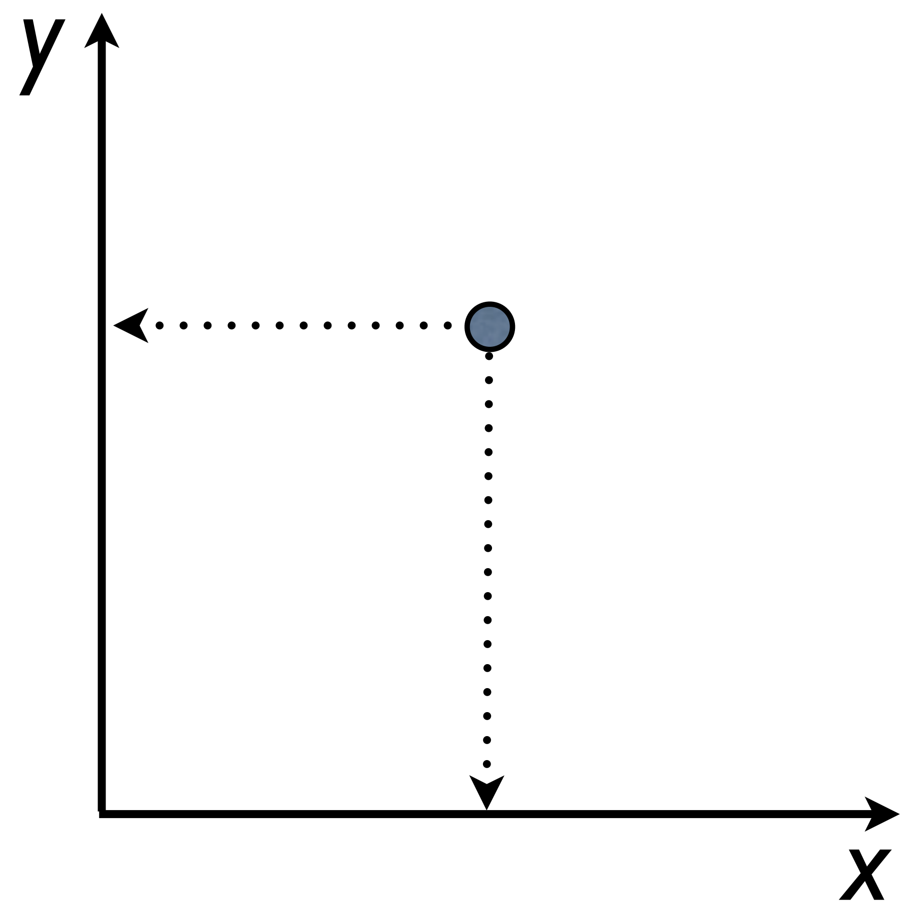
Point, Vectors, and Lines

CS 355: Interactive Graphics and Image Processing

Points



Points can be described by their *Cartesian coordinates*

Vectors

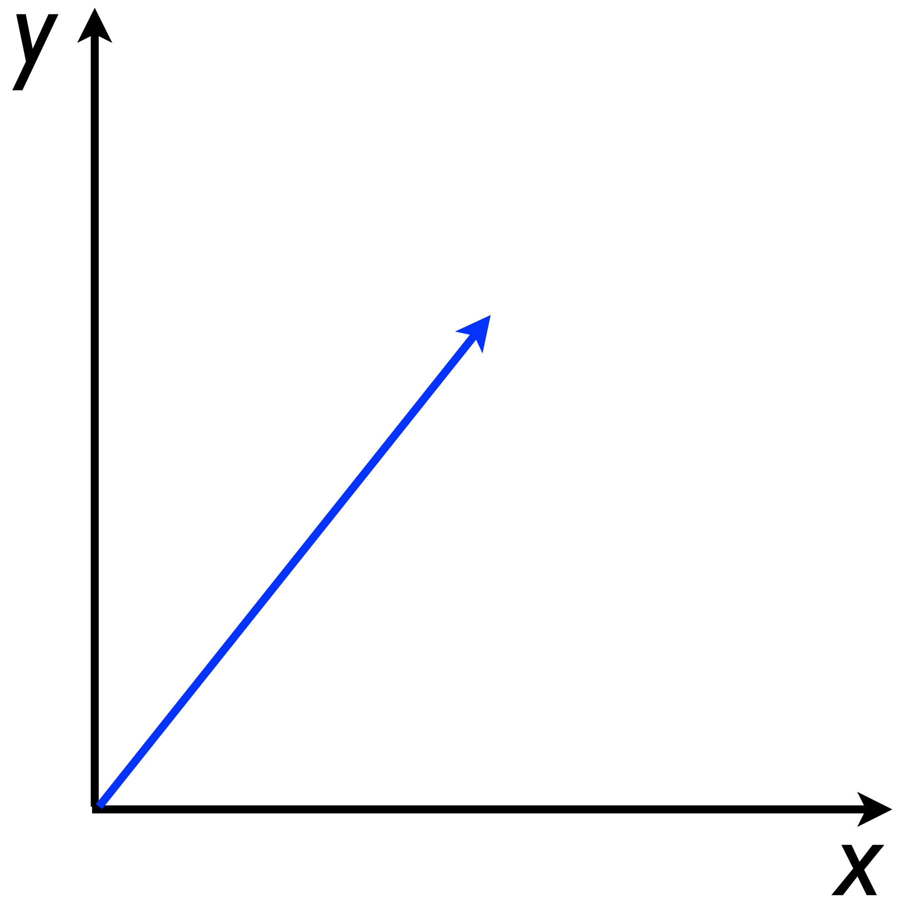
$$\mathbf{a} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad \mathbf{c} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

