

Week 2:

Look, understand and make sense of your data!

Phase 1

Key ideas of this lecture

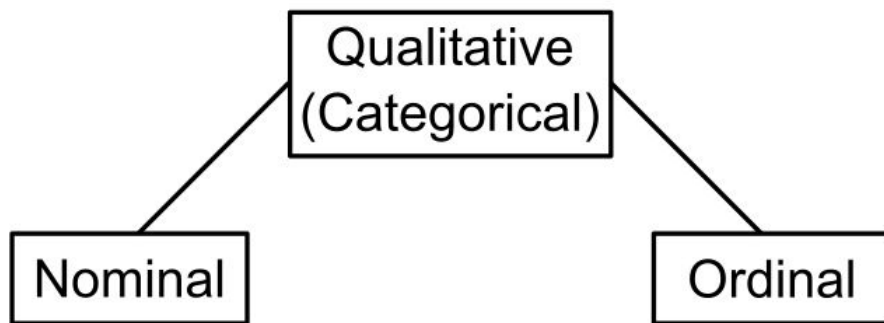
- Descriptive statistics: first step after collecting the data.
- Visualization and summary: better understanding and communication.
- Key concepts: Tendency, variability and shape.

Understand the data: types of variables

It is important to have clear what kind of variables we have in our data.

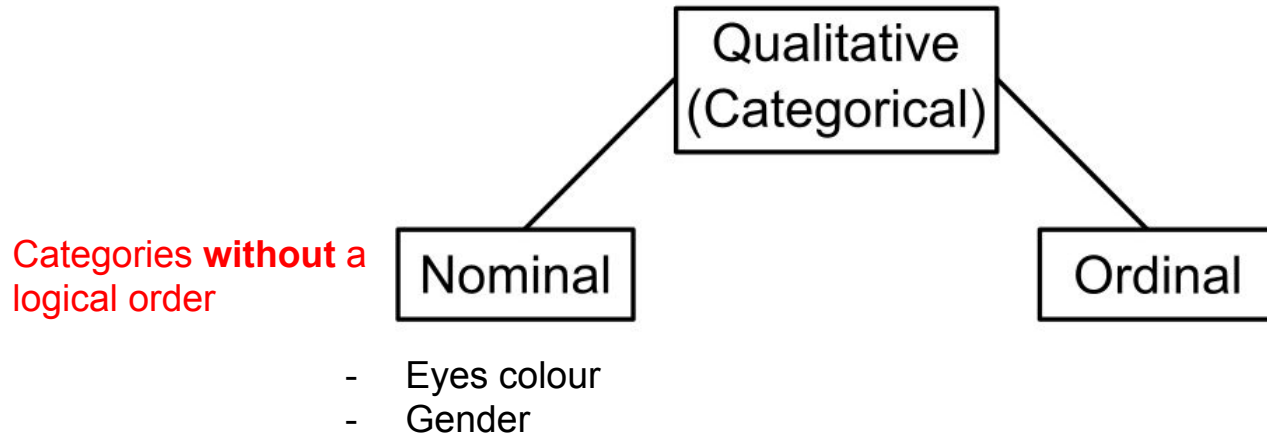
Understand the data: types of variables

Do variables contain only categories or groups for which adding and averaging do not make sense? Then we have **qualitative** data.



