

Chu-Liu-Edmonds Complexity

Chu-Liu-Edmonds' algorithm finds the minimum spanning tree in directed graphs. The algorithm runs in $O(EV)$ time. First, all vertices are considered separate trees and the smallest incoming edge for each vertex is found which takes $O(E)$ time in the worst case. If there are no cycles, then the algorithm is done. If there is a cycle, however, the algorithm breaks the cycle. It does this by looking at the outgoing edge from one vertex that makes the cycle and adds the weight of the outgoing edge to all the weights of the incoming edge of the same vertex. This process seems like it would take $O(EV)$ time. $O(E)$ to update all edges of each vertex in the cycle, and $O(V)$ to this this process for each vertex, assuming worst case in which all vertices are in they cycle. This means a total complexity of $O(E) * O(V)$ or $O(EV)$.