model, there is one ill-defined variance parameter in all variants except for the variant using DFOP for the parent compound.

For the selected combination of the SFORB pathway model with two-component error, the random effect for the rate constant from reversibly bound DMTA to the free DMTA (k_DMTA_bound_free) is not well-defined. Therefore, the fit is updated without assuming a random effect for this parameter.

```
saem_sforb_path_1_tc_reduced <- update(saem_1[["sforb_path_1", "tc"]],
    no_random_effect = "log_k_DMTA_bound_free")
illparms(saem_sforb_path_1_tc_reduced)</pre>
```

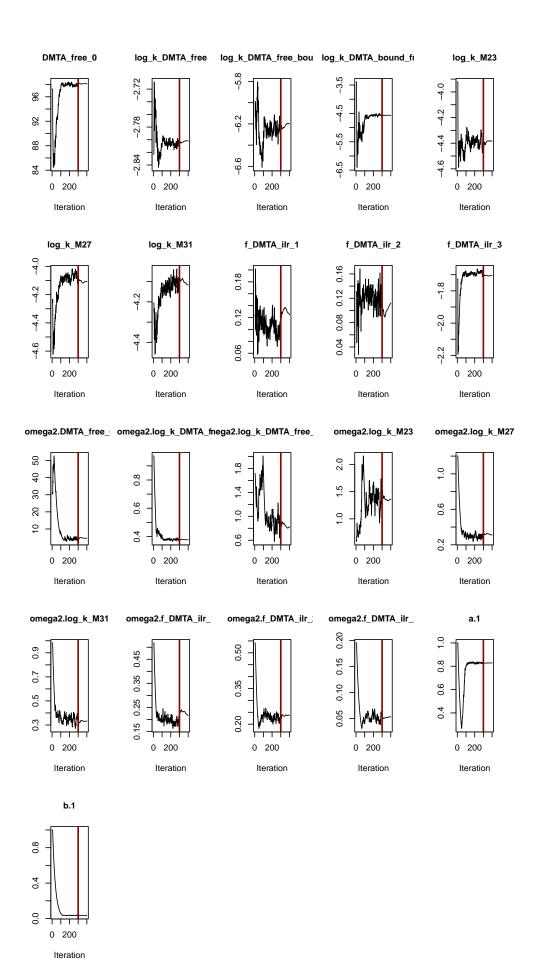
As expected, no ill-defined parameters remain. The model comparison below shows that the reduced model is preferable.

```
anova(saem_1[["sforb_path_1", "tc"]], saem_sforb_path_1_tc_reduced) |> kable(digits = 1)
```

	npar	AIC	BIC	Lik
saem_sforb_path_1_tc_reduced saem_1[["sforb_path_1", "tc"]]		1830.3 1832.9		

The convergence plot of the refined fit is shown below.

```
plot(saem_sforb_path_1_tc_reduced$so, plot.type = "convergence")
```



For some parameters, for example for f_DMTA_ilr_1 and f_DMTA_ilr_2, i.e. for two of the parameters determining the formation fractions of the parallel formation of the three metabolites, some movement of the parameters is still visible in the second phase of the algorithm. However, the amplitude of this movement is in the range of the amplitude towards the end of the first phase. Therefore, it is likely that an increase in iterations would not improve the parameter estimates very much, and it is proposed that the fit is acceptable. No numeric convergence criterion is implemented in saemix.

Alternative check of parameter identifiability

As an alternative check of parameter identifiability (Duchesne et al. 2021), multistart runs were performed on the basis of the refined fit shown above.

```
saem_sforb_path_1_tc_reduced_multi <- multistart(saem_sforb_path_1_tc_reduced,</pre>
 n = 32, cores = 10)
print(saem_sforb_path_1_tc_reduced_multi)
<multistart> object with 32 fits:
 E OK
```

15 17 OK: Fit terminated successfully

E: Error

Out of the 32 fits that were initiated, only 17 terminated without an error. The reason for this is that the wide variation of starting parameters in combination with the parameter variation that is used in the SAEM algorithm leads to parameter combinations for the degradation model that the numerical integration routine cannot cope with. Because of this variation of initial parameters, some of the model fits take up to two times more time than the original fit.

```
par(mar = c(12.1, 4.1, 2.1, 2.1))
parplot(saem_sforb_path_1_tc_reduced_multi, ylim = c(0.5, 2), las = 2)
```

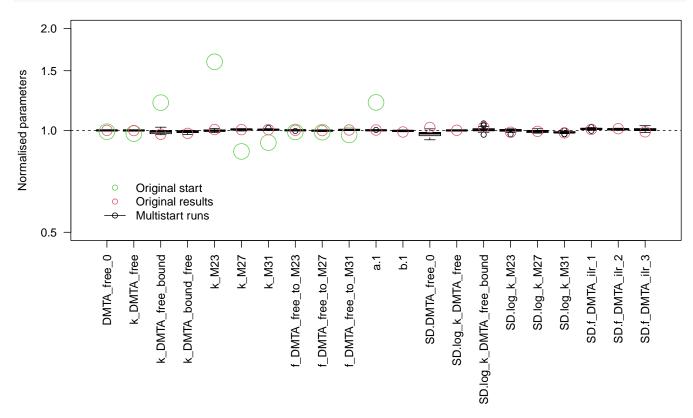


Figure 1: Parameter boxplots for the multistart runs that succeeded

However, visual analysis of the boxplot of the parameters obtained in the successful fits confirms that the results are sufficiently independent of the starting parameters, and there are no remaining ill-defined parameters.

Plots of selected fits

The SFORB pathway fits with full and reduced parameter distribution model are shown below.

plot(saem_1[["sforb_path_1", "tc"]])

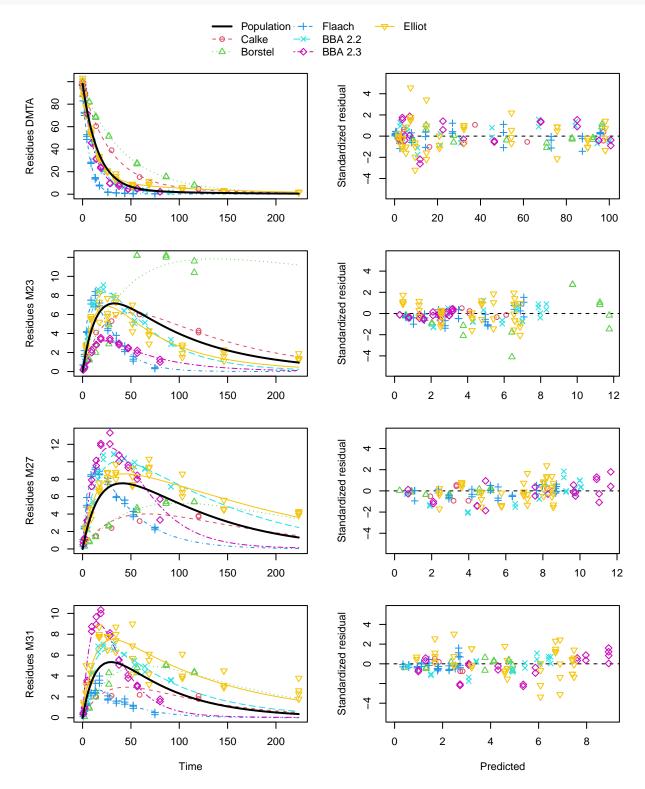


Figure 2: SFORB pathway fit with two-component error $\,$

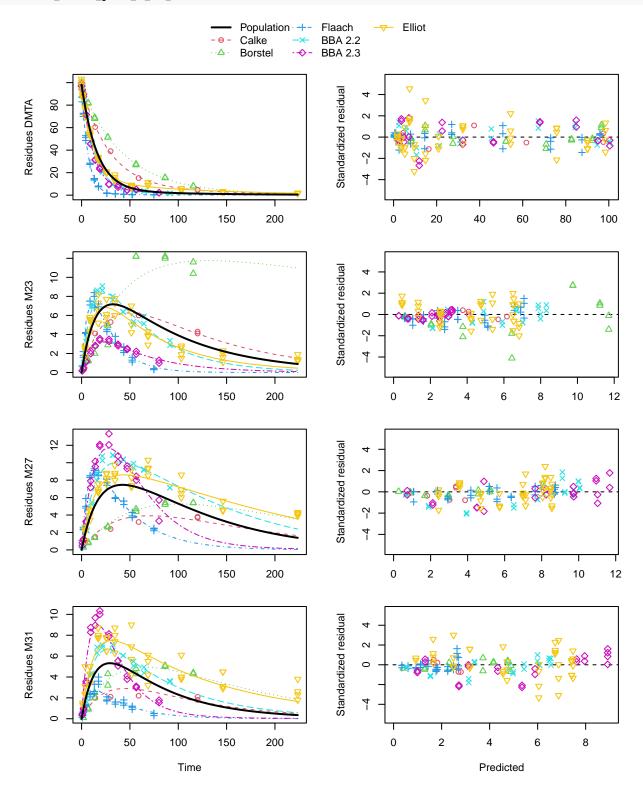


Figure 3: SFORB pathway fit with two-component error, reduced parameter model

Plots of the remaining fits and listings for all successful fits are shown in the Appendix.

Conclusions

Pathway fits with SFO, FOMC, DFOP, SFORB and HS models for the parent compound could be successfully performed.

Acknowledgements

The helpful comments by Janina Wöltjen of the German Environment Agency on earlier versions of this document are gratefully acknowledged.

References

Duchesne, Ronan, Anissa Guillemin, Olivier Gandrillon, and Fabien Crauste. 2021. "Practical Identifiability in the Frame of Nonlinear Mixed Effects Models: The Example of the in Vitro Erythropoiesis." *BMC Bioinformatics* 22 (478). https://doi.org/10.1186/s12859-021-04373-4.

Ranke, Johannes, Janina Wöltjen, Jana Schmidt, and Emmanuelle Comets. 2021. "Taking Kinetic Evaluations of Degradation Data to the Next Level with Nonlinear Mixed-Effects Models." *Environments* 8 (8). https://doi.org/10.3390/environments8080071.

Appendix

Plots of hierarchical fits not selected for refinement

plot(saem_1[["sfo_path_1", "tc"]])