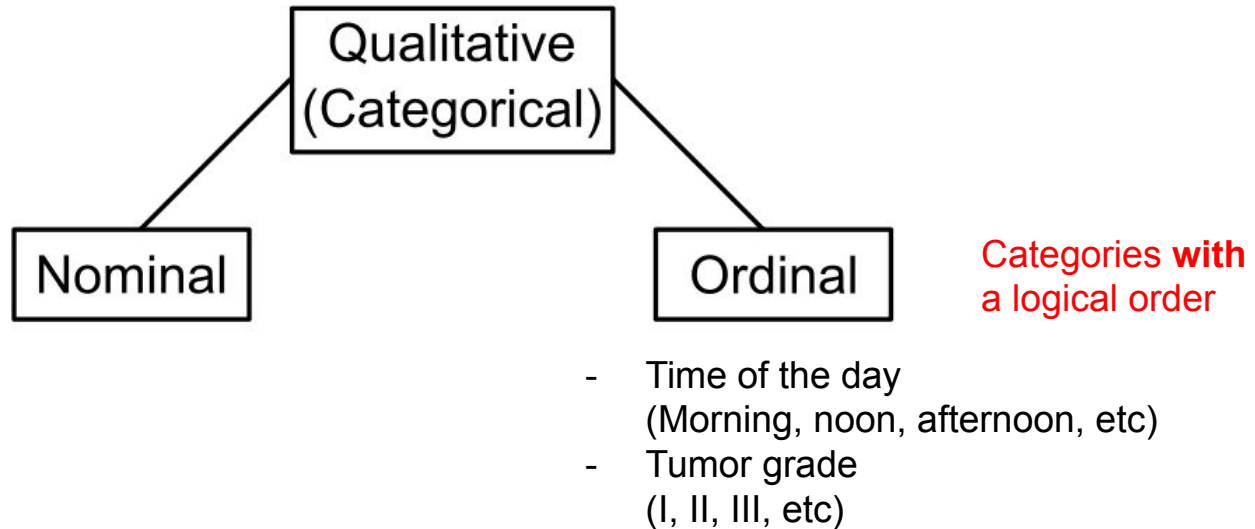
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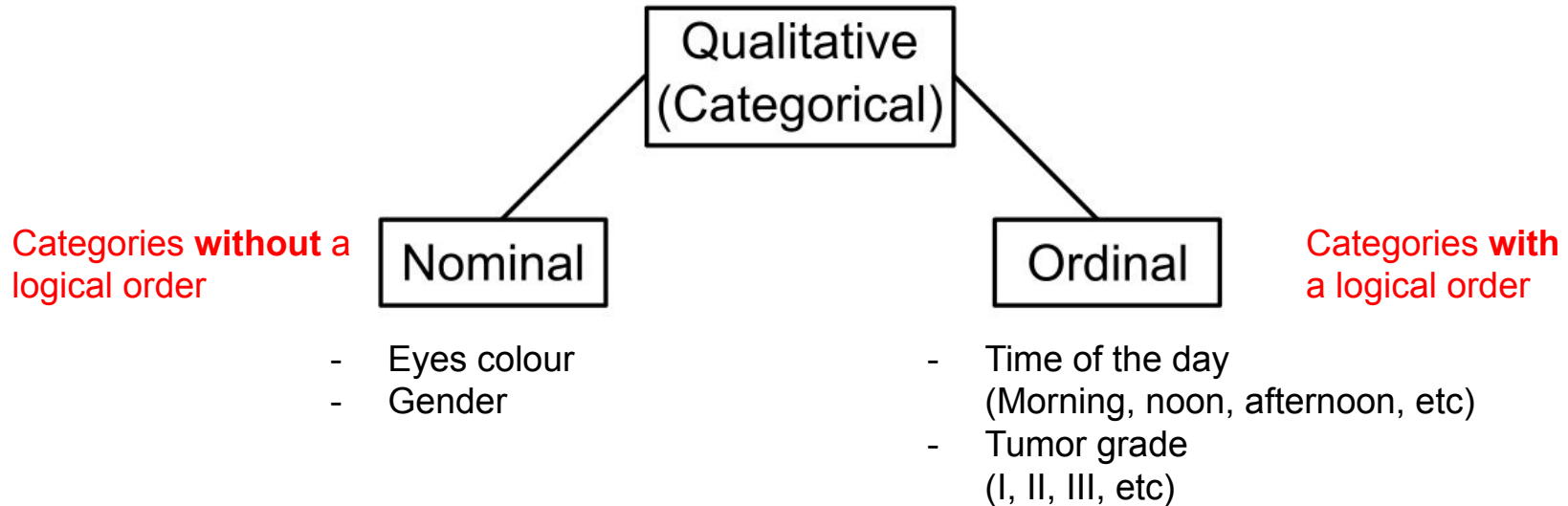
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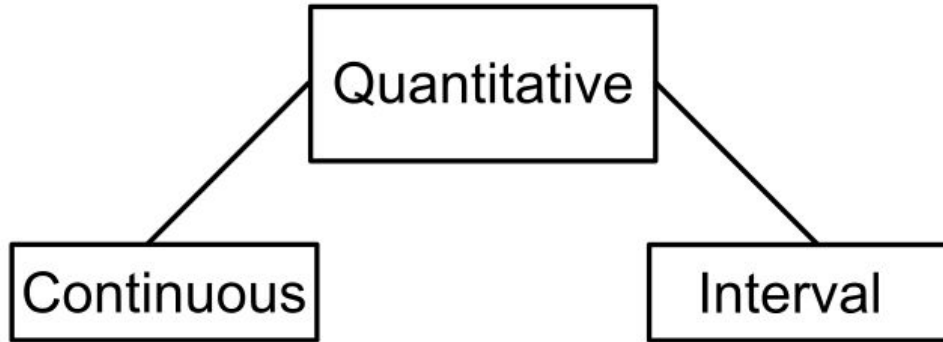
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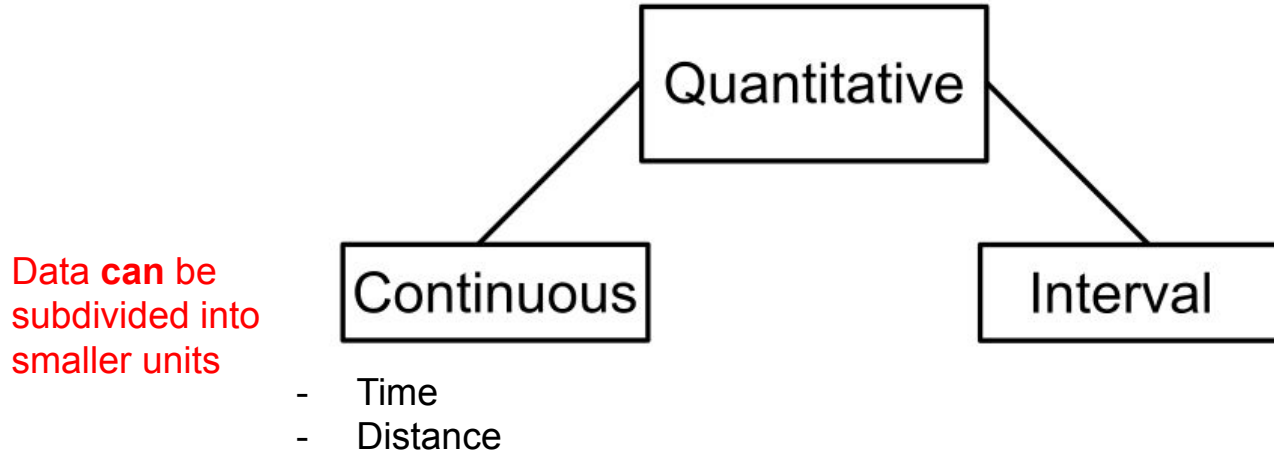
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Do variables take numerical values for which arithmetic operations such as adding and averaging make sense? Then we have **quantitative** data.



