FREQUENCY-DOMAIN EQUALIZATION OF MOBILE RADIO AND TERRESTRIAL BROADCAST CHANNELS

H. Sari, G. Karam, and I. Jeanclaude
SAT, Division Télécommunications
11, rue Watt, B.P. 370, 75626 Paris Cedex 13, France

Abstract: For mobile radio and terrestrial broadcast applications, we compare orthogonal frequency-division multiplexing (OFDM) and single-carrier transmission with frequency-domain equalization. With respect to our earlier results, we include channel coding and frequency-domain interleaving which are necessary for OFDM systems on multipath fading channels characterized by deep notches in the signal spectrum. Our results indicate that a single-carrier system with a frequency-domain equalizer achieves similar performance to coded-OFDM, while alleviating the carrier synchronization and nonlinear distortion problems inherent to this technique.

INTRODUCTION

Mobile radio and terrestrial broadcast channels are affected by multipath fading and require some form of channel equalization to compensate for the resulting signal distortion. The traditional approach to compensate for intersymbol interference (ISI) is time-domain equalization, which usually takes the form of a nonrecursive linear equalizer or a decision-feedback equalizer [1]. The equalizer incorporates an adaptation algorithm required to set the coefficients to their optimum values and update them as the channel varies.

Another approach which was adopted in the European digital audio broadcasting (DAB) project consists of orthogonal frequency-division multiplexing (OFDM) with frequency- and time-domain interleaving, and error correction coding [2]. In this approach, the incoming signal is transmitted using N equally-spaced carriers with a separation of 1/NT Hz, where T designates the symbol period. The number of carriers is chosen such that the channel transfer function is essentially constant over the bandwidth of an individual carrier. The effect of multipath fading then reduces to one attenuation and one phase rotation per carrier. Channel equalization takes the form of a complex multiplier bank in front of the decision circuit at the receiver. With PSK signal formats, amplitude equalization of the channel is not needed in OFDM systems, and furthermore, the phase response can be differentially equalized [3]. The symbols modulating a carrier located in a spectral notch will be strongly affected by additive noise, and channel coding and frequency-domain interleaving are needed to protect them from transmission errors. This technique, called coded-OFDM (COFDM) also forms the basis of all European projects for digital terrestrial TV broadcasting [4] - [6].

In a recent paper [7], the present authors pointed out an important fact that with exception of PSK signal sets, OFDM signaling does not solve the channel equalization problem, but only shifts it from the time domain to the frequency domain. This paper also highlighted a strong analogy between frequency-domain equalization in single-carrier systems and equalized OFDM systems. Another observation was that frequency-domain equalization in single-carrier systems forms an alternative to OFDM signaling on channels with long impulse responses that are difficult to equalize in the time-domain. Next, it was shown in [8] that in the absence of channel coding, single-carrier transmission with a frequency-domain equalizer substantially outperforms equalized OFDM systems, and this result was supported by analytic arguments related to the decision process.

The purpose of the present paper is to extend our previous results to the coded case. In COFDM systems, we also investigate the gain achieved using channel state information (CSI) in the decoder. Simulation results are given for both static ISI channels as well as nonstationay mobile radio channels.

The paper is organized as follows: In the next section, we briefly recall OFDM signaling. In the following section, we present frequency-domain equalization in single-carrier systems and contrast it to OFDM. Next, before presenting our simulation results, we discuss COFDM with frequency-domain equalization or weighted decoding using CSI.

