

# Turbo Frequency Domain Equalization for Single-Carrier Broadband Wireless Systems

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**Abstract**—In this paper, a new class of equalization and channel estimation techniques, using the turbo frequency domain equalization (TFDE), is presented as a promising low-complexity detection method for single-carrier broadband wireless transmissions. Serial modulation (SM), being a direct counterpart of the well-known OFDM modulation, is receiving considerable attention recently, owing to the fact that it delivers comparable performance as OFDM while avoiding the problem of high peak-to-average power ratio. When frequency domain equalization (FDE) is applied, the complexity requirement is low and it becomes feasible to employ iterative processing which relies on decision feedback. This paper considers the turbo principle applied jointly to FDE, channel decoding and channel estimation. The result of this work is a set of effective iterative algorithms which may bring about 2-3dB improvement over the linear FDE method. Furthermore, it is shown that they can provide performance comparable to the time-domain turbo equalization methods but with lower complexity. We then reach the conclusion that SM-based transmission with TFDE is a suitable technology for next generation wireless systems.

**Index Terms**—Turbo frequency domain equalization, single-carrier, channel estimation, broadband wireless systems.

## I. INTRODUCTION

IN future broadband wireless access systems, which will offer bit rates of tens of megabits per second, choosing a suitable air interface technology is essential in order to meet the required quality of wireless service in a hostile radio propagation environment, and under various constraints such as bandwidth and equipment costs. While OFDM is a well recognized candidate as a broadband wireless technology, serial modulation (SM)-based technology has also started to receive considerable attention. It has been shown in [1] that frequency domain equalization (FDE) can be readily applied to SM to yield similar performance as OFDM while demanding essentially the same overall complexity. While OFDM suffers from high peak-to-average power ratio, serial modulation introduces low envelope fluctuations, leading to more efficient uses of power amplifiers. Therefore, serial modulation is particularly suitable for the uplink direction where the user terminals could be equipped with low cost amplifiers and simple transmitters [1].

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Due to the attractive features of SM-FDE as described above, the importance of optimizing a receiver structure based on SM-FDE becomes clear. In the past, nonlinear equalizers such as FDE with time-domain decision feedback were proposed in [1] [2], and they were shown to yield performance improvement over the linear FDE. In [3]-[5], block-iterative frequency domain feedback equalizer was proposed and it was shown to deliver performance very close to the matched filter bound with relatively low complexity. However, these techniques have not considered channel decoding and channel estimation in the equalization process. In [6] [7][8], turbo equalization was considered and implemented in the frequency domain, but again, perfect channel knowledge is assumed to be available at the receiver. In [9] and [10], iterative channel estimation in connection with time domain turbo equalization was proposed and evaluated. And in [11], frequency domain turbo equalization with iterative channel estimation over a succession of blocks was considered. In this paper, our contribution is to examine the iterative process and propose suitable frequency domain turbo equalizers as well as block-based channel estimator for this process. Our channel estimation approach involves iteration within a single block. We term our approach turbo frequency domain equalization (TFDE). Based on TFDE, a novel low-complexity iterative frequency-domain equalizer utilizing either the time-domain or frequency-domain soft decision feedback from the decoder is proposed and compared with the previously proposed frequency domain feedback equalizer. All the iterative frequency domain equalizers considered in our work contain two separate filters, namely the forward and the backward filter, which are jointly optimized in each iteration to mitigate error propagation in the decision feedback process. They therefore have a different structure from that of traditional interference-cancellation based turbo equalizers implemented in the frequency domain, such as [6]. Furthermore, unlike the previous work [2]-[6], [8], [9], and [10] where equalization is performed based on perfect channel knowledge or on iterative channel estimation over many blocks, our work is based on pilot-assisted iterative channel estimation within one block. In order to minimize the channel estimation error effect in the equalizer, we introduce the approach that the calculation of the filter coefficients in the equalizer should take into account the estimation error level, which is provided by the channel estimator. Based on this method, the proposed TFDE can provide near-optimal performance which is close to the equalization performance with perfect channel knowledge.

This paper is organized as follows. In section II, the system model is described. In section III, the turbo frequency domain

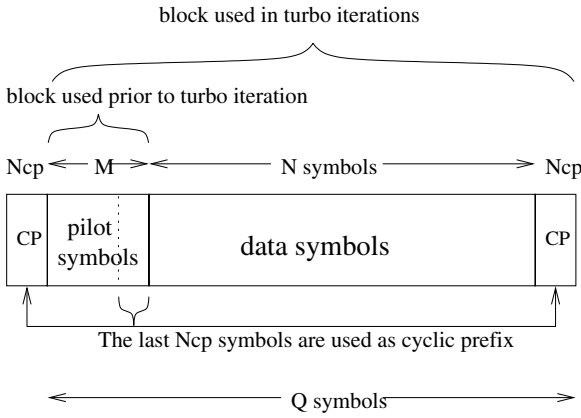


Fig. 1. Packet structure with pilot and data symbols.

equalization (TFDE) techniques are discussed in detail. In section IV, TFDE with iterative channel estimation is described. Finally, the simulation results and concluding remarks are presented in section V and section VI, respectively.

## II. SYSTEM DESCRIPTION

This paper considers packet/block-based transmission. Figure 1 depicts the placement of the pilot and data symbols within a packet. At the transmitter, the input signals are parsed into blocks of  $N$  symbols, given by  $\mathbf{d} = [d_0 \cdots d_{N-1}]^T$ . The block of data symbols is preceded by a training block with  $M$  pilot symbols  $\mathbf{p} = [p_0, p_1, \cdots, p_{M-1}]^T$ . The last  $N_{cp}$  symbols of the training block ( $\mathbf{p}_{cp} = [p_{M-N_{cp}}, p_{M-N_{cp}+1}, \cdots, p_{M-1}]^T$ ) are used as the cyclic prefix (CP), which is placed at the beginning and the end of the packet as shown in Figure 1. The packet can be viewed as consisting of two overlapping blocks. The small block (training block) is used for initial channel estimation before the turbo process. Afterwards, the large block (training+data+CP) is used for joint detection and estimation in the turbo process. We remark that the length of CP ( $N_{cp}$ ) is assumed to be larger than the maximum channel length and less than  $M$ . It is assumed that the system may operate in a burst transmission and therefore one packet can be considered at a time.

