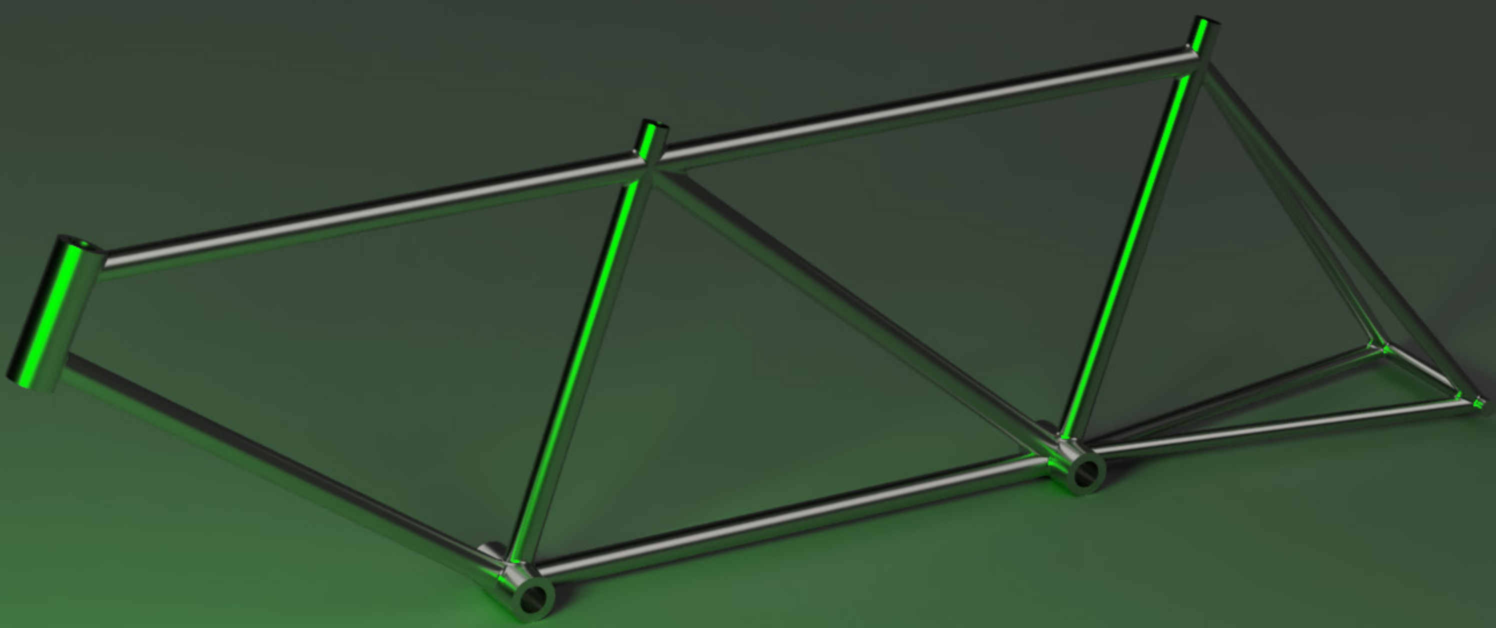
James Howells • [redacted] • Frequency & Fatigue Analysis



BICYCLE FRAME

1. CONTENTS

1. INTRODUCTION

1.1. Task

This report details work designing a tandem bicycle frame and analysing it for structural integrity. Two materials were analysed: an aluminium alloy (7075-T6 was specified) and a titanium alloy (Ti-6Al-4V was chosen). After analysis of fundamental frequency and fatigue life, a second iteration of the frame design was created to improve brief compliance. In all, 10 static studies (for purposes of mesh refinement), four frequency and four fatigue studies were carried out, excluding failed studies or those that had to be re-run.

1.2. Frame Design

The frame had to fit some predefined parameters:

"The overall length of the frame should be between 1.5 and 2 metres.
The diameter of the wheels is 26in and the height of the seat joints from the ground is 800mm.
The crank shells are cylinders with a 50mm external diameter, 10mm thickness and 100mm length. The fork shell has the same profile but a 200mm length."

Specifications for tube thicknesses were also provided: the first iteration had main tubes of 30 mm external, 28 mm internal diameter, with chain- and seatstays of 20 mm with 13 mm internal diameters. A parent-sketch was used for modelling: the frame inherits its geometry from this, and modifications to this modify the frame.