A BRIEF REVIEW OF OFDM

OFDM is a multicarrier transmission technique based on the discrete Fourier transform (DFT). A simplified block diagram of an OFDM system is shown in Fig. 1. Serial-to-parallel and parallel-to-serial conversions inherent in this scheme are dropped for convenience. The transmitted signal is of the form

$$s(t) = \mathrm{Re}\left\{ \sum_{n=-\infty}^{+\infty} b_n f(t - nT) e^{j(\omega_0 t + \varphi)} \right\} \tag{1}$$

where Re(.) denotes real part, f(t) designates the transmit filter impulse response, T is the symbol period, ω_0 is the carrier radian frequency, φ is the carrier phase, and the transmitted $\{b_n\}$ sequence is obtained from the input information sequence $\{a_n\}$ through an N-point inverse DFT (IDFT). In order to distinguish successive DFT blocks, we write the index n in (1) as n = m.N + k with k = 0, 1, ..., N-1, and m integer. The $\{b_n\}$ sequence in (1) is then given by

$$b_k(m) = \frac{1}{N} \sum_{\ell=0}^{N-1} a_\ell(m).e^{j2\pi k\ell/N}, \qquad k = 0, 1, \ldots, N-1. \tag{2}$$

In this 2-index representation, $a_\ell(m)$ represents the ℓth input symbol of the mth IDFT block, and $b_k(m)$ is the kth output sample of the same block. After this transformation, the N parallel output samples are converted into a serial form, lowpass filtered,

0-7803-1820-X/94 $4.00

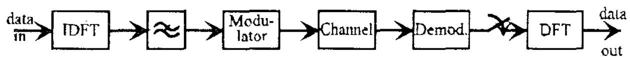

Fig. 1: Simplified block diagram of an OFDM system

and passed to a quadrature modulator which shifts the signal spectrum to center it on the center frequency $f_0 = \omega_0 / 2\pi$.

On the receiver side, the received signal is coherently demodulated, sampled at the symbol rate 1/T, and passed to a DFT operator which converts the signal back to the frequency domain. The demodulator comprises a lowpass filter which limits noise and interference from adjacent channels, without distorting the received signal.

Due to the IDFT operator, the N symbols per DFT block are transmitted as a frequency-division multiplex. The IDFT output has an infinite periodic spectrum whose period is the multiplex formed by the N symbols per block. The transmit filter has the task of selecting one period and suppressing all other periods of the infinite spectrum. This requires an ideal (brick wall) lowpass filter which is not realizable. In practice, the transmit filter is synthesized with some roll-off, and the carriers which fall in the roll-off region are set to zero and do not carry any information. In the sequel, we refer to these carriers as virtual carriers.

An OFDM system typically includes a "guard interval" between transmitted successive IDFT blocks. The guard interval is a circular prefix extension of the blocks at the IDFT operator output, and its length must be larger than the channel impulse response length. Under this condition, the linear convolution performed by the channel becomes identical to circular convolution in the discrete frequency domain using the DFT. Omitting the circular prefix or replacing it by another form of guard interval would significantly degrade performance, as demonstrated in [7]. On the receiver side, the circular prefix is dropped before passing the received signal samples to the DFT operator. Circular prefix extension at the transmitter and dropping at the receiver are not shown in Fig. 1 for simplicity.

We will now examine the channel equalization issue in OFDM. Let h(t) designate the channel impulse response and $H(\omega)$ its Fourier transform, i.e., the channel transfer function. If the number of carriers is sufficiently large, the channel transfer function becomes virtually nonselective within the bandwidth of each individual carrier. Focusing on one particular carrier, the influence of multipath fading reduces to an attenuation and a phase rotation.

Referring back to the channel transfer function $H(\omega)$, we let $H_k = \rho_k . e^{j\theta_k}$ designate its value within the bandwidth of the *k*th carrier. Equalization of the channel requires that at the DFT output in the receiver, the *k*th carrier signal be multiplied by a complex coefficient

$$C_k = 1 / H_k. \tag{3}$$

This is the result of an optimization based on the zero-forcing (ZF) criterion [1] which aims at canceling ISI regardless of the noise level. To minimize the combined effect of ISI and additive noise, the equalizer coefficients can be optimized under the minimum mean-square error (MMSE) criterion. This optimization yields

$$C_k = \frac{H_k^*}{|H_k|^2 + \sigma_n^2 / \sigma_a^2} \tag{4}$$

where σ_n^2 is the variance of additive noise, and σ_a^2 is the variance of the transmitted data symbols. Note that the MMSE solution reduces to the ZF solution for $\sigma_n^2 = 0$.