Figure 2 depicts our system model. Only BPSK modulation is considered during the analysis while QPSK modulation is used for the simulation studies. Extension to other modulations is in principle straightforward but more computationally involved. Assuming symbol-rate ( $1/T_s$ ) sampling, the baseband equivalent received sampled signals can be expressed as,

$$r(n) = \sum_{\nu=0}^{L-1} h(\nu)x(n-\nu) + \eta(n) \quad (1)$$

where  $h(\nu), \nu = 0 \cdots L-1$  are the channel (including the transmit pulse and its matched filter response) coefficients. Now, consider first the received samples due to the training block, it is easily shown that they can be expressed as (after the first CP deletion),

$$\mathbf{r}_p = \mathbf{H}_{M \times M} \mathbf{p} + \mathbf{n}_p \quad (2)$$

where  $\mathbf{n}_p$  denotes the corresponding noise vector, and  $\mathbf{H}_{M \times M}$  is a  $M \times M$  circulant matrix with  $[h(0), \cdots, h(L-$

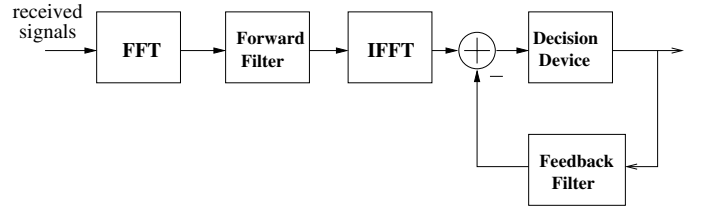


Fig. 2. Non-iterative FDE-TDDF equalizer [1]

$1), \mathbf{0}_{1 \times M-L}]^T$  as its first column. Likewise, for the large block containing both the pilot and data symbols, the received samples, after the CP deletion, can be written as,

$$\mathbf{r} = \mathbf{H}_{Q \times Q} \mathbf{x} + \mathbf{n} \quad (3)$$

where  $Q = M + N + N_{cp}$ ,  $\mathbf{x} = [\mathbf{p} \ \mathbf{d} \ \mathbf{p}_{cp}]^T$ , and  $\mathbf{H}_{Q \times Q}$  is a  $Q \times Q$  circulant matrix with  $[h(0), \cdots, h(L-1), \mathbf{0}_{1 \times Q-L}]^T$  as its first column.

To translate the received signals into the frequency domain, we apply the  $Q$ -point DFT (with  $\mathbf{F}$  as the DFT matrix) as follows,

$$\mathbf{y} = \mathbf{F} \mathbf{r} = \mathbf{D} \mathbf{F} \mathbf{x} + \mathbf{F} \mathbf{n} \quad (4)$$

where we have used the property that the circulant matrix  $\mathbf{H}_{Q \times Q}$  can be decomposed into  $\mathbf{F}^H \mathbf{D} \mathbf{F}$ .  $\mathbf{D}$  is a diagonal matrix with the main diagonal entries given by the channel frequency responses at the frequency bins  $k = 1, \cdots, Q$ , which can be expressed as,

$$h_k = \frac{1}{\sqrt{Q}} \sum_{l=0}^{L-1} h(l) e^{-j2\pi kl/Q}, \quad k = 0, \cdots, Q-1 \quad (5)$$

## III. TURBO FREQUENCY DOMAIN EQUALIZATION: PERFECT CHANNEL KNOWLEDGE

In this section, channel estimation is not considered and perfect channel knowledge is assumed at the receiver. The turbo iteration thus only involves equalization and decoding process. For simplicity, we may neglect the pilot signals and assume that the received signals contain only the data signals,  $\mathbf{y} = \mathbf{D} \mathbf{F} \mathbf{x} + \mathbf{n}$ , where  $\mathbf{x} = \mathbf{d}$  and  $Q$  is replaced by  $N$  in (3)-(5). According to the turbo principle, the optimal channel equalizer is the MAP equalizer, which calculates the log a-posteriori probability (APP) ratio,

$$\underbrace{\log \frac{P(x_m = +1|\mathbf{y})}{P(x_m = -1|\mathbf{y})}}_{\Lambda_m} = \underbrace{\log \frac{P(\mathbf{y}|x_m = +1)}{P(\mathbf{y}|x_m = -1)}}_{\lambda_m^e} + \underbrace{\log \frac{P(x_m = +1)}{P(x_m = -1)}}_{\lambda_m^p} \quad (6)$$

The log APP ( $\Lambda_m$ ) can be expressed as the sum of the log likelihood ratio (LLR) and the a-priori probability ratio. The LLR is termed as the *extrinsic* information ( $\lambda_m^e$ ) delivered by the equalizer, while the a-priori probability ratio is given by the decoder as the *intrinsic* information  $\lambda_m^p$  which is obtained from the previous iteration. The equalizer's extrinsic information is fed to the decoder as the a-priori probability, i.e. the intrinsic information, for the decoder. The decoder

calculates the log APP by taking into account the coding structure and the currently received intrinsic information for the code bits. The LLR calculated by the decoder is considered to be the extrinsic information and is fed to the equalizer as the intrinsic information in the new iteration. Since the optimal MAP equalizer requires exponentially increasing complexity, alternative solutions are needed. Our treatment of turbo equalization here will primarily focus on proposing and analyzing several types of frequency-domain channel equalizers which offer trade-off in performance and complexity. The channel decoder considered here is simply the MAP decoding scheme based on the BCJR algorithm [12] and convolutional code is employed throughout our work.

#### A. Iterative frequency domain equalization with time-domain soft-decision feedback (FDE-TDDF-soft)

The iterative frequency domain equalizer with time-domain decision feedback proposed in this paper can be considered to be the iterative version of the non-iterative FDE-TDDF equalizer proposed in [1]. To proceed, we begin with the description of the non-iterative FDE-TDDF (see Figure 2), which consists of a forward filter operating in the frequency domain and a time-domain backward filter processing the feedback signals. The output value from the non-iterative FDE-TDDF can be expressed as (for the desired symbol  $x_m$ ),

$$\tilde{x}_m = \left( \frac{1}{Q} \sum_{l=0}^{Q-1} w_l y_l e^{j2\pi lm/Q} - \sum_{k \in \chi} g_k \hat{x}_k \right) \quad (7)$$

where  $\hat{x}_k$  denotes the hard decision feedback, and  $\chi$  is the designated set of time-domain taps in the feedback filter, whose coefficients are given by  $g_k, k \in \chi$ . The frequency domain coefficients in the forward filter are denoted as  $w_l, l = 0 \cdots Q-1$ . The filter coefficients are all derived based on the MMSE criterion [1]. If time-domain decisions feedback are not available, the non-iterative FDE-TDDF reduces to the traditional linear FDE, in which the forward filter coefficients can be easily expressed as,

$$w_l = \frac{h_l^*}{|h_l|^2 + \sigma^2}, \quad l = 0, \cdots, Q-1 \quad (8)$$

where  $\sigma^2$  denotes the variance of the noise samples. Although the performance of the non-iterative FDE-TDDF is superior to that of FDE, it suffers from possible error propagation. Here, we propose an iterative FDE-TDDF suitable for the turbo processing considered in this paper. Unlike the original FDE-TDDF where perfect decisions feedback is assumed, the iterative FDE-TDDF takes into account the reliability of the feedback decisions and therefore alleviates the error propagation problem. Furthermore, it is operating on a block basis, therefore canceling both the pre-cursor and the post-cursor intersymbol interference (ISI).

