Linear Frequency Domain Equalization of SOQPSK-TG for Wideband Aeronautical Telemetry Channels

This work explores multipath interference mitigation for a single transmit antenna, single receive antenna system with SOQPSK-TG using a linear single-carrier frequency domain equalization (SCFDE) technique. Linear SCFDE with maximum likelihood (ML) channel estimation is shown to greatly improve system Bit Error Rates (BER). Simulation results also demonstrate that three samples per bit at the receiver are adequate to achieve the best BER performance.

I. INTRODUCTION

Aeronautical telemetry is the communication of vital airborne asset measurements through a wireless downlink channel from an airborne transmitter to a high-gain, tracking ground receiver. Successful experiments require aeronautical telemetry devices that can robustly operate at high wireless communication data rates (10–20 Mbit/s) under adverse physical layer conditions. These experiments are held at numerous test ranges which all affect physical layer communication with severe single-bounce multipath [1–3]. An aeronautical telemetry channel model generated using frequency domain estimation techniques from channel sounding experiments accurately represents this dominant interference source and is the standard wideband frequency model used by the aeronautical telemetry community [4].

The limited frequency spectrum allocated for aeronautical telemetry [5, 6] has driven the telemetry community to develop bandwidth-efficient and power-efficient modulation schemes with higher potential data rates than legacy frequency modulation (FM) waveforms with pulse code modulation (PCM) data encoding [7]. The Telemetry Group (TG) of the Range Commanders Council (RCC) has explicitly defined variants of these modulation schemes as the joint services advanced range telemetry (ARTM) modulation schemes in the IRIG-106 telemetry standard [8]. In this work we focus on channel equalization techniques with shaped offset quadrature phase shift keying (SOQPSK-TG) as defined in IRIG-106.

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Symbol detection performance relative to receiver signal-to-noise power ratio (SNR) with ARTM waveforms corrupted by the aeronautical telemetry channel has been evaluated in [1]–[3]. Rice, et al. demonstrated that SOQPSK-TG obtains twice the spectral efficiency of PCM/FM as required by [8] but at the cost of significant bit detection performance loss under identical multipath channel conditions. Detailed channel model analysis in [11], [12] indicates that phase corruption from single-bounce multipath can hinder detection performance as much as the coherence (effective SNR) between the line-of-sight (LOS) and single-bounce paths, coinciding with previous analysis [2, 3]. Bit error rate (BER) performance without equalization does not monotonically improve as path coherence transitions from being completely destructive to completely constructive.

Multiple techniques [13–17] have been explored to mitigate signal interference observed at a single-antenna receiver corrupted by the aeronautical telemetry channel but none of these methods have successfully recovered SOQPSK-TG symbols across the full range of channel realizations [18]. Single carrier frequency domain equalization (SCFDE) techniques have recently regained attention in the academic community due to their attractive equalization performance gains in highly frequency-selective channel environments and their striking similarity to the popular orthogonal frequency division multiplexing (OFDM) modulation schemes [19–21]. Linear and nonlinear SCFDE specific to continuous phase modulation (CPM) have also recently regained interest within the wireless communications research community [22–24]. SCFDE is ten times less complex to implement than time domain equalization for this application because a time domain equalizer must be at least 100 taps in length to equalize the effective aeronautical telemetry channel.

Our work provides multiple contributions to the field of wireless telemetry communication equalization. Our first contribution originates from the realization that previous techniques applied to the telemetry problem have failed to ameliorate signal corruption observed at the receiver for the full range of channel realizations. We have previously shown [18] that adaptive equalization schemes which attempt to directly define a time domain equalization filter from corrupted training symbols cannot effectively equalize the aeronautical telemetry channel with SOQPSK-TG symbols because the autocorrelation function of the SOQPSK-TG modulation scheme varies rapidly over only a few symbols. Even within a CPI (coherent processing interval) where the channel response is assumed to be fixed, this variation of the signal autocorrelation function precludes the use of adaptive equalization techniques based solely on training symbol error. We are the first to identify this issue for the aeronautical telemetry problem and the first to present a robust equalization solution. Our solution first explicitly estimates the channel response using maximum likelihood (ML) estimation techniques from the training symbols and subsequently equalizes the data symbols within a CPI with the optimal linear minimum mean-square error (MMSE) filter.