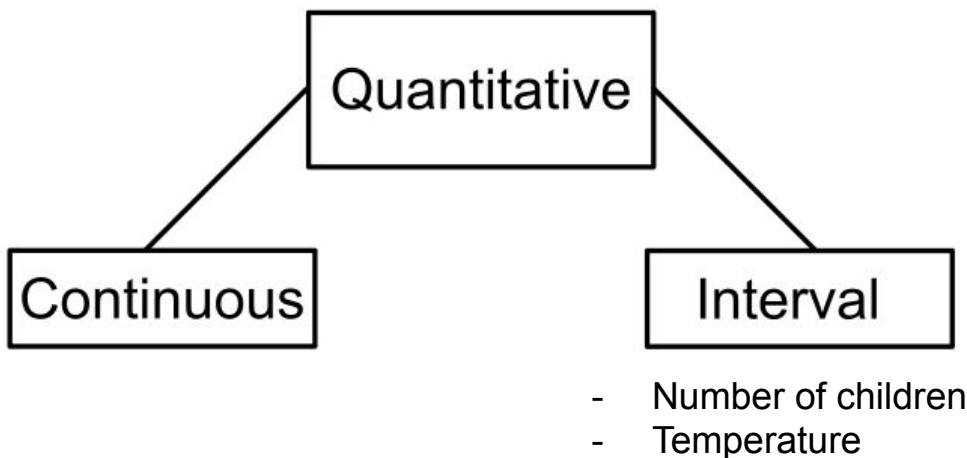
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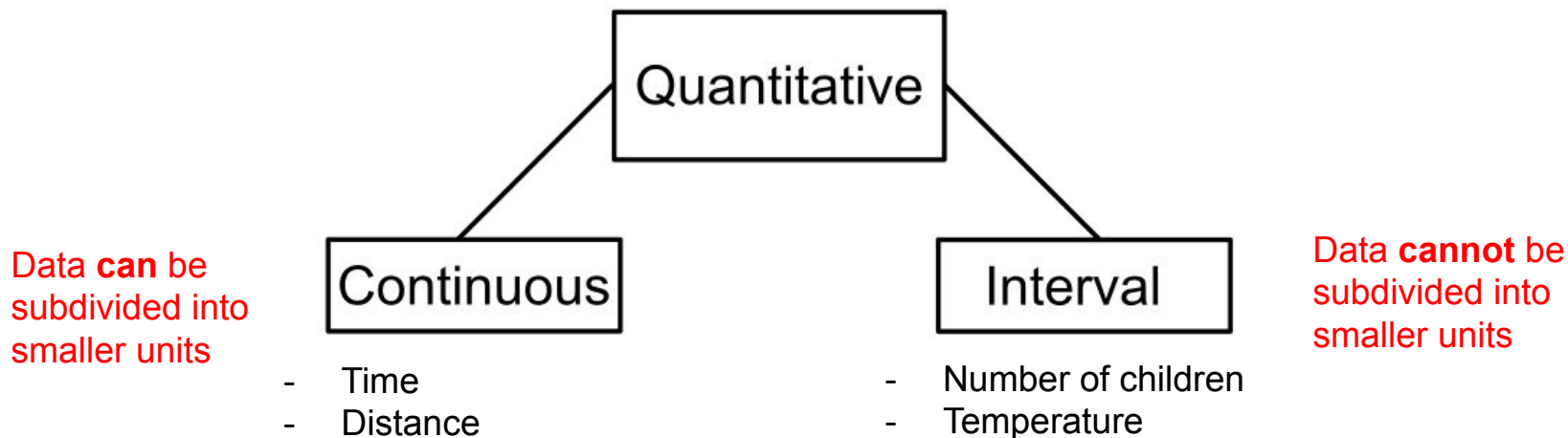
Do variables take numerical values for which arithmetic operations such as adding and averaging make sense? Then we have **quantitative** data.



