Week 5: Introduction to Hypothesis Testing

phase 2

Introduction

- We have previously seen how to estimate parameters from our data.

- The next step is to evaluate how likely such estimations are, given our assumed world (our model).

- This is the basis of null **hypothesis testing**.

Null hypotheses and alternative hypotheses

- The statement being tested is called the **Null Hypothesis,** H_0. Usually H_0 is a statement of *"There is no effect"*.

<u>*Example*</u>*: Drug X did not cause, on average, a physiological response, i.e.*
$<\mathit{Response}>_{\mathit{before}} = <\mathit{Response}>_{\mathit{after}}$

- The **Alternative Hypothesis**, H_a, is every statement that is true instead of the null hypothesis. H_a usually *"There is some effect"* statement.

<u>*Example*</u>*: Drug X did cause, on average, a physiological response, i.e.*
$<\mathit{Response}>_{\mathit{before}} \neq <\mathit{Response}>_{\mathit{after}}$

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<u>Example</u>: Drug X did not increase, on average, the physiological response, i.e.

<Response>$_{before}$ >= <Response>$_{after}$

<u>No effect</u> depends on what you are testing!

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Hypothesis testing is like a jury trial

H_0 : Defendant is innocent.

H_a : Defendant is guilty.

Steps in the trial (Hypothesis Testing):

1. Collect the data.
2. Present the evidence.
3. Make a judgment.

Evidence: "How plausibly could we see these data by chance?"
(Null Hypothesis is true)

Judgment: How unlikely is unlikely enough?

Test statistics

- A test is based on a statistic that estimates the parameter that appears in the hypotheses.
- When H_0 is true, we expect the estimate to take a value near the parameter value specified by H_0. This is the **hypothesized value**.
- Values of the estimate far from the hypothesized value give evidence **against** H_0 . The alternative hypothesis determines which **directions** count against H_0.

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Caution!!!!

This only talks about the probability of the observed data, not the probability of the null hypothesis…

Hypothesis Testing procedure

1. We start with a **null hypothesis** that represents the status quo.
2. We also have an **alternative hypothesis** H_a that represents our research question, i.e. what we're testing for.
3. We conduct a hypothesis test under the **assumption** that the **null** hypothesis is **true**.
4. If the test results suggest that the data do **not** provide **convincing evidence** for the alternative hypothesis, we **stick** with the **null** hypothesis. If they do, then we **reject** the **null** hypothesis **in favor** of the **alternative**.
5. The burden of proof is on the alternative hypothesis.

Hypothesis Testing procedure

How much do we need, and how do we quantify this??

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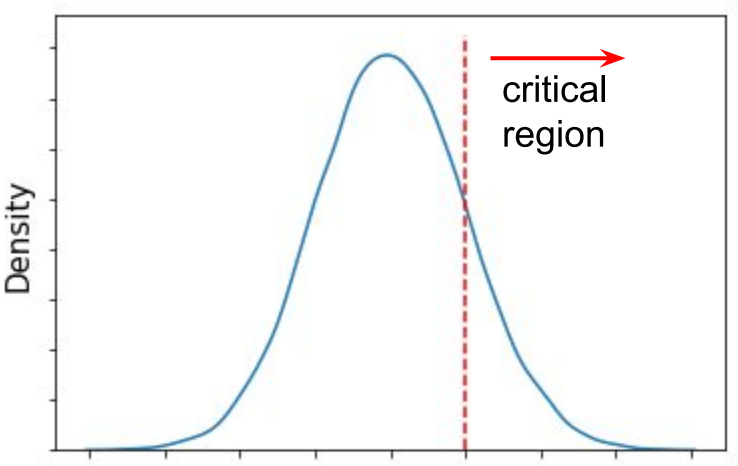
Making decisions

➢ We need to define a **critical region** of values in our test for which we are going to **reject** the null hypothesis.



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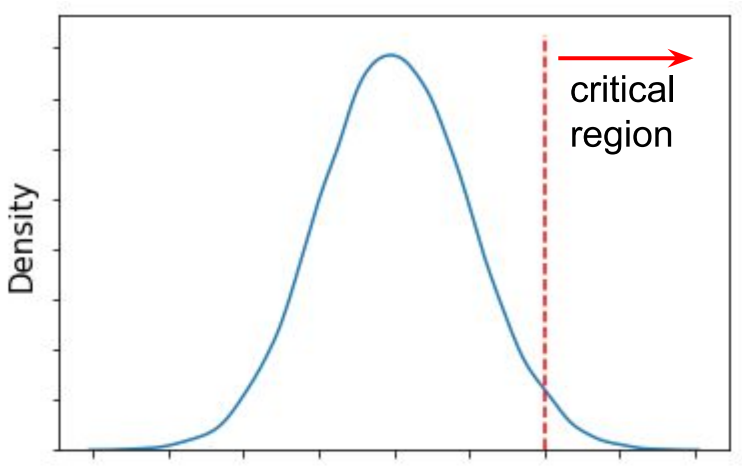

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Making decisions

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- As we have seen, the critical region consists of the most extreme values → the tail of the distribution.