Following channel estimation, the optimal linear MMSE filter may be implemented in the time domain or the frequency domain. The second major contribution of this work is the decision to apply the MMSE filter in the frequency domain through SCFDE to reduce the computational complexity of the equalization process. It is well known that frequency domain equalization is more computationally efficient than time domain equalization when the required length of the equalization filter drives the computational cost of a fast Fourier transform (FFT)/(IFFT) inverse FFT below that of convolution [25], as is the case with the effective aeronautical telemetry channel. We demonstrate through simulation results that our equalization technique substantially improves BER performance for all aeronautical telemetry channel realizations. Detection performance once again becomes dependent on effective SNR with constructive and destructive multipath. For the final contribution of this work we have presented SCFDE BER performance as a function of receiver sample rate.

II. SYSTEM MODEL

The system model utilized in this work describes the SOQPSK-TG modulation scheme transmitted from a single antenna to a single-antenna receiver system. Received signals are corrupted by the effective wideband frequency channel model developed explicitly for the aeronautical telemetry application along with additive noise.

A. Transmitted Signals

The ARTM modulation scheme SOQPSK-TG [7–9] may be explicitly represented as a constrained, partial-response CPM scheme with a ternary symbol alphabet by

$$s(t) = \sqrt{\frac{2E_\mathrm{b}}{T_\mathrm{b}}}\exp[j(\phi(t,b) + \phi_\mathrm{o})] \tag{1}$$

where the information-bearing phase

$$\phi(t,b) = 2\pi h \int_{-\infty}^{t} \sum_{m=-\infty}^{\infty} b_m p(\tau - mT_\mathrm{b}) d\tau \tag{2}$$

has a modulation index $h = 1/2$, bit energy E_b, and a bit period of T_b. ϕ_o is an arbitrary phase offset that will be set to $\phi_\mathrm{o} = 0$ for this work. The information symbols take on values $b \in \{-1, 0, 1\}$ and are determined by

$$b_m = (-1)^{(m+1)} \left[\frac{a_{m-1}(a_m - a_{m-2})}{2}\right] \tag{3}$$

for bit index m and input data bits $a_m \in \{-1, 1\}$. The shaping pulse $p(t)$, which is specific to the TG variant of SOQPSK, combines a spectral raised cosine function with a temporal raised cosine function to significantly reduce sidelobe levels relative to the military standard format SOQPSK-MIL [8]. In our research we utilize the time-averaged power spectrum of the SOQPSK-TG scheme to implement linear frequency domain equalizers. The normalized average power spectrum of the SOQPSK-TG modulation scheme used in our research is displayed in Fig. 1.

B. Channel Model and Received Signals

A generalized model that characterizes the fundamental physical layer channel experienced by RF signals originating from a single-antenna airborne transmitter to a single-antenna ground-based receiver during aeronautical telemetry flight experiments is defined by Rice, et al. in [4]. This model characterizes the dynamic aeronautical telemetry channel as a linear time-invariant (LTI) system within a CPI so that standard analyses and equalization techniques for LTI systems may be performed. The channel modeling efforts in [4] indicate that the aeronautical telemetry physical layer may be represented accurately by a two path system where the LOS return is combined with a strong, single-bounce multipath reflection off the Earth surface and may be represented in continuous time by

$$\tilde{h}(t) = \tilde{\alpha}_0\delta(t - \tilde{\tau}_0) + \tilde{\alpha}_1\delta(t - \tilde{\tau}_1) \tag{4}$$

where $\tilde{\alpha}_0$ is the LOS path complex gain, $\tilde{\alpha}_1$ is the single-bounce multipath complex gain, $\tilde{\tau}_0$ is the delay of the LOS component, and $\tilde{\tau}_1$ is the delay of the single-bounce multipath component, all of which are constant within a CPI. We assume the receiver is delay-synchronized with the LOS component. This allows us to use a normalized version of the channel impulse response:

$$h(t) = \frac{\tilde{h}(t + \tau_0)}{\tilde{\alpha}_0} = \delta(t) + \alpha_1\delta(t - \tau_1) \tag{5}$$

where $\tau_1 = \tilde{\tau}_1 - \tilde{\tau}_0$ represents the relative single-bounce delay and $\alpha_1 = \tilde{\alpha}_1/\tilde{\alpha}_0$ represents the relative strength of the single-bounce reflection. Typical multipath channel parameter values include $|\alpha_1| \in [0.7, 1.0]$, $\angle\alpha_1 \in [0, 2\pi]$, and $\tau_1 \in [20, 100]$ ns [4].