Data **cannot** be subdivided into smaller units.

Understand the data: types of variables

Do variables take numerical values for which arithmetic operations such as adding and averaging make sense? Then we have **quantitative** data.



Let's practice all together!

In a company, what type of data is the following?

- Department?
- Number of years with the company?
- Salary?
- Education (coded as high school, some college, or college degree)?

Understand the data: describe them!

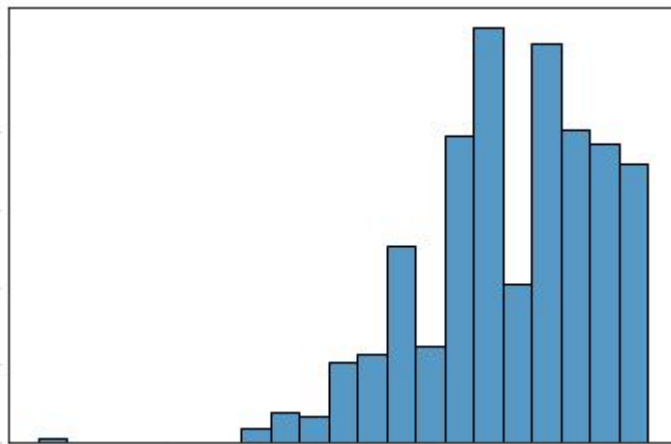
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e.g. **Histograms** to visualize one **continuous** variable.

In ggplot: `geom_histogram()`

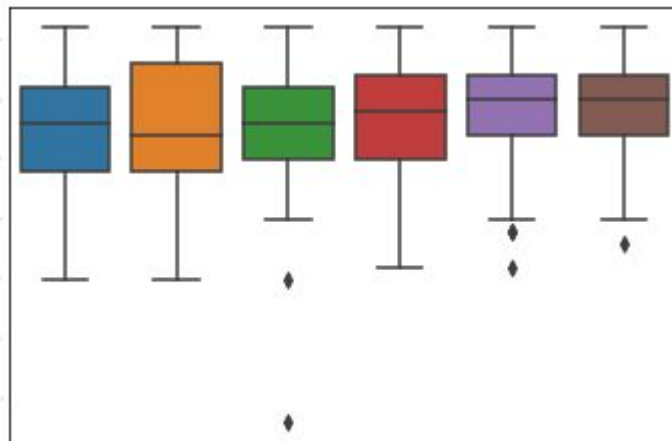


Understand the data: describe them!

- We usually start with visualizing our data to understand them and get a flavour about our research question.

e.g. **Boxplots** to visualize one **continuous** variable versus a **categorical** variable.

In ggplot: `geom_boxplot()`

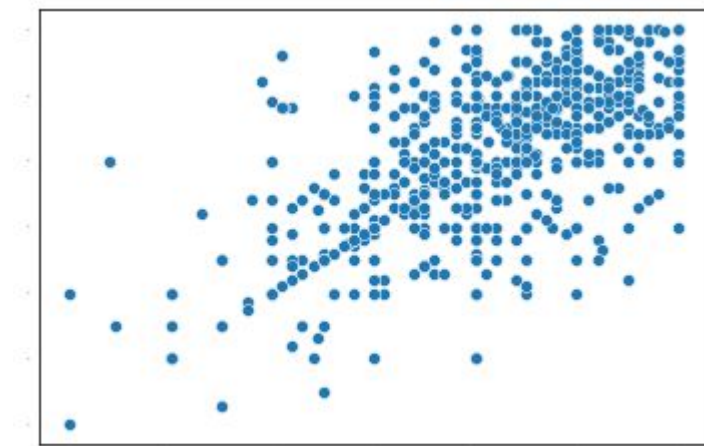


Understand the data: describe them!

- We usually start with visualizing our data to understand them and get a flavour about our research question.

e.g. **Scatterplots** to
visualize one
continuous variable
against another
continuous variable

In ggplot: `geom_point()`



Understand the data: describe them!

