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Proof of Dijkstra's Algorithm by Induction

Let S be the list of vertices on the shortest path from source to u found by Dijkstra's with distances[u] as the total weight of the path.

Base

When |S| = 1, the solution is trivial

Induction

Assume distances[u] is minimized when |S| = k

Now, we choose an edge uv where v has the least weight of any unvisited vertex (connected to S) and distances[v] = distances[u] + weight(uv). distances[v] must be the shortest path from the source to v because if there were a shorter path from source to vertex w then distances[w] < distances[v] and edge uv would not have been chosen. Now |S| = k + 1 and distances[v] is minimized.