Vectors

$$\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$$

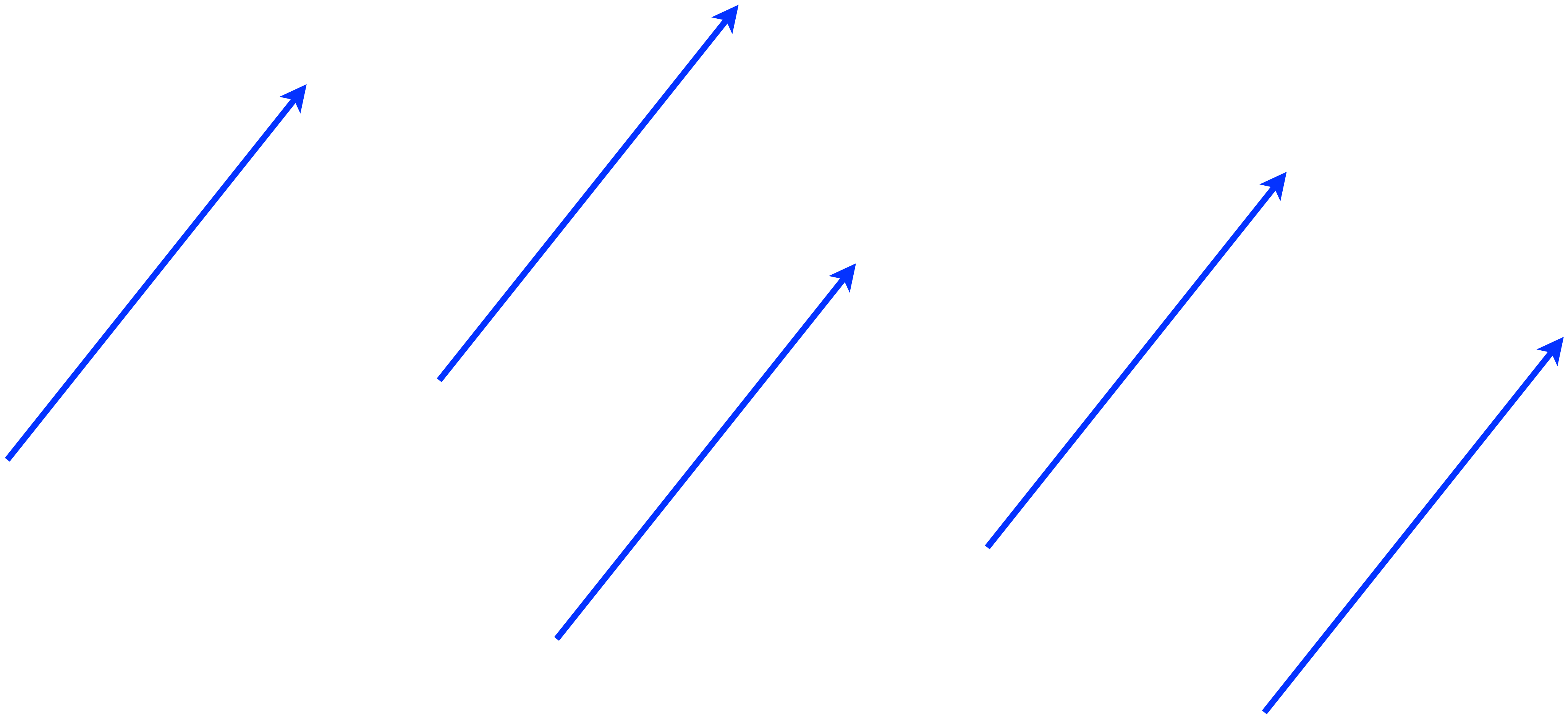
Often use subscripts to denote elements of a vector