The structure of the FDE-TDDF with channel decoding is depicted in Figure 3. The outputs from the FDE-TDDF are de-interleaved prior to entering the decoder, and the outputs from the decoder are interleaved and then fed back to the FDE-TDDF. In the  $i$ -th iteration, the outputs from the FDE-TDDF, denoted as  $\hat{\mathbf{x}}^{(i)} = [\hat{x}_0^{(i)}, \cdots, \hat{x}_{Q-1}^{(i)}]^T$ , can be expressed as,

$$\hat{\mathbf{x}}^{(i)} = \mathbf{F}^H \mathbf{W}^{(i)} \mathbf{y} - \mathbf{G}^{(i)} \hat{\mathbf{x}}^{(i-1)} \quad (9)$$

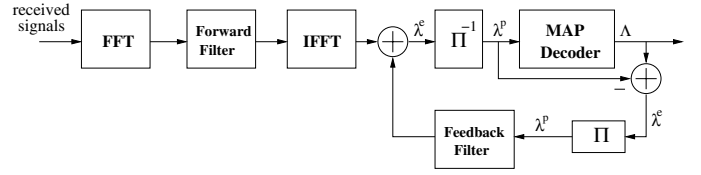


Fig. 3. Iterative FDE-TDDF equalizer

where  $\hat{\mathbf{x}}^{(i-1)} = [\hat{x}_0^{(i-1)}, \cdots, \hat{x}_{Q-1}^{(i-1)}]$  are the *soft* feedback decisions from the previous iteration.  $\mathbf{W}^{(i)} = \text{diag}\{w_0^{(i)}, w_1^{(i)}, \cdots, w_{Q-1}^{(i)}\}$  contains the forward filter coefficients. The backward filter coefficients,  $\{g_1^{(i)}, g_2^{(i)}, \cdots, g_{Q-1}^{(i)}\}$  are contained in a  $Q \times Q$  circulant matrix  $\mathbf{G}^{(i)}$ ,

$$\mathbf{G}^{(i)} = \begin{bmatrix} 0 & g_1^{(i)} & g_2^{(i)} & \cdots & g_{Q-1}^{(i)} \\ g_{Q-1}^{(i)} & 0 & \ddots & \ddots & \ddots \\ \vdots & \ddots & \ddots & \ddots & \ddots \\ g_1^{(i)} & g_2^{(i)} & \cdots & \cdots & 0 \end{bmatrix} \quad (10)$$

Note that as in [1], it is valid to have only a few non-zero taps. If this is the case, only a subset of the feedback decisions are used and computational burden to find  $\mathbf{G}^{(i)}$  is reduced accordingly. Here, for the iterative FDE-TDDF, we consider feeding back the entire block of interfering symbols. Hence, there are  $Q-1$  feedback coefficients. Moreover, the matrix  $\mathbf{G}^{(i)}$  indicates that the time-domain filter coefficients are *time-invariant*, which means that the coefficients remain unchanged in detecting different symbols within  $\mathbf{x}$ .

The forward and backward filter coefficients are found according to the MMSE criterion,  $\min E|\hat{x}_m^{(i)} - x_m|^2$ . The block-averaged mean square error, conditioned on the channel outputs and the results of the previous iteration, can be expressed as,

$$\begin{aligned} \frac{1}{Q} \sum_{m=1}^Q E|\hat{x}_m^{(i)} - x_m|^2 &= \frac{\sigma^2}{Q} \sum_{l=0}^{Q-1} |w_l^{(i)}|^2 + \frac{1}{Q} \sum_{l=0}^{Q-1} |w_l^{(i)}|^2 |h_l|^2 \\ &\quad - \frac{2}{Q} \text{Re}\left\{ \sum_{l=0}^{Q-1} w_l^{(i)} h_l \left[ 1 + \sum_{k=1}^{Q-1} \rho^{(i)} g_k^{(i)} \exp(j2\pi \frac{kl}{Q}) \right] \right\} \\ &\quad + \sum_{k=1}^{Q-1} \underbrace{|\hat{x}_{(m+k) \bmod Q}^{(i-1)}|^2}_{\beta_{(m+k) \bmod Q}^{(i)}} |g_k|^2 + 1 \end{aligned} \quad (11)$$

where  $\beta_m^{(i)} = |\hat{x}_m^{(i-1)}|^2$ ,  $m = 0 \cdots Q-1$  and  $\rho^{(i)}$  is defined as the reliability of the decision feedback given by,

$$\rho^{(i)} = E(x_m^* \hat{x}_m^{(i-1)}) \quad (12)$$

and we have assumed that  $\rho^{(i)}$  is the same for all  $m$ . This is a reasonable assumption, since, in a large block, which undergoes quasi-static fading, the average symbol error probability is approximately the same for each symbol. Also, it is assumed that in (11), the feedback decisions are statistically independent. To simplify the computation of the filter coefficients, we further make the assumption that  $\beta_0^{(i)} = \beta_1^{(i)} = \cdots = \beta_{Q-1}^{(i)} = \bar{\beta}^{(i)}$ , where  $\bar{\beta}^{(i)} = (1/Q) \sum_{k=0}^{Q-1} |\hat{x}_k^{(i-1)}|^2$ . This approximation should not lead to a significant loss in

performance because the reliability of each feedback decision is approximately the same over a long block transmission. By taking the derivative of (11) with respect to  $w_l$  and setting it to zero, we obtain the set of optimal forward filter coefficients,

$$w_l^{(i)} = \frac{h_l^* (1 + (\rho^{(i)})^* \sum_{k=1}^{Q-1} (g_k^{(i)})^* \exp(-j2\pi \frac{kl}{Q}))}{|h_l|^2 + \sigma^2} \quad (13)$$

