Uncertainty quantification for dose-response models

Consider a standard toxicology experiment that produces a response, Y\_i, corresponding to a dose, X\_i, for i=1,…,n independent experiments.  Assume a parametric dose response model

Y\_i = f(X\_i) + e\_i

where f is a non-linear function and the errors e\_i ~ Normal(0,sigma^2).  For example, you many assume the logistic function

f(X) = a + b/[1+exp(-c-dX)]

that is known up to parameters theta = (a,b,c,d).  The objective of the analysis is to estimate theta and thus the dose-response curve f(X).

In many cases, toxicological studies have small sample sizes, say n=5, 10 or 20.   Valid uncertainty quantification (i.e., standard errors, interval estimates, etc) for theta and/or f(X) is challenging for small n.  Your exam is to discuss common methods for generating standard errors for non-linear regression such as the model stated above, and then discuss their validity for small sample sizes.  In addition to this review, conduct a simulation study to evaluate the leading methods identified in your review.  In light of your review and simulation study, make general recommendations for future users (the conclusion that nothing works might be fine).  The full report should be no longer than 10 pages (single spaced, one-inch margins, 11pt font), including figures and tables.  Please submit commented code in a separate file.

Good luck!