Using complex-valued low-pass equivalent notation, the received signal is $y(t) = s(t) * h(t) + w(t)$ where $w(t)$ is a white zero-mean complex-valued Gaussian random process. The receiver applies a

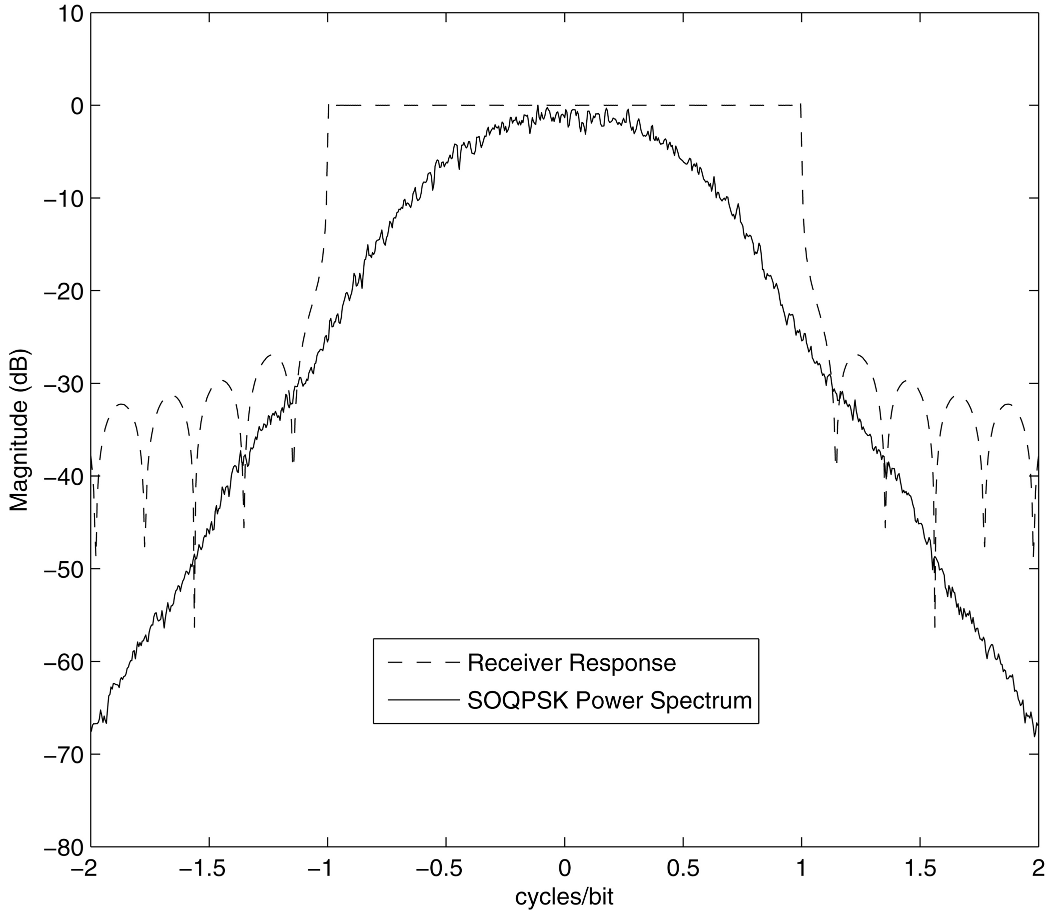

Fig. 1. Normalized average power spectrum for SOQPSK-TG and receiver filter response.

bandlimiting filter, represented by the low-pass filter with impulse response $g(t)$, to produce the signal

$$r(t) = y(t) * g(t) = s(t) * \underbrace{h(t) * g(t)}_{h_{\mathrm{eff}}(t)} + \underbrace{w(t) * g(t)}_{v(t)}. \tag{6}$$