- Everything within the critical region is considered unlikely under the null hypothesis and therefore we are sufficiently confident to reject it →
→ We claim to observe a **significant** result!

We might make mistakes!!!

Any time that we make a decision about whether to trust the null hypothesis or not, we are subject to committing errors!

- Type I error: If we reject H_0 (accept H_a) when in fact H_0 is true.
- Type II error: if we accept H_0 (reject H_a) when in fact H_a is true.

	retain H_0	reject H_0
H_0 is true	$1-\alpha$ (probability of correct retention)	α (type I error rate)
H_0 is false	β (type II error rate)	$1-\beta$ (power of the test)

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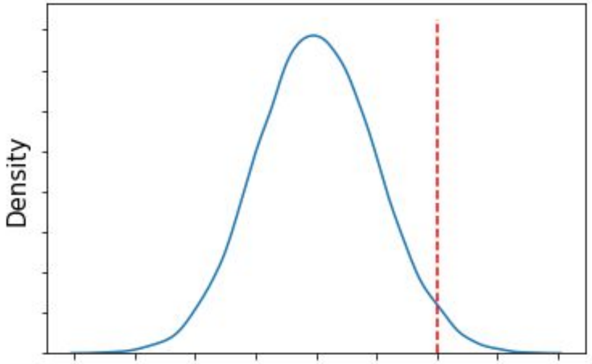
- Any time that we make a decision about whether to trust the null hypothesis or not, we are subject to committing errors!

- **Hypothesis testing** particularly focuses on keeping the **Type I error** rate as **low** as possible, but low **Type II errors are also important** (We'll see this later).

Quantifying evidence: p-values

The p-value is the **probability** that the test statistic would take a value as extreme or more extreme than that actually observed, assuming H_0 is **true**.

Distribution under the null
(e.g. here with a particular mean)



Quantifying evidence: p-values

The p-value is the **probability** that the test statistic would take a value as extreme or more extreme than that actually observed, assuming H_0 is **true**.

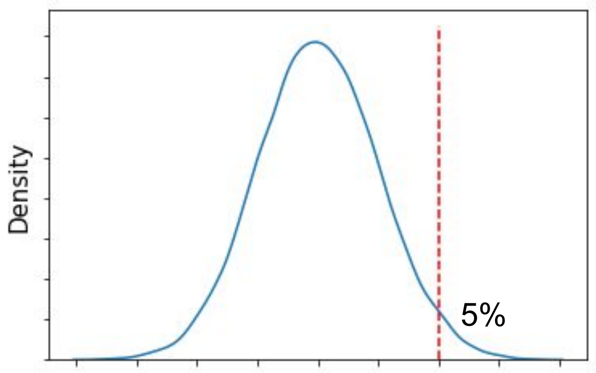
Two possible interpretations:

1. The smaller the p-value, the stronger the evidence AGAINST H_0 provided by the data. (FISHER's)

2. p-values represent the smallest Type I error rate (α) that one is willing to tolerate if you want to reject the null hypothesis. (NEYMAN's)

Quantifying evidence: rejecting the null

➢ In general, any result whose p-value is **below** the threshold α=**0.05** is claimed to be **significant.**



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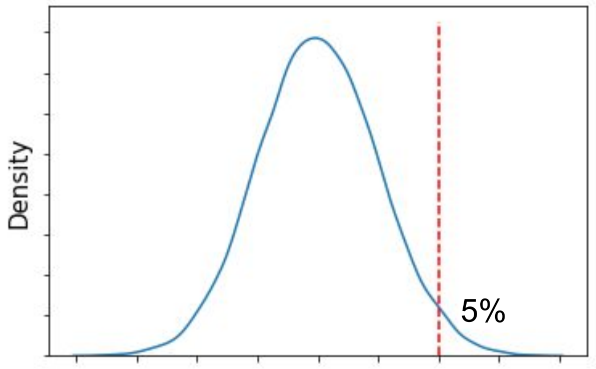

We believe it is sufficiently small to be an evidence against the null (Fisher's), or it's the maximum type I error that we are decided to commit if the null was true (Neyman's).

