



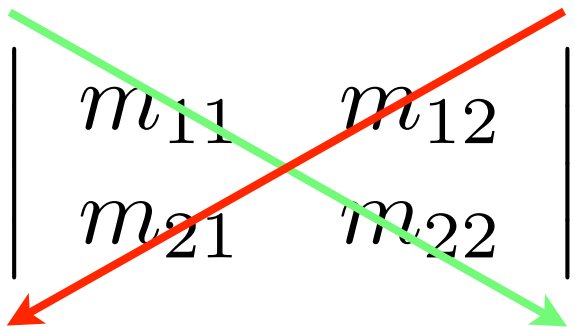
# More Matrix Stuff

CS 355: Interactive Graphics and Image Processing

# Determinant

$$|\mathbf{M}| = \begin{vmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{vmatrix} = m_{11}m_{22} - m_{12}m_{21}$$

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A diagram illustrating the calculation of a 2x2 determinant. A green arrow points from the top-left element  $m_{11}$  to the bottom-right element  $m_{22}$ . A red arrow points from the top-right element  $m_{12}$  to the bottom-left element  $m_{21}$ . The resulting expression  $m_{11}m_{22} - m_{12}m_{21}$  has the first two terms in green and the last two terms in red.

# Determinant

$$|\mathbf{M}| = \begin{vmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{vmatrix}$$

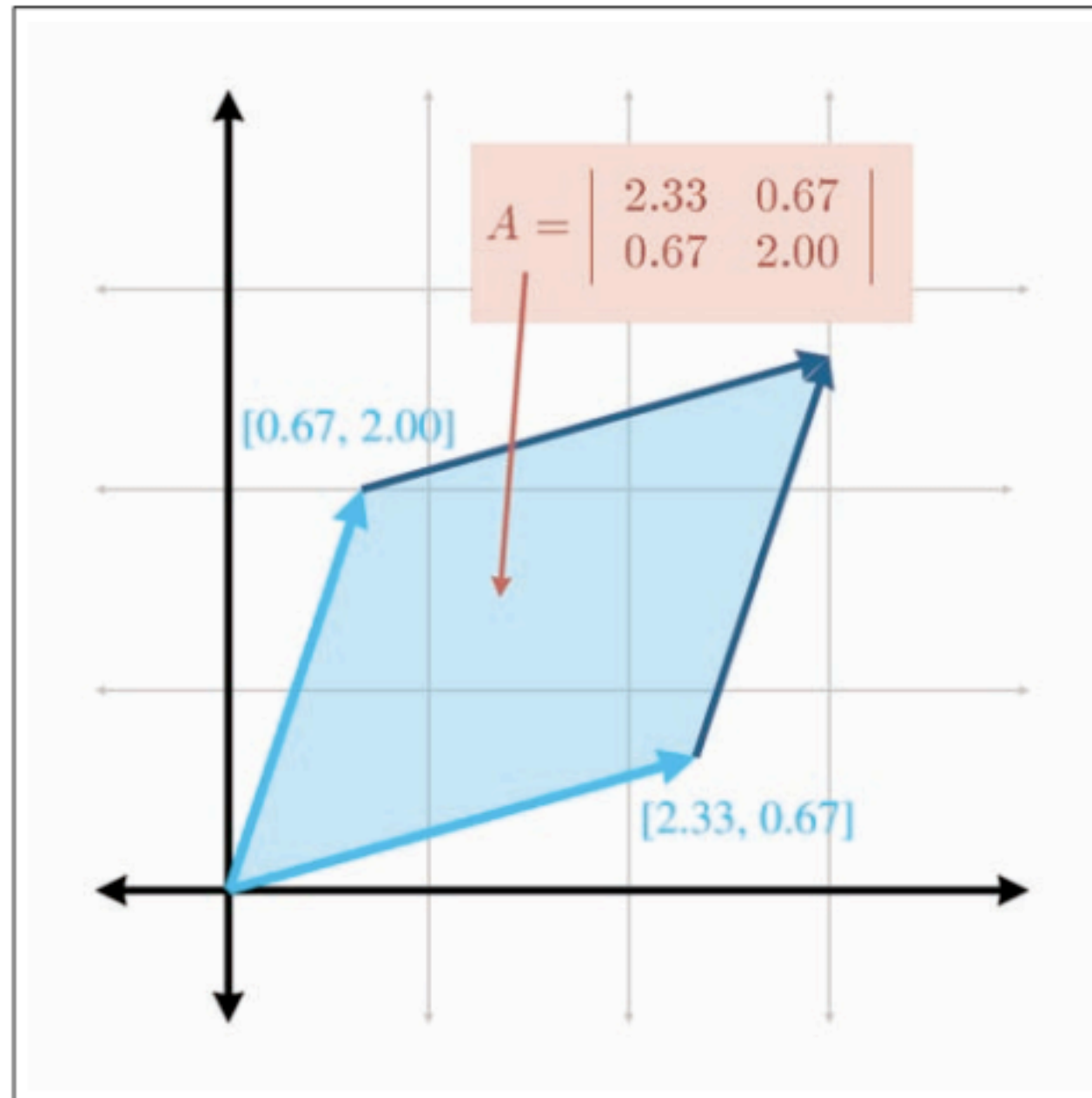
Diagram illustrating the calculation of a 3x3 determinant using Sarrus' rule. The elements are arranged in a 3x3 grid:

$$\begin{matrix} + & + & + & - & - & - \\ m_{11} & m_{12} & m_{13} & m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} & m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} & m_{31} & m_{32} & m_{33} \end{matrix}$$