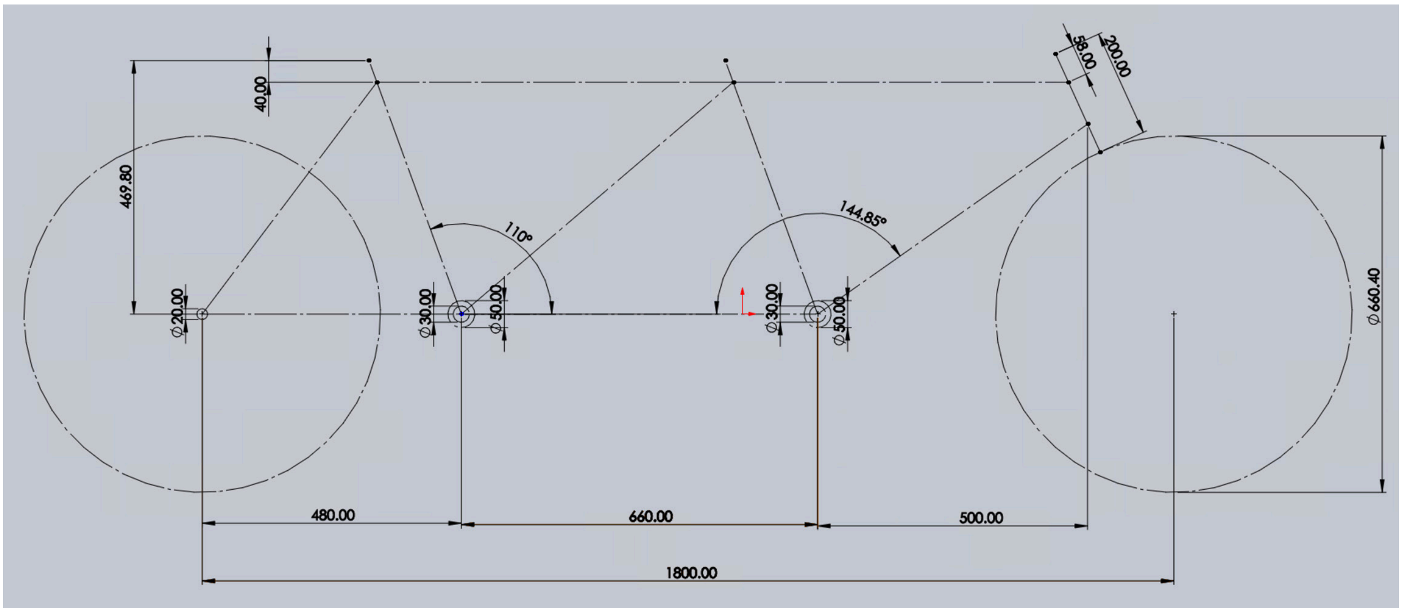


Figure 1: The parent-sketch used to create the model, with dimensions.

Unusually, the frame had a fixed rear axle, rather than an open fork. This axle was added as it was reasoned that, when being ridden, a rear wheel - and thus rear axle - would have to be present. This meant that without an axle, simulations could give unreasonable outcomes - for example the two halves of the rear triangle vibrating out of phase.

2. METHODS

2.1. Boundary Conditions

Fixtures were applied to the front and rear of the frame. The inside of the head tube, where the headset and front fork would go, was fixed, while the rear axle was hinged - as it would be to the wheel.

Figure 2: Renders showing front and back frame fixtures, in blue.

2.2. Loads

The mass of each cyclist, with a safety factor already included, was deemed to be 150 kg. This was applied, using a distributed mass, to the top of both seat tubes:

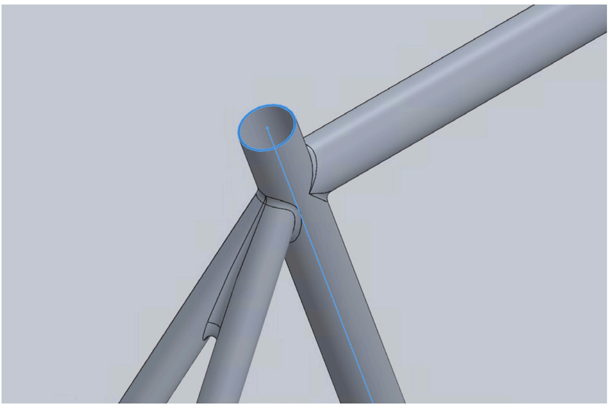


Figure 3: Mass applied to one of the seatposts.

The pedalling forces of the cyclists were modelled as remote loads of 700 N each, acting straight downwards at a point 100 mm out and 200 mm froward from the centre of the bottom bracket. They were applied in-phase with each other (as tandem cyclists have to be). This load was cycled from 0 to 100% in the fatigue analysis.