The low-pass filter output is sampled every T_s seconds to produce the discrete-time sequence $r(nT_s)$. If the sample rate satisfies the sampling theorem for both the SOQPSK signal and the effective channel, then

$$r(nT_s) \approx s(nT_s) * h_{\mathrm{eff}}(nT_s) + v(nT_s) \tag{7}$$

where

$$h_{\mathrm{eff}}(nT_s) = \{h(t) * g(t)\}|_{t=nT_s} \tag{8}$$

is the equivalent discrete-time channel seen by the samples of the SOQPSK signal. An example of $h_{\mathrm{eff}}(nT_s)$ is plotted in Fig. 2 for $h(t) = \delta(t) + 0.95e^{j\pi/2}\delta(t - 5T_s)$ and $g(t)$ given by the windowed low-pass filter whose discrete-time Fourier transform (DTFT) is plotted in Fig. 1, for $-55 \leq n \leq 55$. The receiver response in Fig. 1 is for a 1000 tap sinc filter with a 1 cycle/bit cutoff frequency. The low-pass filter output in the discrete-time frequency domain is

$$R(\omega) = S(\omega)H_{\mathrm{eff}}(\omega) + V(\omega) \tag{9}$$

where $S(\omega)$, $H_{\mathrm{eff}}(\omega)$, and $V(\omega)$ are the discrete Fourier transforms (DFTs) of the samples of the complex baseband transmitted signal, the effective discrete-time channel, and the realization of the noise process $v(nT_s)$, respectively.

In this work realistic parameters are used for the simulations. We assume a bit rate of $1/T_b = 10$ Mbit/s and, for the channel, a relative delay of $\tau_1 = 20$ to 100 ns. We investigate the SCFDE operating at a sample rate $(1/T_s)$ 2 to 5 times the bit rate. A finer sampling period provides the opportunity for improved equalization but does not guarantee the second path delay will be an integer multiple of the sampling period (τ_1/T_s is not necessarily an integer where T_s is the receiver sampling period).

C. Symbol Detection

A common symbol detector is used for all analysis in this paper to emphasize achievable gains with predetection, fractionally-spaced equalizers. Being a CPM signal, an ML sequence estimator (MLSE) is required for optimum detection of SOQPSK-TG signals [26], but for this work we choose to implement symbol-by-symbol detection of the transmitted SOQPSK-TG symbols [3]. This scheme is the lowest complexity detector available and