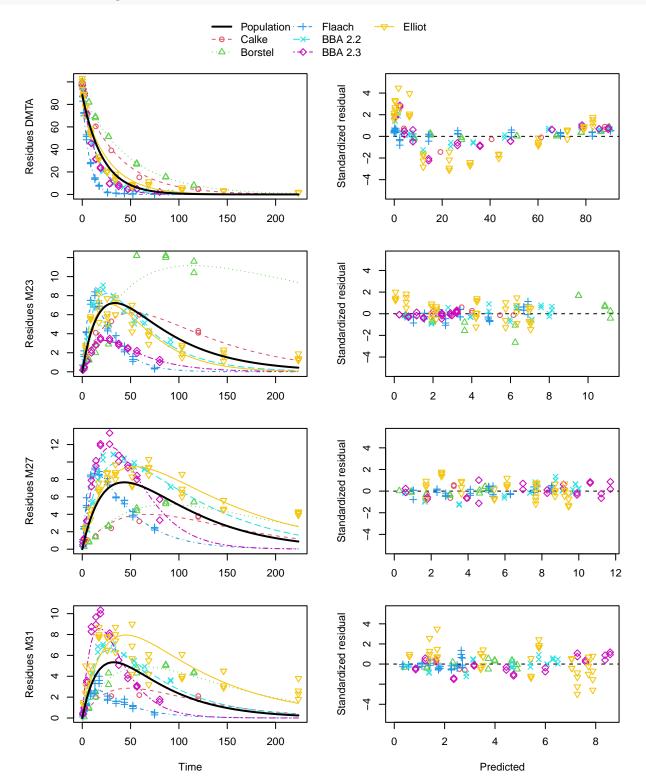


Figure 4: SFO pathway fit with two-component error

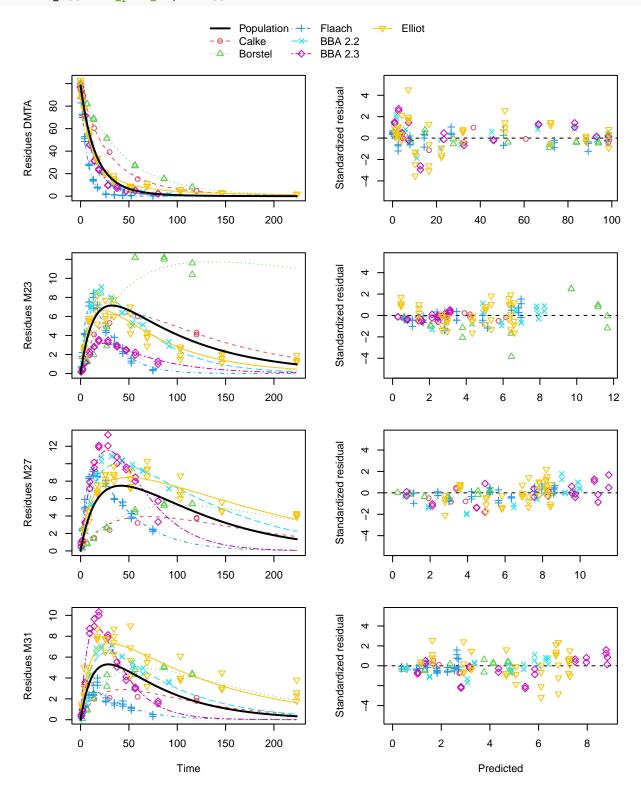


Figure 5: FOMC pathway fit with two-component error

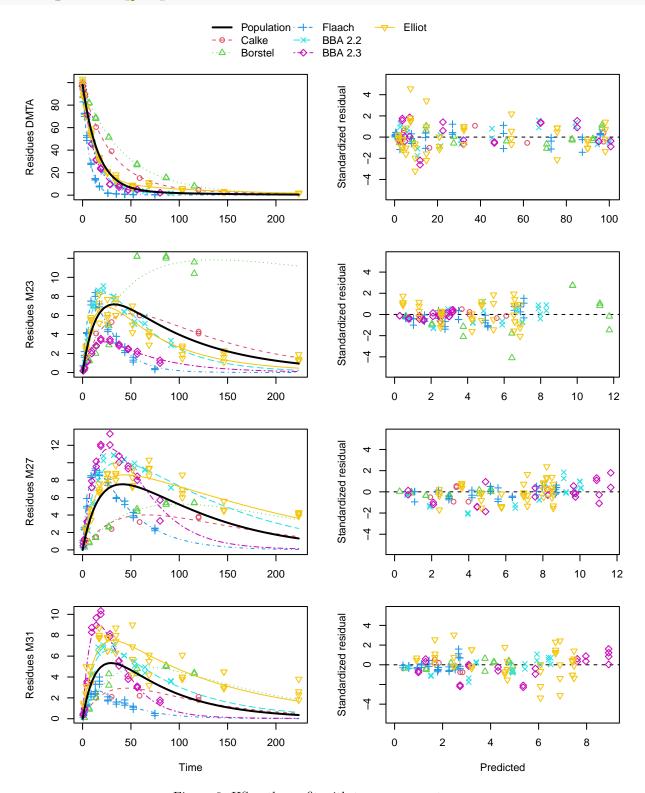


Figure 6: HS pathway fit with two-component error

Hierarchical model fit listings

Fits with random effects for all degradation parameters

Listing 1: Hierarchical SFO path 1 fit with constant variance

```
saemix version used for fitting:
mkin version used for pre-fitting: 1.2.2
R version used for fitting:
Date of fit: Thu Jan 5 15:07:58 2023
Date of summary: Thu Jan 5 16:42:52 2023
d_DMTA/dt = - k_DMTA * DMTA
d_M23/dt = + f_DMTA_to_M23 * k_DMTA * DMTA - k_M23 * M23
563 observations of 4 variable(s) grouped in 6 datasets
Model predictions using solution type deSolve
Fitted in 756.354 s
Using 300, 100 iterations and 9 chains
Variance model: Constant variance
Starting values for degradation parameters:
     DMTA_0 log_k_DMTA
97.2914 -2.8719
                            log_k_M23
                                         log_k_M27
                                                      log_k_M31 f_DMTA_ilr_1
     97.2914
                               -4.4799
                                           -4.1939
                                                        -4.1675
                                                                     0.1423
f_DMTA_ilr_2 f_DMTA_ilr_3
                 -1.7035
      0.1429
Fixed degradation parameter values:
None
Starting values for random effects (square root of initial entries in omega):
             {\tt DMTA\_0~log\_k\_DMTA~log\_k\_M23~log\_k\_M27~log\_k\_M31~f\_DMTA\_ilr\_1}
DMTA O
                       0.0000
                                  0.000
                                            0.000
                                                      0.000
              2.947
                                                                  0.0000
log_k_DMTA
                       0.9413
                                            0.000
                                                                  0.0000
              0.000
                                  0.000
                                                      0.000
log_k_M23
             0.000
                       0.0000
                                  2.331
                                            0.000
                                                                  0.0000
                                                      0.000
log_k_M27
              0.000
                       0.0000
                                  0.000
                                            1.076
                                                      0.000
                                                                  0.0000
log_k_M31
              0.000
                       0.0000
                                  0.000
                                            0.000
                                                      1.004
                                                                  0.0000
f_DMTA_ilr_1 0.000
                       0.0000
                                  0.000
                                            0.000
                                                      0.000
                                                                  0.7349
                       0.0000
f_DMTA_ilr_2 0.000
                                  0.000
                                            0.000
                                                      0.000
                                                                  0.0000
f_DMTA_ilr_3 0.000
                       0.0000
                                  0.000
                                            0.000
                                                      0.000
                                                                  0.0000
             {\tt f\_DMTA\_ilr\_2~f\_DMTA\_ilr\_3}
DMTA_0
                  0.0000
                               0.0000
log_k_DMTA
                   0.0000
                               0.0000
log_k_M23
                   0.0000
                               0.0000
log_k_M27
                   0.0000
                               0.0000
log_k_M31
                   0.0000
                               0.0000
f_DMTA_ilr_1
                   0.0000
                               0.0000
f_DMTA_ilr_2
                   0.7365
                               0.0000
f_DMTA_ilr_3
                   0.0000
                               0.4565
Starting values for error model parameters:
a.1
Results:
Likelihood computed by importance sampling
   AIC BIC logLik
  2292 2288 -1129
Optimised parameters:
                   est.
                           lower
                                   upper
                97.26786 95.67823 98.8575
DMTA_0
log_k_DMTA
               -2.87273 -3.36023 -2.3852
log_k_M23
               -4.18506 -5.00826 -3.3619
log_k_M27
               -3.87596 -4.29723 -3.4547
log_k_M31
               -4.02059 -4.57377 -3.4674
f_DMTA_ilr_1
                0.05974 -0.27557 0.3950
                0.14183 -0.26810 0.5518
f_DMTA_ilr_2
f_DMTA_ilr_3
                -1.60231 -1.75871 -1.4459
                1.57902 1.49964 1.6584
a.1
SD.DMTA_O
                 1.84675 0.64110 3.0524
SD.log_k_DMTA
                0.60889 0.26400 0.9538
SD.log_k_M23
                0.94708 0.33329 1.5609
{\tt SD.log\_k\_M27}
                 0.46988 0.15182 0.7879
                0.59868 0.17038 1.0270
SD.log_k_M31
```

```
SD.f_DMTA_ilr_1 0.38216 0.12846 0.6359
SD.f_DMTA_ilr_2 0.46970 0.16345 0.7760
SD.f_DMTA_ilr_3 0.16279 0.03761 0.2880
            DMTA_0 1__DMTA 1g__M23 1g__M27 1g__M31 f_DMTA__1 f_DMTA__2
log_k_DMTA
           0.0060
log_k_M23
           -0.0037 -0.0007
           -0.0049 -0.0009 -0.0033
-0.0032 -0.0006 -0.0011 0.1030
log_k_M27
log_k_M31
f_DMTA_ilr_1 -0.0011 -0.0002 0.0731 -0.0517 0.0483
f_DMTA_ilr_2 -0.0003 -0.0001 0.0340 -0.0415 -0.1324 -0.0457
f_DMTA_ilr_3 -0.0303 -0.0032 0.0888 0.1419 0.0769 -0.0052 -0.0019
Random effects:
                est. lower upper
               1.8467 0.64110 3.0524
SD.DMTA_O
SD.f_DMTA_ilr_1 0.3822 0.12846 0.6359
SD.f_DMTA_ilr_2 0.4697 0.16345 0.7760
SD.f_DMTA_ilr_3 0.1628 0.03761 0.2880
Variance model:
    est. lower upper
a.1 1.579 1.5 1.658
{\tt Backtransformed\ parameters:}
             est. lower upper 97.26786 95.678234 98.85749
DMTA O
            0.05654 0.034727 0.09207
0.01522 0.006683 0.03467
k_DMTA
k_M23
             0.02073 0.013606 0.03160
k_M27
              0.01794 0.010319 0.03120
k_M31
f_DMTA_to_M23 0.11793
                         NA
                                      NΑ
                                      NA
f_DMTA_to_M27 0.10838
                             NA
f_DMTA_to_M31 0.09503
                             NA
                                      NA
Resulting formation fractions:
              ff
DMTA_M23 0.11793
DMTA_M27 0.10838
DMTA_M31 0.09503
DMTA_sink 0.67867
Estimated disappearance times:
DT50 DT90
DMTA 12.26 40.72
M23 45.54 151.27
M27 33.43 111.05
M31 38.63 128.33
```

Listing 2: Hierarchical SFO path 1 fit with two-component error

```
saemix version used for fitting:
                                   3.2
mkin version used for pre-fitting: 1.2.2
R version used for fitting:
                                  4.2.2
Date of fit: Thu Jan 5 15:08:15 2023
Date of summary: Thu Jan 5 16:42:52 2023
Equations:
d_DMTA/dt = - k_DMTA * DMTA
d_{M27}/dt = + f_{DMTA}/to_{M27} * k_{DMTA} * DMTA - k_{M27} * M27 + k_{M31} * M31
d_M31/dt = + f_DMTA_to_M31 * k_DMTA * DMTA - k_M31 * M31
563 observations of 4 variable(s) grouped in 6 datasets
Model predictions using solution type deSolve
Fitted in 773.377 s
Using 300, 100 iterations and 9 chains
Variance model: Two-component variance function
Starting values for degradation parameters:
     DMTA_0 log_k_DMTA log_k_M23 log_k_M27
95.5662 -2.9048 -3.8130 -4.1600
                                                     log_k_M31 f_DMTA_ilr_1
     95.5662
                                                       -4.1486
f_DMTA_ilr_2 f_DMTA_ilr_3
     0.1385
                 -1.6700
Fixed degradation parameter values:
Starting values for random effects (square root of initial entries in omega):
            DMTA_0 log_k_DMTA log_k_M23 log_k_M27 log_k_M31 f_DMTA_ilr_1
                       0.0000
                                 0.0000
                                           0.000
                                                    0.0000
log_k_DMTA
                       0.9834
                                           0.000
                                                    0.0000
             0.000
                                 0.0000
                                                                 0.0000
log_k_M23
             0.000
                       0.0000
                                 0.6983
                                           0.000
                                                    0.0000
                                                                 0.0000
log_k_M27
             0.000
                       0.0000
                                 0.0000
                                           1.028
                                                    0.0000
                                                                 0.0000
log_k_M31
             0.000
                       0.0000
                                 0.0000
                                           0.000
                                                    0.9841
                                                                 0.0000
f_DMTA_ilr_1 0.000
                       0.0000
                                 0.0000
                                           0.000
                                                    0.0000
                                                                 0.7185
f_DMTA_ilr_2 0.000
                      0.0000
                                 0.0000
                                           0.000
                                                    0.0000
                                                                 0.0000
f_DMTA_ilr_3 0.000
                       0.0000
                                 0.0000
                                           0.000
                                                   0.0000
                                                                 0.0000
            f_DMTA_ilr_2 f_DMTA_ilr_3
DMTA_0
                  0.0000
                               0.0000
log_k_DMTA
                  0.0000
                               0.0000
log_k_M23
                  0.0000
                               0.0000
log_k_M27
                  0.0000
                               0.0000
log_k_M31
                  0.0000
                               0.0000
f_DMTA_ilr_1
                  0.0000
                               0.0000
f_DMTA_ilr_2
                  0.7378
                               0.0000
                  0.0000
f DMTA ilr 3
                               0.4451
Starting values for error model parameters:
a.1 b.1
 1 1
Results:
Likelihood computed by importance sampling
  AIC BIC logLik
 2276 2273 -1120
Optimised parameters:
                 est.
                        lower
DMTA O
               88.3192 83.8656 92.7729
log_k_DMTA
               -3.0530 -3.5686 -2.5373
               -4.0620 -4.9202 -3.2038
log_k_M23
log_k_M27
               -3.8633 -4.2668 -3.4598
log_k_M31
               -3.9731 -4.4763 -3.4699
f_DMTA_ilr_1
               0.1346 -0.2150 0.4841
f_DMTA_ilr_2
               0.1449 -0.2593 0.5491
f_DMTA_ilr_3
               -1.3882 -1.7011 -1.0753
a.1
               0.9156 0.8217 1.0095
b.1
                0.1383 0.1216 0.1550
SD.DMTA_O
                3.7280 -0.6949 8.1508
SD.log_k_DMTA
                0.6431 0.2781 1.0080
SD.log_k_M23
                1.0096 0.3782
                               1.6409
SD.log_k_M27
                0.4583 0.1541 0.7625
SD.log_k_M31
                0.5738 0.1942
                                0.9533
SD.f_DMTA_ilr_1 0.4119 0.1528
                               0.6709
SD.f_DMTA_ilr_2 0.4780 0.1806
SD.f_DMTA_ilr_3 0.3657 0.1383 0.5931
```

```
Correlation:
            DMTA_0 1__DMTA 1g__M23 1g__M27 1g__M31 f_DMTA__1 f_DMTA__2
log_k_DMTA 0.0303
           -0.0229 -0.0032
-0.0372 -0.0049 0.0041
log_k_M23
log_k_M27
log_k_M31
           -0.0245 -0.0032 0.0022 0.0815
f_DMTA_ilr_1 -0.0046 -0.0006 0.0415 -0.0433 0.0324
f_DMTA_ilr_2 -0.0008 -0.0002 0.0214 -0.0267 -0.0893 -0.0361
f_DMTA_ilr_3 -0.1755 -0.0135 0.0423 0.0775 0.0377 -0.0066
Random effects:
                 est.
                       lower upper
SD.DMTA_O
              3.7280 -0.6949 8.1508
SD.f_DMTA_ilr_1 0.4119 0.1528 0.6709
SD.f_DMTA_ilr_2 0.4780 0.1806 0.7754
SD.f_DMTA_ilr_3 0.3657 0.1383 0.5931
Variance model:
est. lower upper a.1 0.9156 0.8217 1.009
b.1 0.1383 0.1216 0.155
{\tt Backtransformed\ parameters:}
                         lower
                 est.
                                   upper
             88.31924 83.865625 92.77286
DMTA O
k_DMTA
              0.04722 0.028196 0.07908
              0.01721 0.007298 0.04061
0.02100 0.014027 0.03144
0.01882 0.011375 0.03112
k M23
k_M27
k M31
f_DMTA_to_M23 0.12077

f_DMTA_to_M27 0.12077
                        NA
NA
                                      NΑ
                                      NA
f_DMTA_to_M31 0.11123
                           NA
                                      NA
Resulting formation fractions:
             ff
DMTA M23 0.1461
DMTA_M27 0.1208
DMTA_M31 0.1112
DMTA_sink 0.6219
Estimated disappearance times:
     DT50 DT90
DMTA 14.68 48.76
M23 40.27 133.76
M27 33.01 109.65
M31 36.84 122.38
```