The ZF criterion does not have a solution if the channel transfer function has spectral nulls in the signal bandwidth. Inversion of the channel transfer function requires an infinite gain and leads to infinite noise enhancement at those frequencies corresponding to spectral nulls. In general, the MMSE solution is more efficient, as it makes a trade-off between residual ISI (in the form of gain and phase mismatchs) and noise enhancement. This is particularly attractive for channels with spectral nulls or deep amplitude depressions.

Channel equalization in OFDM systems thus takes the form of a complex multiplier bank at the DFT output in the receiver. If the modulation used is a PSK signal format, the channel does not need amplitude equalization, because the information is entirely carried by the signal phase. In addition, phase equalization can be made differential, provided that differential encoding is used at the transmitter. There are two possible techniques to differentially equalize the transmission channel. The first technique consists of using the point with the same index in the previous DFT block as phase reference. The second uses the received signal point on an adjacent carrier in the same DFT block as reference. Both techniques are described in some detail in [7]. The first differential channel equalization technique assumes that the channel response does not significantly vary over two consecutive DFT blocks, and is therefore sensitive to channel nonstationarity. The key assumption underlying the second technique is that there is little difference between the respective attenuations and phase rotations of two adjacent carriers, which is valid for large N. Clearly, the first technique is better suited to stationary or slowly time-varying channels, whereas the second is better suited to rapidly time-varying channels.

FREQUENCY-DOMAIN EQUALIZATION

Analyzing the operation principle of OFDM, the present authors noticed a striking resemblance to frequency-domain channel equalization for traditional single-carrier systems, a concept proposed over two decades ago [9]. Frequency-domain equalization is illustrated in Fig. 2a which shows the baseband-equivalent model of a single-carrier system employing this equalization technique. The received signal samples are passed to an N-point DFT, each output sample is multiplied by a complex coefficient C_i, and the output is passed to an IDFT to transform the signal back to the time domain. Now, if we take the system sketched in Fig. 2a and place it between an IDFT operator and a DFT operator, we obtain an OFDM system incorporating a frequency-domain equalizer. Obviously, the DFT and IDFT operators at the output end cancel each other, and the system simplifies to what we see in Fig. 2b. This is precisely the schematic diagram of the equalized OFDM system discussed in the previous section. Figs. 2a and 2b thus give evidence of the strong similarities of OFDM signaling and frequency-domain equalization in single-carrier systems. In both cases, time/-

frequency and frequency/time transformations are made. The difference is that in OFDM systems, both channel equalization and receiver decisions are performed in the frequency domain, whereas in single-carrier systems the receiver decisions are made in the time domain although channel equalization is performed in the frequency domain.

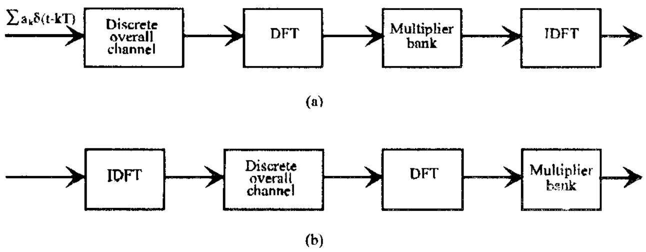

Fig. 2: Frequency-domain channel equalization. (a): Base band-equivalent model of a single-carrier system with a frequency-domain equalizer, (b): Baseband-equivalent model of an equalized OFDM system.