By substituting the optimal  $w_l^{(i)}$ ,  $l = 0, \dots, Q-1$  into (11), the backward filter coefficients can be readily found. After some calculus, we obtain the following,

$$|\rho^{(i)}|^2 \mathbf{V}^{(i)} \mathbf{g}^{(i)} = -\mathbf{v}^{(i)} \rho^{(i)} \quad (14)$$

where

$$\begin{aligned} \mathbf{g}^{(i)} &= [g_1^{(i)}, g_2^{(i)}, \dots, g_{Q-1}^{(i)}]^T \\ \mathbf{v}^{(i)} &= [v_1^{(i)}, v_2^{(i)}, \dots, v_{Q-1}^{(i)}]^T \\ \mathbf{V}^{(i)} &= \begin{bmatrix} v_0^{(i)} & v_{-1}^{(i)} & \dots & v_{-(Q-2)}^{(i)} \\ v_1^{(i)} & v_0^{(i)} & v_{-1}^{(i)} & \ddots \\ \ddots & \ddots & \ddots & \ddots \\ v_{Q-2}^{(i)} & \ddots & \ddots & v_0^{(i)} \end{bmatrix} \end{aligned} \quad (15)$$

and

$$v_k^{(i)} = \frac{1}{Q} \sum_{l=0}^{Q-1} \underbrace{|h_l|^2 (\bar{\beta}^{(i-1)} / (\rho^{(i)})^2 - 1) + \sigma^2 \bar{\beta}^{(i-1)} / (\rho^{(i)})^2}_{\psi_l^{(i)}} \cdot \exp(-j2\pi \frac{lk}{Q}) \quad (16)$$

As seen in (14), the computation of feedback coefficients requires the calculation of a Hermitian Toeplitz matrix inverse. This is also the case for the non-iterative DFE in [1]. In appendix A, we outline a method to simplify the calculation of the matrix inverse.

With the optimal filter coefficients, the inputs to the decoder in terms of the LLRs can now be calculated as follows. First, the soft output from the FDE-TDDF in the  $i$ -th iteration can be written as,

$$\tilde{x}_m^{(i)} = \gamma^{(i)} x_m + \epsilon_m^{(i)} \quad (17)$$

where

$$\gamma^{(i)} = \frac{1}{Q} \sum_{l=0}^{Q-1} w_l^{(i)} h_l \quad (18)$$

$$\begin{aligned} E(|\epsilon_m^{(i)}|^2) &= \frac{1}{Q} \sum_{l=0}^{Q-1} |\alpha_l^{(i)}|^2 \left( \frac{\sigma^2}{|h_l|^2 + \sigma^2} + \left( \frac{\bar{\beta}^{(i)}}{(\rho^{(i)})^2} - 1 \right) \right) \\ &\quad - \frac{\bar{\beta}^{(i)}}{(\rho^{(i)})^2} + 2\gamma^{(i)} - |\gamma^{(i)}|^2 \end{aligned} \quad (19)$$

Assuming that the  $\epsilon_m^{(i)}$  is a zero-mean Gaussian random variable, the extrinsic information can be expressed as,

$$\lambda_m^e = \frac{2\tilde{x}_m^{(i)} \gamma^{(i)}}{E(|\epsilon_m^{(i)}|^2)} \quad (20)$$

Next, we consider the generation of soft feedback decisions for the FDE-TDDF using the extrinsic information provided

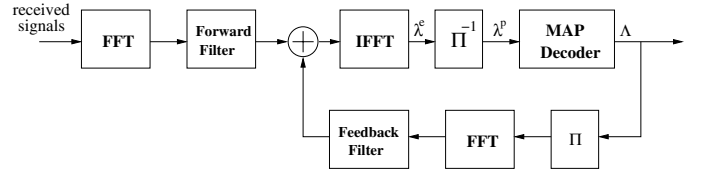


Fig. 4. Iterative FDE-FDDF equalizer

by the decoder. As in [6] [13], the soft feedback decisions can be expressed as  $\hat{x}_m = \tanh(0.5\lambda_m^p)$ . The reliability value ( $\rho$ ) can then be calculated using the following approximation,

$$\begin{aligned} \rho_m &= E(\hat{x}_m x_m^*) = E(E(x_m) x_m^*) \\ &= E(x_m) E(x_m)^* = |\hat{x}_m|^2 \\ \rho &= \frac{1}{Q} \sum_{m=0}^{Q-1} \rho_m \end{aligned} \quad (21)$$

where we have used the approximation  $E(x_m) = \hat{x}_m$ , which is a common assumption in various turbo detection techniques [13]. It turns out that the expression (21) used for calculating  $\rho$  is identical to the definition of  $\beta$ , so we may henceforth assume that  $\beta = \rho$ .

#### B. Iterative Frequency Domain Equalization with frequency-domain soft-decision feedback (FDE-FDDF-soft)

The iterative frequency domain equalizer with hard decision feedback in the frequency domain (FDE-FDDF) was proposed in [3]-[5]. The same equalizer based on soft decision feedback was also recently studied in [4]. Like the FDE-TDDF, FDE-FDDF is particularly suitable for SM systems since the complexity is comparable to equalizing an OFDM system. In this section, we consider the FDE-FDDF in conjunction with the decoding operation in the iterative process (see Figure 4). Moreover, the treatment of the FDE-FDDF-soft here is slightly different from those in [4] and [5], as we derive the filter coefficients based on the MMSE criterion in the *time domain*, as opposed to maximizing the SINR over the received signals in each frequency bin. Hence, the method proposed here is essentially equivalent to the FDE-TDDF described in the previous section and they are both based on the same criterion (in the time domain) using the decision feedback equalizer structure.

According to Figure 4, the outputs of the FDE-FDDF can be expressed as (for the  $i$ -th iteration),

$$\tilde{\mathbf{x}}^{(i)} = \mathbf{F}^H (\mathbf{W}^{(i)} \mathbf{y} - \mathbf{B}^{(i)} \mathbf{F} \tilde{\mathbf{x}}^{(i)}) \quad (22)$$

where  $\mathbf{W}^{(i)} = \text{diag}\{w_0^{(i)}, w_1^{(i)}, \dots, w_{Q-1}^{(i)}\}$  and  $\mathbf{B}^{(i)} = \text{diag}\{b_0^{(i)}, b_1^{(i)}, \dots, b_{Q-1}^{(i)}\}$  are the forward and backward filter coefficients, respectively<sup>1</sup>. The block-averaged mean square

<sup>1</sup>It can be shown that the optimization problem presented here is mathematically equivalent to the FDE-TDDF (9) if we include  $g_0^{(i)}$  in  $\mathbf{g}^{(i)}$ , with the constraint, via Lagrange multiplier, that  $g_0^{(i)} = 0$ .