Vectors



Vectors can also be thought of
in terms of Cartesian coordinates

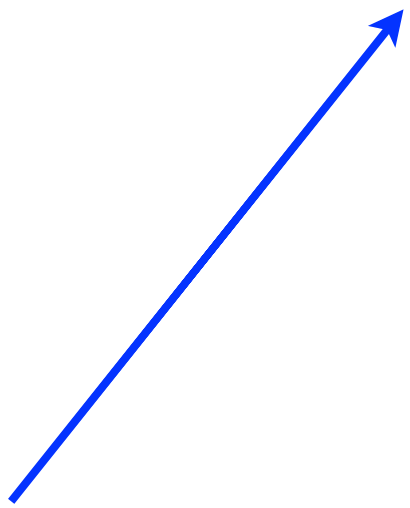
Vectors



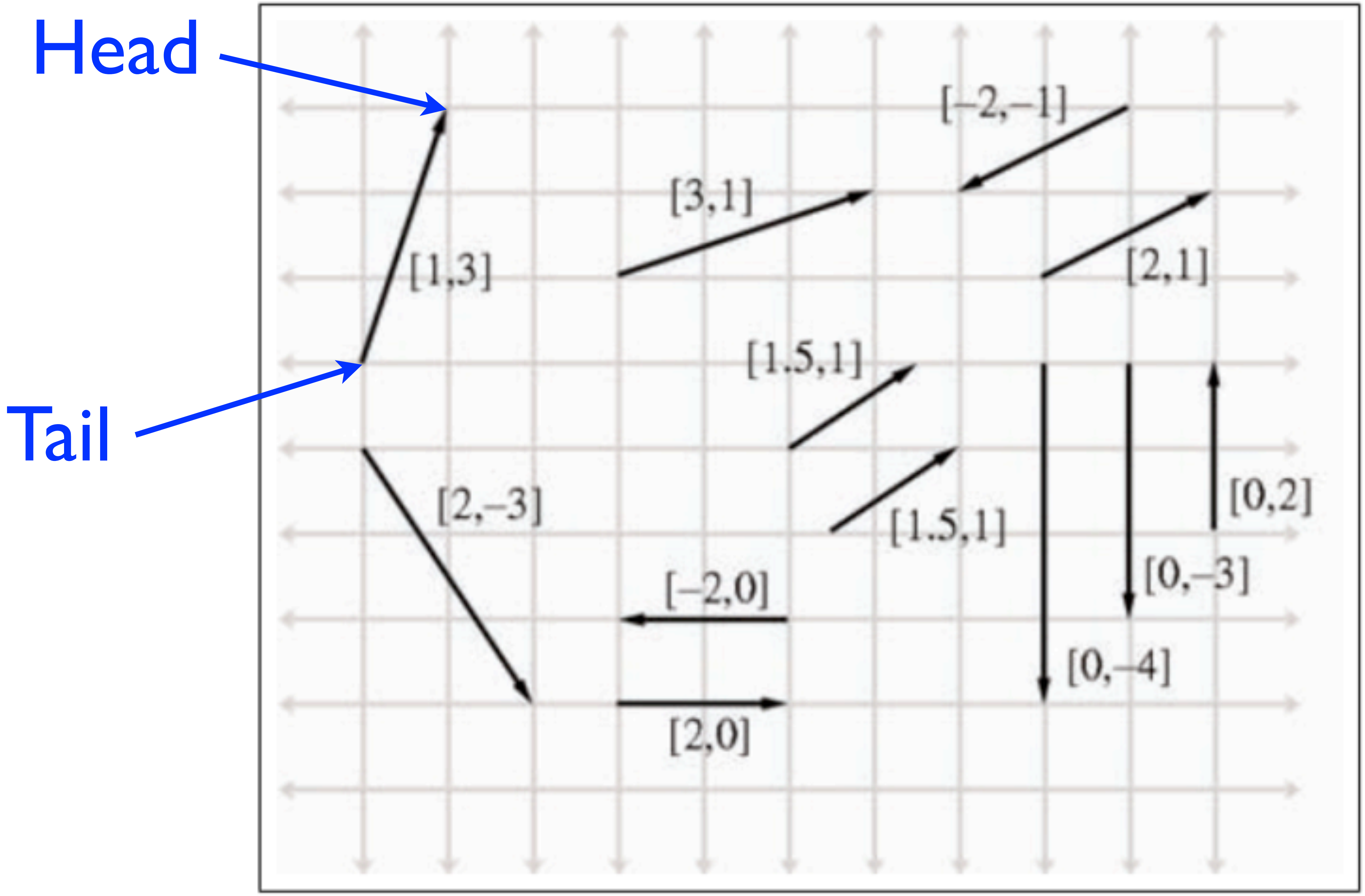
Vectors can be thought of as directional quantities without a specific location

Vectors

- *Direction*
- *Magnitude (length)*



Vectors as Displacements

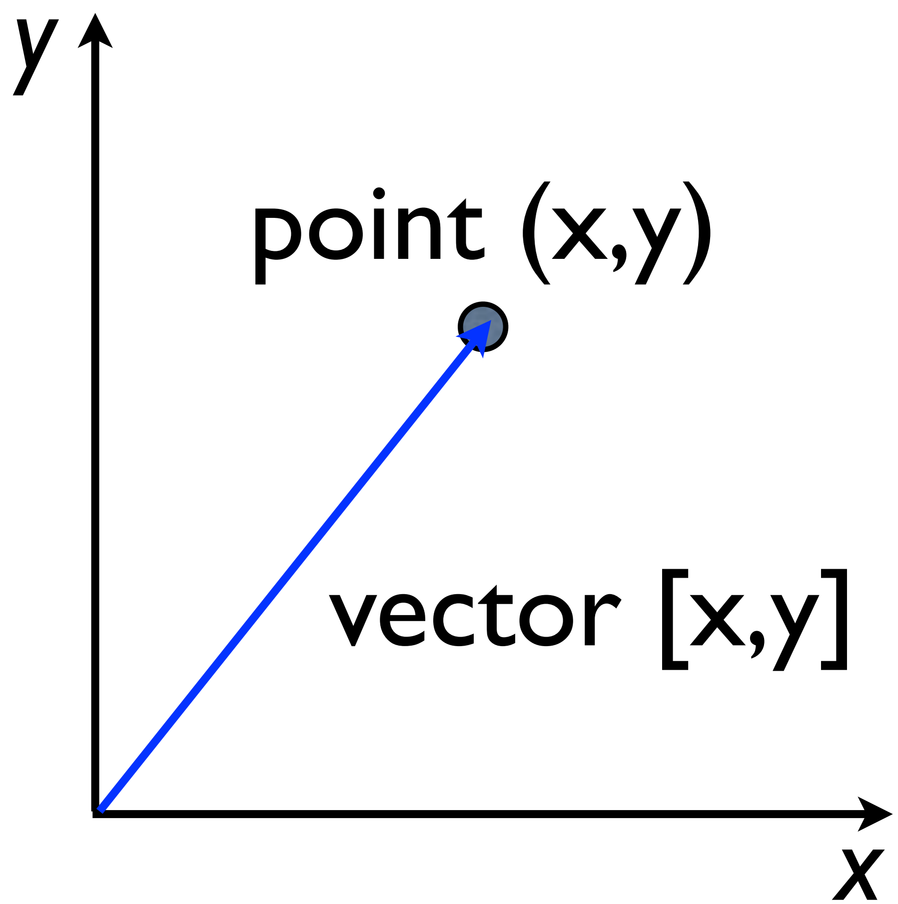


Zero Vector

- The *zero vector* is all zeroes
- No displacement
- Magnitude is 0
- Direction is undefined