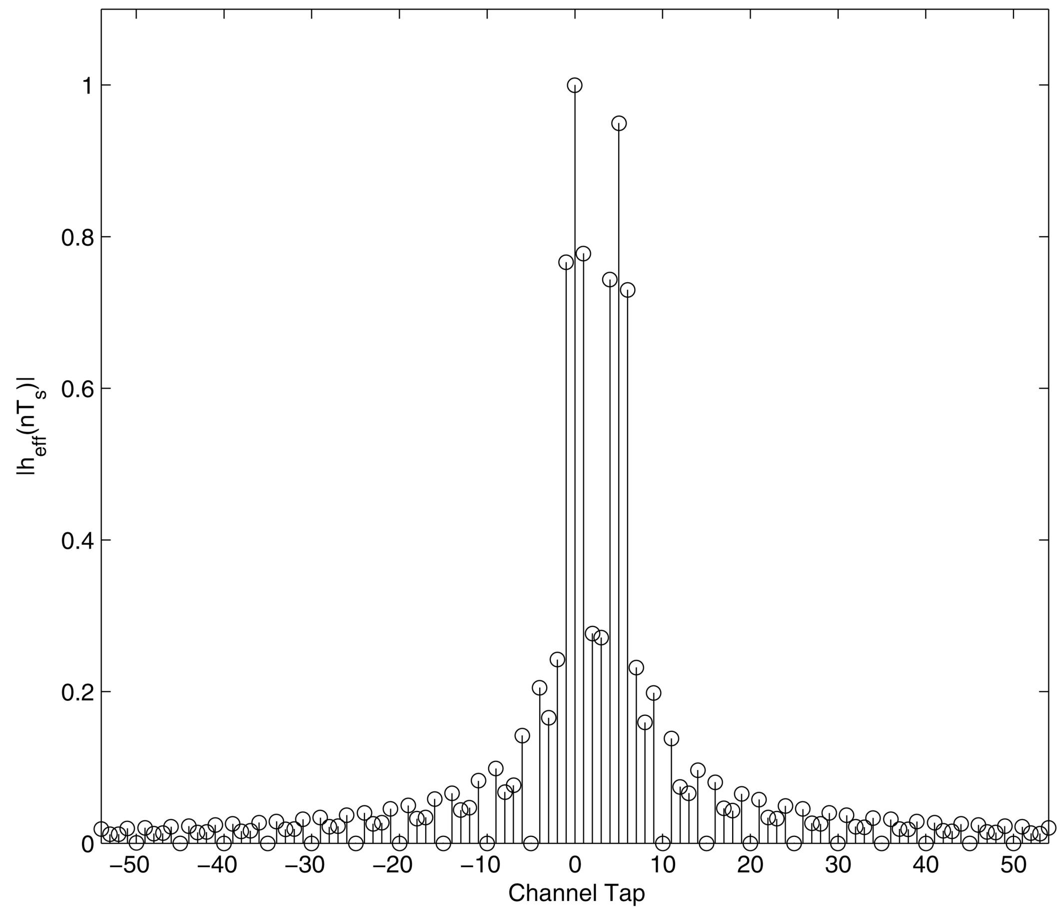

Fig. 2. Example of effective aeronautical telemetry channel normalized magnitude response $|h_{\text{eff}}|$. Simulation parameters include $|\alpha_1| = 0.95$, $\angle\alpha_1 = 0.5\pi$, $\tau_1 = 5T_s$ and a sample rate of 5 samples/bit.

has been shown to deteriorate detection performance by only 2 dB relative to the optimum detector [27].

III. LINEAR FREQUENCY DOMAIN EQUALIZATION

The equalization scheme investigated in this work first estimates the channel response using an ML technique [28] from training symbols within a CPI and subsequently equalizes the data symbols with the optimal linear MMSE filter. The computational cost of implementing the equalizer determines the selection of either a time domain filter or a frequency domain filter for the MMSE equalizer. Figure 2 displays the normalized magnitude response of a representative effective aeronautical telemetry channel. From the figure it is clear that if one was to approximate the channel using truncation, at least 100 samples of the channel would be required to adequately represent the response and a time domain equalizer would therefore require at least 100 taps. A comparison of the computational complexity of time domain (convolution) application of the equalizer and FDE (FFT/IFFT with a single point-by-point multiplication) reveals over a ten times reduction in complexity with frequency domain equalization relative to time domain equalization [25].

The linear MMSE filter in the frequency domain for a general transmit signal has been widely explored in wireless communications [19–21, 26] but this work represents the first application of SCFDE for the aeronautical telemetry application with the SOQPSK-TG waveforms. Linear convolution analysis techniques may be used with frequency domain equalizers when signal transmission is performed in a block-by-block fashion and a cyclic prefix of length N_{CP} that is longer than the channel response added to each transmitted data block of length N_{FFT}. At the receiver this cyclic prefix is removed before transformation to the frequency domain using the FFT operator.

In this section we present the linear MMSE SCFDE specific to the SOQPSK-TG modulation scheme. To obtain the optimum linear frequency domain filter in the mean-square sense, one must minimize the error cost function

$$J = \mathbb{E}\{\|S(\omega) - F(\omega)R(\omega)\|^2\} \tag{10}$$

through optimal selection of the linear filter $F(\omega)$. The resultant linear MMSE filter is given by

$$F_{\text{opt}}(\omega) = \frac{H^*_{\text{eff}}(\omega)}{|H_{\text{eff}}(\omega)|^2 + [\text{SNR}(\omega)]^{-1}} \tag{11}$$