The determinant is calculated as the sum of the products of the elements along the green arrows (positive terms) minus the sum of the products of the elements along the red arrows (negative terms):

$$m_{11}m_{22}m_{33} + m_{12}m_{23}m_{31} + m_{13}m_{21}m_{32} - m_{13}m_{22}m_{31} - m_{12}m_{21}m_{33} - m_{11}m_{23}m_{32}$$

# Geometric Interpretation



# Properties of Determinants

$$|\mathbf{I}| = 1$$

$$|\mathbf{AB}| = |\mathbf{A}| |\mathbf{B}|$$

$$|\mathbf{M}^T| = |\mathbf{M}|$$

$$|\mathbf{M}^{-1}| = \frac{1}{|\mathbf{M}|}$$

# Linear Independence

A set of vectors is said to be *linearly dependent* if at least one of them can be expressed as a linear combination (weighted sum) of the others:

$$\mathbf{v}_j = \sum_{i \neq j} w_i \mathbf{v}_i$$

If not linearly dependent, then *linearly independent*

# Singular Matrices

$$|\mathbf{M}^{-1}| = \frac{1}{|\mathbf{M}|}$$

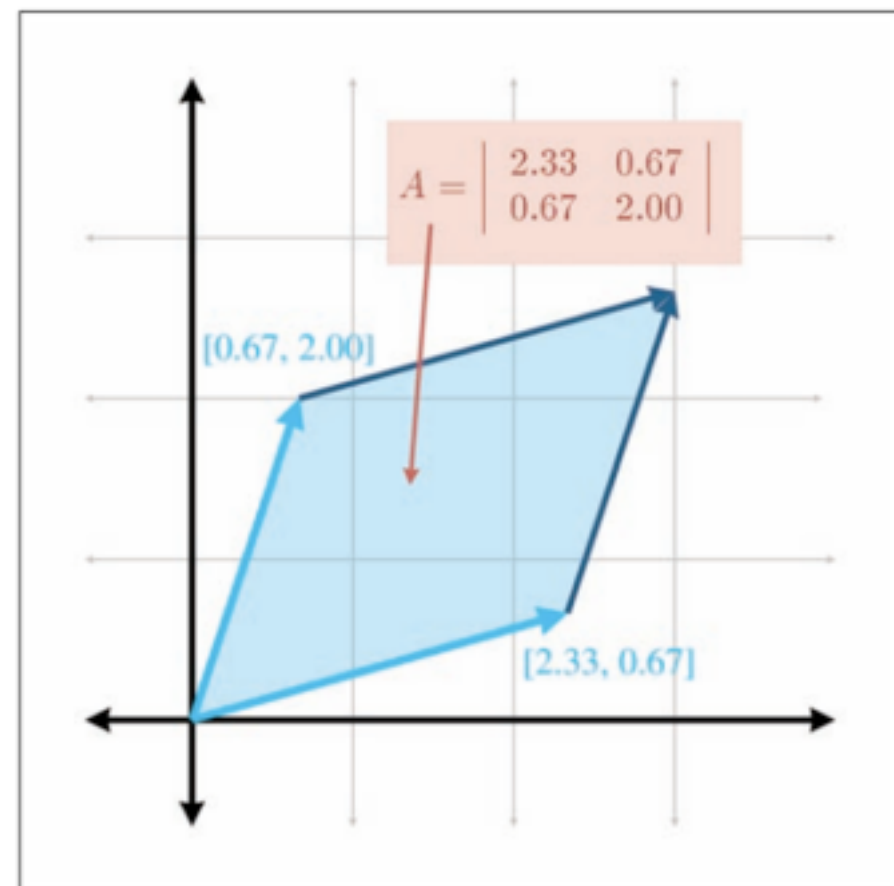
But what if  $|\mathbf{M}| = 0$ ?

A matrix whose determinant is zero has no inverse and is said to be *singular*



# Singular Matrices

- What does a singular matrix mean geometrically?
- *The rows are linearly dependent*



# Matrix Rank

- The *rank* of a matrix is the number of linearly independent rows
- When used as transforms, matrices with *full rank* transform to the full space
- Singular matrices have *insufficient rank* and collapse to a corresponding subspace

# Orthogonal Matrices

- Two matrices are said to be *orthogonal* iff

$$\mathbf{M}\mathbf{M}^T = \mathbf{I}$$

- Implies rows are orthonormal vectors

# Orthonormal Matrices

- Orthonormal matrices are also easily invertible:

$$\mathbf{M}^{-1} = \mathbf{M}^T$$

- Implies

$$|\mathbf{M}| = |\mathbf{M}^{-1}| = 1$$

# Orthonormal Matrices

- All rotation matrices are orthonormal

AND

- All orthonormal matrices are rotations!

# Coming up...

- 3D!