$$\mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Points and Vectors



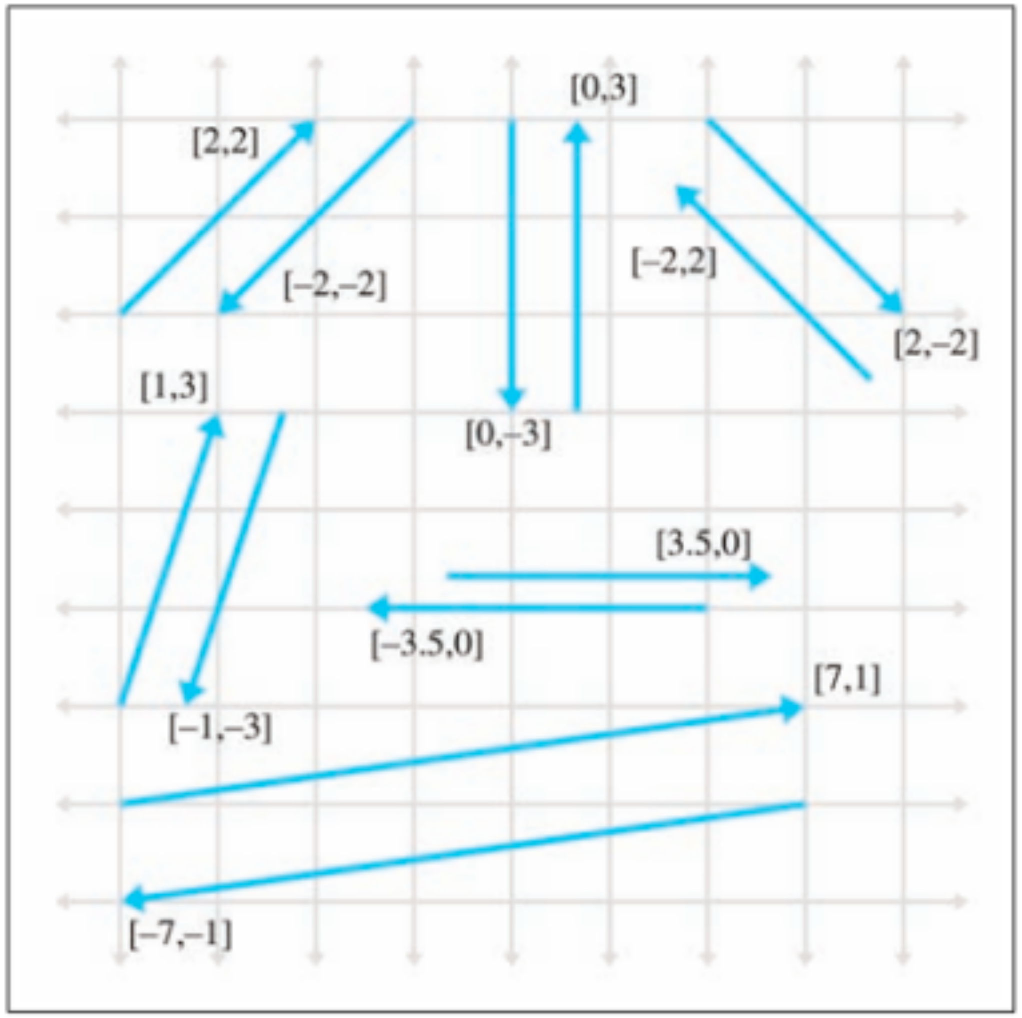
Points and vectors are different but related

Interchangeable (but be careful)

Negating Vectors

$$\mathbf{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$-\mathbf{v} = \begin{bmatrix} -x \\ -y \\ -z \end{bmatrix}$$