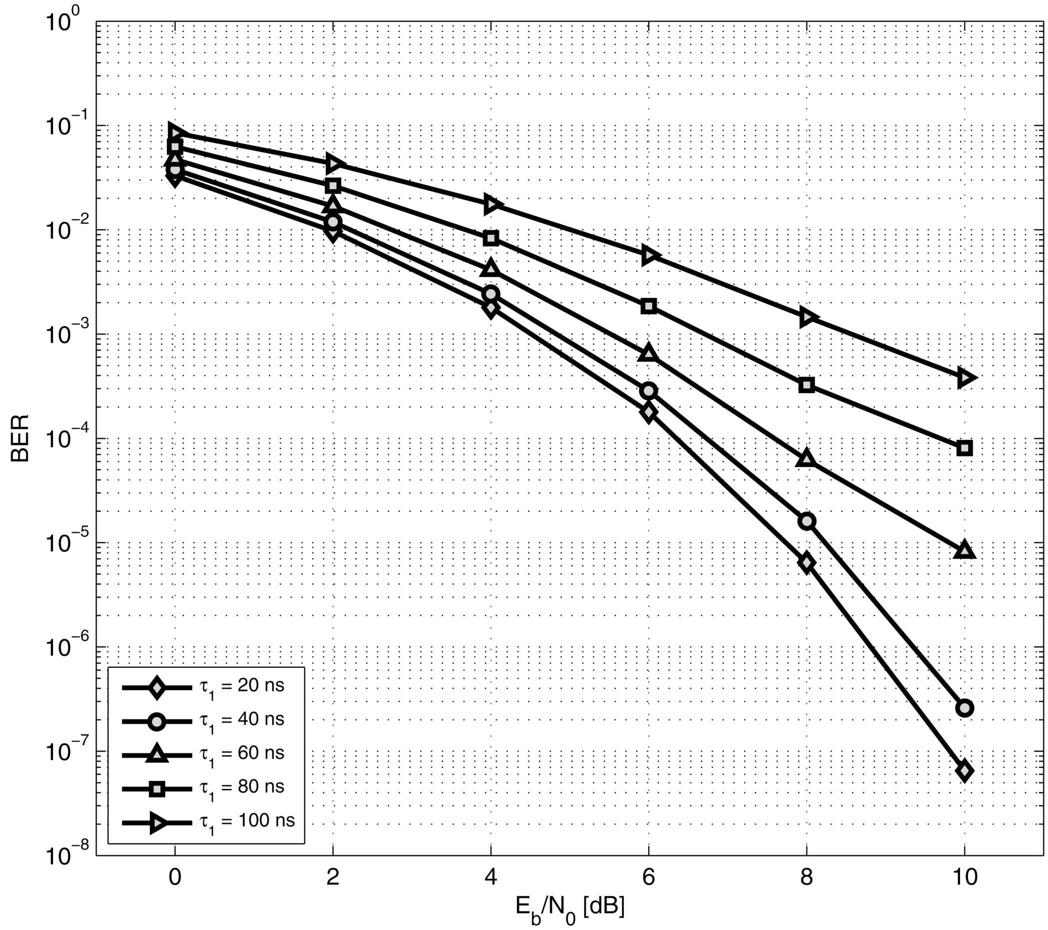

Fig. 3. BER versus SNR. Simulation parameters include $|\alpha_1| = 0.95$ and $\angle\alpha_1 = \pi/3$.

where $\mathrm{SNR}(\omega) = \mathbb{E}\{|S(\omega)|^2\}/\sigma^2$. Equalization with an approximation to the linear MMSE filter in (11) is performed in a block-by-block fashion by segmenting incoming receive signal data into length $N_{\mathrm{FFT}} + N_{\mathrm{CP}}$ blocks followed by cyclic prefix removal and transformation to the frequency domain using the FFT operator. The MMSE filter is applied in the sample frequency domain followed by a transformation to the sample time domain using the IFFT operation to produce the equalized signal in the time domain.

The approximate linear MMSE SCFDE may be written as

$$F_{\mathrm{opt}}(\omega) = \frac{H^*_{\mathrm{eff}}(\omega)}{|H_{\mathrm{eff}}(\omega)|^2 + [\widehat{\mathrm{SNR}}(\omega)]^{-1}} \tag{12}$$

where $\omega = (2\pi k/T_s N_{\mathrm{FFT}})$, $k = 0,1,\ldots,(N_{\mathrm{FFT}} - 1)$ and $\widehat{\mathrm{SNR}}(\omega) = \Psi(\omega)/\sigma^2$. $\Psi(\omega)$ is an approximation to $\mathbb{E}\{|S(\omega)|^2\}$ by the ensemble average spectrum of $|S(\omega)|^2$. The resultant output in the time domain,

$$r_{\mathrm{opt}}(n) = \mathrm{IFFT}\{F_{\mathrm{opt}}(\omega)R(\omega)\} \tag{13}$$

is used for symbol detection. Following application of $F_{\mathrm{opt}}(\omega)$ to the received signal, the corrupted signal is equalized but the noise spectrum becomes nonwhite. The MLSE symbol detector for symbols corrupted by white noise may be found using the Forney model [29] while the MLSE detector for colored noise may be found using the Ungerboeck model [30, 31]. In this work we have chosen to implement symbol-by-symbol detection to avoid the highly complex MLSE for SOQPSK-TG. The SCFDE in (12) may be implemented in telemetry receivers designed specifically for SOQPSK-TG signals and is an important contribution for the aeronautical telemetry community.

