

Proof

M is maximum if and only if there is no augmenting path.

First prove If M is maximum then it has no augmenting path:

If M has an augmenting path, then we switch the edges along this augmenting path in and out of M, which results in a matching with 1 extra edge, therefore the previous M was not maximum.

Next prove if there is no augmenting path, then M is maximum:

This statement is logically equivalent to the following: if M is not maximum, then there is an augmenting path. Now we prove the contrapositive (and therefore prove the argument).

This contrapositive can be proven the same as above. If there is an augmenting path and we alternatively add the edges in this path to M, this results in 1 added edge in new-M over old-M which means old-M was not maximum.