Multiple Coordinate Spaces (Revisited)

CS 355: Interactive Graphics and Image Processing

Forward (Drawing)

- Object Coordinates

Lab #2

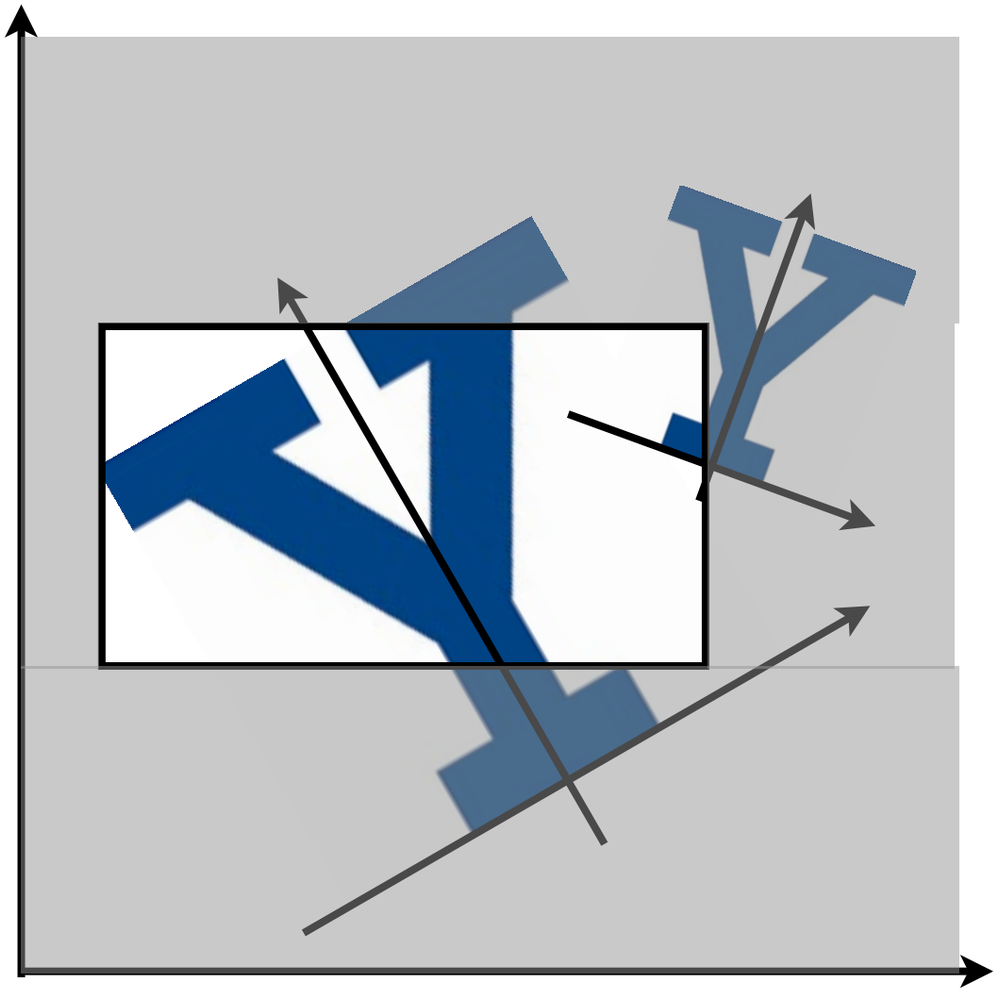
$\mathbf{O}_i$

- World Coordinates

Lab #3

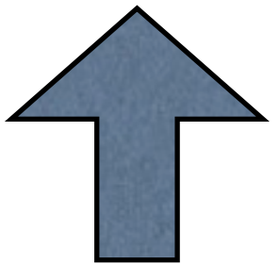
$\mathbf{V}$

- Viewing Coordinates



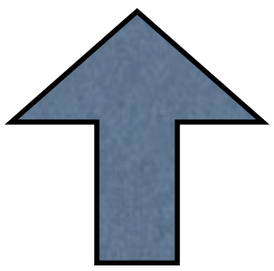
Backward (Selecting)