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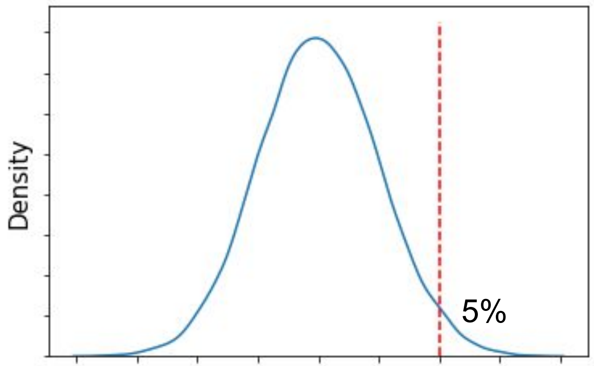

0.05 is just a **SUBJECTIVE** threshold; it does not have any special meaning!!!!

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Quantifying evidence: p-values

<u>Let’s discuss:</u> Would you say that a result $p=0.5$ means that the null hypothesis is ten times more likely than $p=0.05$?

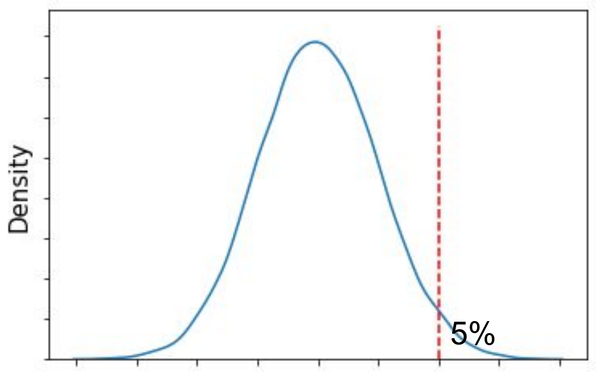
Quantifying evidence: p-values

Let's discuss: Would you say that a result p=0.5 means that the null hypothesis is ten times more likely than p=0.05?

- **NOOOOOOOOOO** and **NEVER** interpret p-values this way!
- P-values are just a probability about the observed data under an assumed model; it does not quantify the probability of a hypothesis!

Quantifying evidence: p-values

➢ As we said earlier, the critical region represents the tail area of a distribution.



Quantifying evidence: p-values

- As we said earlier, the critical region represents the tail area of a distribution.

- But distributions have two tails (tests have **directionality**!).

e.g. Is a certain parameter different from zero?

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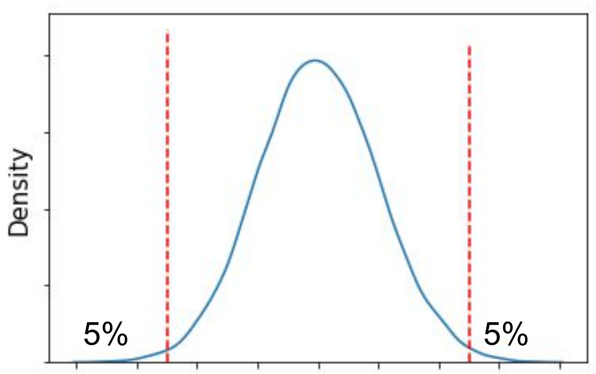

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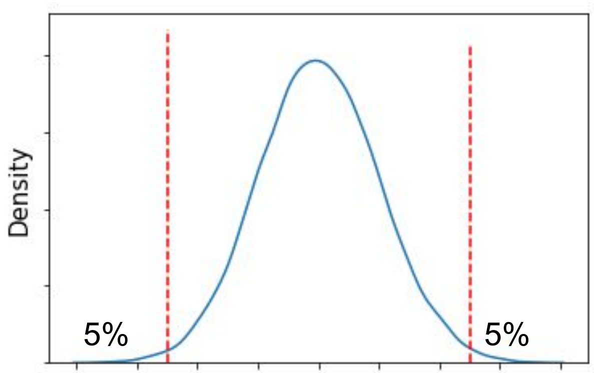

This increases our Type I Error rate!

We now reject twice as many null hypotheses!