error can now be expressed as,

$$\begin{aligned} \frac{1}{Q} \sum_{m=0}^{Q-1} E|\tilde{x}_m^{(i)} - x_m|^2 &= \frac{\sigma^2}{Q} \sum_{l=0}^{Q-1} |w_l^{(i)}|^2 + \frac{1}{Q} \sum_{l=0}^{Q-1} |w_l^{(i)}|^2 |h_l|^2 \\ &\quad - \frac{2}{Q} \text{Re}\left\{ \sum_{l=0}^{Q-1} w_l^{(i)} h_l [1 + b_l^* \rho] \right\} + \frac{\rho}{Q} \sum_{l=1}^{Q-1} |b_l|^2 \\ &\quad + \frac{2}{Q} \text{Re}\left\{ \sum_{l=0}^{Q-1} b_l \right\} + 1 \end{aligned} \quad (23)$$

where we have assumed that  $|\hat{x}_m^{(i)}|^2 = \bar{\beta} = \rho, \forall m$ . To avoid self-subtraction of the desired symbol by its previous estimate, the mean square error is minimized with respect to  $w_l^{(i)}, l = 0 \dots Q-1$  and  $b_l^{(i)}, l = 0 \dots Q-1$  subject to the constraint  $\sum_{l=0}^{Q-1} b_l = 0$ . Following the same procedure as in the FDE-TDDF, the forward filter's coefficients are found to be,

$$w_l^{(i)} = \frac{h_l^* (1 + b_l)}{\sigma^2 + |h_l|^2}, \quad l = 0 \dots Q-1 \quad (24)$$

Substituting the above values into (23) and by using the Lagrange multiplier method, the backward filter's coefficients are found to be,

$$b_l^{(i)} = \frac{\lambda(\sigma^2 + |h_l|^2) - \sigma^2}{(\sigma^2 + |h_l|^2) - \rho|h_l|^2} \quad (25)$$

where

$$\lambda = \sigma^2 \frac{\sum_{l=0}^{Q-1} \frac{1}{(\sigma^2 + |h_l|^2) - \rho|h_l|^2}}{\sum_{l=0}^{Q-1} \frac{\sigma^2 + |h_l|^2}{(\sigma^2 + |h_l|^2) - \rho|h_l|^2}} \quad (26)$$

The inputs to the decoder, in terms of the LLRs, can be expressed as,

$$\tilde{x}_m^{(i)} = \gamma^{(i)} x_m + \epsilon_m^{(i)} \quad (27)$$

where  $\gamma^{(i)} = \frac{1}{Q} \sum w_l h_l$ , and

$$\begin{aligned} E(|\epsilon_m^{(i)}|^2) &= \frac{\sigma^2}{Q} \sum_{l=0}^{Q-1} \frac{|1 + b_l \rho|^2}{\sigma^2 + |h_l|^2} + \frac{1}{Q} (\rho - \rho^2) \sum_{l=0}^{Q-1} |b_l|^2 \\ &\quad - 1 + 2\gamma^{(i)} - (\gamma^{(i)})^2 \end{aligned} \quad (28)$$

### C. Iterative Frequency Domain Equalization with frequency-domain hard-decision feedback (FDE-FDDF-hard)

For completeness, we also include the results obtained in [3] for the FDE-FDDF with hard decision feedback, and apply the results with decoding. We first express the SINR at the output of the FDE-FDDF as (29), suppressing the iteration number. The forward filter and backward filter coefficients, which maximize the SINR, are obtained as follows [3],

$$w_l = \frac{\text{SNR} \cdot h_l^*}{1 + \text{SNR}(1 - \rho^2) \cdot (|h_l|^2)} \quad (30)$$

where  $\text{SNR} = E_s/N_o$ , and

$$b_l = \rho(w_l h_l - \gamma) \quad (31)$$

The inputs to the decoder, in terms of the LLRs, can be expressed as,

$$\lambda_m^e = \frac{2\tilde{x}_m \text{SNR}}{\gamma} \quad (32)$$

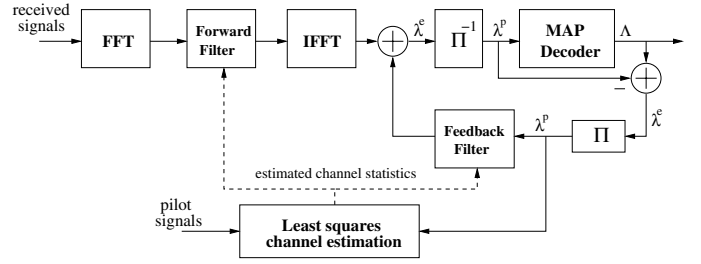


Fig. 5. Iterative FDE-TDDF-CE equalizer

Next, the hard feedback decisions for the FDE-FDDF can be generated using the information provided by the decoder. However, unlike the FDE-TDDF, simulation study has shown that using the APP ( $\Lambda_m$ ) from the decoder yields better performance than using the extrinsic information ( $\lambda_m^e$ ). This is due to the fact that the MAP decoder algorithm minimizes the bit error probability (BEP) with the hard outputs based on APP. The hard decisions can be simply expressed as  $\hat{x}_m = \text{sign}\{\Lambda_m\}$ . For the reliability coefficient ( $\rho$ ) associated with the hard outputs, it can be shown that [5],

$$\rho = 1 - 2P_e \quad (33)$$

where  $P_e$  is the average BEP.

## IV. TURBO FREQUENCY DOMAIN EQUALIZATION WITH CHANNEL ESTIMATION

In this section, a more realistic scenario where perfect channel knowledge is not available at the receiver is tackled. Pilot-assisted channel estimation over a single block is considered in conjunction with the turbo equalization based on the FDE-TDDF-soft<sup>2</sup>. The proposed structure is depicted in Figure 5, in which it can be seen that the channel estimation is an integral part of the turbo process. During each iteration, channel estimation is accomplished by utilizing the combination of known pilot signals and the current estimates of the data symbols obtained from the decoder in the previous iteration<sup>3</sup>. The outputs from the channel estimator, i.e., the estimated channel coefficients, are fed to the forward and backward filters. As the equalizer delivers new information to the decoder, the turbo process continues as the outputs from the decoder will be fed to both the channel equalizer and the channel estimator again in the next iteration.

Unlike the traditional turbo equalization methods, a novel feature is incorporated in our proposed method, which is now explained as follows. During each iteration, the unknown channel coefficients are treated as random quantities by the FDE-TDDF, and the filter coefficients are derived according to the mean and the variance of these channel coefficients. The channel estimator estimates the mean and the variance of the channel coefficients with the help of the pilot and the data symbols. This method is therefore different from traditional

<sup>2</sup>Although not considered further here, the channel estimation process could be improved by weighted averaging over successive blocks. Restricting estimation to a single block would be required if there is frequency-hopping between blocks.

