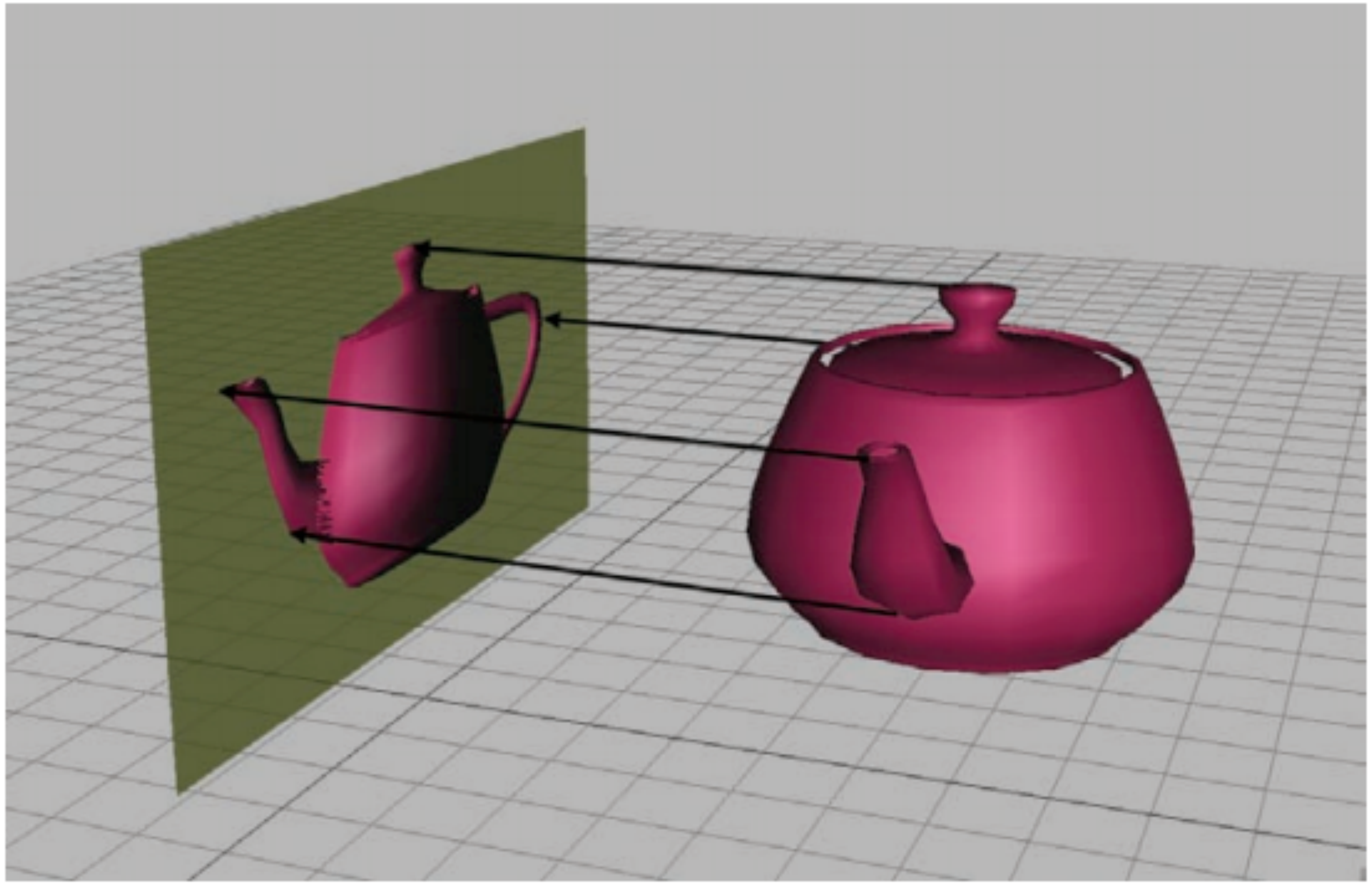
Cameras and Projection

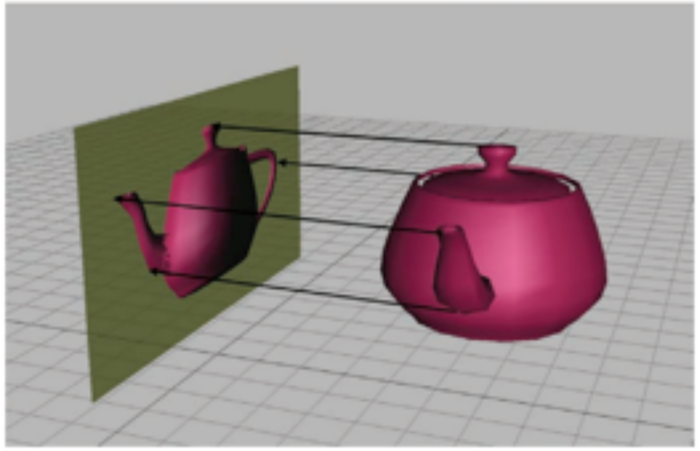
CS 355: Interactive Graphics and Image Processing