IV. SIMULATION RESULTS

System detection performance with the linear MMSE SCFDE was evaluated using simulations with realistic aeronautical telemetry channel parameter values. BER over 500 simulation runs were averaged for each simulation case presented. Figure 3 displays system BER with $|\alpha_1| = 0.95$ and second-path phase of $\angle\alpha_1 = \pi/3$. The delay of the second path was varied from 20 to 100 ns in order to span realistic aeronautical telemetry ground bounce delays. Figure 4 displays system BER with $|\alpha_1| = 0.95$ and second-path phase of $\angle\alpha_1 = \pi/2$ where the delay of the second path was also varied from 20 to 100 ns. The receiver impulse response $g(t)$ used in simulation was a 1000 tap sinc filter with a cutoff

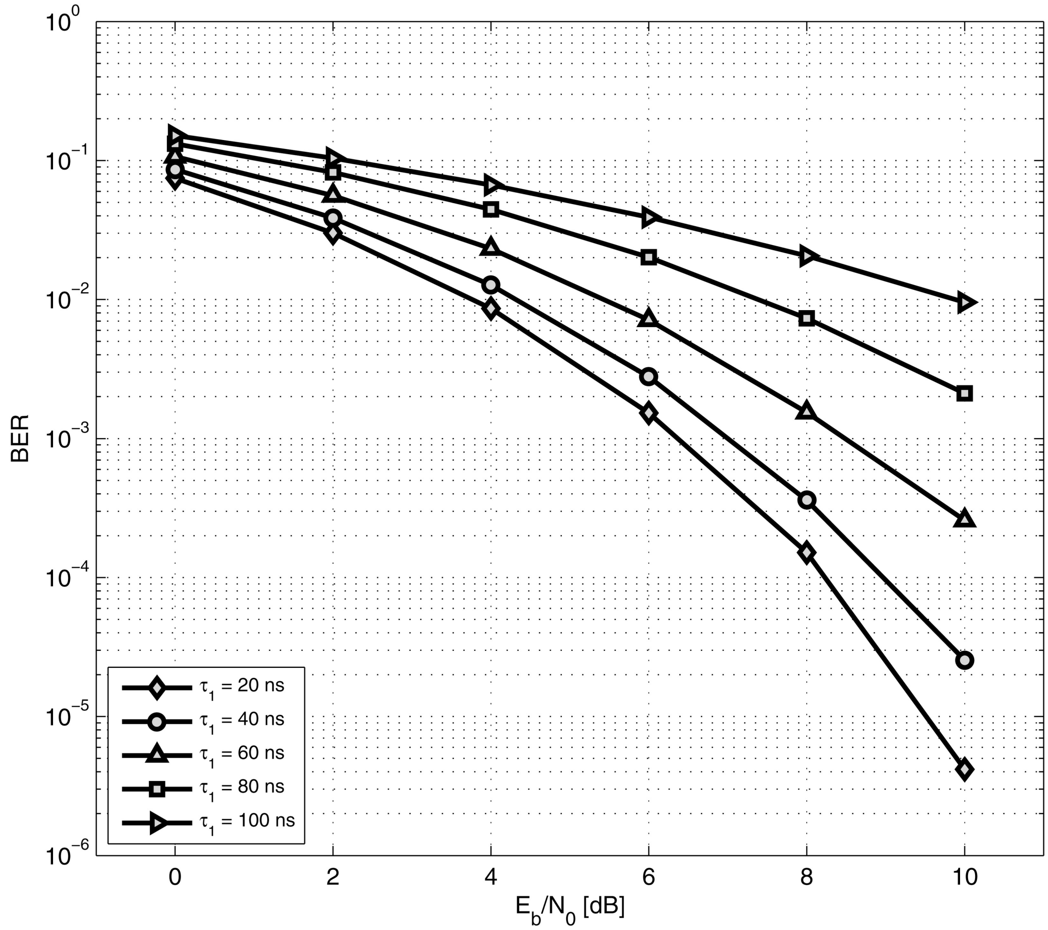

Fig. 4. BER versus SNR. Simulation parameters include $|\alpha_1| = 0.95$ and $\angle\alpha_1 = \pi/2$.

frequency of 1 cycle/bit and a magnitude response of unity within the SOQPSK-TG signal band as depicted in Fig. 1. Figures 3 and 4 clearly indicate that linear MMSE SCFDE can successfully equalize the aeronautical telemetry channel across relevant aeronautical telemetry channel realizations. It was also observed through simulations that when the relative second-path phase approaches π (completely destructive interference), there is little to no signal information remaining in the received signal to equalize. The linear MMSE SCFDE provides significant BER improvement when the relative second-path phase is in the range $\angle\alpha_1 = [0,(3/4)\pi]$.