From purely channel equalization capability standpoint, both systems are equivalent, assuming they use the same DFT block length. They have, however, an essential difference: Since the receiver decisions in OFDM are independently made on different carriers, those corresponding to carriers located in a region with a deep amplitude depression will be unreliable. If we assume that one hundredth of the N modulated carriers are affected by a spectral null or a deep spectral notch, the system will have a residual error rate on the order of 10^{-2}. (This is due to the fact that the error probability of decisions corresponding to carriers located in a deep spectral notch will be close to 0.5.) On a static channel, one can get out of this problem by assigning "virtual carriers" to the frequency band with a spectral notch, but this technique does not lend itself to radio channels affected by time-varying multipath fading events. In that situation, the frequent errors randomly occurring in different parts of the signal spectrum need to be corrected using a powerful error correction coding technique.

Although the operation of a single-carrier system with a frequency-domain equalizer looks similar to that of OFDM, a closer inspection reveals that this technique substantially alleviates the detection problem. Once the channel is equalized in the frequency domain, the signal is transformed back to the time domain and the receiver decisions are based on the signal energy transmitted over the entire channel bandwidth. In other words, the SNR value which dictates performance (assuming that residual ISI is negligible) corresponds to the SNR averaged over the entire channel bandwidth. Consequently, the performance degradation due to a deep notch in the signal spectrum remains small with respect to that suffered by OFDM. The .implication of this is that on multipath fading channels, single-carrier systems with a frequency-domain equalizer substantially outperforms OFDM systems.

CODED OFDM

In the previous sections, the focus was on uncoded-OFDM in which the receiver decisions are made independently on each carrier. Our analysis suggested that without channel coding, OFDM is virtually unusable on multipath fading channels with deep notches occurring in the signal spectrum. It was indicated that performance of uncoded-OFDM is essentially dictated by the lowest SNR value in the signal bandwidth, whereas the IDFT operator in single-carrier systems performs perfect SNR averaging over the entire channel bandwidth.

From these considerations, it is clear that the OFDM technique requires channel coding to protect the system from transmission errors which would be frequently occurring in the absence of coding. In a block coding and "hard-decision" decoding approach, a natural choice is to select the code block length equal to the DFT block length of the OFDM system. An OFDM system with such a coding and decoding scheme will be efficient if the code used can correct the errors per block with a high probability.

A better coding approach consists of convolutional coding with frequency-domain interleaving and "soft-decision" decoding. This allows to perform SNR averaging, and the resulting system approaches the performance of single-carrier transmission with frequency-domain equalization. What is needed is an interleaver which uniformly distributes the low-SNR samples over the channel bandwidth and a convolutional code with a large Hamming distance.

The receiver performs maximum-likelihood sequence decoding using the well-known Viterbi algorithm which searches for the most likely path (the path with the smallest metric, or Euclidian distance, from the received noisy and distorted signal) in the code trellis. In an equalized OFDM system, the branch metrics over the *n*th DFT block period are of the form

$$D(n) = \sum_k |y_k(n) - a_k(n)|^2 \tag{5}$$

where $\{y_k(n)\}$ represents the equalized signal sequence, and $\{a_k(n)\}$ is the data sequence associated to a particular path in the trellis.

A still better approach in COFDM systems is to use channel state information (CSI) in computing the branch metrics:

$$D'(n) = \sum_k |x_k(n) - \rho_k(n) a_k(n)|^2 \tag{6}$$

where $\{x_k(n)\}$ represents the unequalized signal sequence, and $\{\rho_k(n)\}$ is the sequence of channel attenuation parameters during the *n*th DFT block. The metric D'(n) can also be expressed as

$$D'(n) = \sum_k \rho_k^2(n) |y_k(n) - a_k(n)|^2, \tag{7}$$

a form closely related to the branch metrics of (5). This expression shows that the branch metrics computed using CSI can also be interpreted as the metrics computed using the equalized signal samples and weighting each local metric by the corresponding squared channel attenuation factor. In other words, a small weighting factor is associated to local metrics with low reliability, and a large weighting factor is associated to local metrics with high reliability.