Listing 3: Hierarchical FOMC path 1 fit with constant variance

```
saemix version used for fitting:
                                     3.2
mkin version used for pre-fitting: 1.2.2
R version used for fitting:
                                   4.2.2
Date of fit: Thu Jan 5 15:09:48 2023
Date of summary: Thu Jan 5 16:42:52 2023
d_DMTA/dt = - (alpha/beta) * 1/((time/beta) + 1) * DMTA
d_M23/dt = + f_DMTA_to_M23 * (alpha/beta) * 1/((time/beta) + 1) * DMTA
          - k_M23 * M23
d_M27/dt = + f_DMTA_to_M27 * (alpha/beta) * 1/((time/beta) + 1) * DMTA
          - k_M27 * M27 + k_M31 * M31
d_M31/dt = + f_DMTA_to_M31 * (alpha/beta) * 1/((time/beta) + 1) * DMTA
          - k_M31 * M31
Data:
563 observations of 4 variable(s) grouped in 6 datasets
Model predictions using solution type deSolve
Fitted in 866.372 s
Using 300, 100 iterations and 9 chains
Variance model: Constant variance
Starting values for degradation parameters:
     DMTA_0
               log_k_M23 log_k_M27
                                         log_k_M31 f_DMTA_ilr_1 f_DMTA_ilr_2
                  -3.8778
                               -4.2808
                                            -4.2073
     98.1960
                                                         0.1409
f_DMTA_ilr_3
                log_alpha
                              log_beta
     -1.7285
                   2.1896
Fixed degradation parameter values:
Starting values for random effects (square root of initial entries in omega):
            DMTA_0 log_k_M23 log_k_M27 log_k_M31 f_DMTA_ilr_1 f_DMTA_ilr_2
DMTA_0
              2.663
                      0.0000
                                  0.000
                                            0.000
                                                        0.0000
                                                                     0.0000
log_k_M23
              0.000
                      0.7521
                                  0.000
                                            0.000
                                                        0.0000
                                                                     0.0000
log_k_M27
              0.000
                      0.0000
                                  1.205
                                            0.000
                                                        0.0000
                                                                     0.0000
log_k_M31
              0.000
                     0.0000
                                 0.000
                                           1.063
                                                        0.0000
                                                                     0.0000
f_DMTA_ilr_1
              0.000
                      0.0000
                                 0.000
                                            0.000
                                                        0.7374
                                                                     0.0000
f_DMTA_ilr_2
                     0.0000
                                 0.000
                                            0.000
                                                        0.0000
                                                                     0.7375
              0.000
f_DMTA_ilr_3 0.000
                      0.0000
                                 0.000
                                            0.000
                                                        0.0000
                                                                     0.0000
log_alpha
              0.000
                      0.0000
                                 0.000
                                            0.000
                                                        0.0000
                                                                     0.0000
              0.000
                      0.0000
                                 0.000
                                            0.000
                                                        0.0000
                                                                     0.0000
log_beta
            {\tt f\_DMTA\_ilr\_3~log\_alpha~log\_beta}
DMTA_0
                  0.0000
                              0.000
                                      0.000
log_k_M23
                   0.0000
                              0.000
                                      0.000
log_k_M27
                   0.0000
                              0.000
                                      0.000
log_k_M31
                   0.0000
                              0.000
                                      0.000
f_DMTA_ilr_1
                   0.0000
                              0.000
                                      0.000
                   0.0000
                              0.000
                                      0.000
f_DMTA_ilr_2
f_DMTA_ilr_3
                   0.4598
                              0.000
                                      0.000
                   0.0000
                              1.762
                                      0.000
log_alpha
log_beta
                   0.0000
                              0.000
                                      1.463
Starting values for error model parameters:
a.1
 1
Results:
Likelihood computed by importance sampling
  AIC BIC logLik
  2099 2095 -1030
Optimised parameters:
                  est.
                          lower
DMTA 0
                98.2099 96.82991 99.5898
log_k_M23
               -4.3443 -5.33209 -3.3565
log_k_M27
               -3.9589 -4.39124 -3.5265
log_k_M31
                -4.0469 -4.56847 -3.5253
f_DMTA_ilr_1
                0.0782 -0.28002 0.4364
f_DMTA_ilr_2
                0.1123 -0.30050 0.5250
f_DMTA_ilr_3
               -1.6557 -1.82793 -1.4836
                 2.2802 1.47312 3.0873
log_alpha
                 5.0466 4.19543 5.8978
log_beta
                 1.2813 1.20298
                                 1.3596
SD.DMTA_O
                1.5989 0.55414 2.6436
SD.log_k_M23
                 1.1470 0.41798 1.8760
SD.log_k_M27
                 0.4919 0.16775 0.8160
SD.log_k_M31
                 0.5796 0.18013 0.9791
```

```
SD.f_DMTA_ilr_1 0.4202 0.15421 0.6862
SD.f_DMTA_ilr_2 0.4838 0.17898 0.7885
SD.f_DMTA_ilr_3 0.1940 0.06269 0.3252
SD.log_alpha
               0.8910 0.28062 1.5014
SD.log_beta
                0.9401 0.29616 1.5839
            DMTA_0 lg__M23 lg__M27 lg__M31 f_DMTA__1 f_DMTA__2 f_DMTA__3
log_k_M23
            -0.0060
log_k_M27
            -0.0072 -0.0008
log_k_M31
           -0.0045 0.0002 0.0897
f_DMTA_ilr_1 -0.0009 0.0468 -0.0396 0.0397
f_DMTA_ilr_2 -0.0004  0.0230 -0.0336 -0.1056 -0.0404
f_DMTA_ilr_3 -0.0259 0.0567 0.1049 0.0523 -0.0142
                                                    0.0028
log_alpha -0.0556 0.0088 0.0096 0.0048 0.0004
                                                              0.0132
                                                    0.0013
log_beta
            -0.0588 0.0092 0.0101 0.0051 0.0005
                                                   0.0013
                                                             0.0145
            log_lph
log_k_M23
log_k_M27
log_k_M31
f_DMTA_ilr_1
f_DMTA_ilr_2
f_DMTA_ilr_3
log_alpha
            0.2191
log_beta
Random effects:
                      lower upper
                est.
SD.DMTA_O
             1.5989 0.55414 2.6436
SD.f_DMTA_ilr_1 0.4202 0.15421 0.6862
SD.f_DMTA_ilr_2 0.4838 0.17898 0.7885
SD.f_DMTA_ilr_3 0.1940 0.06269 0.3252
SD.log_alpha 0.8910 0.28062 1.5014
SD.log_beta
              0.9401 0.29616 1.5839
Variance model:
    est. lower upper
a.1 1.281 1.203 1.36
{\tt Backtransformed\ parameters:}
                 est.
                         lower
DMTA O
             98.20985 96.829913 99.58980
              0.01298 0.004834 0.03486
k_M23
k_M27
              0.01908 0.012385
                                0.02941
k_M31
               0.01748 0.010374 0.02944
f_DMTA_to_M23
              0.11318
                            NA
                                      NA
f_DMTA_to_M27 0.10134
                             NA
                                      NA
f_DMTA_to_M31 0.09334
                            NA
                                      NA
alpha
             9.77858 4.362842 21.91707
           155.49513 66.381971 364.23646
Resulting formation fractions:
             ff
DMTA_M23 0.11318
DMTA_M27 0.10134
DMTA_M31 0.09334
DMTA_sink 0.69214
Estimated disappearance times:
DT50 DT90 DT50back
DMTA 11.42 41.28 12.43
M23 53.40 177.39
M27 36.32 120.65
M31 39.66 131.75
```

Listing 4: Hierarchical FOMC path 1 fit with two-component error

```
saemix version used for fitting:
                                      3.2
mkin version used for pre-fitting: 1.2.2
R version used for fitting:
                                    4.2.2
Date of fit: Thu Jan 5 15:09:53 2023
Date of summary: Thu Jan 5 16:42:52 2023
d_DMTA/dt = - (alpha/beta) * 1/((time/beta) + 1) * DMTA
d_M23/dt = + f_DMTA_to_M23 * (alpha/beta) * 1/((time/beta) + 1) * DMTA
           - k_M23 * M23
d_M27/dt = + f_DMTA_to_M27 * (alpha/beta) * 1/((time/beta) + 1) * DMTA
           - k_M27 * M27 + k_M31 * M31
d_M31/dt = + f_DMTA_to_M31 * (alpha/beta) * 1/((time/beta) + 1) * DMTA
           - k_M31 * M31
Data:
563 observations of 4 variable(s) grouped in 6 datasets
Model predictions using solution type deSolve
Fitted in 871.866 s
Using 300, 100 iterations and 9 chains
Variance model: Two-component variance function
Starting values for degradation parameters:
     DMTA_0
               log_k_M23
                            log_k_M27
                                         log_k_M31 f_DMTA_ilr_1 f_DMTA_ilr_2
                  -3.9376
                                            -4.0881
     97.9192
                               -4.1632
                                                         0.1262
f_DMTA_ilr_3
                log_alpha
                              log_beta
Fixed degradation parameter values:
Starting values for random effects (square root of initial entries in omega):
            DMTA_0 log_k_M23 log_k_M27 log_k_M31 f_DMTA_ilr_1 f_DMTA_ilr_2
DMTA_0
              5.883
                       0.0000
                                  0.000
                                            0.000
                                                        0.0000
                                                                     0.0000
log_k_M23
              0.000
                       0.7991
                                  0.000
                                            0.000
                                                        0.0000
                                                                     0.0000
log_k_M27
              0.000
                       0.0000
                                  1.196
                                            0.000
                                                        0.0000
                                                                     0.0000
log_k_M31
              0.000
                      0.0000
                                  0.000
                                            1.033
                                                        0.0000
                                                                     0.0000
f_DMTA_ilr_1
              0.000
                       0.0000
                                  0.000
                                            0.000
                                                        0.7173
                                                                     0.0000
f_DMTA_ilr_2
                     0.0000
                                  0.000
                                            0.000
                                                        0.0000
                                                                     0.7387
              0.000
f_DMTA_ilr_3 0.000
                      0.0000
                                  0.000
                                            0.000
                                                        0.0000
                                                                     0.0000
log_alpha
              0.000
                      0.0000
                                  0.000
                                            0.000
                                                        0.0000
                                                                     0.0000
              0.000
                      0.0000
                                  0.000
                                            0.000
                                                        0.0000
                                                                     0.0000
log_beta
             {\tt f\_DMTA\_ilr\_3~log\_alpha~log\_beta}
DMTA_0
                   0.0000
                              0.000
                                       0.000
log_k_M23
                   0.0000
                              0.000
                                       0.000
log_k_M27
                   0.0000
                              0.000
                                       0.000
log_k_M31
                   0.0000
                              0.000
                                       0.000
f_DMTA_ilr_1
                   0.0000
                              0.000
                                       0.000
                   0.0000
                              0.000
                                       0.000
f_DMTA_ilr_2
f_DMTA_ilr_3
                   0.4557
                              0.000
                                       0.000
                   0.0000
                              1.973
                                       0.000
log_alpha
log_beta
                   0.0000
                              0.000
                                       1.736
Starting values for error model parameters:
a.1 b.1
 1 1
Results:
Likelihood computed by importance sampling
  AIC BIC logLik
  1940 1935 -949.8
Optimised parameters:
                    est.
                            lower
DMTA 0
                98.40215 96.54263 100.2617
log_k_M23
               -4.42549 -5.47959 -3.3714
log_k_M27
               -4.09561 -4.57796 -3.6133
log_k_M31
                -4.11539 -4.63602
                                   -3.5948
f_DMTA_ilr_1
                0.10845 -0.26103
                                    0.4779
f_DMTA_ilr_2
                0.11247 -0.28749
                                   0.5124
f_DMTA_ilr_3
               -1.70891 -1.91873
                                   -1.4991
                1.97268 1.23049
log_alpha
                                    2.7149
log_beta
                 4.69673 3.77267
                                    5.6208
                 0.84402 0.76903
                                    0.9190
a.1
                 0.05663 0.04736
                                    0.0659
b.1
SD.DMTA_O
                 0.50302 -5.02389
                                    6.0299
SD.log_k_M23
                 1.23709 0.46698
                                    2.0072
                 0.56292 0.20599
                                    0.9198
SD.log_k_M27
```

```
SD.log_k_M31 0.60563 0.21756 0.9937
SD.f_DMTA_ilr_1 0.44620 0.17729
                                  0.7151
SD.f_DMTA_ilr_2 0.48080 0.18949
                                 0.7721
SD.f_DMTA_ilr_3 0.24795 0.09702
                                 0.3989
SD.log_alpha 0.82800 0.27453 1.3815
SD.log_beta
               1.05702 0.38076
                                 1.7333
Correlation:
           DMTA_0 lg__M23 lg__M27 lg__M31 f_DMTA__1 f_DMTA__2 f_DMTA__3
log_k_M23
            -0.0339
log_k_M27
            -0.0462 0.0042
log_k_M31
           -0.0294 0.0028 0.0680
f_DMTA_ilr_1 -0.0033 0.0256 -0.0272 0.0245
f_DMTA_ilr_2 -0.0014  0.0135 -0.0222 -0.0680 -0.0269
f_DMTA_ilr_3 -0.1744 0.0355 0.0692 0.0334 -0.0105
                                                     0.0024
0.0014
                                                               0.0463
                                                              0.0451
log_beta
                                                    0.0013
            log_lph
log_k_M23
log_k_M27
log_k_M31
f_DMTA_ilr_1
f_DMTA_ilr_2
f_DMTA_ilr_3
log_alpha
             0.1810
log_beta
Random effects:
                est. lower upper
SD.DMTA_O
              0.5030 -5.02389 6.0299
SD.f_DMTA_ilr_1 0.4462 0.17729 0.7151
SD.f_DMTA_ilr_2 0.4808 0.18949 0.7721
SD.f_DMTA_ilr_3 0.2480 0.09702 0.3989
SD.log_alpha 0.8280 0.27453 1.3815
SD.log_beta 1.0570 0.38076 1.7333
SD.log_beta
Variance model:
est. lower upper a.1 0.84402 0.76903 0.9190
b.1 0.05663 0.04736 0.0659
{\tt Backtransformed\ parameters:}
                 est.
                          lower
DMTA_0
              98.40215 96.542632 100.26166
               0.01197 0.004171 0.03434
0.01665 0.010276 0.02696
k_M23
k_M27
k_M31
               0.01632 0.009696 0.02747
               0.11076 NA
0.09501 NA
0.08938 NA
f_DMTA_to_M23
                                       NA
f_DMTA_to_M27
              0.09501
                             NA
                                       NA
f_DMTA_to_M31 0.08938
                             NA
                                       NA
alpha
              7.18989 3.422914 15.10247
             109.58857 43.495928 276.10984
beta
Resulting formation fractions:
             ff
DMTA_M23 0.11076
DMTA_M27 0.09501
DMTA_M31 0.08938
DMTA_sink 0.70486
Estimated disappearance times:
    DT50 DT90 DT50back
DMTA 11.09 41.37
M23 57.92 192.39
                      NA
M27 41.64 138.33
                       NA
M31 42.47 141.09
```