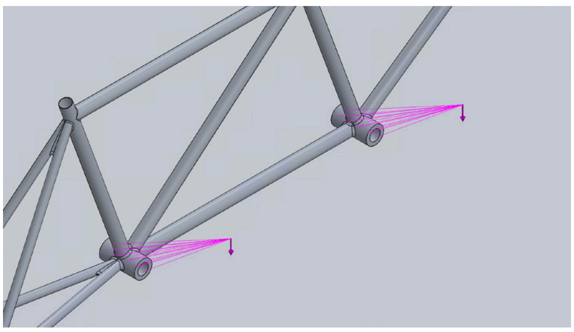


Figure 4: Remote load applied to the bottom brackets.

2.3. Simulation Parameters

2.3.1. Materials

Two frame materials were tried for each simulation: aluminium and titanium. The aluminium (7075-T6) had a pre-defined S-N curve, which was used, but no titanium alloys had curves available.
Ti-6Al-4V titanium was chosen due to its wide availability (it is a commonly used alloy), so fatigue data were available, although they had to be converted from graphical to numerical.

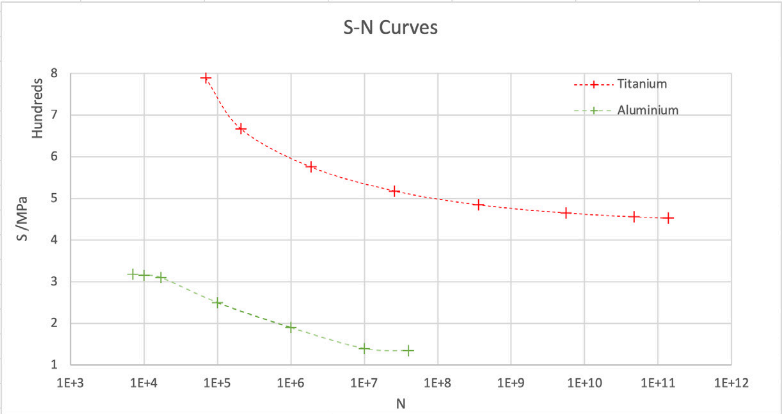


Figure 5: S-N curves for titanium[1, 2] and aluminium.

2.3.2. Frequency Simulation

The loading of a structure affects its mode shapes, so it was important to apply correct loads to the frame for the fatigue study. The load simulating mass of the cyclist was kept but the pedalling forces were not, as it would be the pedalling riders that would cause the frame to oscillate.

In order for the distributed masses of the cyclists to have any effect, a gravitational acceleration of 9.81 ms^{-2} downwards was added.

2.3.3. Fatigue Simulation

Both continuous and cyclic loading affect fatigue performance, as both contribute to stresses. Therefore, it was important to categorise the forces correctly: riders were assumed to be sitting on their seats constantly (so the force exerted by their mass was constant), and the only oscillating load was the pedalling force.

This pedalling force was varied from 0 to 700 N over 1,000,000 loading cycles, while keeping cyclist masses at 300 kg total, in order to determine fatigue life.

3. RESULTS

3.1. Mesh Refinement

To make a suitable mesh, a mesh convergence study was carried out by trying multiple mesh sizes and running a fully-loaded static simulation using each.

The aspect ratio, Jacobian ratio and meshing times were considered. The aim was to ensure that as many elements as possible had an aspect ratio of less than three, and as few as possible had them over 10. All Jacobian ratios were brought as close to one - an undistorted tetrahedron - as possible, and Jacobian ratio for all elements needed to be kept below 40.

Table 1: Results for different mesh sizes:

Mesh Size	# Elements	% Elemets with Aspect Ratio <3	% Elemets with Aspect Ratio >10	Time to Mesh /mm:ss	Max. Stress /MPa
10	43,456	14.3	30.6	0:10	126
8	73,449	24.3	10.2	0:17	179
6	120,243	26.4	0.0873	0:21	22
4	279,110	53.1	0.00645	0:41	286
3.75	325,252	64.2	0.00461	0:44	299
3.5	378,950	78.4	0.00317	0:42	315
3.25	462,756	94.3	0.00346	0:57	311
3	550,257	98.3	0.00309	0:47	323
2.75	661,212	99.6	0.00257	0:53	Simulation Failed
2	1,323,113	99.9	0.000831	1:51	Simulation Failed

After the mesh convergence study, a 3.25 mm global mesh size was chosen: this took into account the limitations of the hardware, as well as the mesh convergence. There was only a 4.35% difference between the 4 and 3.75 mm meshes, with 5.08% separating the 3.75 and 3.5 mm meshes - but the smallest convergence was from the 3.5 to 3.25 mm meshes, with a change of -1.29%. Given the changes were so minimal, any of these would do well - but the percentage of elements with a low aspect ration was far higher for the 3.25 mm mesh, so it was used (the 3 mm mesh was rejected because it made simulations take too long).