- We usually start with visualizing our data to understand them and get a flavour about our research question.
- However, we need to make sense of what we see. This is done by summarizing the data in a compact, easily-understood way. This is usually called **descriptive statistics** (or exploratory data analysis, EDA).

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In contrast, **inferential statistics** is a follow-up step, aiming at drawing conclusions about what we do not know.

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- However, we need to make sense of what we see. This is done by summarizing the data in a compact, easily-understood way. This is usually called **descriptive statistics** (or exploratory data analysis, EDA).
- When summarizing the data, we generally talk about **tendency, variability and shape**.

Descriptive vs Inferential statistics: an analogy

- When you taste a spoonful of soup and decide the spoonful you tasted isn't salty enough, that's exploratory analysis.
- If you **generalize** and **conclude** that your entire soup needs salt, that's an inference.
- For your inference to be valid, the spoonful you tasted (the sample) needs to be representative of the entire pot (the population) If the soup is not well stirred, it doesn't matter how large a spoon you have, it will still not taste right. If the soup is well stirred, a small spoon will suffice to test the soup.

Central Tendency

- In most situations, the first thing that you'll want to calculate is a measure of central tendency. That is, you'd like to know something about the “average” or the “middle” of where your data lies.
- The most commonly used measures are the **mean**, **median** and **mode**; occasionally people will also report a trimmed mean.

A measure of central tendency: The mean

- For a sample X consisting of N observed values (X_1, X_2, \dots, X_N) , the **sample mean**, denoted as $\langle X \rangle$, is calculated as

$$\langle X \rangle = \frac{1}{N}(X_1 + X_2 + \dots + X_N) = \frac{1}{N} \sum_{i=1}^N X_i$$

- The **population mean** is also computed the same way but is denoted as μ . It is often not possible to calculate μ since population data are rarely available.
- The sample mean is a **sample statistic**, i.e. a quantity computed from the sample. It serves as an estimate of the population mean, which, yet not perfect, is usually a pretty good estimate (unless the sample is not a good representative of the population).

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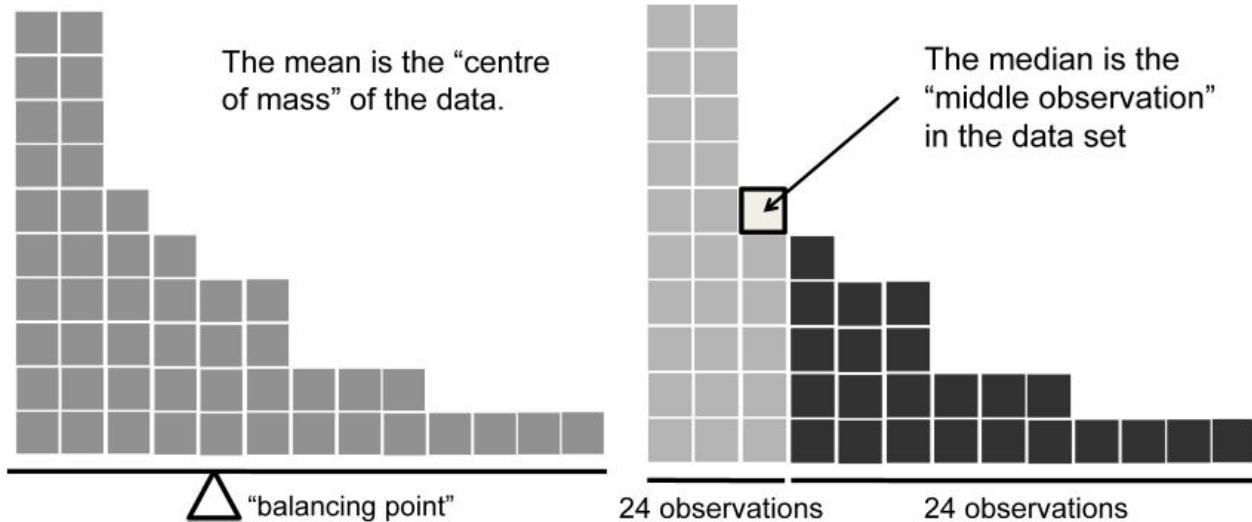
- The **population mean** is also computed the same way. It is often not possible to calculate the population mean μ . It is often not possible to calculate the population mean μ .
- The **sample mean** is a good estimate of the population mean, which, yet not a perfect estimate (unless the sample is not a good representation of the population).

We'll come back in week 4 to a more thorough explanation about the difference between sample and population!

A measure of central tendency: The median

- Another widely used measure of tendency is the median.
- The median of a sample of observations is just the **middle value**. For example:
 - In 1, 2, **3**, 4, 5, the median is 3
 - In 1, 2, **3, 4**, 4, 5, the median is $(3+4) / 2 = 3.5$

Mean vs Median



Other central tendency measures

- Trimmed mean:
 - It is the mean calculated by dropping extreme values (on both ends).
 - It is useful when **outliers** are presented in the data.