Listing 5: Hierarchical DFOP path 1 fit with constant variance

```
saemix version used for fitting:
                                       3.2
mkin version used for pre-fitting: 1.2.2
R version used for fitting:
                                     4.2.2
Date of fit: Thu Jan 5 15:10:48 2023
Date of summary: Thu Jan 5 16:42:52 2023
* DMTA
d_M23/dt = + f_DMTA_to_M23 * ((k1 * g * exp(-k1 * time) + k2 * (1 - g))
* exp(-k2 * time)) / (g * exp(-k1 * time) + (1 - g) * exp(-k2 * time))) * DMTA - k_M23 * M23 * M23 * M27/dt = + f_DMTA_to_M27 * ((k1 * g * exp(-k1 * time) + k2 * (1 - g)
* exp(-k2 * time)) / (g * exp(-k1 * time) + (1 - g) * exp(-k2 * time))) * DMTA - k_M27 * M27 + k_M31 * M31 d_M31/dt = + f_DMTA_to_M31 * ((k1 * g * exp(-k1 * time) + k2 * (1 - g)
           * \exp(-k2 * time)) / (g * \exp(-k1 * time) + (1 - g) *
            exp(-k2 * time))) * DMTA - k_M31 * M31
563 observations of 4 variable(s) grouped in 6 datasets
Model predictions using solution type deSolve
Fitted in 926.756 s
Using 300, 100 iterations and 9 chains
Variance model: Constant variance
Starting values for degradation parameters:
                                          log_k_M31 f_DMTA_ilr_1 f_DMTA_ilr_2
      DMTA_0 log_k_M23 log_k_M27
     98.5020
                    -4.1092
                                  4.4920
                                              -4.3713
                                                             0.1367
f_DMTA_ilr_3
                   log_k1
                                 log_k2
                                             g_qlogis
     -1.7427
                   -2.3315
                                 -3.7194
Fixed degradation parameter values:
Starting values for random effects (square root of initial entries in omega):
             DMTA_0 log_k_M23 log_k_M27 log_k_M31 f_DMTA_ilr_1 f_DMTA_ilr_2
DMTA_0
              2.422
                        0.0000
                                    0.000
                                              0.000
                                                           0.0000
                                                                         0.0000
log_k_M23
               0.000
                        0.4611
                                    0.000
                                              0.000
                                                           0.0000
                                                                         0.0000
log_k_M27
               0.000
                        0.0000
                                              0.000
                                                           0.0000
                                                                         0.0000
                                    1.152
log_k_M31
               0.000
                        0.0000
                                    0.000
                                              1.047
                                                           0.0000
                                                                         0.0000
                                                           0.7396
f_DMTA_ilr_1 0.000
                        0.0000
                                    0.000
                                              0.000
                                                                         0.0000
f_DMTA_ilr_2 0.000
                        0.0000
                                    0.000
                                              0.000
                                                           0.0000
                                                                         0.7319
f_DMTA_ilr_3 0.000
                       0.0000
                                    0.000
                                              0.000
                                                           0.0000
                                                                         0.0000
               0.000
                        0.0000
                                    0.000
                                              0.000
                                                           0.0000
                                                                         0.0000
log_k1
               0.000
                       0.0000
                                    0.000
                                                           0.0000
                                                                         0.0000
                                              0.000
log_k2
                       0.0000
              0.000
                                    0.000
                                              0.000
                                                           0.0000
                                                                         0.0000
g_qlogis
             f\_DMTA\_ilr\_3 \ log\_k1 \ log\_k2 \ g\_qlogis
                    0.0000 0.000 0.000
DMTA 0
                                             0.000
log_k_M23
                    0.0000 0.000 0.000
                                             0.000
log_k_M27
                    0.0000 0.000 0.000
                                             0.000
log_k_M31
                    0.0000 0.000 0.000
                                             0.000
                    0.0000 0.000
                                    0.000
                                             0.000
f_DMTA_ilr_1
                    0.0000 0.000
f DMTA ilr 2
                                    0.000
                                             0.000
                    0.4627 0.000 0.000
f_DMTA_ilr_3
                                             0.000
log_k1
                    0.0000
                           1.464
                                    0.000
                                             0.000
                    0.0000 0.000
log_k2
                                    1.491
                                             0.000
g_qlogis
                    0.0000 0.000 0.000
                                             5.149
Starting values for error model parameters:
a.1
 1
Results:
Likelihood computed by importance sampling
   AIC BIC logLik
  2039 2034 -998.4
Optimised parameters:
                             lower
                                      upper
                 98.10945 96.63826 99.5806
DMTA 0
log_k_M23
                -4.32896 -5.27027 -3.3876
log_k_M27
                -4.01882 -4.47638 -3.5613
                 -4.09749 -4.63229 -3.5627
log_k_M31
f_DMTA_ilr_1
                  0.09446 -0.27614 0.4651
f_DMTA_ilr_2
                  0.12385 -0.29707 0.5448
f_DMTA_ilr_3
                 -1.67130 -1.85702 -1.4856
```

```
-2.89772 -3.83174 -1.9637
log_k1
log_k2
              -3.58026 -4.50361 -2.6569
              0.20649 -3.37843 3.7914
g_qlogis
               1.17980 1.11928 1.2403
SD.DMTA_O
               1.74178 0.64708 2.8365
SD.log_k_M23
                1.09824 0.39902 1.7975
SD.log_k_M27
                0.52185 0.17817 0.8655
SD.log_k_M31
                0.59267 0.18116 1.0042
SD.f_DMTA_ilr_1 0.43199 0.15538 0.7086
SD.f_DMTA_ilr_2 0.49162 0.17972 0.8035
SD.f_DMTA_ilr_3 0.20921 0.06744 0.3510
           1.10830 0.43293 1.7837
1.09078 0.42140 1.7602
SD.log_k1
SD.log_k2
SD.g_qlogis
              3.34706 0.62689 6.0672
Correlation:
            DMTA_0 lg__M23 lg__M27 lg__M31 f_DMTA__1 f_DMTA__2 f_DMTA__3
            -0.0048
log_k_M23
log_k_M27
            -0.0063 0.0002
log_k_M31
            -0.0039 0.0005 0.0962
f_DMTA_ilr_1 -0.0008  0.0600 -0.0378  0.0406
f_DMTA_ilr_2 -0.0002  0.0303 -0.0373 -0.1124 -0.0333
f_DMTA_ilr_3 -0.0209 0.0680 0.1038 0.0560 0.0032
                                                     0.0087
            0.0294 0.0006 -0.0010 -0.0003 0.0004
                                                     0.0001
                                                              -0.0062
log_k1
          0.0083 0.0028 0.0070 0.0056 0.0000
                                                     -0.0015
                                                               0.0044
log_k2
           -0.0287 -0.0035 -0.0050 -0.0044 -0.0007 log_k1 log_k2
                                                     0.0008
                                                               0.0007
g_qlogis
log_k_M23
log_k_M27
log_k_M31
f DMTA ilr 1
f_DMTA_ilr_2
f_DMTA_ilr_3
log_k1
             0.0542
log_k2
g_qlogis
           -0.1621 -0.1702
Random effects:
                 est. lower upper
SD.DMTA O
              1.7418 0.64708 2.8365
SD.log_k_M23 1.0982 0.39902 1.7975
SD.f_DMTA_ilr_1 0.4320 0.15538 0.7086
SD.f_DMTA_ilr_2 0.4916 0.17972 0.8035
SD.f_DMTA_ilr_3 0.2092 0.06744 0.3510
           1.1083 0.43293 1.7837
SD.log_k1
SD.log_k2
              1.0908 0.42140 1.7602
SD.g_qlogis 3.3471 0.62689 6.0672
  est. lower upper
a.1 1.18 1.119 1.24
Backtransformed parameters:
                est. lower
DMTA_0
             98.10945 96.638257 99.58064
k_M23
             0.01318 0.005142 0.03379
k_M27
              0.01797 0.011375 0.02840
              0.01661 0.009732 0.02836
k_M31
                       NA
NA
f_DMTA_to_M23 0.11358
                                     NA
f_DMTA_to_M27 0.09938
f_DMTA_to_M31 0.09129
                            NA
              0.05515 0.021672 0.14034
k2
              0.02787 0.011069 0.07016
              0.55144 0.032976 0.97793
g
Resulting formation fractions:
              ff
DMTA_M23 0.11358
DMTA_M27 0.09938
DMTA_M31 0.09129
DMTA_sink 0.69576
Estimated disappearance times:
    DT50 DT90 DT50back DT50_k1 DT50_k2
DMTA 16.76 61.31
                   18.46 12.57 24.87
                    NA
M23 52.59 174.69
                              NA
                                      NA
M27 38.56 128.10
M31 41.72 138.59
                       NΑ
                              NA
                                      NΑ
                      NA
                              NA
                                      NA
```