In addition to this, stress concentrations were analysed and local mesh control added to increase simulation accuracy without adding elements unnecessarily.

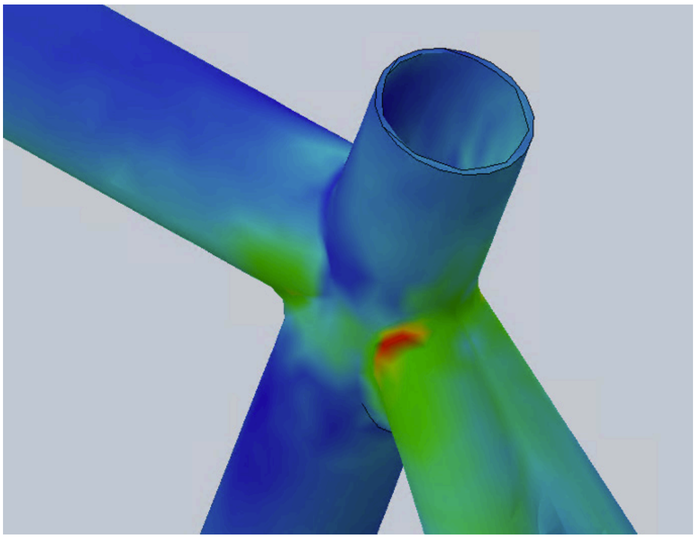


Figure 6: Some areas, especially in filleted regions, had high stress concentrations.

Mesh was locally controlled around all filleted areas: a local mesh of size 2 mm was added - smaller sizes were tried, but these overwhelmed the hardware, completely filling the available storage with cache files during simulation.

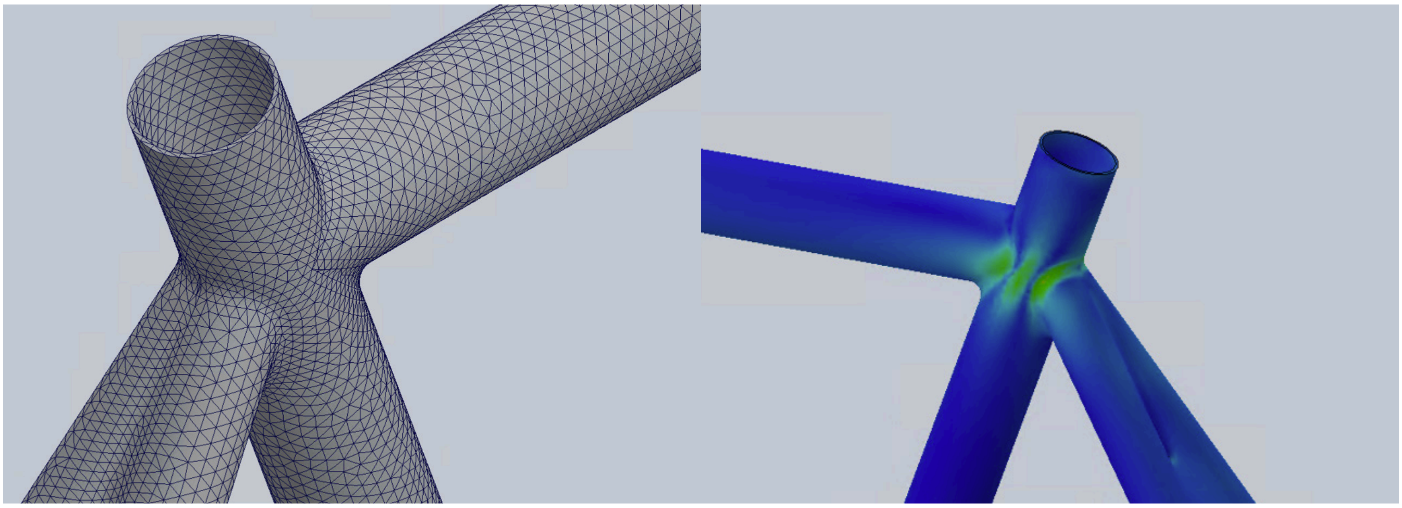


Figure 7: Mesh control was used to make elements at joins smaller, which reduced stress concentrations.