A second set of simulation results illustrate the linear MMSE SCFDE detection performance with respect to receiver sampling rate. Average BER for receivers with 2, 3, 4, and 5 samples per bit are presented in Fig. 5. It is clear from the figure there is little to no difference between equalization performance with 3, 4 or 5 samples per bit. Performance is deteriorated only when processing with 2 samples per bit.

V. CONCLUSIONS

Aeronautical telemetry is the communication downlink of test measurements from an airborne platform or asset to a ground receiver. Receiver digital symbol detection performance during flight experiments can be severely deteriorated when a strong single-bounce multipath interference accompanies the LOS waveform. In this work we have shown that linear MMSE SCFDE implemented specifically for SOQPSK-TG can substantially improve BER performance for the aeronautical telemetry application. Our results also indicate that three samples per bit at the receiver are adequate to achieve the best BER performance following linear MMSE equalization.

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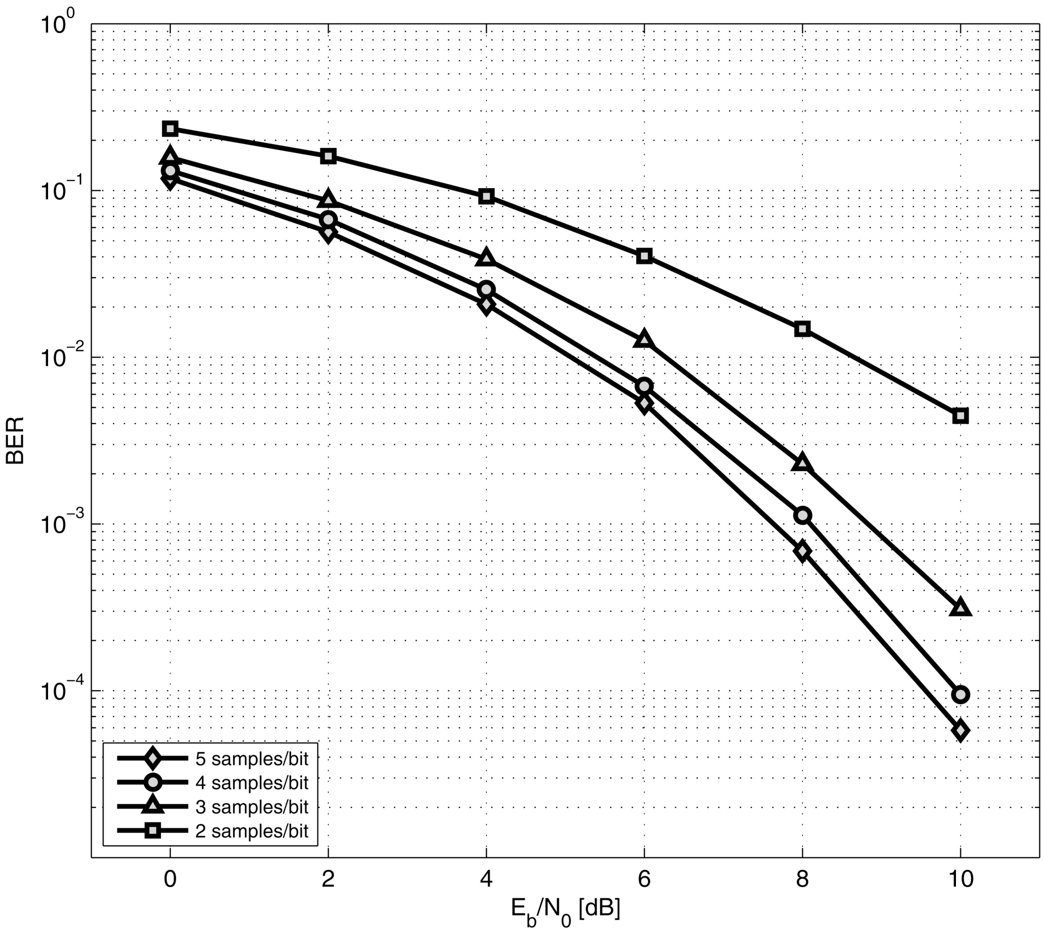

Fig. 5. BER versus SNR. Simulation parameters include $|\alpha_1| = 0.95$, $\angle\alpha_1 = \pi/2$ and $\tau_1 = 20$ ns.

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