Listing 6: Hierarchical DFOP path 1 fit with two-component error

```
saemix version used for fitting:
                                        3.2
mkin version used for pre-fitting: 1.2.2
R version used for fitting:
                                      4.2.2
Date of fit: Thu Jan 5 15:11:03 2023
Date of summary: Thu Jan 5 16:42:52 2023
* DMTA
d_M23/dt = + f_DMTA_to_M23 * ((k1 * g * exp(-k1 * time) + k2 * (1 - g))
* exp(-k2 * time)) / (g * exp(-k1 * time) + (1 - g) * exp(-k2 * time))) * DMTA - k_M23 * M23 * M23 * M27/dt = + f_DMTA_to_M27 * ((k1 * g * exp(-k1 * time) + k2 * (1 - g)
* exp(-k2 * time)) / (g * exp(-k1 * time) + (1 - g) * exp(-k2 * time))) * DMTA - k_M27 * M27 + k_M31 * M31 d_M31/dt = + f_DMTA_to_M31 * ((k1 * g * exp(-k1 * time) + k2 * (1 - g)
           * \exp(-k2 * time)) / (g * \exp(-k1 * time) + (1 - g) *
            exp(-k2 * time))) * DMTA - k_M31 * M31
563 observations of 4 variable(s) grouped in 6 datasets
Model predictions using solution type deSolve
Fitted in 941.485 s
Using 300, 100 iterations and 9 chains
Variance model: Two-component variance function
Starting values for degradation parameters:
                                           log_k_M31 f_DMTA_ilr_1 f_DMTA_ilr_2
      DMTA_0 log_k_M23 log_k_M27
     98.7132
                    -3.9216
                                  4.3306
                                               -4.2442
                                                             0.1376
f_DMTA_ilr_3
                    log_k1
                                 log_k2
                                             g_qlogis
     -1.7554
                   -2.2352
                                 -3.7758
Fixed degradation parameter values:
Starting values for random effects (square root of initial entries in omega):
              DMTA_0 log_k_M23 log_k_M27 log_k_M31 f_DMTA_ilr_1 f_DMTA_ilr_2
DMTA_0
              3.291
                        0.0000
                                    0.000
                                               0.000
                                                           0.0000
                                                                         0.0000
log_k_M23
               0.000
                        0.7768
                                    0.000
                                               0.000
                                                           0.0000
                                                                         0.0000
log_k_M27
               0.000
                        0.0000
                                    1.209
                                               0.000
                                                           0.0000
                                                                         0.0000
log_k_M31
               0.000
                        0.0000
                                    0.000
                                               1.055
                                                           0.0000
                                                                         0.0000
                                                           0.7396
f_DMTA_ilr_1 0.000
                        0.0000
                                    0.000
                                               0.000
                                                                         0.0000
f_DMTA_ilr_2 0.000
                        0.0000
                                    0.000
                                               0.000
                                                           0.0000
                                                                         0.7324
f_DMTA_ilr_3 0.000
                        0.0000
                                    0.000
                                               0.000
                                                           0.0000
                                                                         0.0000
               0.000
                        0.0000
                                    0.000
                                               0.000
                                                           0.0000
                                                                         0.0000
log_k1
               0.000
                        0.0000
                                    0.000
                                                           0.0000
                                                                         0.0000
                                               0.000
log_k2
               0.000
                        0.0000
                                    0.000
                                               0.000
                                                           0.0000
                                                                         0.0000
g_qlogis
              f\_DMTA\_ilr\_3 \ log\_k1 \ log\_k2 \ g\_qlogis
                    0.0000 0.000
DMTA 0
                                     0.00
                                             0.000
log_k_M23
                    0.0000 0.000
                                             0.000
                                     0.00
log_k_M27
                    0.0000 0.000
                                     0.00
                                             0.000
                    0.0000 0.000
log_k_M31
                                     0.00
                                             0.000
                    0.0000 0.000
                                     0.00
                                             0.000
f_DMTA_ilr_1
                    0.0000 0.000
f DMTA ilr 2
                                     0.00
                                             0.000
                    0.4505 0.000
f_DMTA_ilr_3
                                     0.00
                                             0.000
log_k1
                    0.0000
                            1.274
                                     0.00
                                             0.000
                    0.0000 0.000
log_k2
                                     2.23
                                             0.000
g_qlogis
                    0.0000 0.000
                                     0.00
                                             4.387
Starting values for error model parameters:
a.1 b.1
 1 1
Results:
Likelihood computed by importance sampling
   AIC BIC logLik
  1880 1875 -917.9
Optimised parameters:
                     est.
                             lower
                                        upper
                 97.87025 95.66344 100.07706
DMTA 0
log_k_M23
                 -4.34365 -5.25761 -3.42969
log_k_M27
                 -4.09030 -4.56320 -3.61739
                 -4.12995 -4.64427
log_k_M31
                                     -3.61563
f_DMTA_ilr_1
                  0.11371 -0.25718
                                     0.48459
f_DMTA_ilr_2
                  0.12477 -0.27937
f_DMTA_ilr_3
                 -1.69151 -1.88761 -1.49541
```

```
log_k1
              -2.98891 -3.67142 -2.30639
log_k2
              -4.52624 -5.99605 -3.05644
              2.49480 -0.34432 5.33392
g_qlogis
a.1
b.1
              0.82333 0.75859
                                 0.88807
               0.03652 0.03040
                                0.04264
SD.DMTA_O
               2.32169 0.49719
                                 4.14619
SD.log_k_M23 1.08750 0.41815
                                1.75686
SD.log_k_M27
               0.54747 0.19530
                                0.89965
               0.59008 0.20283
SD.log_k_M31
                                 0.97733
SD.f_DMTA_ilr_1 0.44731 0.17709
                                 0.71753
SD.f_DMTA_ilr_2 0.48496 0.19043
                                0.77950
SD.f_DMTA_ilr_3 0.23105 0.08645
                                 0.37564
               0.83930 0.35424
SD.log_k1
                                1.32436
SD.log_k2
               1.23217 0.05037
                                 2.41397
            2.70695 0.66629
                                4.74762
SD.g_qlogis
Correlation:
            DMTA_0 lg__M23 lg__M27 lg__M31 f_DMTA__1 f_DMTA__2 f_DMTA__3
log_k_M23
            -0.0154
log_k_M27
            -0.0211 0.0035
log_k_M31
            -0.0139 0.0025 0.0811
f_DMTA_ilr_1 -0.0026 0.0281 -0.0295 0.0265
f_DMTA_ilr_2 -0.0008  0.0146 -0.0261 -0.0777 -0.0272
f_DMTA_ilr_3 -0.0743 0.0384 0.0794 0.0411 -0.0098
                                                   0.0015
          0.0256 -0.0040 -0.0058 -0.0032 -0.0005
log_k1
                                                   -0.0006
                                                            -0.0113
log_k2
            0.0169 0.0142 0.0089 0.0091 0.0014
                                                   -0.0029
                                                             0.0019
            -0.0382 0.0028 0.0052 0.0010 0.0002
                                                   0.0021
                                                            0.0137
g_qlogis
           log_k1 log_k2
log_k_M23
log_k_M27
log_k_M31
f_DMTA_ilr_1
f DMTA ilr 2
f_DMTA_ilr_3
log_k1
            0.0659
log_k2
           -0.0926 -0.3433
g_qlogis
Random effects:
               est. lower upper
est. lower upper SD.DMTA_0 2.3217 0.49719 4.1462
SD.f_DMTA_ilr_1 0.4473 0.17709 0.7175
SD.f_DMTA_ilr_2 0.4850 0.19043 0.7795
SD.f_DMTA_ilr_3 0.2310 0.08645 0.3756
SD.log_k1 0.8393 0.35424 1.3244
SD.log_k2
              1.2322 0.05037 2.4140
SD.g_qlogis
              2.7070 0.66629 4.7476
Variance model:
      est. lower upper
a.1 0.82333 0.7586 0.88807
b.1 0.03652 0.0304 0.04264
Backtransformed parameters:
                est. lower
                                  upper
DMTA_0
             97.87025 95.663437 100.07706
             0.01299 0.005208 0.03240
k_M23
k_M27
              0.01673 0.010429
                                0.02685
              0.01608 0.009617 0.02690
k_M31
f_DMTA_to_M23 0.11329
                      NA
                                     NA
f_DMTA_to_M27 0.09646
                           NA
                                     NA
f_DMTA_to_M31 0.08973
                            NΑ
                                     NA
              0.05034 0.025440 0.09962
k2
              0.01082 0.002489
                                0.04706
              0.92378 0.414760 0.99520
g
Resulting formation fractions:
             ff
DMTA_M23 0.11329
DMTA_M27 0.09646
DMTA_M31 0.08973
DMTA_sink 0.70052
Estimated disappearance times:
    DT50 DT90 DT50back DT50 k1 DT50 k2
DMTA 14.95 55.0 16.56 13.77 64.05
                  NA
                          NA
M23 53.36 177.3
                                    NA
M27 41.42 137.6
                     NA
                            NA
                                    NΑ
M31 43.10 143.2
                            NA
                    NA
                                    NA
```

Listing 7: Hierarchical SFORB path 1 fit with constant variance

```
saemix version used for fitting:
                                      3.2
mkin version used for pre-fitting: 1.2.2
R version used for fitting:
                                     4.2.2
Date of fit: Thu Jan 5 15:10:47 2023
Date of summary: Thu Jan 5 16:42:52 2023
d_DMTA_free/dt = - k_DMTA_free * DMTA_free - k_DMTA_free_bound *
           DMTA_free + k_DMTA_bound_free * DMTA_bound
d_DMTA_bound/dt = + k_DMTA_free_bound * DMTA_free - k_DMTA_bound_free *
           DMTA bound
\label{eq:dm23} $$d_M23/dt = + f_DMTA\_free\_to_M23 * k_DMTA\_free * DMTA\_free - k_M23 * M23$
d_M27/dt = + f_DMTA_free_to_M27 * k_DMTA_free * DMTA_free - k_M27 * M27
           + k_M31 * M31
d_M31/dt = + f_DMTA_free_to_M31 * k_DMTA_free * DMTA_free - k_M31 * M31
563 observations of 4 variable(s) grouped in 6 datasets
Model predictions using solution type deSolve
Fitted in 925.232 s
Using 300, 100 iterations and 9 chains
Variance model: Constant variance
Starting values for degradation parameters:
          DMTA_free_0
                           log_k_DMTA_free log_k_DMTA_free_bound
              98.5280
                                     -2.5260
                                                            -3.7382
                                   log_k_M23
                                                          log_k_M27
log_k_DMTA_bound_free
               -1.7548
                                      -3.8933
                                                             -4.2881
                                                       f_DMTA_ilr_2
            log_k_M31
                                f_DMTA_ilr_1
                4.2144
                                      0.1393
                                                             0.1359
         f_DMTA_ilr_3
              -1.7391
Fixed degradation parameter values:
Starting values for random effects (square root of initial entries in omega):
                      DMTA_free_O log_k_DMTA_free log_k_DMTA_free_bound
DMTA_free_0
                             2.391
                                            0.0000
log_k_DMTA_free
                             0.000
                                            0.8891
                                                                    0.000
log_k_DMTA_free_bound
                             0.000
                                            0.0000
                                                                    4.519
log_k_DMTA_bound_free
                             0.000
                                             0.0000
                                                                    0.000
log_k_M23
                             0.000
                                             0.0000
                                                                    0.000
log_k_M27
                             0.000
                                             0.0000
                                                                    0.000
log_k_M31
                             0.000
                                            0.0000
                                                                    0.000
f DMTA ilr 1
                             0.000
                                             0.0000
                                                                    0.000
f DMTA ilr 2
                             0.000
                                            0.0000
                                                                    0.000
f_DMTA_ilr_3
                             0.000
                                            0.0000
                                                                    0.000
                      log_k_DMTA_bound_free log_k_M23 log_k_M27 log_k_M31
DMTA_free_0
                                       0.000
                                                 0.0000
                                                            0.000
                                                                       0.000
log_k_DMTA_free
                                       0.000
                                                 0.0000
                                                            0.000
                                                                       0.000
                                                            0.000
log_k_DMTA_free_bound
                                       0.000
                                                 0.0000
                                                                       0.000
log_k_DMTA_bound_free
                                       3.192
                                                 0.0000
                                                            0.000
                                                                       0.000
                                                 0.7268
                                                            0.000
                                                                       0.000
log_k_M23
                                       0.000
                                                                       0.000
log_k_M27
                                       0.000
                                                 0.0000
                                                            1.167
                                                 0.0000
log_k_M31
                                       0.000
                                                            0.000
                                                                       1.041
f_DMTA_ilr_1
                                       0.000
                                                 0.0000
                                                            0.000
                                                                       0.000
                                                                       0.000
{\tt f\_DMTA\_ilr\_2}
                                       0.000
                                                 0.0000
                                                            0.000
f_DMTA_ilr_3
                                       0.000
                                                 0.0000
                                                            0.000
                                                                       0.000
                       {\tt f\_DMTA\_ilr\_1~f\_DMTA\_ilr\_2~f\_DMTA\_ilr\_3}
DMTA_free_0
                             0.0000
                                           0.0000
                                                        0.0000
log_k_DMTA_free
                             0.0000
                                           0.0000
                                                        0.0000
log_k_DMTA_free_bound
                             0.0000
                                           0.0000
                                                        0.0000
log_k_DMTA_bound_free
                             0.0000
                                           0.0000
                                                        0.0000
log_k_M23
                             0.0000
                                           0.0000
                                                        0.0000
log_k_M27
                             0.0000
                                           0.0000
                                                        0.0000
log_k_M31
                             0.0000
                                           0.0000
                                                        0.0000
f_DMTA_ilr_1
                             0.7452
                                           0.0000
                                                        0.0000
f_DMTA_ilr_2
                             0.0000
                                           0.7306
                                                        0.0000
f_DMTA_ilr_3
                             0.0000
                                           0.0000
                                                        0.4521
Starting values for error model parameters:
a.1
Results:
Likelihood computed by importance sampling
  AIC BIC logLik
```

```
NA NA
Optimised parameters:
                             est.
                                    lower
                                            upper
                         98.27188 96.80788 99.7359
DMTA_free_0
log_k_DMTA_free
                         -2.69691 -3.13017 -2.2636
log_k_DMTA_free_bound
                         -4.82084 -6.64366 -2.9980
log_k_DMTA_bound_free
                        -3.97953 -6.52678 -1.4323
                         -4.35772 -5.32473 -3.3907
log_k_M23
                         -3.99549 -4.42448 -3.5665
log_k_M27
log_k_M31
                         -4.09809 -4.61809 -3.5781
f_DMTA_ilr_1
                         0.07078 -0.27912 0.4207
                         0.13180 -0.28124 0.5448
f_DMTA_ilr_2
f_DMTA_ilr_3
                         -1.66881 -1.83891 -1.4987
                         1.17123 1.12495 1.2175
a.1
SD.DMTA_free_0
                         1.72001 0.62279 2.8172
SD.log_k_DMTA_free
                         0.53476 0.22553 0.8440
SD.log_k_DMTA_free_bound 2.12596 0.78120 3.4707
SD.log_k_DMTA_bound_free 2.58759 0.70645 4.4687
SD.log_k_M23
                         1.13179 0.41916 1.8444
                         0.49159 0.17088 0.8123
SD.log_k_M27
SD.log_k_M31
                         0.58329 0.18688 0.9797
SD.f_DMTA_ilr_1
                         0.41413 0.15577
                                           0.6725
SD.f_DMTA_ilr_2
                         0.48909 0.18581 0.7924
                         0.19445 0.06599 0.3229
SD.f DMTA ilr 3
Correlation:
                      {\tt DMTA\_O~lg\_DMTA\_~lg\_k\_DMTA\_f\_~lg\_k\_DMTA\_b\_~lg\_\_M23}
log_k_DMTA_free
                      0.0227
log_k_DMTA_free_bound 0.0337 0.0390
log_k_DMTA_bound_free 0.0204 0.0226
                                        0.1104
log_k_M23
                      -0.0054 -0.0016
                                                     0.0017
                                       -0.0002
                      -0.0057 -0.0013
log_k_M27
                                       -0.0006
                                                     0.0030
                                                                  -0.0005
                                        0.0002
                                                                  0.0001
log_k_M31
                      -0.0034 -0.0007
                                                     0.0028
                     -0.0008 -0.0002
                                                                  0.0416
f_DMTA_ilr_1
                                        0.0002
                                                     0.0008
                     -0.0004 -0.0002 -0.0004
-0.0224 -0.0052 -0.0052
f_DMTA_ilr_2
                                                     -0.0007
                                                                   0.0202
f_DMTA_ilr_3
                                                     -0.0006
                                                                  0.0503
                     lg__M27 lg__M31 f_DMTA__1 f_DMTA__2
log_k_DMTA_free
log_k_DMTA_free_bound
log_k_DMTA_bound_free
log_k_M23
log_k_M27
log_k_M31
                      0.0850
f_DMTA_ilr_1
                     -0.0372 0.0339
f_DMTA_ilr_2
                      -0.0298 -0.0941 -0.0331
f_DMTA_ilr_3
                      0.0958 0.0500 -0.0117
                                               -0.0004
Random effects:
                          est. lower upper
SD.DMTA_free_0
                        1.7200 0.62279 2.8172
SD.log_k_DMTA_free
                       0.5348 0.22553 0.8440
SD.log_k_DMTA_free_bound 2.1260 0.78120 3.4707
SD.log_k_DMTA_bound_free 2.5876 0.70645 4.4687
                     1.1318 0.41916 1.8444
SD.log_k_M23
SD.log_k_M27
                        0.4916 0.17088 0.8123
SD.log_k_M31
                        0.5833 0.18688 0.9797
SD.f_DMTA_ilr_1
                        0.4141 0.15577 0.6725
SD.f_DMTA_ilr_2
                        0.4891 0.18581 0.7924
                        0.1945 0.06599 0.3229
SD.f_DMTA_ilr_3
Variance model:
    est. lower upper
a.1 1.171 1.125 1.218
Backtransformed parameters:
                      est.
                               lower
                                         upper
DMTA_free_0
                   98.27188 96.807877 99.73588
k_DMTA_free
                   0.06741 0.043710 0.10397
k_DMTA_free_bound 0.00806 0.001302 0.04989
k_DMTA_bound_free
                   0.01869 0.001464 0.23876
                    0.01281
                             0.004870 0.03368
k M23
k_M27
                    0.01840
                            0.011980 0.02825
k_M31
                    0.01660
                           0.009872 0.02793
f_DMTA_free_to_M23  0.11229  0.069099  0.10722
f_DMTA_free_to_M27  0.10160  0.069099  0.10722
f_DMTA_free_to_M31 0.09089 0.069099 0.10722
Estimated Eigenvalues of SFORB model(s):
DMTA_b1 DMTA_b2 DMTA_g
0.07801 0.01615 0.82864
Resulting formation fractions:
```