Orthographic Projection



Orthographic projection involves no perspective

Orthographic Projection

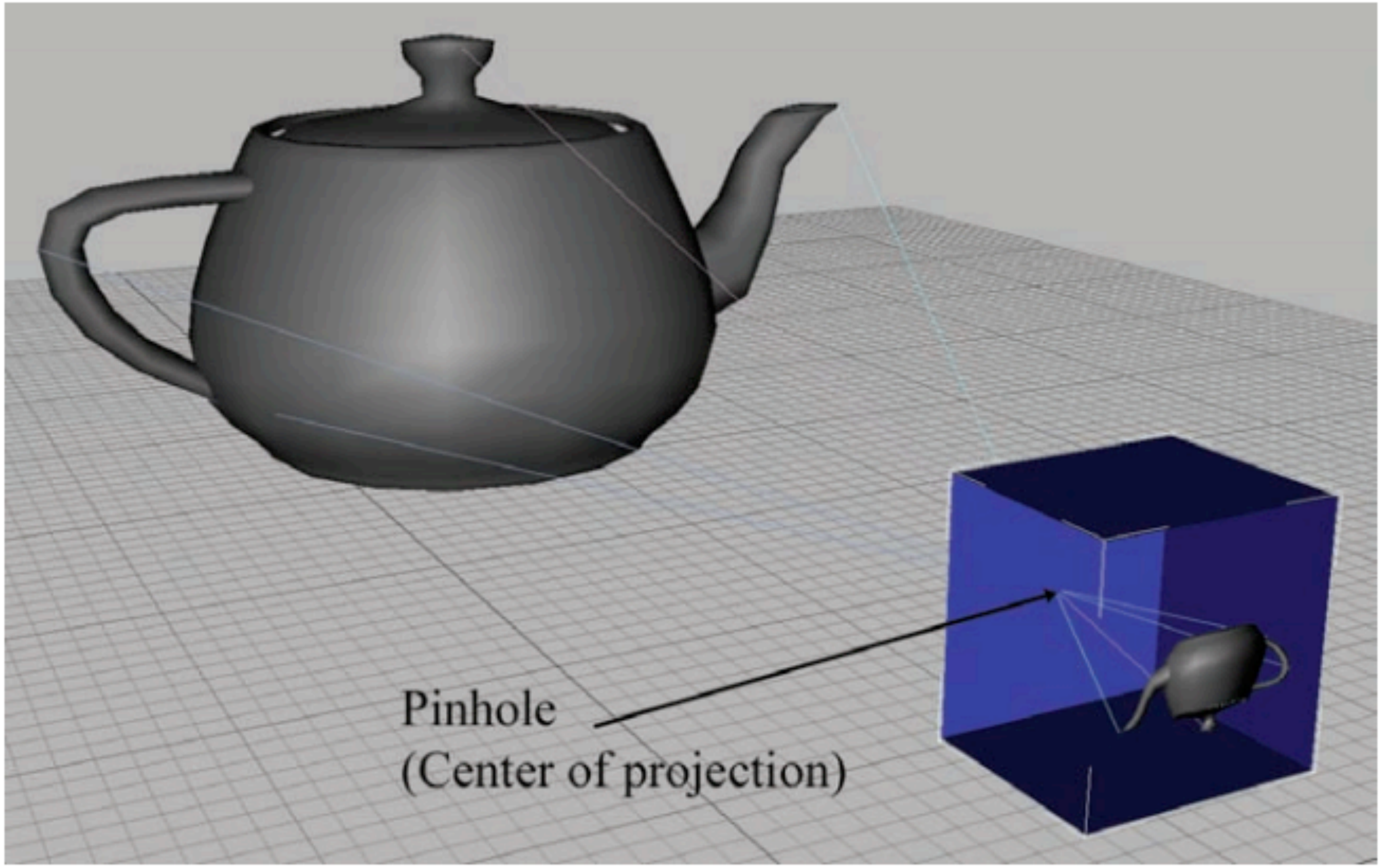


3D point in homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

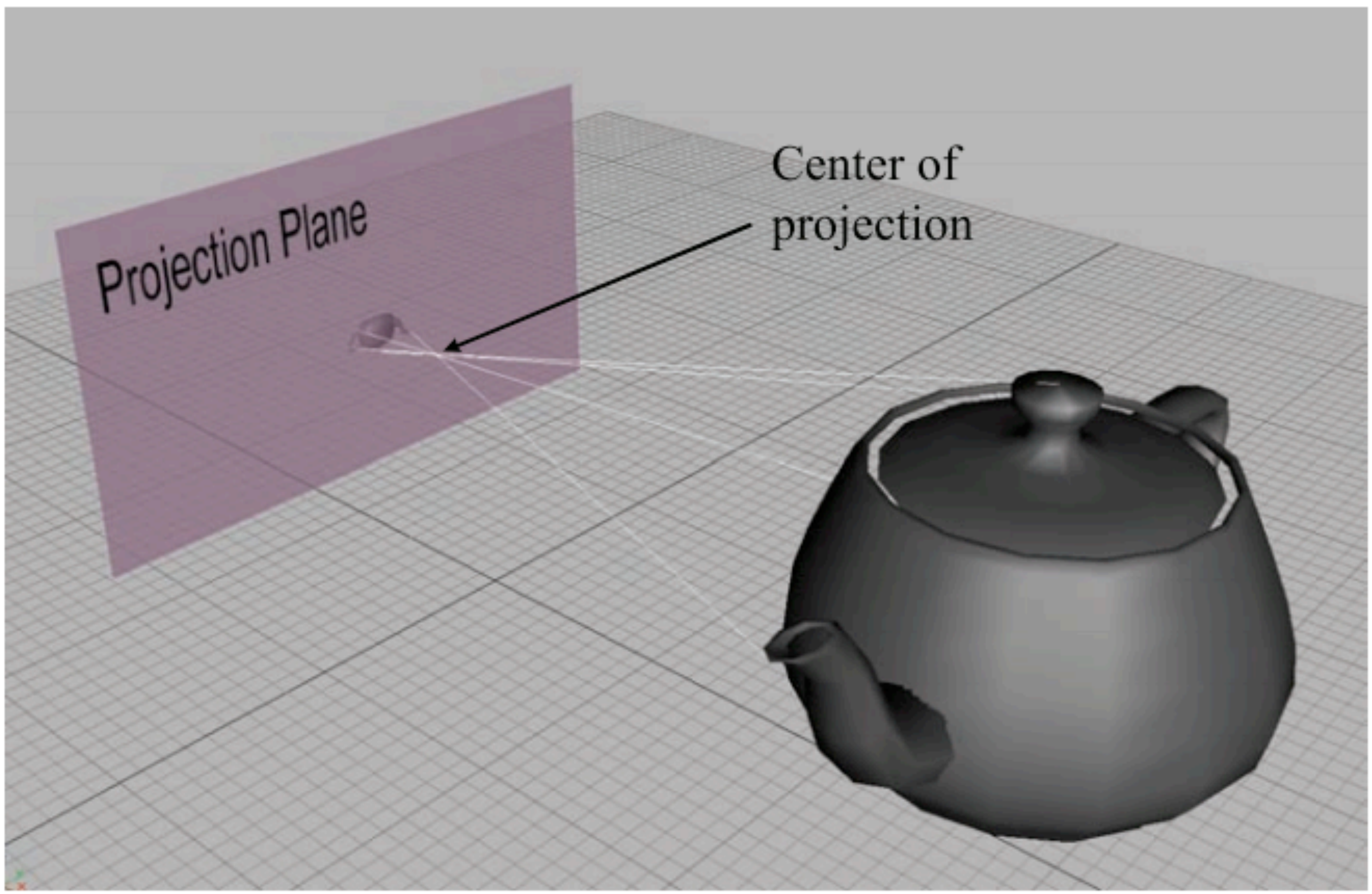
Orthographic projection involves no perspective