<sup>3</sup>If pilot symbols are transmitted only at the beginning of a sequence of blocks, each subsequent block could use the previous block's channel estimate instead of pilot symbols in the first iteration.

$$\text{SINR} = \frac{|\gamma|^2}{\frac{1}{Q} \sum_{l=0}^{Q-1} (|h_l|^2 \sigma^2 + (1 - \rho^2) |w_l h_l - \gamma|^2 + |b_l - \rho(w_l h_l - \gamma)|^2)}, \quad \text{where} \quad \gamma = \frac{1}{Q} \sum_{l=0}^{Q-1} w_l h_l \quad (29)$$

iterative approaches where the channel coefficients are treated as deterministic and their estimated values are taken as true values in deriving the filter coefficients. As the variance of the channel coefficient can be considered as the channel estimation error level, the new method may mitigate error propagation by taking into account the potential error level fed into the equalizer.

#### A. Least squares channel estimation

Channel estimation is conducted in the time domain based on the least square criterion, and the knowledge of the maximum channel length ( $L$ ) is assumed to be available or else it is assumed to be equal to the length of the cyclic prefix ( $N_{cp}$ ). It turns out that time-domain channel estimation can exploit this knowledge to yield more accurate channel estimates. The resulting complexity in implementing these matrix time domain equations is high (on the order of  $N_{cp}^3$  complex multiplies). An alternative method with significantly less complexity is to employ least mean square (LMS) iteration with overlap-save frequency domain methods. Blocks of length  $Q$  would be subdivided into overlapping sub-blocks of length  $2N_{cp}$ , and the number of iterations would be on the order of  $Q/N_{cp}$ . Each overlap-save iteration would require on the order of  $10N_{cp} \log_2 N_{cp}$  complex multiplies and adds [14]. But for simplicity in illustrating our new approach, only time-domain channel estimation is investigated in the sequel.

Prior to the first iteration, when the estimates of data symbols are unavailable, the observation window is based on  $\mathbf{r}_p$ , which contains only the pilot signals. The received signals in the time domain can be expressed as,

$$\mathbf{r}_p = \mathbf{\Gamma}_p \mathbf{h}_t + \mathbf{n}_p \quad (34)$$

where  $\mathbf{\Gamma}_p$  is a  $M \times N_{cp}$  Toeplitz matrix with the first row and the first column given by the cyclic prefix symbols  $[p_0 p_{M-1} p_{M-2} \cdots p_{M-N_{cp}+1}]$  and training symbols  $[p_0 p_1 \cdots p_{M-1}]^T$ , respectively. Also,  $\mathbf{h}_t = [h(0), h(1), \cdots, h(N_{cp} - 1)]$  denotes the time domain channel impulse response vector. The channel impulse response estimates are given by,

$$\hat{\mathbf{h}}_t = (\mathbf{\Gamma}_p^H \mathbf{\Gamma}_p)^{-1} \mathbf{\Gamma}_p^H \mathbf{r}_p \quad (35)$$

$$= \mathbf{h}_t + (\mathbf{\Gamma}_p^H \mathbf{\Gamma}_p)^{-1} \mathbf{\Gamma}_p^H \mathbf{n}_p \quad (36)$$

which are interpreted as the mean values of the channel coefficients by the FDE-TDDF. The channel covariance matrix can be approximated by,

$$E\{(\hat{\mathbf{h}}_t - \mathbf{h}_t)(\hat{\mathbf{h}}_t - \mathbf{h}_t)^H\} = \sigma^2 (\mathbf{\Gamma}_p^H \mathbf{\Gamma}_p)^{-1} \quad (37)$$

In the frequency domain, the channel estimates can be obtained by  $\hat{\mathbf{h}} = \mathbf{F} \hat{\mathbf{h}}_t$ . The frequency domain channel coefficients all have the same covariance given by

$$E|h_l - \hat{h}_l|^2 = \sigma^2 \text{trace}\{(\mathbf{\Gamma}_p^H \mathbf{\Gamma}_p)^{-1}\}, \quad l = 0 \cdots N - 1 \quad (38)$$

For simplicity, we neglect the cross-correlation between the channel coefficients as they are not required in the computations of the filter coefficients in the FDE-TDDF.

During the turbo iterations when the data symbol estimates ( $\hat{x}_m, m = 0 \cdots N - 1$ ) are utilized for the channel estimation, the observation window is enlarged to cover the entire block (see Figure 1),

$$\mathbf{r} = \mathbf{\Gamma} \mathbf{h} + \mathbf{n} \quad (39)$$

where  $\mathbf{\Gamma}$  is a  $Q \times N_{cp}$  Toeplitz matrix with the first row given by  $[p_0 p_{M-1} p_{M-2} \cdots p_{M-N_{cp}+1}]$  and the first column given by  $[p_0 \cdots p_{M-1}, \hat{x}_0 \cdots \hat{x}_{N-1}, p_{M-N_{cp}+1}, \cdots, p_{M-1}]^T$ , respectively. Likewise, the least square estimation can be expressed as follows,

$$\hat{\mathbf{h}}_t = (\mathbf{\Gamma}^H \mathbf{\Gamma})^{-1} \mathbf{\Gamma}^H \mathbf{r} \quad (40)$$

and the channel covariance matrix is given by  $\sigma^2 (\mathbf{\Gamma}^H \mathbf{\Gamma})^{-1}$ .  $\hat{\mathbf{h}}_t$  is the  $N_{cp}$ -dimensional channel impulse response estimate. It is zero-padded to length  $Q$  and subjected to an FFT to get the  $Q$ -dimensional frequency response estimate  $\hat{\mathbf{h}}$ . The frequency domain channel coefficients all have the same covariance given by

$$E|h_l - \hat{h}_l|^2 = \sigma^2 \text{trace}\{(\mathbf{\Gamma}^H \mathbf{\Gamma})^{-1}\}, \quad l = 0 \cdots Q - 1 \quad (41)$$