- Mode:
 - It is the most frequent value.
 - It is particularly important with **categorical** data.

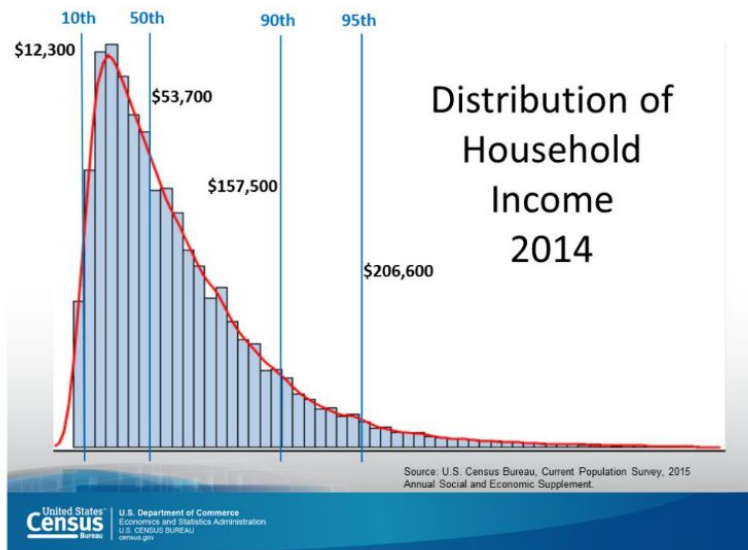
Which one to use?

It depends a little on what type of data you've got and what you're trying to achieve. As a rough guide:

- **Nominal categorical** data: The mode.
- **Ordinal categorical** data: The median.
- **Quantitative (discrete and continuous)** data: The mean and/or the median.

Which one to use? US Household income example

To estimate the typical household income in the US, would you be more interested in the mean or median income?

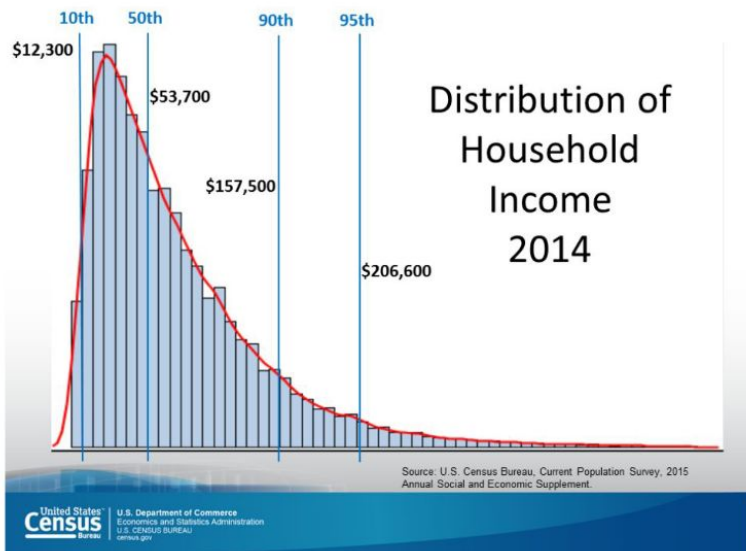


Median: \$53,700

Mean: \$75,738

Which one to use? US Household income example

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Median: \$53,700

Mean: \$75,738

Sometimes, the mean is not the best tendency measure (e.g. when distributions are **asymmetrical**, i.e. **always** visualize your data to decide!!!!)

Variability of the data

When we talk about variability in the data, we mean to address the following questions:

- That is, how “**spread out**” are the data?
- How “**far**” away from the **mean or median** do the observed values tend to be?

Variability of the data

It **complements** the information provided by the central tendency:

	# people at Sally's book club	# people at Maria's book club
Week 1	8	1
Week 2	10	18
Week 3	11	10
Week 4	9	2
Week 5	12	19
Mean	$= \frac{8 + 10 + 11 + 9 + 12}{5} = 10$	$= \frac{1 + 18 + 10 + 2 + 19}{5} = 10$
<i>Standard Deviation</i>	$= \sqrt{\frac{(8-10)^2 + (10-10)^2 + (11-10)^2 + (9-10)^2 + (12-10)^2}{4}} \approx 1.6$	$= \sqrt{\frac{(1-10)^2 + (18-10)^2 + (10-10)^2 + (2-10)^2 + (19-10)^2}{4}} \approx 8.5$

A measure of variability: Variance

The **variance** is sometimes referred to as the “mean square deviation”; that is, the average of all points with respect to the mean sample in square units:

$$s^2 = \frac{1}{N} \sum_{i=1}^N (X_i - \langle X \rangle)^2$$

A measure of variability: Standard Deviation

- Sometimes also called the “root mean squared deviation” or RMSD, it is the **square root of the variance**:

$$s = \sqrt{\frac{1}{N} \sum_{i=1}^N (X_i - \langle X \rangle)^2}$$

- As a result, in contrast to the variance, the standard deviation is in **the same units** as the data themselves.
- (One) **interpretation**: 68% of the data fall within 1 standard deviation of the mean, 95% within 2 standard deviation of the mean, and 99.7% within 3 standard deviations of the mean. (Only for Gaussian distributions!)