Perspective Projection

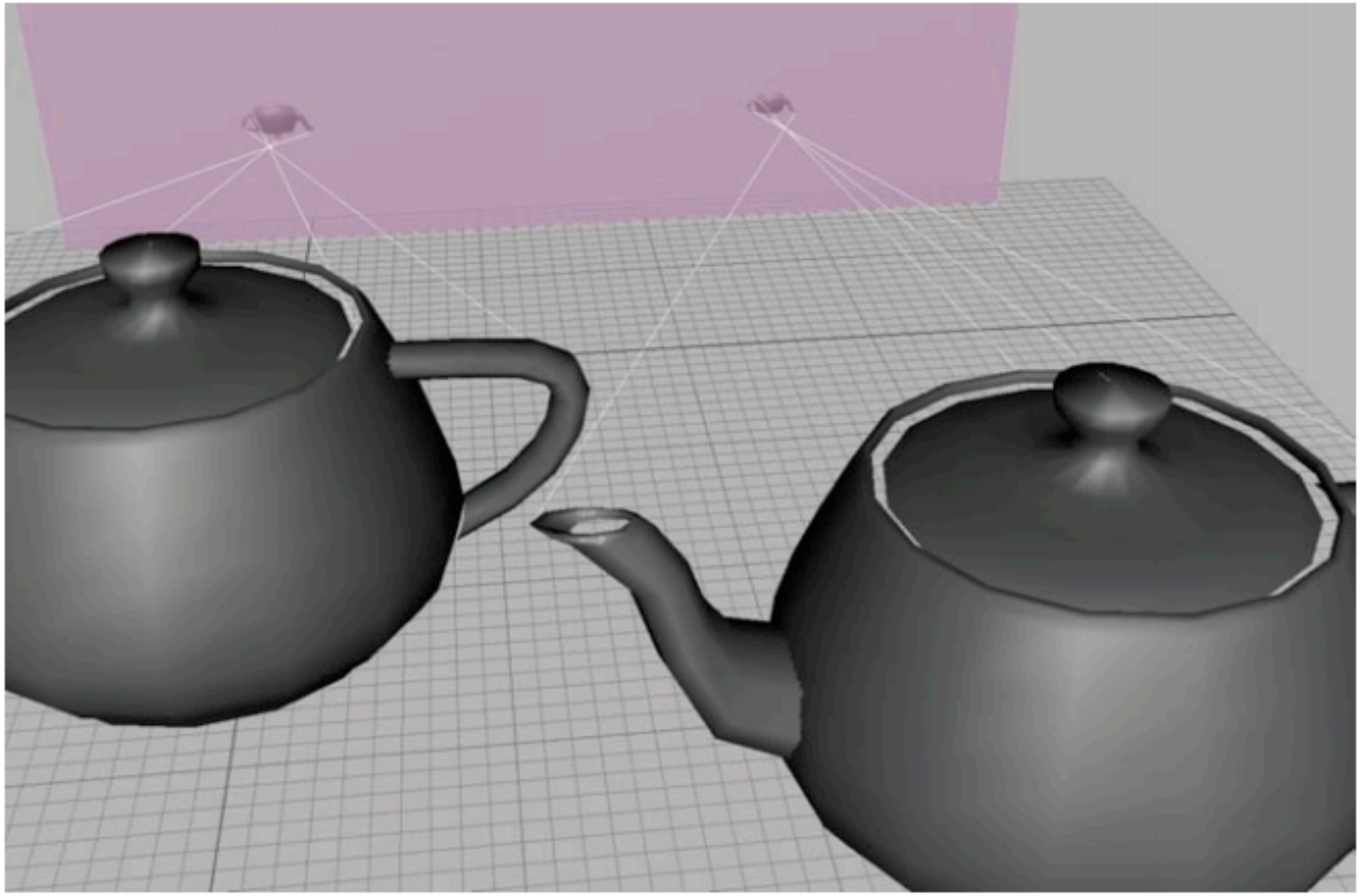


Many graphics systems assume a
simple *pinhole camera model*