Weighting can be interpreted as the dual of equalizing the channel in the sense that equalization consists of amplifying an attenuated received signal to match it to the nominal decision levels, whereas weighting consists of matching the decision levels to the received signal attenuation. Weighting clearly avoids the noise enhancement inherent to equalized OFDM systems and appears as the best strategy in branch metric computations.

COMPUTER SIMULATION RESULTS

Performance comparison of OFDM and single-carrier transmission was carried out using a number of channels. In this section, we first report the results corresponding to two static channels, which we refer to as Channel A and Channel B, respectively. Channel A corresponds to a mild amplitude distortion. Its discrete impulse response is represented by (0.04, -0.05, 0.07, -0.21, -0.5, 0.72, 0.36, 0.0, 0.21, 0.03, 0.07). Channel B represents a stronger amplitude distortion, and its transfer function has a 25 dB notch in the signal spectrum. Its discrete impulse response is (0.74, -0.42, 0.083, 0.049, -0.12, 0.01). The amplitude response of Channel B is shown in Figure 3, and that of Channel A can be found in Figure 6.4.8a on p. 573 of [1].

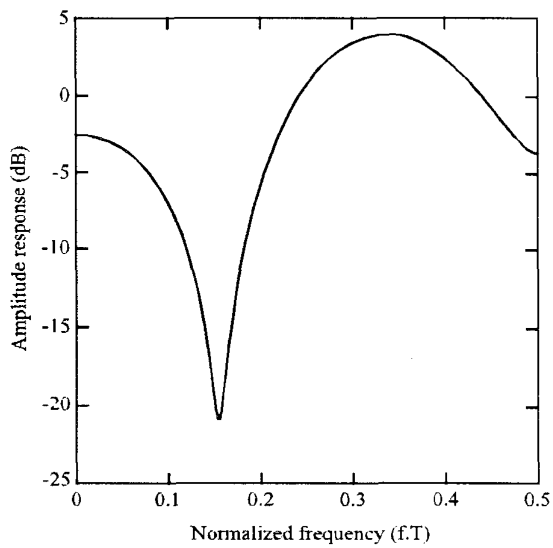

Fig. 3: Amplitude response of channel B

In these simulations, we used an OFDM system with N = 1024 carriers and a single-carrier system with a 1024-tap frequency-domain equalizer. In both cases, we used a circular prefix of minimum length, i.e., the prefix length was 10 for Channel A and 5 for Channel B. The equalizer optimization was performed under the MMSE criterion. (It was verified by means of computer simulations that this optimization leads to significantly better performance than the zero-forcing criterion.)

The first set of simulations were performed using uncoded systems. Next, we included the k = 7, rate-1/2 convolutional code which has become a "de-facto" industry standard for channel coding. In the coded case, the receiver comprised a "soft-decision" Viterbi algorithm for maximum-likelihood decoding. The interleaver used in COFDM was a block interleaver represented by a matrix of 16 columns and 8 rows, where the input symbols are written by rows and read by columns. The deinterleaver simply performs the inverse operation. With this interleaver/deinterleaver pair, two symbols transmitted at two adjacent frequencies are separated by 16 symbols at the Viterbi decoder input.

The simulation results are given in figure 4 for Channel A and in Figure 5 for Channel B. The dashed curves correspond to OFDM and the solid-line curves correspond to single-carrier transmission with frequency-domain equalization. Each figure also includes a dotted curve which corresponds to COFDM with weighted decoding. On both cannels, we observe that in the absence of channel coding, single-carrier transmission with frequency-domain equalization substantially outperforms OFDM signaling. The second basic observation is that the convolutional code used in the second simulation runs leads to a tremendous improvement, particularly with OFDM signaling. With convolutional coding, frequency-domain equalization, and maximum-likelihood decoding, performance of OFDM becomes very close to that of single-carrier transmission. Finally, with weighted maximum-likelihood decoding, COFDM leads to a slightly improved performance than single-carrier transmission.