Listing 8: Hierarchical SFORB path 1 fit with two-component error

```
saemix version used for fitting:
                                      3.2
mkin version used for pre-fitting: 1.2.2
R version used for fitting:
                                     4.2.2
                Thu Jan 5 15:10:44 2023
Date of fit:
Date of summary: Thu Jan 5 16:42:52 2023
d_DMTA_free/dt = - k_DMTA_free * DMTA_free - k_DMTA_free_bound *
           DMTA_free + k_DMTA_bound_free * DMTA_bound
d_DMTA_bound/dt = + k_DMTA_free_bound * DMTA_free - k_DMTA_bound_free *
           DMTA bound
\label{eq:dm23} $$d_M23/dt = + f_DMTA\_free\_to_M23 * k_DMTA\_free * DMTA\_free - k_M23 * M23$
d_M27/dt = + f_DMTA_free_to_M27 * k_DMTA_free * DMTA_free - k_M27 * M27
           + k_M31 * M31
d_M31/dt = + f_DMTA_free_to_M31 * k_DMTA_free * DMTA_free - k_M31 * M31
563 observations of 4 variable(s) grouped in 6 datasets
Model predictions using solution type deSolve
Fitted in 922.452 s
Using 300, 100 iterations and 9 chains
Variance model: Two-component variance function
Starting values for degradation parameters:
          DMTA_free_0
                           log_k_DMTA_free log_k_DMTA_free_bound
              97.2597
                                     -2.8229
                                   log_k_M23
                                                          log_k_M27
log_k_DMTA_bound_free
              -3.3880
                                      -3.9216
                                                            -4.2555
            log_k_M31
                                                      f_DMTA_ilr_2
                                f_DMTA_ilr_1
               4.2054
                                      0.1243
                                                             0.1306
         f_DMTA_ilr_3
              -1.7266
Fixed degradation parameter values:
Starting values for random effects (square root of initial entries in omega):
                      DMTA_free_O log_k_DMTA_free log_k_DMTA_free_bound
DMTA_free_0
                             5.586
                                            0.0000
log_k_DMTA_free
                             0.000
                                            0.9851
                                                                    0.000
log_k_DMTA_free_bound
                             0.000
                                            0.0000
                                                                    1.309
log_k_DMTA_bound_free
                             0.000
                                            0.0000
                                                                    0.000
log_k_M23
                             0.000
                                            0.0000
                                                                    0.000
log_k_M27
                             0.000
                                            0.0000
                                                                    0.000
log_k_M31
                             0.000
                                            0.0000
                                                                    0.000
f DMTA ilr 1
                             0.000
                                            0.0000
                                                                    0.000
                                                                    0.000
f DMTA ilr 2
                             0.000
                                            0.0000
f_DMTA_ilr_3
                             0.000
                                            0.0000
                                                                    0.000
                      log_k_DMTA_bound_free log_k_M23 log_k_M27 log_k_M31
DMTA_free_0
                                       0.000
                                                 0.0000
                                                            0.000
                                                                     0.0000
log_k_DMTA_free
                                       0.000
                                                 0.0000
                                                            0.000
                                                                     0.0000
                                                                     0.0000
log_k_DMTA_free_bound
                                       0.000
                                                 0.0000
                                                            0.000
                                                            0.000
log_k_DMTA_bound_free
                                       4.482
                                                 0.0000
                                                                     0.0000
                                                 0.7768
                                                            0.000
                                                                     0.0000
log_k_M23
                                       0.000
                                                                     0.0000
log_k_M27
                                       0.000
                                                 0.0000
                                                            1.096
                                                            0.000
                                                 0.0000
                                                                     0.9909
log_k_M31
                                       0.000
f_DMTA_ilr_1
                                       0.000
                                                 0.0000
                                                            0.000
                                                                     0.0000
                                                                     0.0000
{\tt f\_DMTA\_ilr\_2}
                                       0.000
                                                 0.0000
                                                            0.000
f_DMTA_ilr_3
                                       0.000
                                                0.0000
                                                            0.000
                                                                     0.0000
                      {\tt f\_DMTA\_ilr\_1~f\_DMTA\_ilr\_2~f\_DMTA\_ilr\_3}
DMTA_free_0
                             0.0000
                                          0.0000
                                                        0.0000
log_k_DMTA_free
                             0.0000
                                          0.0000
                                                        0.0000
log_k_DMTA_free_bound
                             0.0000
                                          0.0000
                                                        0.0000
log_k_DMTA_bound_free
                             0.0000
                                          0.0000
                                                        0.0000
log_k_M23
                             0.0000
                                          0.0000
                                                        0.0000
log_k_M27
                             0.0000
                                          0.0000
                                                        0.0000
log_k_M31
                             0.0000
                                          0.0000
                                                        0.0000
f_DMTA_ilr_1
                             0.7196
                                          0.0000
                                                        0.0000
f_DMTA_ilr_2
                             0.0000
                                          0.7359
                                                        0.0000
f_DMTA_ilr_3
                             0.0000
                                          0.0000
                                                        0.4417
Starting values for error model parameters:
a.1 b.1
Results:
Likelihood computed by importance sampling
   AIC BIC logLik
```

```
1833 1828 -894.4
Optimised parameters:
                             est.
                                     lower
                                               upper
                        98.131566 95.97820 100.28493
DMTA free 0
log_k_DMTA_free
                        -2.803276 -3.29545 -2.31110
log_k_DMTA_free_bound
                        -6.220056 -7.02080 -5.41932
log_k_DMTA_bound_free
                        -4.601324 -4.84240 -4.36025
                        -4.409080 -5.42254 -3.39562
log_k_M23
                        -4.073777 -4.52470 -3.62285
log_k_M27
log_k_M31
                        -4.123967 -4.63101 -3.61693
f_DMTA_ilr_1
                         0.096016 -0.26295
                                             0.45498
                        0.122320 -0.27368
f_DMTA_ilr_2
                                             0.51832
f_DMTA_ilr_3
                        -1.694475 -1.88172
                                            -1.50723
                         0.829893 0.76551
                                             0.89428
a.1
                         0.035087 0.02892
b.1
                                             0.04125
                         2.251174 0.46307
SD.DMTA_free_0
                                             4.03928
SD.log_k_DMTA_free
                         0.614338 0.26609
                                             0.96258
SD.log_k_DMTA_free_bound 0.904557 0.30542
                                             1.50370
SD.log_k_DMTA_bound_free 0.005574 -3.83190
                                             3.84305
SD.log_k_M23
                         1.199822 0.45907
                                             1.94058
SD.log_k_M27
                         0.528851 0.19554
                                             0.86216
                         0.590868 0.21289
SD.log_k_M31
                                             0.96885
SD.f_DMTA_ilr_1
                         0.434070 0.17309
                                             0.69505
SD.f DMTA ilr 2
                         0.477091 0.18914
                                             0.76505
SD.f_DMTA_ilr_3
                         0.222161 0.08468
                                             0.35964
Correlation:
                     DMTA__O lg__DMTA_ lg_k_DMTA_f_ lg_k_DMTA_b_ lg__M23
log k DMTA free
                      0.0162
log_k_DMTA_free_bound 0.0582 0.0141
log_k_DMTA_bound_free 0.0518 0.0139
                                       0.1538
                                                    -0.0020
log_k_M23
                     -0.0136 -0.0023
                                      -0.0170
                     -0.0166 -0.0025
                                                     0.0002
log_k_M27
                                      -0.0154
                                                                  0.0023
                     -0.0109 -0.0016
log_k_M31
                                      -0.0095
                                                     0.0021
                                                                  0.0017
                     -0.0019 -0.0003
                                      -0.0012
f_DMTA_ilr_1
                                                     0.0001
                                                                  0.0247
                     -0.0004 -0.0001
f DMTA ilr 2
                                      -0.0006
                                                    -0.0018
                                                                  0.0127
f DMTA ilr 3
                     -0.0694 -0.0058 -0.0242
                                                    -0.0119
                                                                  0.0329
                     \label{lg_M27} $\tt lg\_M31\ f\_DMTA\_1\ f\_DMTA\_2$
log_k_DMTA_free
log_k_DMTA_free_bound
log_k_DMTA_bound_free
log_k_M23
log_k_M27
log_k_M31
                     0.0650
f_DMTA_ilr_1
                     -0.0258 0.0237
f_DMTA_ilr_2
                     -0.0221 -0.0660 -0.0257
f_DMTA_ilr_3
                      0.0664 0.0322 -0.0111
                                                0.0013
Random effects:
SD.DMTA_free_0
                        2.251174 0.46307 4.0393
SD.log_k_DMTA_free
                        0.614338 0.26609 0.9626
SD.log_k_DMTA_free_bound 0.904557 0.30542 1.5037
SD.log_k_DMTA_bound_free 0.005574 -3.83190 3.8431
SD.log_k_M23
                     1.199822 0.45907 1.9406
SD.log_k_M27
                        0.528851 0.19554 0.8622
SD.log_k_M31
                        0.590868 0.21289 0.9688
SD.f_DMTA_ilr_1
                        0.434070 0.17309 0.6950
                        0.477091 0.18914 0.7650
SD.f_DMTA_ilr_2
SD.f_DMTA_ilr_3
                        0.222161 0.08468 0.3596
Variance model:
      est. lower
                     upper
a.1 0.82989 0.76551 0.89428
b.1 0.03509 0.02892 0.04125
Backtransformed parameters:
                       est.
                                lower
                                          upper
DMTA_free_0
                  98.131566 9.598e+01 100.28493
k_DMTA_free
                   0.060611 3.705e-02
                                        0.09915
k_DMTA_free_bound 0.001989 8.931e-04
                                        0.00443
k_DMTA_bound_free 0.010039 7.888e-03
                                        0.01278
k M23
                   0.012166 4.416e-03
                                        0.03352
k_M27
                   0.017013 1.084e-02
                                        0.02671
                   0.016180 9.745e-03
k M31
                                        0.02687
f_DMTA_free_to_M23  0.111529  6.530e-02
                                        0.10607
f DMTA free to M27 0.097368 6.530e-02
                                        0.10607
f_DMTA_free_to_M31 0.089710 6.530e-02
                                        0.10607
Estimated Eigenvalues of SFORB model(s):
DMTA_b1 DMTA_b2 DMTA_g
0.062977 0.009661 0.955617
```