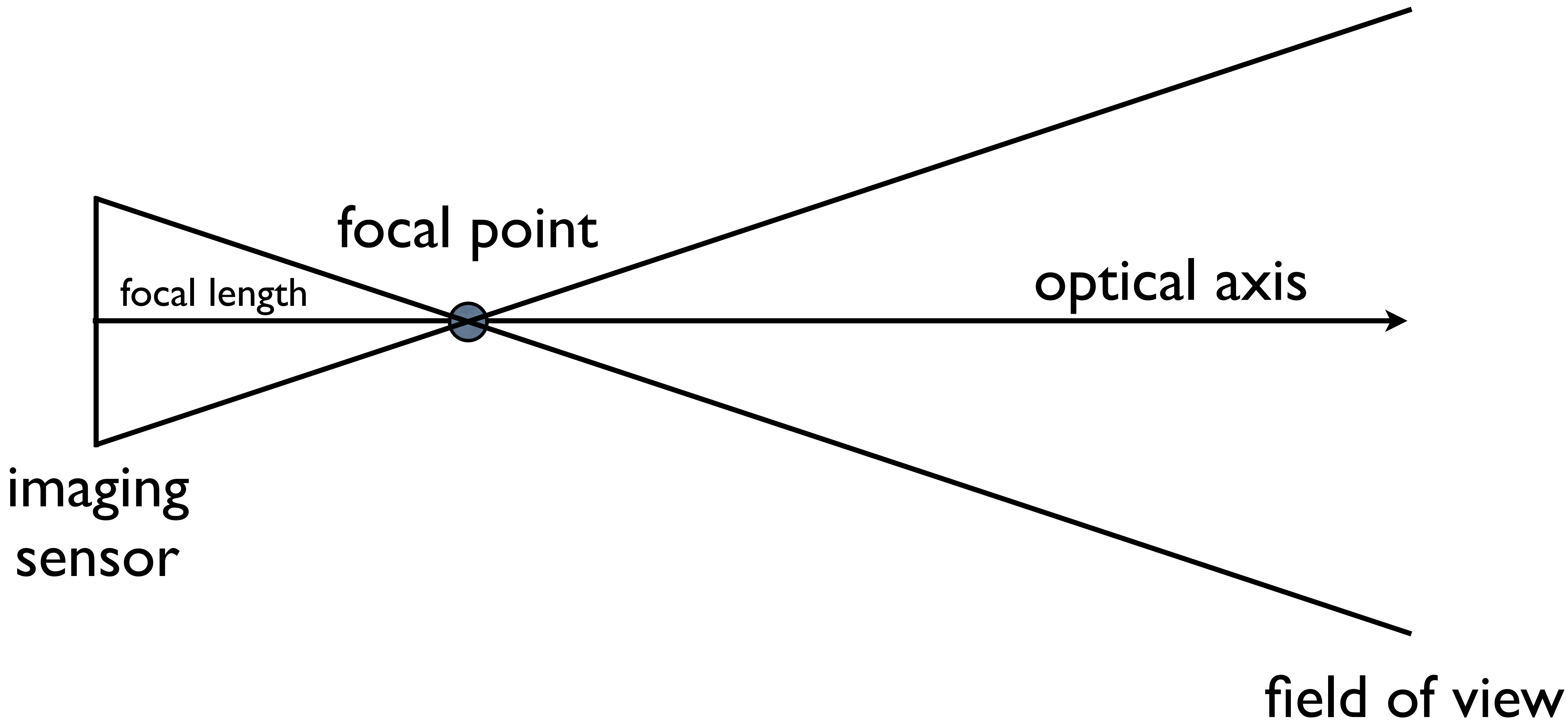
Perspective Projection



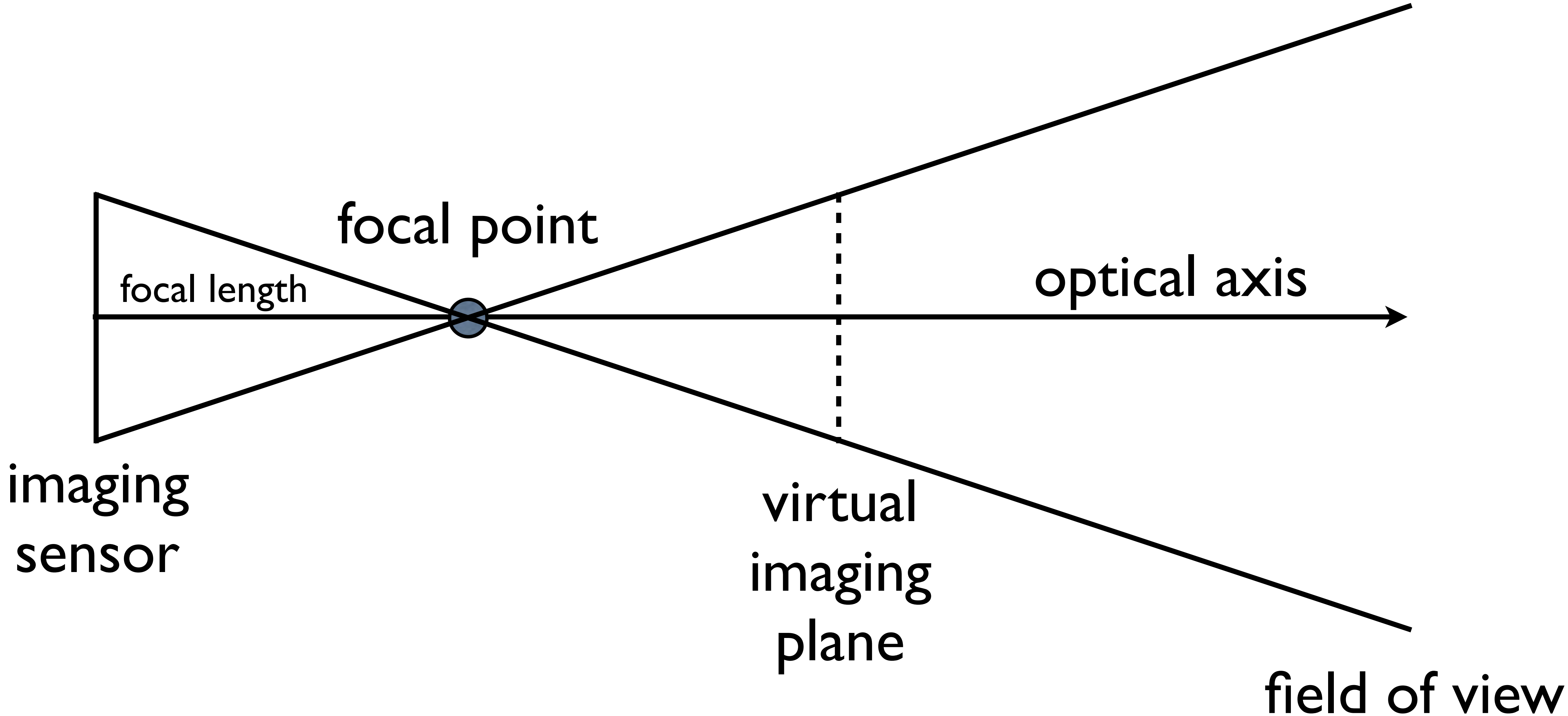
Perspective Projection