#### B. Frequency domain equalization with time-domain decision feedback and channel estimation (FDE-TDDF-CE)

When channel estimation is considered, frequency domain equalization with time-domain decision feedback (FDE-TDDF-soft) is employed in the proposed turbo process. The extension to FDE-FDDF-soft is in principle straightforward. Here, we derive the FDE-TDDF's filter coefficients taking into account the random channel coefficients and the information provided by the channel estimator. The derivation is identical to that in section III-A, except that, the expected squared error value (MMSE) is taken over the noise, the interference, and the random frequency domain channel coefficients whose first and second order statistics are given by ,

$$\begin{aligned} E\{h_l\} &= \hat{h}_l \\ E\{|h_l - E\{h_l\}|^2\} &= E\{|h_l - \hat{h}_l|^2\} \end{aligned} \quad (42)$$

where  $l = 0, \cdots, N + N_{cp} - 1$ . The above values can be found from (40) and (41). The MMSE can now be written as (suppressing the iteration index and using the approximation that  $\beta = \rho$ ),

$$\begin{aligned} E(|e_m|^2) &= \frac{1}{N} \sum_{l=0}^{N-1} |w_l|^2 (|\hat{h}_l|^2 + \Delta) - \\ &\frac{2}{N} \text{Re}\left\{ \sum_{l=0}^{N-1} w_l \hat{h}_l \left[ 1 + \rho \sum_{k=1}^{N-1} g_k \exp(j2\pi \frac{kl}{N}) \right] \right\} \\ &+ \sum_{k=1}^{N-1} \rho |g_k|^2 + 1 + \frac{\sigma^2}{N} \sum_{l=0}^{N-1} |w_l|^2 \end{aligned} \quad (43)$$

where, for simplicity, we denote the  $E\{|h_l - \hat{h}_l|^2\}$  by  $\Delta$ . The optimal forward filter coefficients can be subsequently obtained as,

$$w_l = \frac{\hat{h}_l^* \left(1 + \rho^* \sum_{k=1}^{N-1} g_k^* \exp(j2\pi \frac{kl}{N})\right)}{(|\hat{h}_l|^2 + \Delta) + \sigma^2} \quad (44)$$

Substituting the forward filter coefficients into (43), the MMSE becomes,

$$E(|e_m|^2) = \frac{1}{N} \sum_{l=0}^{N-1} |\alpha_l|^2 \left( \xi_l + \left(\frac{1}{\rho} - 1\right) \right) + \left(1 - \frac{1}{\rho}\right) \quad (45)$$

where

$$\xi_l = \frac{\sigma^2 |\hat{h}_l|^2 + (\sigma^2 + \Delta)^2}{(|\hat{h}_l|^2 + \sigma^2 + \Delta)^2} \quad (46)$$

and the backward filter coefficients are given by (14) using a new expression of  $v_k$ ,

$$v_k = \frac{1}{N} \sum_{l=0}^{N-1} \left\{ \xi_l + \left(\frac{1}{\rho} - 1\right) \right\} \exp(-j2\pi lk/N) \quad (47)$$

Next, it is necessary to calculate the extrinsic information sent to the decoder when the channel coefficients are treated as random. Recall that the outputs from the equalizer can be given as,

$$\tilde{x}_m = \gamma x_m + \epsilon_m \quad (48)$$

Since, unlike the case with deterministic channel coefficients, both  $\gamma$  and  $\epsilon_m$  are random, and therefore, a closed-form expression for the LLR is difficult to be obtained. For simplicity, it is therefore assumed that  $\gamma$  is replaced with  $\hat{\gamma}$  (i.e., the mean value of  $\gamma$ ), while  $\epsilon$  remains random. It can be shown that, by averaging over the random channel coefficients,

$$E(|\epsilon_m|^2) = \frac{1}{N} \sum_{l=0}^{N-1} |\alpha_l|^2 \left( \xi_l + \left(\frac{1}{\rho} - 1\right) \right) + \left(1 - \frac{1}{\rho}\right) - \frac{1}{\rho} + 2\hat{\gamma} - |\hat{\gamma}|^2 \quad (49)$$

where  $\xi_l$  is defined in (46). Hence, the LLR can be easily expressed as,

$$\lambda_m^e = \frac{2\tilde{x}_m \hat{\gamma}}{E(|\epsilon_m|^2)} \quad (50)$$

## V. SIMULATION RESULTS

The bit error rate (BER) performance of the proposed turbo frequency domain equalization techniques are simulated based on the static SUI-5 model [15] and the static urban macro channel [16] using the simulation parameters listed in Table 2. The rate 1/2 convolutional code with generator (133, 171) is employed, and the encoded binary symbols are interleaved randomly and mapped to the QPSK constellation.

First, assume that perfect channel knowledge is available at the receiver. In Figure 6, the performance curves for the FDE-TDDF-soft, the FDE-FDDF-soft and the FDE-FDDF-hard in the SUI-5 channel are shown. The number of iterations is set to three. In this case, all three equalizers yield similar BER performance. The gain based on these iterative methods is about 1 dB in comparison with the basic linear FDE method

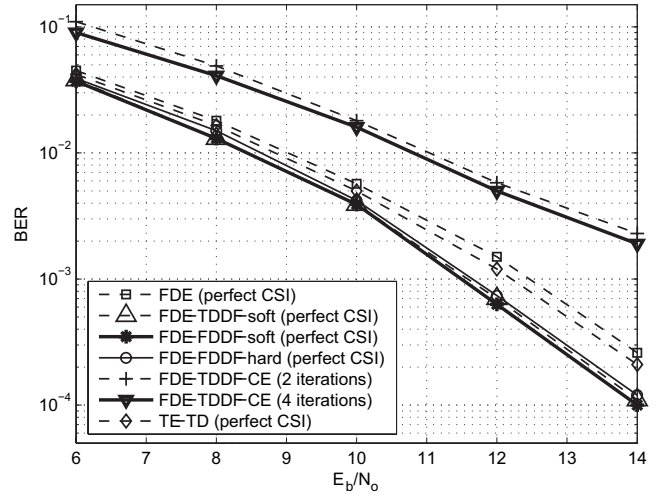


Fig. 6. Performance of various turbo frequency domain equalization techniques (SUI-5 channel)

TABLE I  
URBAN MACRO CHANNEL [16]

Delays[μs]	0.0 0.01 0.03 0.36 0.37 0.385 0.25 0.26 0.28 1.04
	1.045 1.065 2.73 2.74 2.76 4.6 4.61 4.625
Power[dB]	-3.0 -5.22 -6.98 -5.22 -7.44 -9.2 -4.72 -6.94 -8.7 -8.19
	-10.41 -12.17 -12.05 -14.27 -16.03 -15.50 -17.72 -19.48

(i.e., FDE-TDDF/FDDF with no iterations). In Figure 6, we also include the results using a standard time-domain turbo equalization (TE-TD) method as proposed in [6] where this method is named the MMSE-LE. The number of time domain taps in this MMSE-LE equalizer is ( $N_{FF} = 60, N_{BF} = 10$ ), which is designed to capture the paths with rather long delays. Although with large number of taps, its performance is slightly inferior to both the FDE-TDDF and FDE-FDDF, as indicated in Figure 6. The main reason is that FDE-TDDF/FDDF is block-iterative, meaning that the entire block is processed in each iteration to yield better equalization performance. In the same figure, we also include the results for the FDE-TDDF-CE. With iterative channel estimation, the performance gain remains satisfactory, as it is only 1 – 2 dB from the basic linear FDE method with perfect CSI.