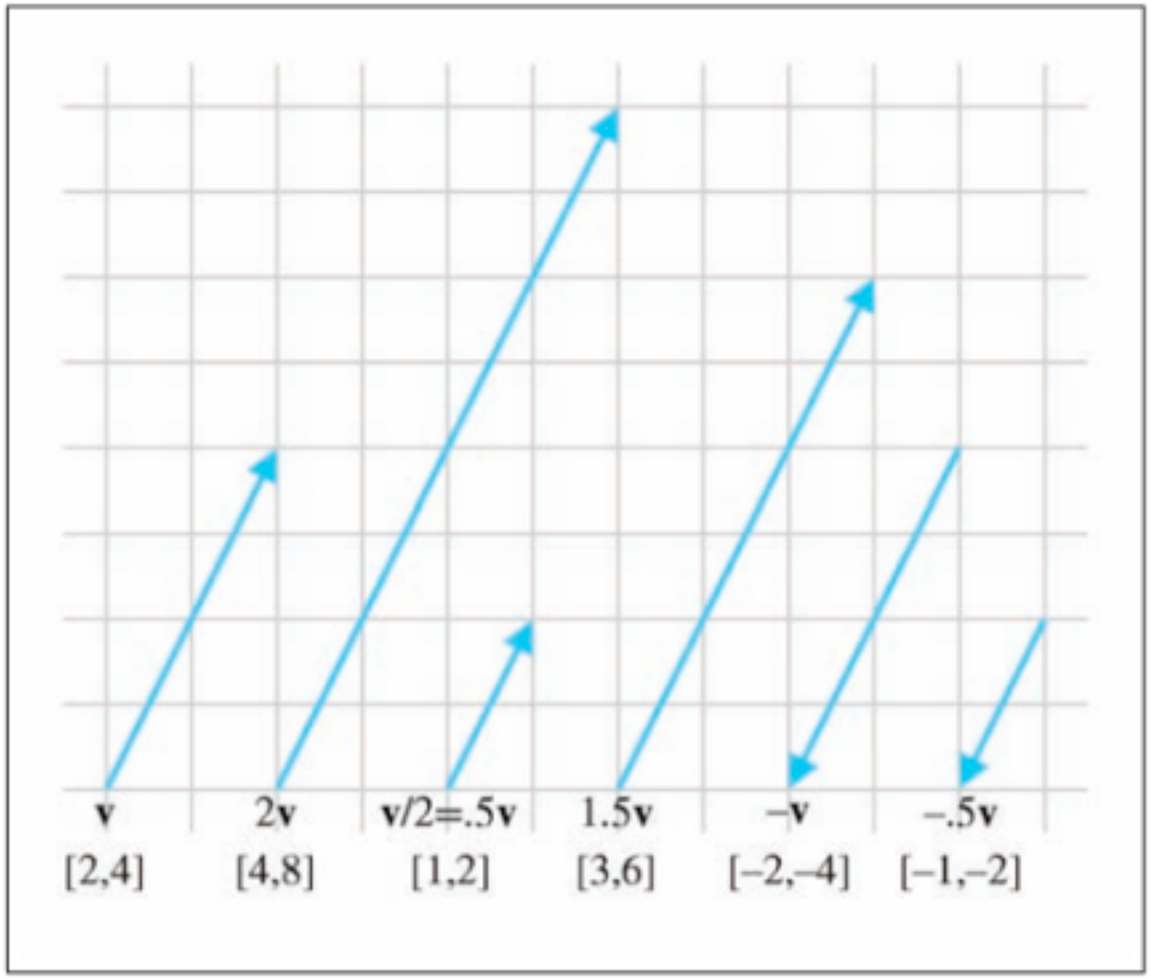


The negative of a vector has the same magnitude in the opposite direction

Scaling Vectors

$$\mathbf{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$k\ \mathbf{v} = \begin{bmatrix} k\ x \\ k\ y \\ k\ z \end{bmatrix}$$



Multiplying by a constant multiplies each element
-- multiplies magnitude, same (or opposite) direction

Adding Vectors

$$\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

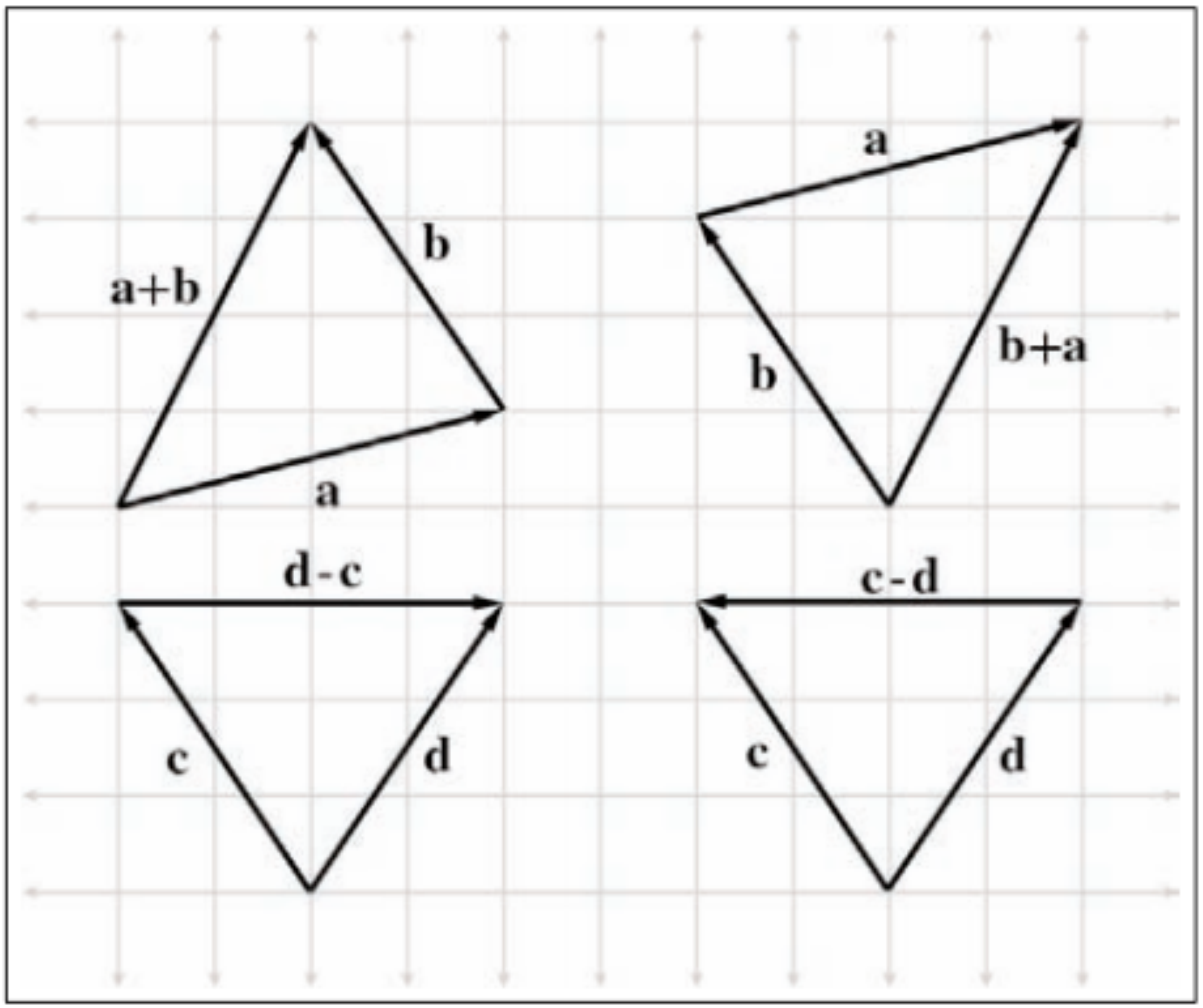
$$\mathbf{a} + \mathbf{b} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \\ a_3 + b_3 \end{bmatrix}$$

