

Proof of Max-Flow Min-Cut Theory

Lemma 1: There is no extra edge in the min-cut

Proof: If there were an extra edge in the min-cut, then there exists another cut that would exclude the extra edge, since this cut excludes the extra edge, it must have smaller capacity which means any cut with an extra edge cannot be the min-cut.

Lemma 2: All edges in the min-cut must be parts of augmenting paths

Proof: If there is an edge that is cut and doesn't affect an augmenting path, it is an extra edge, which can't exist in a min-cut.

Lemma 3: Each edge in the min-cut is the minimum weighted edge in each augmenting path

Proof: If there was an edge that's not the min, there would be a smaller edge that is part of another cut which would be smaller.

We know that all edges in the min-cut must be part of augmenting paths in a graph. We also know that each edge in the min-cut is the minimum-weighted edge in each augmenting path. Since the max-flow is limited by the minimum-weighted edge of each augmenting path, that means max flow equals the sum of the minimum-weighted edges. Since min-cut is the set of the minimum weighted edges, that must mean the sum of the capacities of the min-cut set is equal to the maximum flow possible through a graph from S to T, where S is the source node and T is the target node.