Next, Figure 7 contains similar results for the urban macro channel model, where the results for TE-TD are also included for comparison. In this case, both iterative methods yield about 2 – 3 dB performance gain over the basic linear FDE method and about 1.5 – 2.5 dB for the TE-TD method. The relatively large improvement using the iterative methods in the urban macro channel may be due to its rich multipath profile with excessive number of multipaths. On the other hand, the SUI-5 channel has a rather poor multipath profile, yielding less improvement using the iterative methods. Finally, the results based on channel estimation, i.e., the FDE-TDDF-CE, are also shown. The performance is superior as the FDE-TDDF-CE performance is very close to the basic linear FDE method with perfect CSI. Notice that the loss due to imperfect CSI is only around 2 – 3 dB when compared with the FDE-TDDF with perfect CSI. We re-iterate that channel estimation is carried out

TABLE II  
TRANSMISSION PARAMETERS

Parameter	urban macro	SUI-5
Symbol rate	16.25 Msps	5 Msps
Symbols per block	832	512
Prefix symbols	80	60
Pulse filtering	Root-raised Cosine	Root-raised Cosine
Roll-off	0.23	0.23
Pilot sequence length	128	64
Doppler frequency	0	0

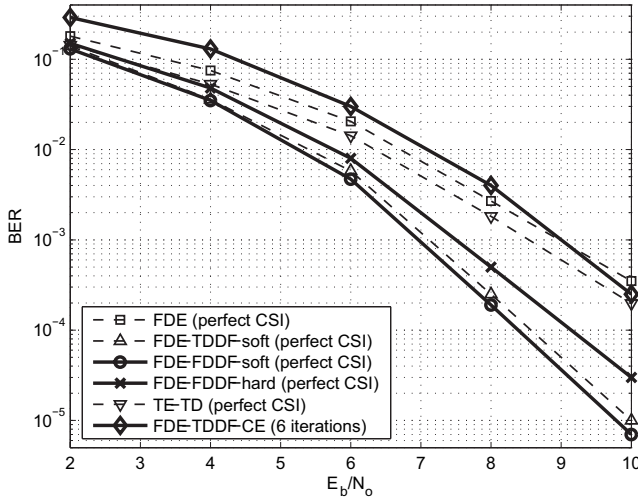


Fig. 7. Performance of various turbo frequency domain equalization techniques (urban macro channel)

within one block. Weighted averaging over successive blocks would yield even better performance.

The complexity of the FDE-TDDF and FDE-FDDF can be estimated as follows. For the FDE-TDDF, both the forward and feedback filtering require only element-wise multiplications. Hence, the main computational burden lies in the derivation of the filters' coefficients. For the forward filter, it is dominated by the  $\mathcal{O}(Q)$  complex multiplications and additions. For the feedback filter's coefficients,  $\mathcal{O}(Q^2)$  complex multiplications and additions are required due to the calculations involved in (15) and (A-5). Taking the FFTs into account, the total complexity of the FDE-TDDF is therefore  $\mathcal{O}(Q^2)$ . For the FDE-FDDF, the complexity is lowered than the FDE-TDDF, as both filters' coefficients require only  $\mathcal{O}(Q)$  complex multiplications and additions. The total complexity is therefore dominated by the FFTs and is shown to be  $\mathcal{O}(Q \log Q)$ . As mentioned in Section IV A, with overlap-save frequency domain processing, channel complexity would be  $\mathcal{O}(Q \log N_{cp})$ .

## VI. CONCLUSIONS

A set of new turbo frequency domain equalization (TFDE) techniques suitable for single-carrier signal detection is investigated in this paper. In TFDE, the turbo principle is applied jointly to frequency domain equalization, decoding and single-block channel estimation. In particular, two novel frequency domain equalizers, one based on time domain

decision feedback (FDE-TDDF) and one based on frequency domain decision feedback (FDE-FDDF) are studied in detail. Furthermore, when pilot-assisted channel estimation is considered, the frequency domain equalizer FDE-TDDF has been successfully modified to mitigate the estimation error effect in the equalization process. In the BER simulation, all these techniques are shown to have the advantages of low complexity while producing excellent performance in severe frequency-selective fading channels.

## APPENDIX I

First, we define the following  $Q \times Q$  matrix as follows (for simplicity, the index ( $i$ ) is suppressed),

$$\mathcal{V} = \begin{bmatrix} v_0 & v_{-1} & \cdots & v_{-(Q-1)} \\ v_1 & v_0 & v_{-1} & \ddots \\ \vdots & \ddots & \ddots & \ddots \\ v_{Q-1} & \ddots & \ddots & v_0 \end{bmatrix} \quad (\text{A-1})$$

$$= \begin{bmatrix} & & & v_{-(Q-1)} \\ & \mathbf{V} & & v_{-(Q-2)} \\ & & & \vdots \\ v_{Q-1} & \cdots & \cdots & v_0 \end{bmatrix} \quad (\text{A-2})$$

It can be shown that  $\mathcal{V}$  is a circulant matrix and its inverse can be expressed as,

$$\mathcal{V} = \mathbf{F}(\text{diag}\{\psi_1, \dots, \psi_Q\})\mathbf{F}^H \quad (\text{A-3})$$

$$\mathcal{V}^{-1} = \mathbf{F}(\text{diag}\{1/\psi_1, \dots, 1/\psi_Q\})\mathbf{F}^H \quad (\text{A-4})$$

Hence, the inverse of  $\mathcal{V}$  is simply a circulant matrix with the first column given by  $\frac{1}{Q}\mathbf{F}[1/\psi_1, \dots, 1/\psi_Q]^T$ . And the inverse of  $\mathbf{V}$  can be easily obtained from the following relationship,

$$\mathbf{V}^{-1} = [\mathcal{V}^{-1}]_{1:Q-1,1:Q-1} - [\mathcal{V}^{-1}]_{1:Q-1,Q}[\mathcal{V}^{-1}]_{Q,1:Q-1}/[\mathcal{V}^{-1}]_{Q,Q} \quad (\text{A-5})$$

where the notation  $[\mathbf{A}]_{a:b,c:d}$  denotes the submatrix formed by the  $a$ th to  $b$ th rows and the  $c$ th to  $d$ th columns in matrix  $\mathbf{A}$ . The  $(Q, Q)$ -th element of the matrix  $\mathcal{V}^{-1}$  is denoted as  $[\mathcal{V}^{-1}]_{Q,Q}$ .

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