Subtracting Vectors

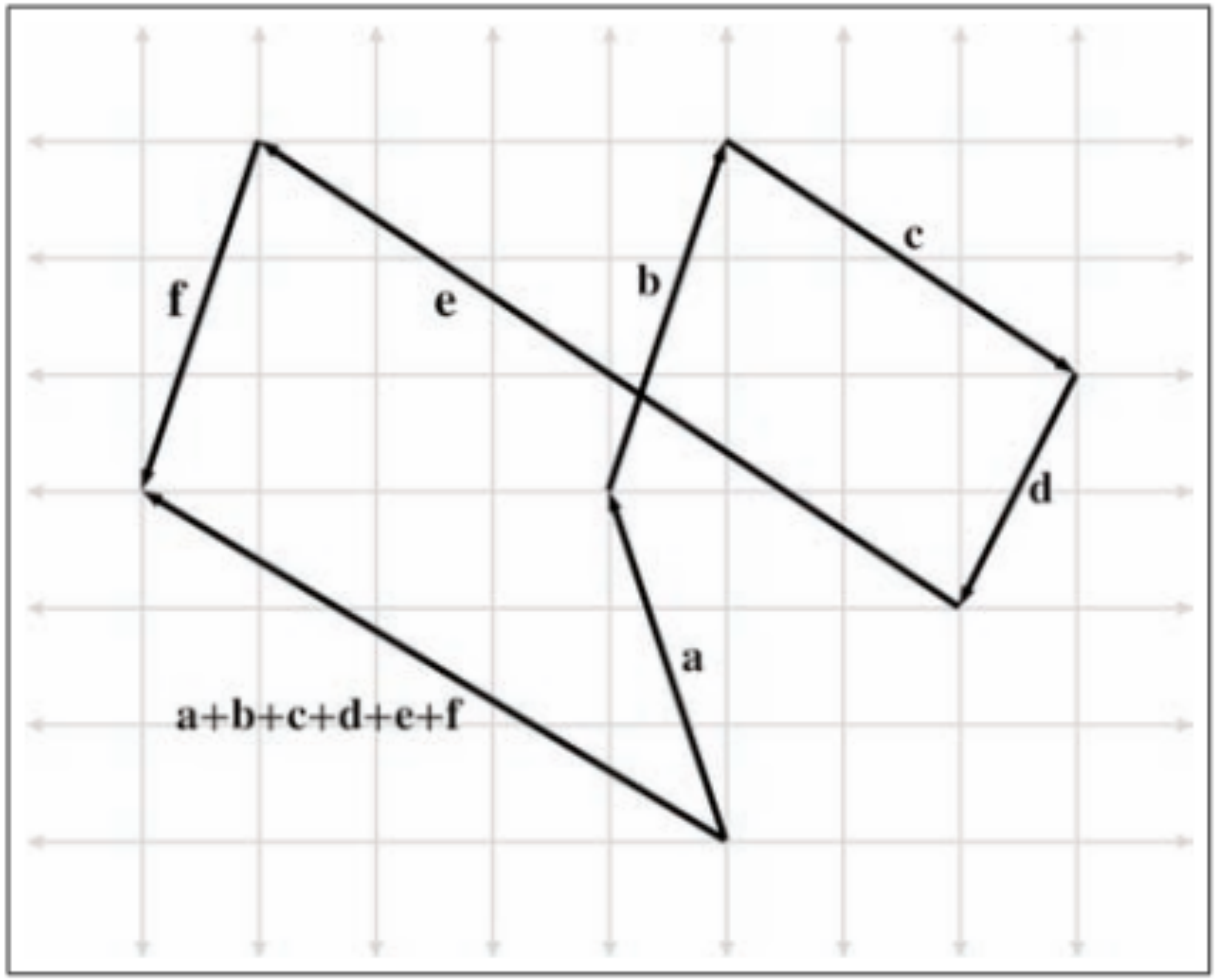
$$\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\mathbf{a} - \mathbf{b} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} - \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} a_1 - b_1 \\ a_2 - b_2 \\ a_3 - b_3 \end{bmatrix}$$