Listing 9: Hierarchical HS path 1 fit with constant variance

```
saemix version used for fitting:
                                      3.2
mkin version used for pre-fitting: 1.2.2
R version used for fitting:
                                    4.2.2
Date of fit: Thu Jan 5 15:10:37 2023
Date of summary: Thu Jan 5 16:42:52 2023
Equations:
d_DMTA/dt = - ifelse(time <= tb, k1, k2) * DMTA
d_M23/dt = + f_DMTA_to_M23 * ifelse(time <= tb, k1, k2) * DMTA - k_M23
          * M23
d_M27/dt = + f_DMTA_to_M27 * ifelse(time <= tb, k1, k2) * DMTA - k_M27
          * M27 + k_M31 * M31
d_M31/dt = + f_DMTA_to_M31 * ifelse(time <= tb, k1, k2) * DMTA - k_M31
           * M31
Data:
563 observations of 4 variable(s) grouped in 6 datasets
Model predictions using solution type deSolve
Fitted in 915.146 s
Using 300, 100 iterations and 9 chains
Variance model: Constant variance
Starting values for degradation parameters:
      DMTA_0
               log_k_M23
                            log_k_M27
                                          log_k_M31 f_DMTA_ilr_1 f_DMTA_ilr_2
                  -3.8929
                                -4.2833
                                             -4.2120
     98.1794
                                                           0.1387
f_DMTA_ilr_3
                   log_k1
                                log_k2
                                             log_tb
     -1.7320
                  -2.6017
                                -3.5984
Fixed degradation parameter values:
Starting values for random effects (square root of initial entries in omega):
             \label{eq:decomposition} \mbox{DMTA\_O log\_k\_M23 log\_k\_M27 log\_k\_M31 f\_DMTA\_ilr\_1 f\_DMTA\_ilr\_2}
DMTA_0
              2.953
                       0.0000
                                  0.000
                                             0.00
                                                         0.0000
                                                                      0.0000
log_k_M23
              0.000
                       0.7318
                                  0.000
                                             0.00
                                                         0.0000
                                                                      0.0000
log_k_M27
              0.000
                       0.0000
                                  1.161
                                             0.00
                                                         0.0000
                                                                      0.0000
log_k_M31
              0.000
                       0.0000
                                  0.000
                                             1.03
                                                         0.0000
                                                                      0.0000
f_DMTA_ilr_1
              0.000
                       0.0000
                                  0.000
                                             0.00
                                                         0.7464
                                                                      0.0000
f_DMTA_ilr_2
                      0.0000
                                  0.000
                                             0.00
                                                         0.0000
                                                                      0.7316
              0.000
f_DMTA_ilr_3 0.000
                       0.0000
                                  0.000
                                             0.00
                                                         0.0000
                                                                      0.0000
log_k1
              0.000
                       0.0000
                                  0.000
                                             0.00
                                                         0.0000
                                                                      0.0000
              0.000
                       0.0000
                                  0.000
                                                         0.0000
                                                                      0.0000
log_k2
                                             0.00
                       0.0000
              0.000
                                  0.000
                                                         0.0000
                                                                      0.0000
log_tb
                                             0.00
             f_DMTA_ilr_3 log_k1 log_k2 log_tb
                  0.0000 0.0000 0.0000 0.000
DMTA O
log_k_M23
                   0.0000 0.0000 0.0000 0.000
log_k_M27
                   0.0000 0.0000 0.0000 0.000
log_k_M31
                   0.0000 0.0000 0.0000 0.000
                   0.0000 0.0000 0.0000 0.000
f_DMTA_ilr_1
f_DMTA_ilr_2
                   0.0000 0.0000 0.0000 0.000
f_DMTA_ilr_3
                   0.4555 0.0000 0.0000 0.000
log_k1
                   0.0000 0.6228 0.0000 0.000
log_k2
                   0.0000 0.0000 0.3525 0.000
                   0.0000 0.0000 0.0000 1.511
log_tb
Starting values for error model parameters:
a.1
 1
Results:
Likelihood computed by importance sampling
  AIC BIC logLik
  2024 2020 -991.1
Optimised parameters:
                    est.
                            lower
DMTA_0
                98.01978 96.44819 99.5914
log_k_M23
                -4.34266 -5.30344 -3.3819
log_k_M27
                -4.01771 -4.46462 -3.5708
log_k_M31
                -4.10032 -4.62327 -3.5774
f_DMTA_ilr_1
                 0.08931 -0.27364 0.4523
f_DMTA_ilr_2
                 0.12507 -0.28841 0.5385
f_DMTA_ilr_3
                -1.67136 -1.85691 -1.4858
                -2.66051 -3.01910 -2.3019
log_k1
                -3.78451 -4.03983 -3.5292
log_k2
log_tb
                 2.82494 1.58415 4.0657
                 1.17672 1.13019 1.2232
SD.DMTA_O
                 1.86733 0.70082 3.0338
```

```
SD.f_DMTA_ilr_1 0.42928 0.16136 0.6972
SD.f_DMTA_ilr_2 0.48811 0.18409 0.7921
SD.f_DMTA_ilr_3 0.21445 0.07543 0.3535
SD.f_DMTA_____

SD.log_k1 0.41452 0.15400 ...

SD.log_k2 0.24919 0.05181 0.4466

en log tb 1.35920 0.45938 2.2590
Correlation:
            DMTA_0 lg__M23 lg__M27 lg__M31 f_DMTA__1 f_DMTA__2 f_DMTA__3
            -0.0053
log_k_M23
log_k_M27
             -0.0051 -0.0004
            -0.0034 0.0001 0.0884
log_k_M31
f_DMTA_ilr_1 -0.0008  0.0414 -0.0386  0.0353
f_DMTA_ilr_2 -0.0002  0.0208 -0.0308 -0.0982 -0.0365
f_DMTA_ilr_3 -0.0186  0.0479  0.0925  0.0464 -0.0140
                                                       0.0021
          0.0190 -0.0024 -0.0020 -0.0012 0.0003
                                                      -0.0003
                                                                -0.0037
log_k1
log_k2
            -0.0003 0.0022 0.0062 0.0051 0.0006
                                                      -0.0012
                                                                0.0035
            -0.0007 -0.0007 -0.0003 -0.0001 -0.0005
                                                                -0.0009
                                                      0.0000
log_tb
            log_k1 log_k2
log_k_M23
log_k_M27
log_k_M31
f_DMTA_ilr_1
f DMTA ilr 2
f_DMTA_ilr_3
log_k1
             -0.0008
log_k2
             -0.0984 -0.0179
log_tb
Random effects:
                 est. lower upper
SD.DMTA 0
               1.8673 0.70082 3.0338
SD.log_k_M23 1.1246 0.41614 1.8330
SD.log_k_M27 0.5105 0.17612 0.8449
SD.log_k_M31 0.5852 0.18634 0.9841
SD.f_DMTA_ilr_1 0.4293 0.16136 0.6972
SD.f_DMTA_ilr_2 0.4881 0.18409 0.7921
SD.f_DMTA_ilr_3 0.2145 0.07543 0.3535
           0.4145 0.15450 0.6745
SD.log_k1
SD.log_k2
               0.2492 0.05181 0.4466
SD.log_tb
             1.3592 0.45938 2.2590
Variance model:
    est. lower upper
a.1 1.177 1.13 1.223
{\tt Backtransformed\ parameters:}
                 est. lower
DMTA_0
              98.01978 96.448195 99.59136
k_M23
              0.01300 0.004974 0.03398
k_M27
              0.01799 0.011509 0.02813
k_M31
               0.01657 0.009821 0.02795
f_DMTA_to_M23 0.11322 NA NA
f_DMTA_to_M27 0.09979
                             NA
                                      NA
f_DMTA_to_M31 0.09120
                             NA
k1
              0.06991 0.048845 0.10007
              0.02272 0.017600 0.02933
             16.85989 4.875122 58.30747
Resulting formation fractions:
              ff
DMTA_M23 0.11322
DMTA_M27 0.09979
DMTA_M31 0.09120
DMTA_sink 0.69579
Estimated disappearance times:
      DT50 DT90 DT50back DT50_k1 DT50_k2
DMTA 9.914 66.33 19.97 9.914 30.51
                      NA
NA
M23 53.311 177.10
                                NA
                                        NA
M27 38.521 127.96
                                NA
                                        NA
M31 41.838 138.98
                       NA
                                NA
                                        NA
```

Listing 10: Hierarchical HS path 1 fit with two-component error

```
saemix version used for fitting:
                                    3.2
mkin version used for pre-fitting: 1.2.2
R version used for fitting:
                                   4.2.2
Date of fit: Thu Jan 5 15:11:14 2023
Date of summary: Thu Jan 5 16:42:52 2023
d_DMTA/dt = - ifelse(time <= tb, k1, k2) * DMTA
d_M23/dt = + f_DMTA_to_M23 * ifelse(time <= tb, k1, k2) * DMTA - k_M23
          * M23
d_M27/dt = + f_DMTA_to_M27 * ifelse(time <= tb, k1, k2) * DMTA - k_M27
          * M27 + k_M31 * M31
d_M31/dt = + f_DMTA_to_M31 * ifelse(time <= tb, k1, k2) * DMTA - k_M31
          * M31
Data:
563 observations of 4 variable(s) grouped in 6 datasets
Model predictions using solution type deSolve
Fitted in 952.309 s
Using 300, 100 iterations and 9 chains
Variance model: Two-component variance function
Starting values for degradation parameters:
     DMTA_0
               log_k_M23
                            log_k_M27
                                         log_k_M31 f_DMTA_ilr_1 f_DMTA_ilr_2
                               -4.1230
                  -3.8918
                                           -4.1315
     96.0836
                                                         0.1151
f_DMTA_ilr_3
                  log_k1
                               log_k2
                                            log_tb
     -1.6682
                  -2.8580
                               -3.4085
Fixed degradation parameter values:
Starting values for random effects (square root of initial entries in omega):
            DMTA_0 log_k_M23 log_k_M27 log_k_M31 f_DMTA_ilr_1 f_DMTA_ilr_2
DMTA_0
             6.474
                       0.000
                                 0.000
                                           0.000
                                                       0.0000
                                                                    0.0000
log_k_M23
             0.000
                        0.786
                                 0.000
                                           0.000
                                                       0.0000
                                                                    0.0000
log_k_M27
             0.000
                       0.000
                                 0.985
                                           0.000
                                                       0.0000
                                                                    0.0000
log_k_M31
             0.000
                       0.000
                                 0.000
                                           0.913
                                                       0.0000
                                                                    0.0000
f_DMTA_ilr_1
             0.000
                       0.000
                                 0.000
                                           0.000
                                                       0.7081
                                                                    0.0000
f_DMTA_ilr_2
                       0.000
                                 0.000
                                           0.000
                                                       0.0000
                                                                    0.7444
             0.000
f_DMTA_ilr_3 0.000
                       0.000
                                 0.000
                                           0.000
                                                       0.0000
                                                                    0.0000
                                 0.000
                                                       0.0000
log_k1
             0.000
                       0.000
                                           0.000
                                                                    0.0000
             0.000
                       0.000
                                 0.000
                                           0.000
                                                       0.0000
                                                                    0.0000
log_k2
             0.000
                       0.000
                                 0.000
                                           0.000
                                                       0.0000
                                                                    0.0000
log_tb
            f_DMTA_ilr_3 log_k1 log_k2 log_tb
DMTA O
                 0.0000 0.000 0.000 0.000
log_k_M23
                  0.0000 0.000 0.000 0.000
log_k_M27
                  0.0000 0.000 0.000 0.000
log_k_M31
                  0.0000 0.000 0.000 0.000
                  0.0000 0.000 0.000
f_DMTA_ilr_1
                                       0.000
f_DMTA_ilr_2
                  0.0000 0.000 0.000 0.000
f_DMTA_ilr_3
                  0.4377 0.000 0.000
                                       0.000
                          1.081
log_k1
                  0.0000
                                 0.000 0.000
log_k2
                  0.0000 0.000
                                 1.097
                                       0.000
                  0.0000 0.000 0.000 1.524
log_tb
Starting values for error model parameters:
a.1 b.1
 1
     1
Results:
Likelihood computed by importance sampling
  AIC BIC logLik
  1831 1827 -893.7
Optimised parameters:
                    est.
                            lower
                                     upper
DMTA_0
               97.127306 94.96753 99.28708
log_k_M23
               -4.340939 -5.27262 -3.40926
log_k_M27
               -4.079298 -4.54430 -3.61429
log_k_M31
               -4.125264 -4.63566 -3.61487
f_DMTA_ilr_1
                0.108318 -0.25702 0.47366
f_DMTA_ilr_2
                0.128873 -0.27187 0.52962
f_DMTA_ilr_3
               -1.672627 -1.86802 -1.47723
               -2.858335 -3.35349 -2.36318
log_k1
                -3.749355 -4.02909 -3.46962
log_k2
log_tb
                3.442549 3.39860 3.48650
                0.808860 0.74423 0.87349
a.1
                0.040018 0.03339 0.04665
```

```
SU.DMTA_0 2.163341 0.30906 4.01762
SD.log_k_M23 1.108478 0.427F6 1.7
SD.log_k_M27
SD.log_k_M27
               0.545110 0.20147 0.88875
SD.log_k_M31
                0.594829 0.21460 0.97506
SD.f_DMTA_ilr_1 0.442524 0.17725 0.70780
SD.f_DMTA_ilr_2 0.483605 0.19250 0.77471
SD.f_DMTA_ilr_3  0.232158  0.08913  0.37518
                0.618000 0.26758 0.96842
SD.log_k1
                0.307633 0.10361 0.51165
SD.log_k2
                0.006502 -0.14501 0.15802
SD.log_tb
Correlation:
            DMTA_0 lg__M23 lg__M27 lg__M31 f_DMTA__1 f_DMTA__2 f_DMTA__3
log_k_M23
            -0.0199
            -0.0209 0.0042
log_k_M27
log_k_M31
            -0.0138 0.0030 0.0661
f_DMTA_ilr_1 -0.0025 0.0243 -0.0261 0.0227
f_DMTA_ilr_2 -0.0006  0.0125 -0.0212 -0.0648 -0.0244
f_DMTA_ilr_3 -0.0822 0.0349 0.0664 0.0329 -0.0104
                                                    0.0013
            0.0186 -0.0034 -0.0031 -0.0020 -0.0004
                                                    -0.0001
                                                             -0.0068
log k1
            -0.0077 0.0050 0.0069 0.0050 0.0007
                                                    -0.0004
                                                              0.0070
log_k2
            -0.0804 0.0054 0.0079 0.0035 0.0004
log_tb
                                                    0.0017
                                                              0.0249
            log_k1 log_k2
log_k_M23
log_k_M27
log_k_M31
f DMTA ilr 1
f_DMTA_ilr_2
f_DMTA_ilr_3
log_k1
            -0.0026
log_k2
           -0.0111 -0.1647
log_tb
Random effects:
                          lower upper
                  est.
SD.DMTA_O
              2.163341 0.30906 4.0176
SD.f_DMTA_ilr_1 0.442524 0.17725 0.7078
SD.f_DMTA_ilr_2 0.483605 0.19250 0.7747
SD.f_DMTA_ilr_3 0.232158 0.08913 0.3752
          0.618000 0.26758 0.9684
SD.log_k1
SD.log_k2
              0.307633 0.10361 0.5117
SD.log_tb
              0.006502 -0.14501 0.1580
Variance model:
     est. lower upper
a.1 0.80886 0.74423 0.87349
b.1 0.04002 0.03339 0.04665
Backtransformed parameters:
                est.
                      lower
                                 upper
DMTA_0
             97.12731 94.96753 99.28708
k_M23
             0.01302 0.00513 0.03307
k_M27
              0.01692 0.01063 0.02694
k_M31
              0.01616 0.00970 0.02692
                      NA NA
f_DMTA_to_M23 0.11479
                       NA
NA
f_DMTA_to_M27 0.09848
                                   NA
f_DMTA_to_M31 0.09080
              0.05736 0.03496 0.09412
k2
              0.02353 0.01779 0.03113
             31.26657 29.92214 32.67141
tb
Resulting formation fractions:
             ff
DMTA_M23 0.11479
DMTA_M27 0.09848
DMTA_M31 0.09080
DMTA_sink 0.69593
Estimated disappearance times:
     DT50 DT90 DT50back DT50_k1 DT50_k2
DMTA 12.08 52.9 15.92 12.08 29.45
M23 53.22 176.8
                   NA
                             NA
                                    NA
M27 40.97 136.1
                     NA
                             NA
                                    NA
M31 42.89 142.5
                             NA
                    NA
                                    NA
```