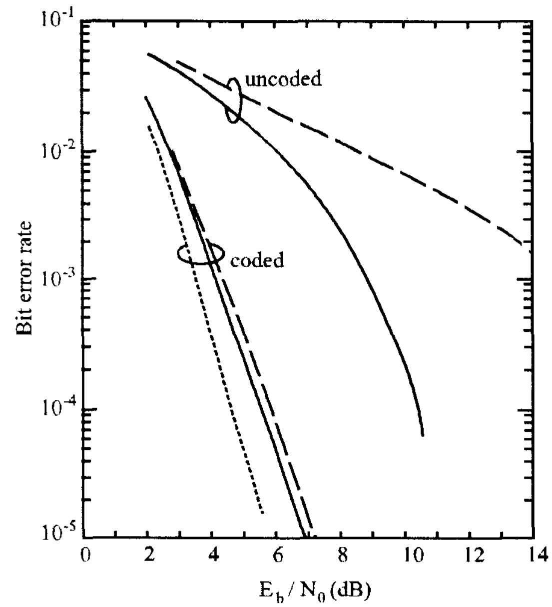

Fig. 4: Bit error rate performance of different systems on channel A

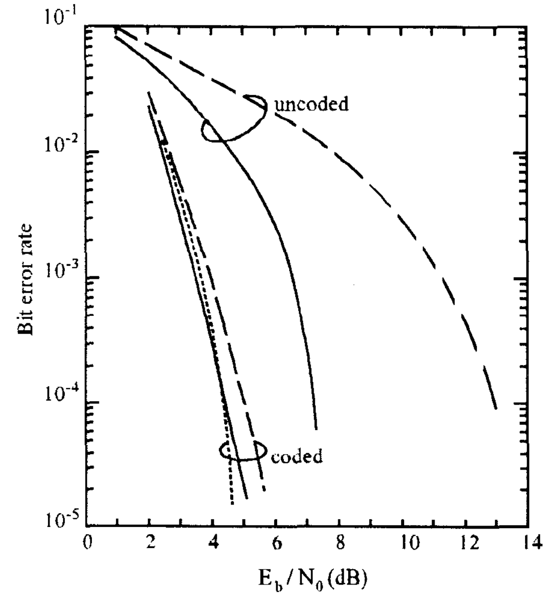

Fig. 5: Bit error rate performance of different systems on channel B

Next we simulated a nonstationary radio channel with an exponential multipath intensity profile which spans 7 symbol durations. The average power of the *k*th sample of the discrete impulse response is given by $P(k) = (1/\Delta).e^{-kT/\Delta}$, where Δ is calculated such that P(7) is 30 dB below P(0). Each of the 7 samples of the channel impulse response is the output of a low-pass filter driven by a complex zero-mean Gaussian noise. The input noise variance is normalized to unity, and the filter output is weighted so as to obtain the predetermined average power. The transfer function of the low-pass filters which generate the channel impulse response samples is an approximation of

$$F(f) = \begin{cases} \dfrac{1}{\pi f_d} \cdot \dfrac{1}{\sqrt{1-f^2/f_d^2}}, & |f| \le f_d \\ 0, & \text{elsewhere} \end{cases} \tag{8}$$

where f_d is the Doppler frequency which characterizes the degree of channel nonstationarity.

The simulations were carried out using a 128-carrier OFDM system and a single-carrier system with a 128-tap frequency-domain equalizer. A circular prefix of length 6 was inserted between transmitted 128-symbol blocks to solve the circular convolution problem. In both systems, the equalizer was optimized under the MMSE criterion.