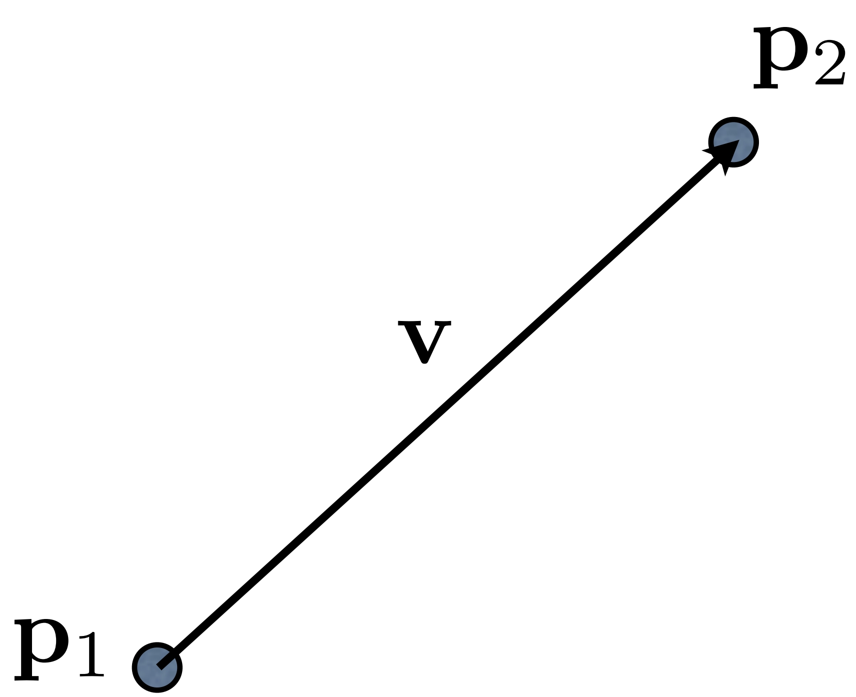
Geometric Interpretation



Geometric Interpretation

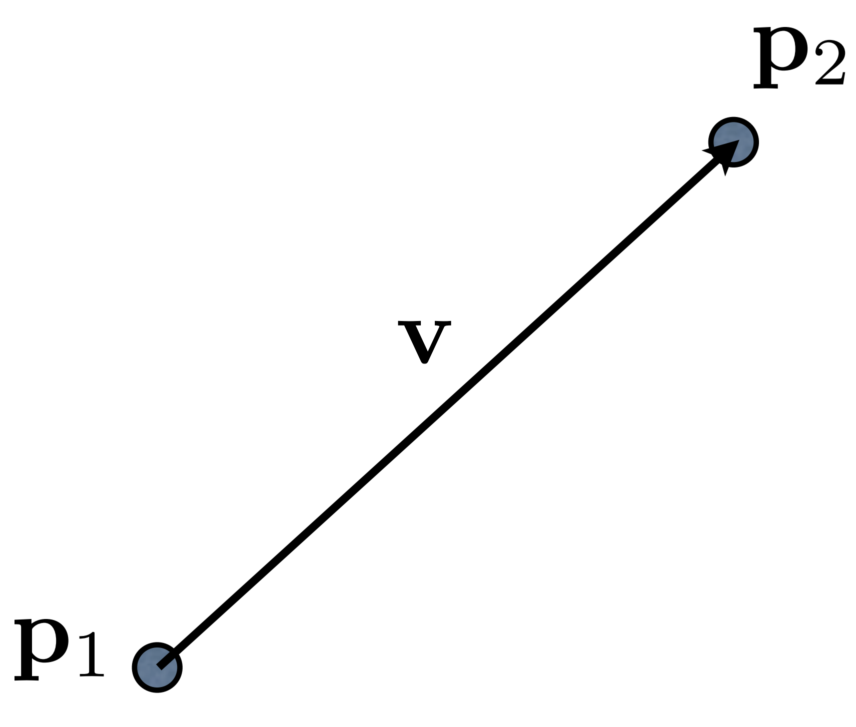


Displacement Vectors



$$\mathbf{p}_1 + \mathbf{v} = \mathbf{p}_2$$

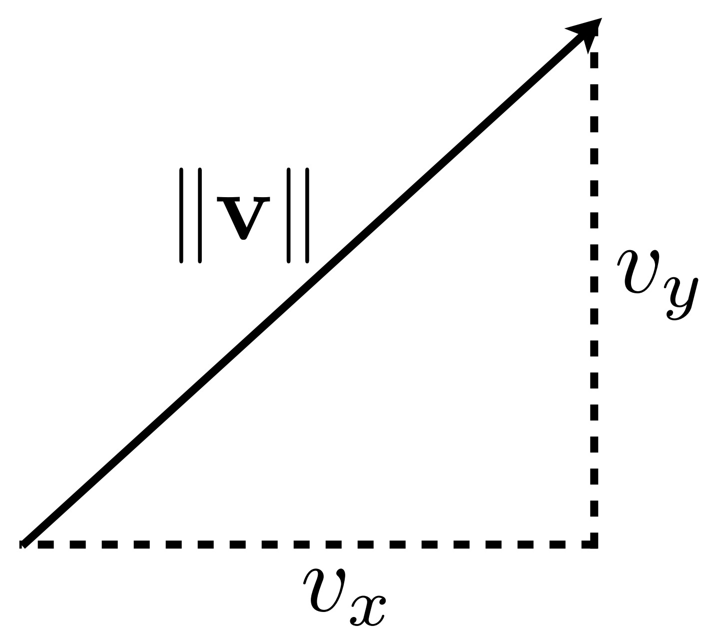
Displacement Vectors



$$\mathbf{v} = \mathbf{p}_2 - \mathbf{p}_1$$

Magnitude

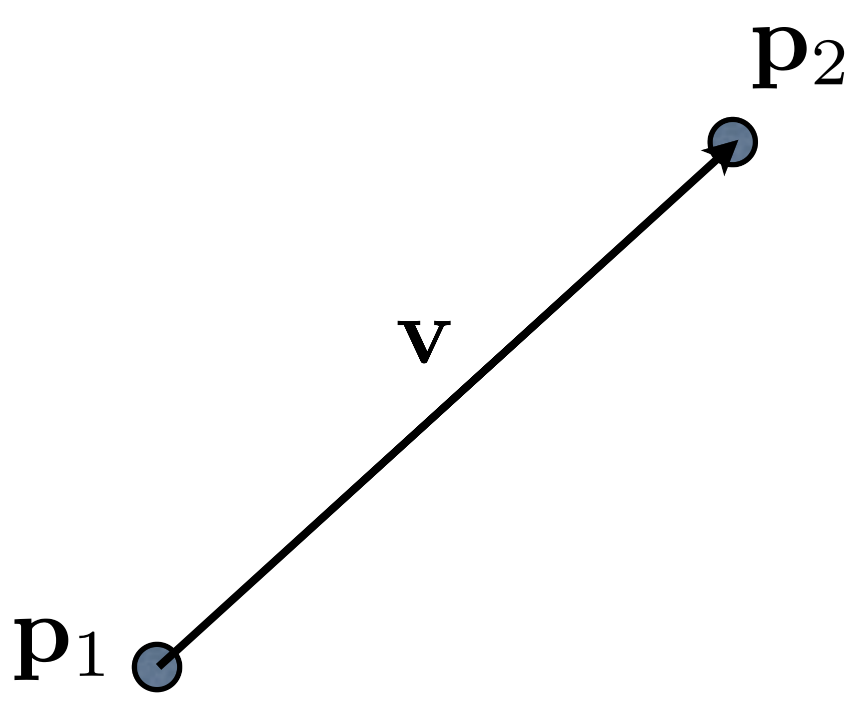
- The magnitude (length) of a vector can be calculated using Pythagorean theorem
- Sometimes called the *norm* of the vector
- Note: there are other vector norms, but assume this unless stated otherwise



$$\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2 + \cdots + v_n^2}$$

“magnitude”, “length”, or “norm” of v