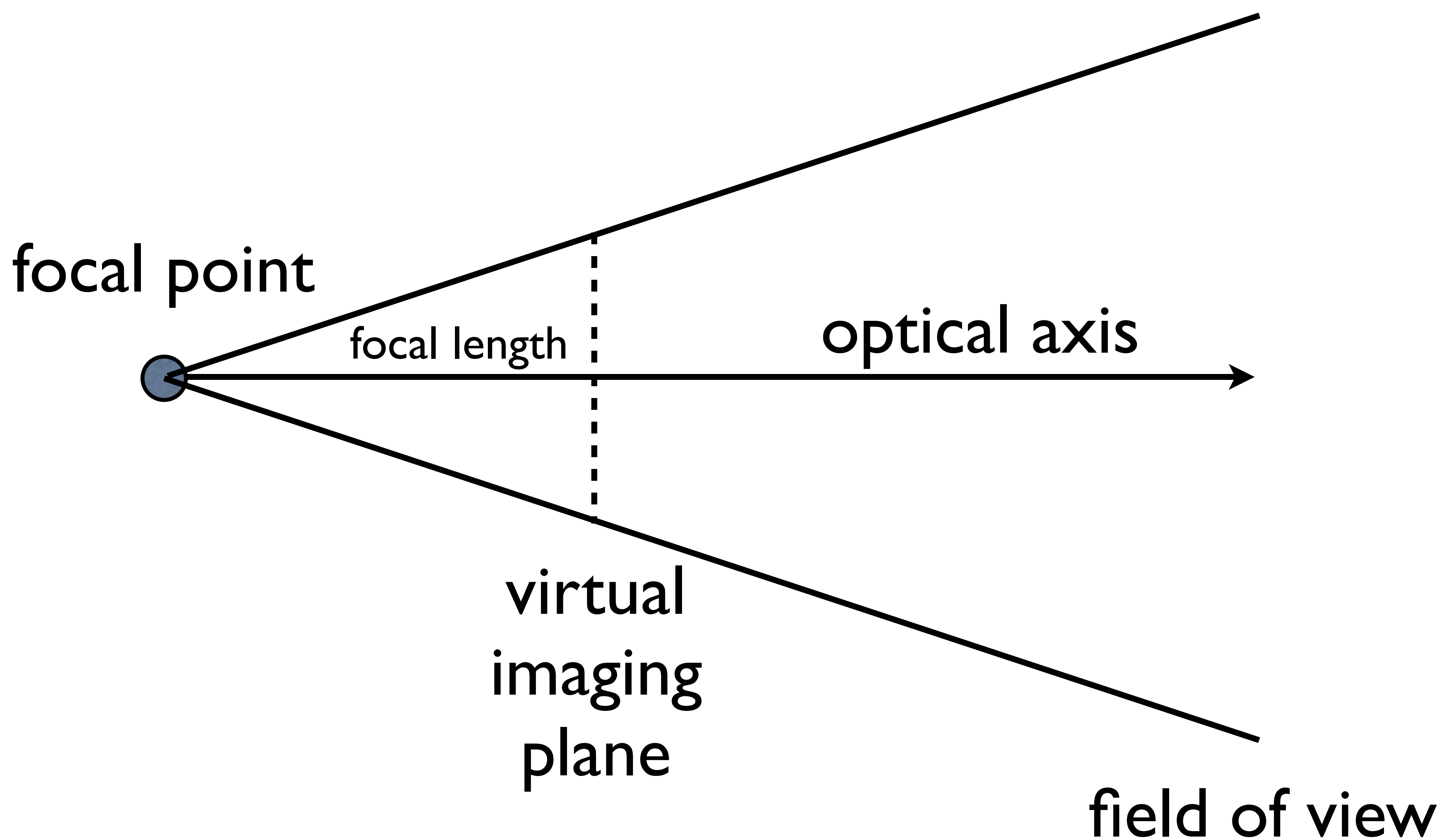
Pinhole Cameras



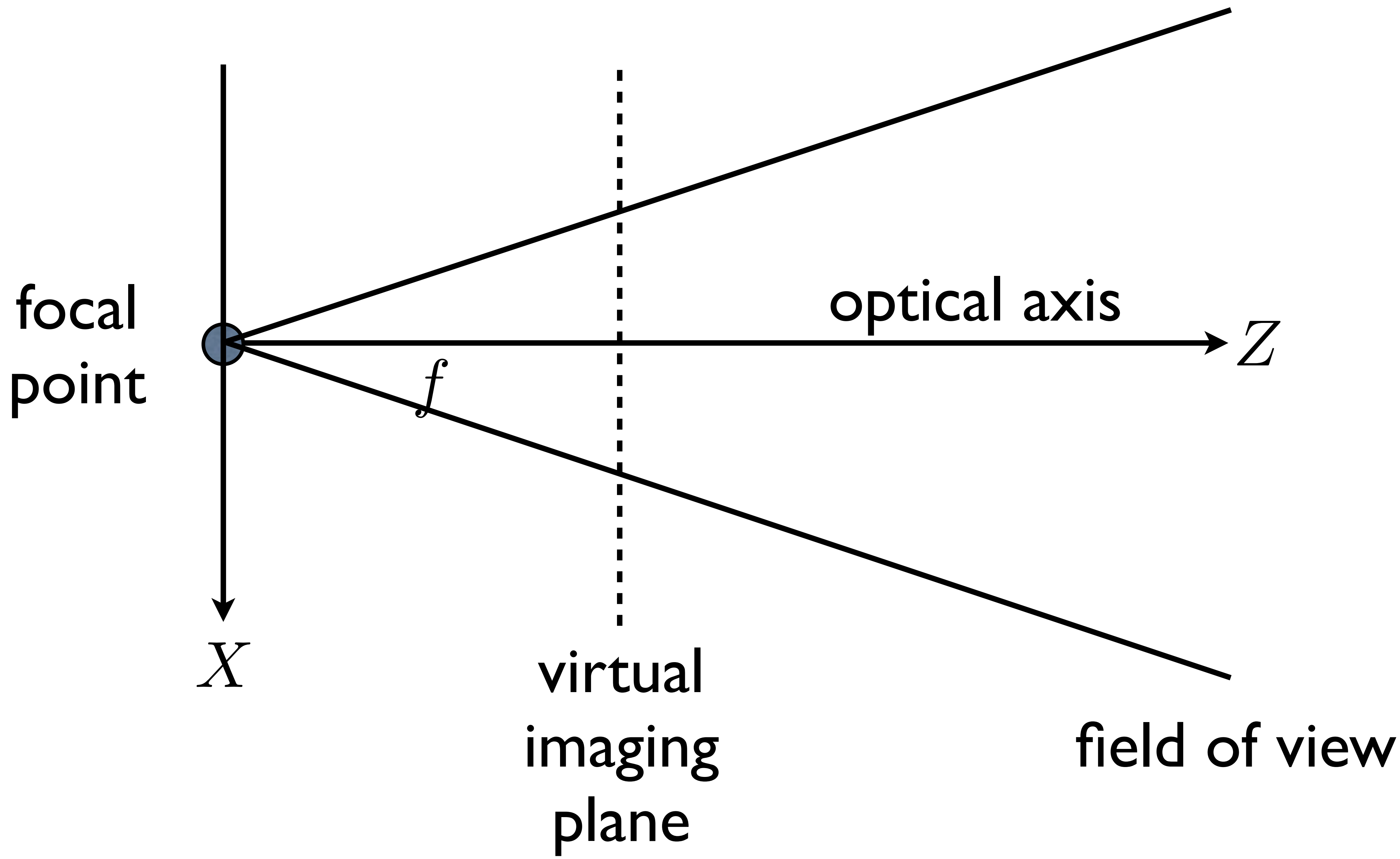
Geometric Model