In the simulation runs, identification of the channel transfer function was performed by transmitting one predetermined symbol block known from the receiver, once every 4 blocks. The results corresponding to $f_d.T = 2.5 \times 10^{-5}$ are depicted in Fig. 6. We can make the same observations here as for static channels, i.e., without channel coding, single-carrier QPSK significantly outperforms OFDM, but with coding, both systems give similar results. Note that the residual BER which appears in the case of uncoded systems is one order of magnitude lower with single-carrier QPSK than with OFDM.

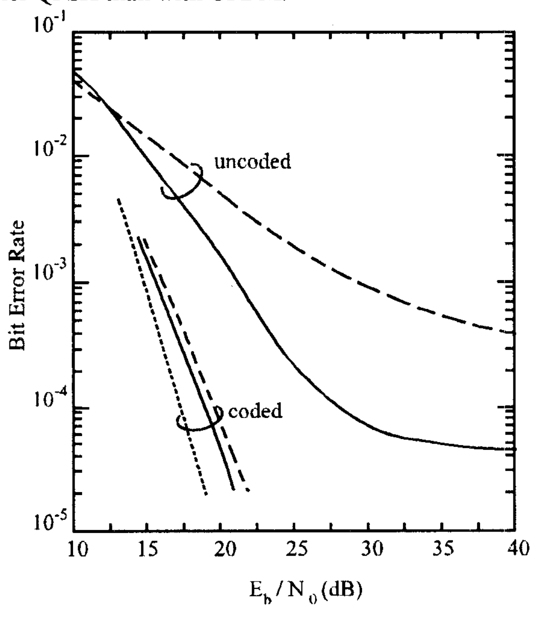

Fig. 6: Bit error rate performance of coded OFDM and single-carrier QPSK on nonstationary fading channels (channel identification every 4 frames)

CONCLUSIONS

This paper has analyzed and compared two potential transmission techniques for multipath propagation channels with long impulse responses that are difficult to equalize in the time-domain. It was shown that in the absence of channel coding, a single-carrier system with a frequency-domain equalizer substantially outperforms an OFDM system using the same DFT block length. With channel coding, interleaving, and maximum-likelihood decoding, OFDM approaches and eventually surpasses the performance of a single-carrier system with a frequency-domain equalizer, but the latter technique has the advantage of low sensitivity to nonlinear signal distortion, and significantly alleviates the carrier synchronization problems of OFDM pointed out in [7].

REFERENCES

[1] J.G. Proakis, "Digital Communications," McGraw Hill, New York, 1989.

[2] M. Alard and R. Lassalle, "Principles of modulation and channel coding for digital broadcasting for mobile receivers," EBU Review, no. 224, pp. 3-25, August 1987.

[3] S. B. Weinstein and P. M. Ebert, "Data transmission by frequency-division multiplexing using the discrete Fourier transform," IEEE Trans. Commun. Techn., vol. COM-19, no. 5, pp. 628-634, October 1971.

[4] U. Reimers, "European perspectives in digital television broadcasting – Conclusions of the Working Group in digital television," EBU Technical Review, no. 256, pp. 3-8, 1993.

[5] P. Appelquist, "HD-DIVINE, a Scandinavian terrestrial HDTV system," EBU Technical Review, no. 256, pp. 16-19, 1993.

[6] B. Marti et al., "European activities on digital television broadcasting – From company to cooperative projects," EBU Technical Review, no. 256, pp. 20-29, 1993.

[7] H. Sari, G. Karam, and I. Jeanclaude, "Channel equalization and carrier synchronization in OFDM systems," presented at the 1993 Tirrenia Int. Workshop on Digital Communications, September 1993, Tirrenia (Pisa), Italy.

[8] H. Sari, G. Karam and I. Jeanclaude, "An analysis of orthogonal frequency-division multiplexing for mobile radio applications," VTC'94 Conf. Rec., vol. 3, pp. 1635-1639, June 1994, Stockholm.

[9] T. Walzman and M. Schwartz, "Automatic equalization using the discrete Fourier domain," IEEE Trans. Inform. Theory, vol. IT-19, pp. 59-68, January 1973.