Improved fit of the SFORB pathway model with two-component error

Listing 11: Hierarchical SFORB pathway fit with two-component error

```
saemix version used for fitting:
mkin version used for pre-fitting: 1.2.2
R version used for fitting:
Date of fit: Thu Jan 5 15:32:42 2023
Date of summary: Thu Jan 5 16:42:52 2023
d_DMTA_free/dt = - k_DMTA_free * DMTA_free - k_DMTA_free_bound *
           DMTA_free + k_DMTA_bound_free * DMTA_bound
d_DMTA_bound/dt = + k_DMTA_free_bound * DMTA_free - k_DMTA_bound_free *
           DMTA_bound
d_M23/dt = + f_DMTA_free_to_M23 * k_DMTA_free * DMTA_free - k_M23 * M23
d_M27/dt = + f_DMTA_free_to_M27 * k_DMTA_free * DMTA_free - k_M27 * M27
           + k_M31 * M31
d_M31/dt = + f_DMTA_free_to_M31 * k_DMTA_free * DMTA_free - k_M31 * M31
563 observations of 4 variable(s) grouped in 6 datasets
Model predictions using solution type deSolve
Fitted in 1286.938 s
Using 300, 100 iterations and 9 chains
Variance model: Two-component variance function
Starting values for degradation parameters:  \\
                            log_k_DMTA_free log_k_DMTA_free_bound
          DMTA free 0
              97.2597
                                    -2.8229
                                                           -5.9851
log_k_DMTA_bound_free
                                  log_k_M23
                                                         log_k_M27
                                    -3.9216
              -3.3880
                                                           -4.2555
                               f_DMTA_ilr_1
                                                     f_DMTA_ilr_2
            log_k_M31
              -4.2054
                                     0.1243
                                                            0.1306
         f_DMTA_ilr_3
              -1.7266
Fixed degradation parameter values:
Starting values for random effects (square root of initial entries in omega):
                      DMTA_free_0 log_k_DMTA_free log_k_DMTA_free_bound
DMTA_free_0
                            5.586
                                           0.0000
log_k_DMTA_free
                            0.000
                                           0.9851
                                                                   0.000
log_k_DMTA_free_bound
                            0.000
                                           0.0000
                                                                   1.309
log_k_DMTA_bound_free
                            0.000
                                           0.0000
                                                                   0.000
log_k_M23
                            0.000
                                           0.0000
                                                                   0.000
log_k_M27
                            0.000
                                           0.0000
                                                                   0.000
log_k_M31
                            0.000
                                           0.0000
                                                                   0.000
f_DMTA_ilr_1
                            0.000
                                            0.0000
                                                                   0.000
f_DMTA_ilr_2
                            0.000
                                           0.0000
                                                                   0.000
                                           0.0000
f_DMTA_ilr_3
                            0.000
                                                                   0.000
                      log_k_DMTA_bound_free log_k_M23 log_k_M27 log_k_M31
DMTA_free_0
                                      0.000
                                                0.0000
                                                           0.000
                                                                    0.0000
log_k_DMTA_free
                                       0.000
                                                0.0000
                                                           0.000
                                                                    0.0000
log_k_DMTA_free_bound
                                       0.000
                                                0.0000
                                                           0.000
                                                                    0.0000
log_k_DMTA_bound_free
                                       4.482
                                                0.0000
                                                           0.000
                                                                    0.0000
log_k_M23
                                       0.000
                                                0.7768
                                                           0.000
                                                                    0.0000
log_k_M27
                                       0.000
                                                0.0000
                                                           1.096
                                                                    0.0000
                                       0.000
log_k_M31
                                                0.0000
                                                           0.000
                                                                    0.9909
f_DMTA_ilr_1
                                       0.000
                                                0.0000
                                                           0.000
                                                                    0.0000
f_DMTA_ilr_2
                                       0.000
                                                0.0000
                                                           0.000
                                                                    0.0000
f_DMTA_ilr_3
                                       0.000
                                                0.0000
                                                           0.000
                                                                    0.0000
                      f_DMTA_ilr_1 f_DMTA_ilr_2 f_DMTA_ilr_3
                            0.0000
DMTA_free_0
                                          0.0000
                                                       0.0000
log_k_DMTA_free
                            0.0000
                                          0.0000
                                                       0.0000
log_k_DMTA_free_bound
                            0.0000
                                          0.0000
                                                       0.0000
log_k_DMTA_bound_free
                            0.0000
                                          0.0000
                                                       0.0000
log_k_M23
                            0.0000
                                          0.0000
                                                       0.0000
log_k_M27
                            0.0000
                                          0.0000
                                                       0.0000
                            0.0000
                                          0.0000
                                                       0.0000
log k M31
f_DMTA_ilr_1
                            0.7196
                                          0.0000
                                                       0.0000
f_DMTA_ilr_2
                            0.0000
                                          0.7359
                                                       0.0000
f DMTA ilr 3
                            0.0000
                                          0.0000
                                                       0.4417
Starting values for error model parameters:
a.1 b.1
Results:
```

```
Likelihood computed by importance sampling
   AIC BIC logLik
  1830 1826 -894.2
Optimised parameters:
                            est.
                                    lower
                                              upper
DMTA_free_0
                        98.12951 96.04142 100.21760
log_k_DMTA_free
                        -2.80215 -3.29420 -2.31011
                        -6.20032 -7.00324 -5.39740
log_k_DMTA_free_bound
log_k_DMTA_bound_free
                        -4.56658 -4.80300 -4.33015
log_k_M23
                        -4.38232 -5.36504 -3.39961
log_k_M27
                        -4.10689 -4.58275 -3.63102
                        -4.11686 -4.61305 -3.62067
log_k_M31
f_DMTA_ilr_1
                        0.12460 -0.25920
                                            0.50840
                        0.11236 -0.29215
f_DMTA_ilr_2
                                            0.51686
                        -1.70444 -1.89852 -1.51037
f_DMTA_ilr_3
                        0.82880 0.76216 0.89543
a.1
                         0.03539 0.02900
b.1
                                            0.04178
SD.DMTA_free_0
                         2.14737 0.38706
                                            3.90767
SD.log_k_DMTA_free
                         0.61415 0.26600
                                            0.96230
SD.log_k_DMTA_free_bound 0.90747 0.30720
                                            1.50774
SD.log_k_M23
                         1.16637 0.44805
                                            1.88469
                         0.55720 0.20526
                                            0.90915
SD.log_k_M27
SD.log_k_M31
                         0.57721 0.20682
                                            0.94760
SD.f_DMTA_ilr_1
                         0.46475 0.18610
                                            0.74340
SD.f_DMTA_ilr_2
                         0.48722 0.19310
                                            0.78135
SD.f_DMTA_ilr_3
                         0.23092 0.08873
                                            0.37312
Correlation:
                     DMTA__0 lg__DMTA_ lg_k_DMTA_f_ lg_k_DMTA_b_ lg__M23
log k DMTA free
                      0.0170
log_k_DMTA_free_bound 0.0607 0.0143
log_k_DMTA_bound_free 0.0544 0.0142
                                       0.1550
log_k_M23
                     -0.0143 -0.0023 -0.0172
                                                    -0.0021
                     -0.0169 -0.0025 -0.0153
                                                     0.0001
log_k_M27
                                                                  0.0023
                     -0.0115 -0.0016
                                      -0.0096
                                                     0.0021
                                                                  0.0018
log_k_M31
                     -0.0022 -0.0003
                                       -0.0015
f_DMTA_ilr_1
                                                     0.0001
                                                                  0.0232
                     -0.0003 -0.0001 -0.0005
-0.0697 -0.0056 -0.0234
                                                    -0.0018
f DMTA ilr 2
                                                                  0.0123
f_DMTA_ilr_3
                                                    -0.0118
                                                                  0.0317
                     lg__M27 lg__M31 f_DMTA__1 f_DMTA__2
log_k_DMTA_free
log_k_DMTA_free_bound
log_k_DMTA_bound_free
log_k_M23
log_k_M27
log_k_M31
                      0.0658
                     -0.0244 0.0253
f_DMTA_ilr_1
f_DMTA_ilr_2
                     -0.0220 -0.0667 -0.0279
f_DMTA_ilr_3
                     0.0643 0.0290 -0.0130
                                                0.0039
Random effects:
                          est. lower upper
SD.DMTA_free_0
                        2.1474 0.38706 3.9077
SD.log_k_DMTA_free
                       0.6142 0.26600 0.9623
SD.log_k_DMTA_free_bound 0.9075 0.30720 1.5077
               1.1664 0.44805 1.8847
SD.log_k_M23
SD.log_k_M27
                        0.5572 0.20526 0.9091
SD.log_k_M31
                        0.5772 0.20682 0.9476
SD.f_DMTA_ilr_1
                        0.4648 0.18610 0.7434
                        0.4872 0.19310 0.7813
SD.f_DMTA_ilr_2
SD.f_DMTA_ilr_3
                        0.2309 0.08873 0.3731
Variance model:
      est. lower upper
a.1 0.82880 0.7622 0.89543
b.1 0.03539 0.0290 0.04178
Backtransformed parameters:
                       est.
                                lower
                                          upper
DMTA_free_0
                  98.129511 9.604e+01 1.002e+02
k_DMTA_free
                   0.060679 3.710e-02 9.925e-02
k_DMTA_free_bound 0.002029 9.089e-04 4.528e-03
k_DMTA_bound_free 0.010393 8.205e-03 1.317e-02
k M23
                   0.012496 4.677e-03 3.339e-02
k_M27
                   0.016459 1.023e-02 2.649e-02
                   0.016296 9.922e-03 2.676e-02
k M31
f_DMTA_free_to_M23  0.112408  6.387e-02  1.056e-01
f DMTA free to M27 0.094247 6.387e-02 1.056e-01
f_DMTA_free_to_M31  0.089695  6.387e-02  1.056e-01
Estimated Eigenvalues of SFORB model(s):
DMTA_b1 DMTA_b2 DMTA_g
0.063108 0.009993 0.954272
```


Session info

R version 4.2.2 Patched (2022-11-10 r83330) Platform: x86_64-pc-linux-gnu (64-bit) Running under: Debian GNU/Linux bookworm/sid

Matrix products: default

BLAS: /usr/lib/x86_64-linux-gnu/openblas-serial/libblas.so.3

LAPACK: /usr/lib/x86_64-linux-gnu/openblas-serial/libopenblas-r0.3.21.so

locale:

- [1] LC CTYPE=de DE.UTF-8 LC NUMERIC=C
- [3] LC_TIME=de_DE.UTF-8 LC_COLLATE=de_DE.UTF-8
 [5] LC_MONETARY=de_DE.UTF-8 LC_MESSAGES=de_DE.UTF-8
- [7] LC_PAPER=de_DE.UTF-8 LC_NAME=C
 [9] LC_ADDRESS=C LC_TELEPHONE=C
- [11] LC_MEASUREMENT=de_DE.UTF-8 LC_IDENTIFICATION=C

attached base packages:

- [1] parallel stats graphics grDevices utils datasets methods
- [8] base

other attached packages:

- [1] saemix_3.2 npde_3.3 knitr_1.41 mkin_1.2.2
- [5] rmarkdown_2.19 nvimcom_0.9-133

loaded via a namespace (and not attached):

[1]	deSolve_1.34	zoo_1.8-11	tidyselect_1.2.0	xfun_0.35
[5]	lattice_0.20-45	colorspace_2.0-3	vctrs_0.5.1	generics_0.1.3
[9]	htmltools_0.5.4	yam1_2.3.6	utf8_1.2.2	rlang_1.0.6
[13]	pkgbuild_1.4.0	pillar_1.8.1	glue_1.6.2	DBI_1.1.3
[17]	lifecycle_1.0.3	stringr_1.5.0	munsell_0.5.0	gtable_0.3.1
[21]	codetools_0.2-18	evaluate_0.19	inline_0.3.19	callr_3.7.3
[25]	fastmap_1.1.0	ps_1.7.2	<pre>lmtest_0.9-40</pre>	fansi_1.0.3
[29]	highr_0.9	scales_1.2.1	<pre>gridExtra_2.3</pre>	ggplot2_3.4.0
[33]	digest_0.6.31	stringi_1.7.8	processx_3.8.0	dplyr_1.0.10
[37]	grid_4.2.2	cli_3.5.0	tools_4.2.2	magrittr_2.0.3
[41]	tibble_3.1.8	crayon_1.5.2	pkgconfig_2.0.3	<pre>prettyunits_1.1.1</pre>
[45]	assertthat_0.2.1	R6_2.5.1	mclust_6.0.0	nlme_3.1-161
$\Gamma \wedge \cap \Gamma$				

[49] compiler_4.2.2

Hardware info

CPU model: AMD Ryzen 9 7950X 16-Core Processor

MemTotal: 64940452 kB