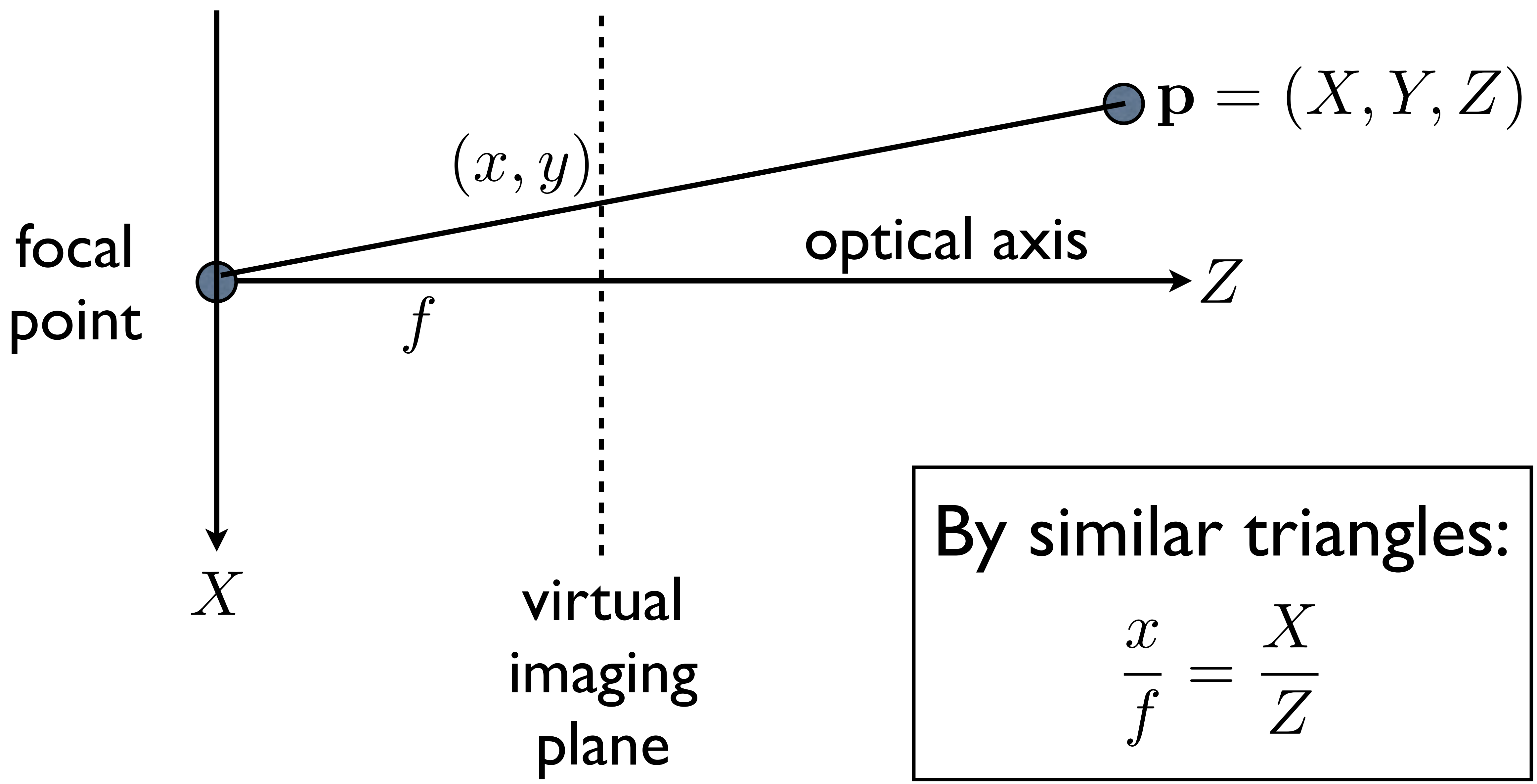
Geometric Model



Camera Coordinates



Projection



By similar triangles:

$$\frac{x}{f} = \frac{X}{Z}$$

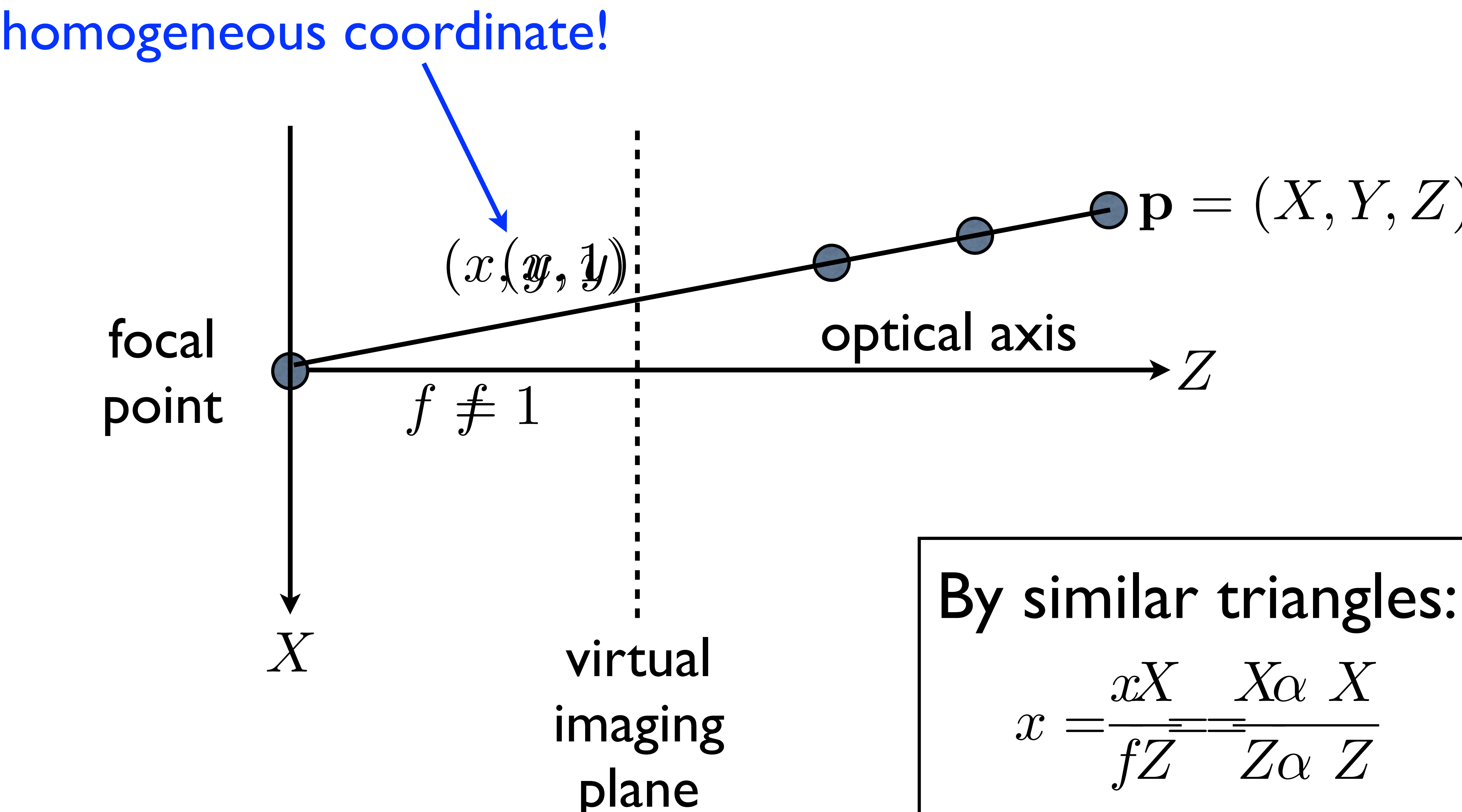
Projection

$$\frac{x}{f} = \frac{X}{Z} \qquad \frac{y}{f} = \frac{Y}{Z}$$

$$(x, y) = \left(\frac{fX}{Z}, \frac{fY}{Z} \right)$$

Note: this is the projected coordinate in real-world units. To get actual pixel location, have to scale by pixel density and apply offset to image origin (more on this later...)

Projection



By similar triangles:

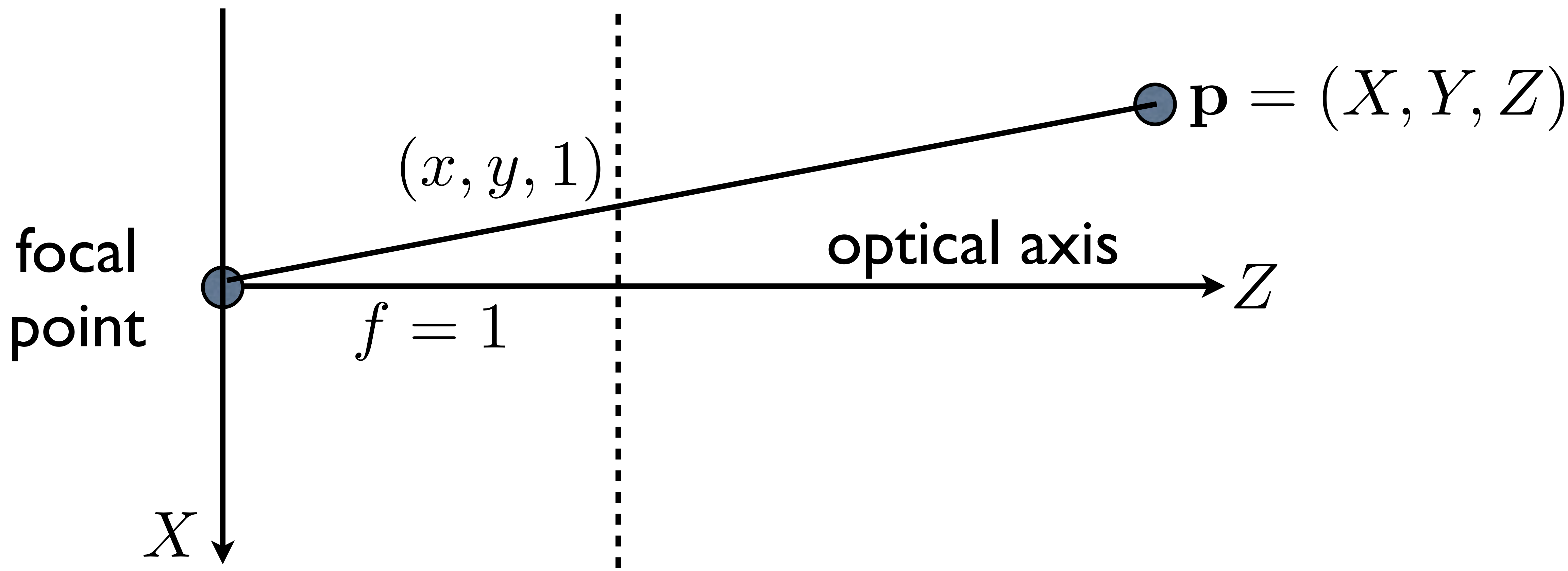
$$x = \frac{xX}{fZ} = \frac{X\alpha}{Z\alpha} = \frac{X}{Z}$$

Homogenous Coordinates

- Homogeneous coordinates are used to represent *all 3D points along the ray* that falls on the *same 2D projection*

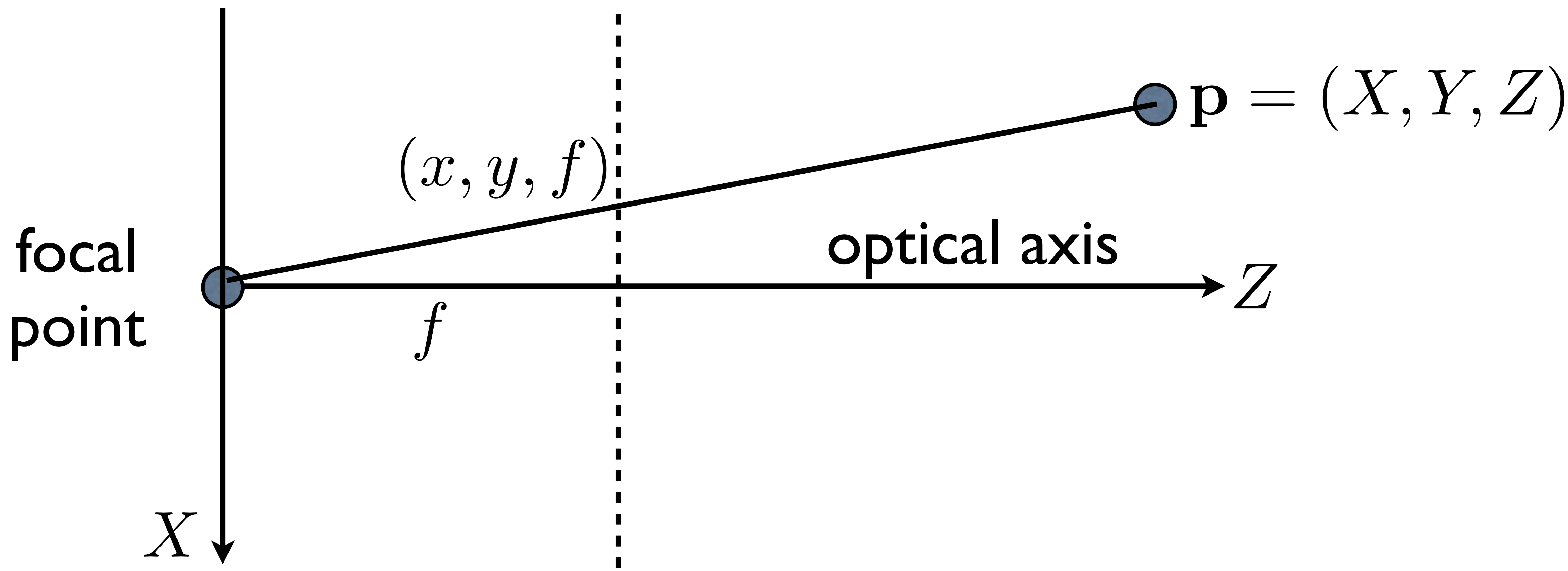
$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \sim \begin{bmatrix} \alpha\ x \\ \alpha\ y \\ \alpha \end{bmatrix}$$

Perspective Projection



$$\begin{bmatrix} x \\ y \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} X/Z \\ Y/Z \\ 1 \\ 1 \end{bmatrix} \sim \begin{bmatrix} X \\ Y \\ Z \\ Z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Perspective Projection



$$\begin{bmatrix} \boxed{\begin{matrix} x \\ y \\ f \end{matrix}} \\ 1 \end{bmatrix} = \begin{bmatrix} fX/Z \\ fY/Z \\ f \\ 1 \end{bmatrix} \sim \begin{bmatrix} X \\ Y \\ Z \\ Z/f \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & \boxed{1/f} & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Alternative Form

One way (some implementation advantages):

$$\begin{bmatrix} x \\ y \\ f \\ 1 \end{bmatrix} = \begin{bmatrix} fX/Z \\ fY/Z \\ f \\ 1 \end{bmatrix} \sim \begin{bmatrix} X \\ Y \\ Z \\ Z/f \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Another way (some conceptual advantages):

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} fX/Z \\ fY/Z \\ 1 \end{bmatrix} \sim \begin{bmatrix} X \\ Y \\ Z/f \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Coming up...

- World to camera transformations
- Specifying camera pose