Distance

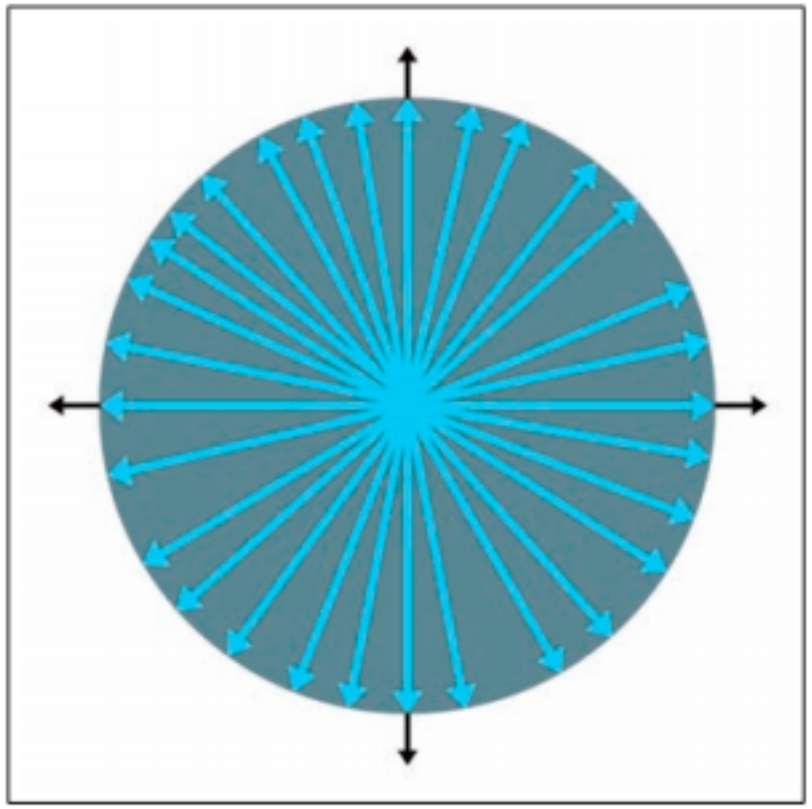


$$\|\mathbf{v}\| = \|\mathbf{p}_2 - \mathbf{p}_1\| = \sqrt{(p_2[x] - p_1[x])^2 + (p_2[y] - p_1[y])^2}$$

Unit Vectors

- A "unit vector" has a length of one
- Useful to describe direction when we don't care about magnitude

$$\|v\| = 1$$



Normalizing

- Sometimes we want to *normalize* a vector to have the same direction but unit length
- Key: just divide it by its own length
- Can't do this for the zero vector of course

$$\frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{\mathbf{v}}{\sqrt{v_1^2 + v_2^2 + \cdots + v_n^2}}$$

Next time...

- More on points, vectors, and lines:
  - Dot products
  - Cross products
  - Lines