- Object Coordinates



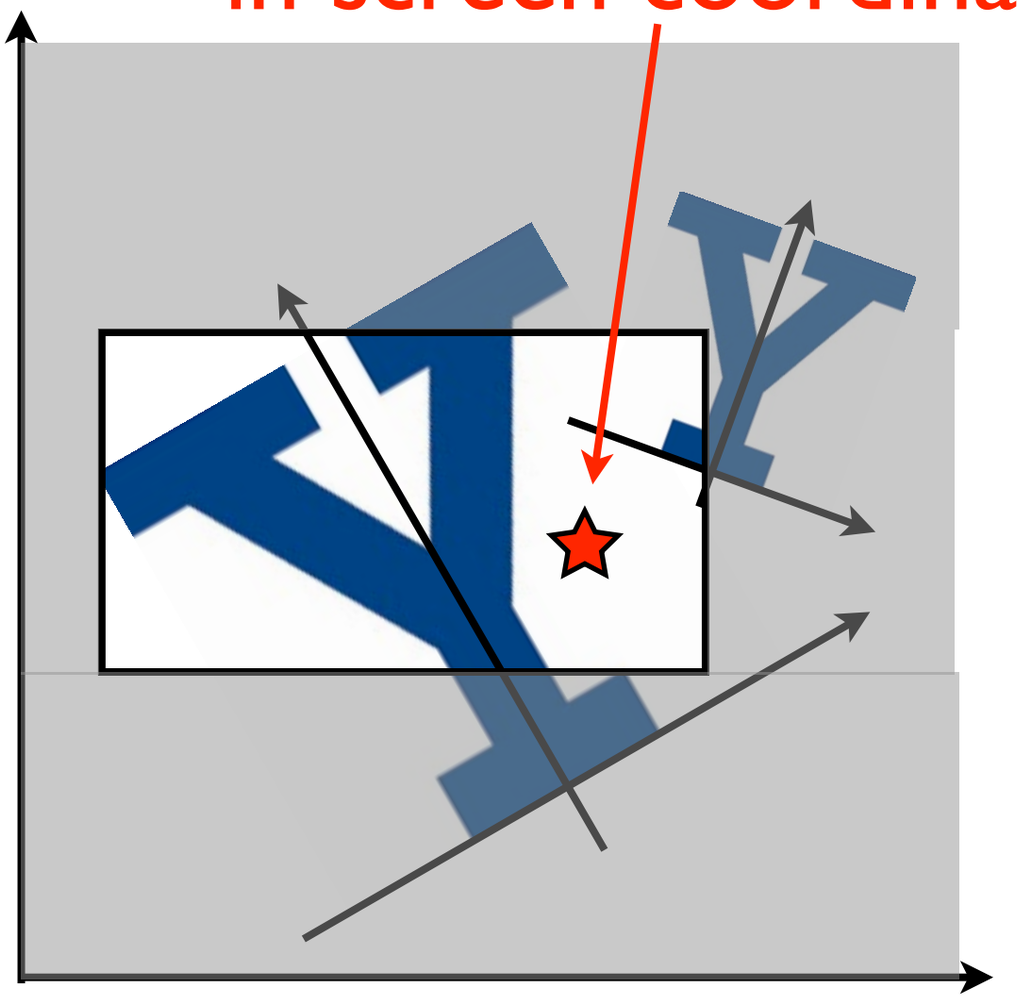
$$\mathbf{O}_i^{-1}$$

- World Coordinates



$$\mathbf{V}^{-1}$$

- Viewing Coordinates



Object to World

- To transform a point from <u>object space</u> to the <u>world space</u>:
  - **Rotate** to orientation
  - **Translate** to position

order

$$\mathbf{O}_i = \mathbf{T}(\mathbf{c}_i)\ \mathbf{R}(\theta_i)$$

$$\mathbf{R}(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T}(\mathbf{c}) = \begin{bmatrix} 1 & 0 & c_x \\ 0 & 1 & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