Other variability measures

- Average absolute deviation (AAD):

$$AAD = \frac{1}{N} \sum_{i=1}^N |X_i - \langle X \rangle|$$








- Median absolute deviation (MAD):

$$MAD = \text{median}(|X_1 - \langle X \rangle|, |X_2 - \langle X \rangle|, \dots, |X_N - \langle X \rangle|)$$

- Range: Biggest value minus the smallest value.
- Interquartile range: Difference between the **25th quantile** and the **75th quantile**. (What is a quantile? We'll see this later)

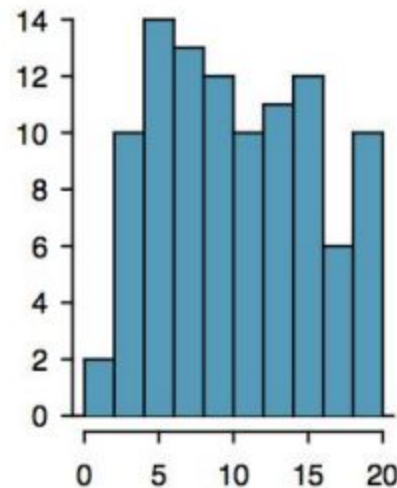
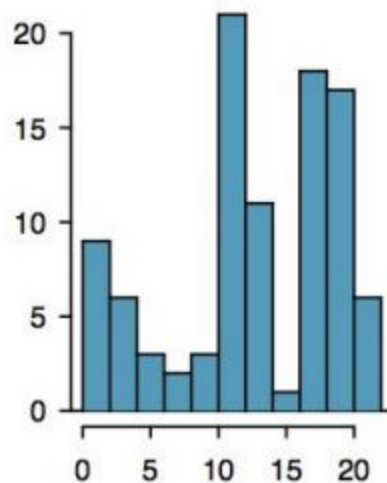
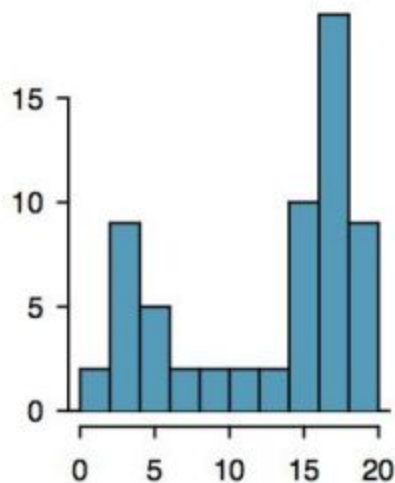
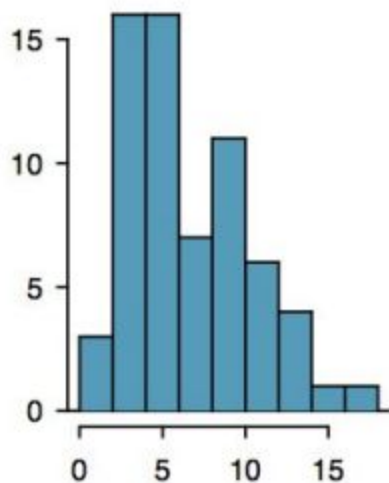
Which one to use?

(Personal) ranking of use:

- Standard Deviation    
- IQR   
- AAD, MAD  
- Variance 
- Range 

Shape of the data: Modality

Do our data exhibit a single prominent peak (unimodal), several prominent peaks (bimodal/multimodal), or no apparent peaks (uniform)? ✓✓✓✓



Shape of the data: Skewness

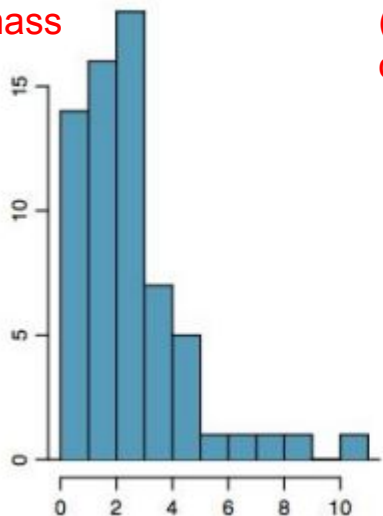
Skewness is a measure of **asymmetry** of the data

$$skewness(X) = \frac{1}{N\hat{\sigma}^3} \sum_{i=1}^N (X - \langle X_i \rangle)^3$$