Quantifying evidence: p-values

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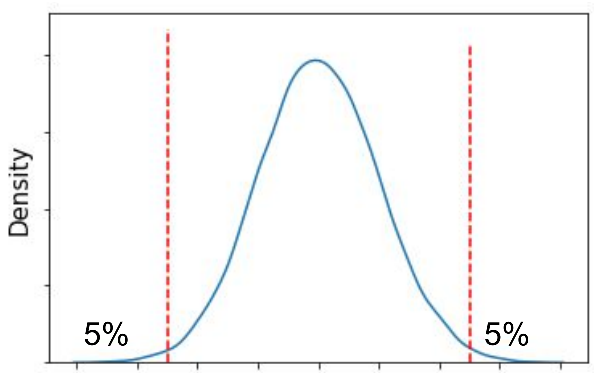

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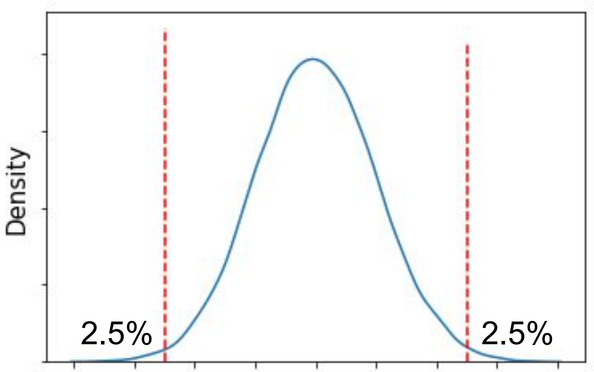


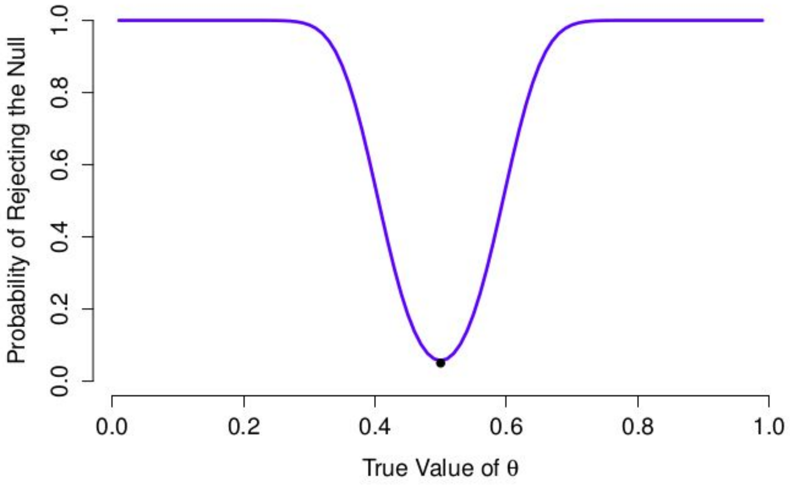
This keeps our total Type I error rate the same.

Controlling type II errors

- ➢ A secondary goal of hypothesis testing is to try to minimise the Type II error rate (β) or, maximise the **power** of the test (1-β).
- ➢ How can we increase the power of the test?

<u>Big effect sizes.</u>

- **The bigger the effect, the easier to reject the null.**
- This is usually something out of our control.



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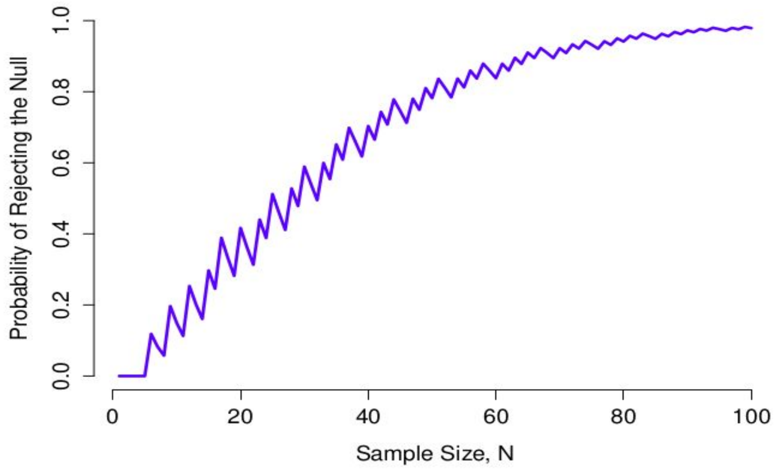
➢ A secondary goal of hypothesis testing is to try to minimise the Type II error rate (β) or, maximise the **power** of the test (1-β).

➢ How can we increase the power of the test?

<u>Big sample sizes.</u>

- **The more data, the easier to pick up any differences.**

- This is something that **we can control when designing the experiment.**



Recap

- Null hypothesis testing is a framework for **quantifying evidence**.
- We quantify this evidence to be able to **make a decision about the null**!
- We may need to **adjust** our degree of decision based on our tested alternative hypothesis.
- We **generally** talk about **Type I error**, but **Type II** errors are also **important**, particularly in **designing a study**!