World to Object

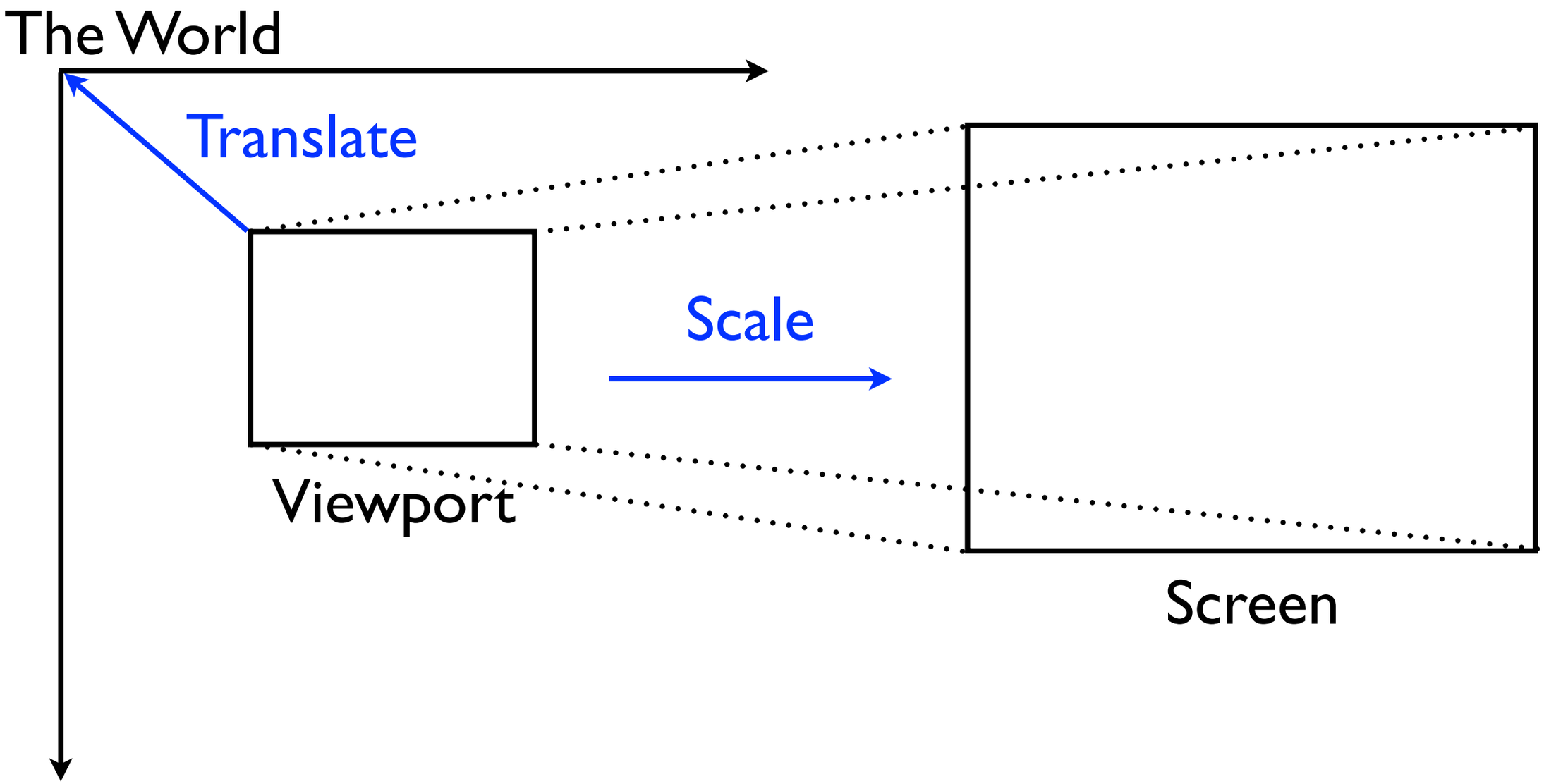
- To transform a point from world space to object space:
  - Translate from position to origin
  - Rotate back from orientation

$$\mathbf{O}_i^{-1} = \mathbf{R}(-\theta_i)\ \mathbf{T}(-\mathbf{c}_i)$$

$$\mathbf{R}(-\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T}(-\mathbf{c}) = \begin{bmatrix} 1 & 0 & -c_x \\ 0 & 1 & -c_y \\ 0 & 0 & 1 \end{bmatrix}$$

Viewports



World to View

- To transform a point from world space to viewing space:

$$\mathbf{V} = \mathbf{S}(f)\ \mathbf{T}(-\mathbf{p})$$

order ←

- **Translate** from origin of viewport to origin of the world

$$\mathbf{T}(-\mathbf{p}) = \begin{bmatrix} 1 & 0 & -p_x \\ 0 & 1 & -p_y \\ 0 & 0 & 1 \end{bmatrix}$$

- **Scale** by the zoom factor

$$\mathbf{S}(f) = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

View to World

- To transform a point from world space to viewing space:

$$\mathbf{V}^{-1} = \mathbf{T}(\mathbf{p})\ \mathbf{S}(1/f)$$

- **Translate** from origin of viewport to origin of the world

$$\mathbf{T}(\mathbf{p}) = \begin{bmatrix} 1 & 0 & p_x \\ 0 & 1 & p_y \\ 0 & 0 & 1 \end{bmatrix}$$

- **Scale** by the zoom factor

$$\mathbf{S}(1/f) = \begin{bmatrix} 1/f & 0 & 0 \\ 0 & 1/f & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Drawing

- Transform from object space to world space

order

$$\mathbf{M} = \mathbf{V}\ \mathbf{O}_i$$

- Transform from world space to view space

$$\mathbf{p}_{\text{view}} = \mathbf{M}\ \mathbf{p}_{\text{object}}$$

Selecting

- Transform from view space to world space

order

$$\mathbf{M}^{-1} = \mathbf{O}_i^{-1}\ \mathbf{V}^{-1}$$

- Transform from world space to object space

$$\mathbf{p}_{\text{object}} = \mathbf{M}^{-1}\ \mathbf{p}_{\text{view}}$$

Coming up...

- 3D!