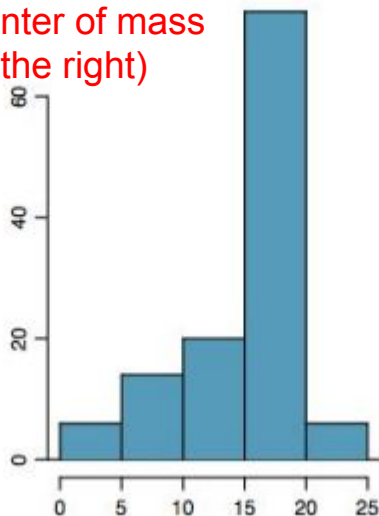
Positive

(center of mass
on the left)

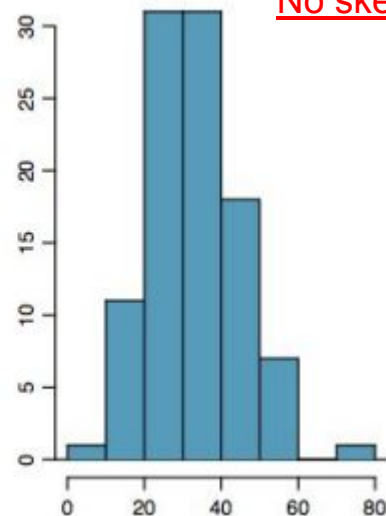


Negative

(center of mass
on the right)



No skew

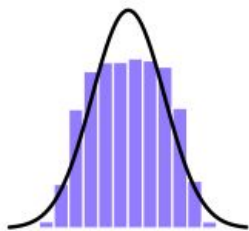


Shape of the data: Kurtosis

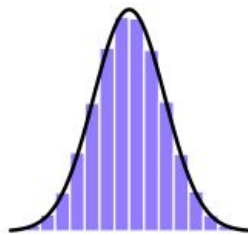
Kurtosis is a measure of the
“**pointiness**” of a the data ✓✓

$$kurtosis(X) = \frac{1}{N\hat{\sigma}^4} \sum_{i=1}^N (X - \langle X_i \rangle)^4 - 3$$

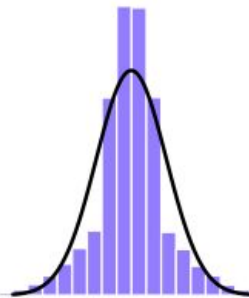
Platykurtic
("too flat")



Mesokurtic

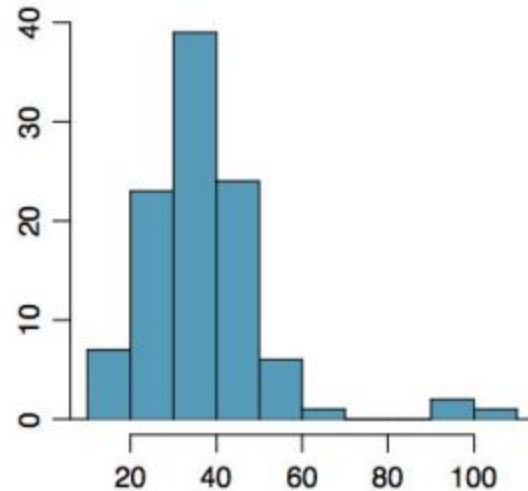
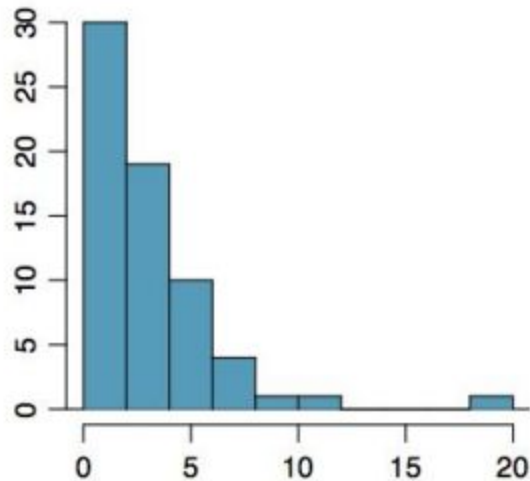


Leptokurtic
("too pointy")



Shape of the data: Outliers

Do we suspect to have unusual observations or potential outliers in our data?



Recap

- **Always start by understanding your data!**
- One way of doing this is by **visualizing** (e.g. using R ggplot) and **summarizing** (e.g. using R dplyr) them.
- Descriptive statistics compress data to make it easier to understand and communicate (crucial in research!!!!)
- We generally want to talk about measures of tendency, variability and shape, but be **aware** of your data properties before selecting a specific measure!