The refined mesh had a minimum Jacobian ratio of 1, and a maximum of 7.2, as well as 93.9% of elements having aspect ratios less than 3.

3.2. Sanity Checks

Sanity checking was used to make sure results of the simulations were appropriate.

3.2.1. Mesh, Loads and Fixtures

To check the mesh, loads and fixtures, a static study was carried out and resultant forces checked:

Table 3: SolidWorks loads vs applied loads.

Static Simulation Results				Loads Applied			
Sum X	**Sum Y**	**Sum Z**	**Resultant**	**Mass Applied**	**g applied**	**Force Appplied**	**Resultant**
-0.73	4372.22	-0.18	4372.22	300.00	9.81	1400	4343.00

The difference between the resultant forces calculated by SolidWorks (left) and applied loads (right) is 0.67%, so it was concluded that the model is set up correctly and simulations will provide results to a good degree of accuracy.

3.2.2. Frequency Study

Visual sanity checking was used to check for issues with the frequency simulations. The first aluminium frame is taken as an example below, but this process was applied to all frequency simulations.

The first time the simulation was run, only 5 mode shapes were calculated. Sanity checking revealed there was a problem, because none of the mode shapes were sensible:

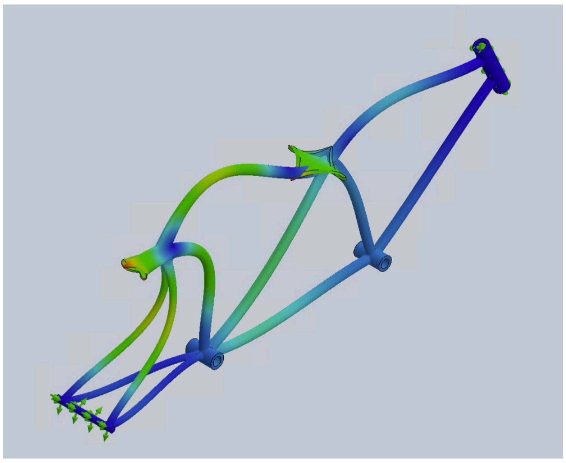


Figure 14: The first five frequencies had unusual mode shapes, with very pronounced deformation.

The simulation was re-run to calculate 10 mode shapes, but this too proved insufficient. A third run of the simulation, for 20 mode shapes, finally revealed the true natural frequency. This process took five hours due to high simulation times.

Mode shape #20 was confirmed to be correct by the shift in order of magnitude: mode shapes 1-19 all had maximum amplitude (AMPRES) values in the 10^{-1} or 10^{-2} range, while #20 had an AMPRES an order of magnitude higher, at 1.18. It was also the first mode shape not to involve extremely large and unexpected distortions of the tubes themselves.

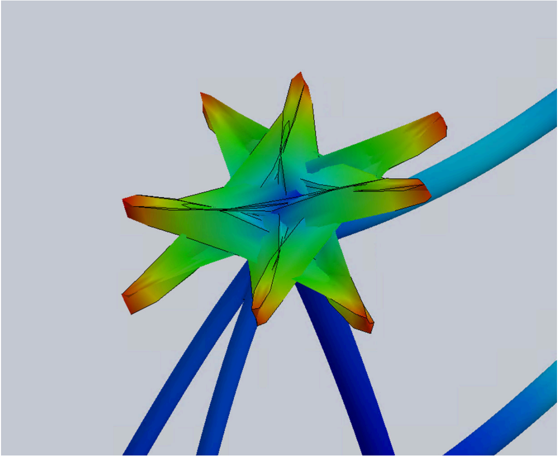


Figure 15: The unexpected distortion of tubes.

3.2.3. Fatigue Study

Sanity checks for the fatigue study were simpler: Gerber stress correction[3] was used to convert stresses to their zero-based equivalents, then read off the appropriate S-N curve. The aluminium version of frame 2 is used to illustrate as it was the only frame that suffered fatigue.

$$\Delta\sigma_{Gerber} = \frac{\Delta\sigma_{actual}}{1-\left(\frac{\sigma_m}{\sigma_{ult}}\right)^2} = \frac{342.6-184.9}{1-\left(\frac{\frac{342.6-184.9}{2}}{570}\right)^2} = 160.8\ MPa$$

$$\Delta\sigma_{Gerber} = \sigma^{corrected}{}_{max} - \sigma^{corrected}{}_{min} \qquad \sigma_m = \sigma_{mean} = \frac{\sigma_{max}+\sigma_{min}}{2}$$

$$\Delta\sigma_{actual} = \sigma_{max} - \sigma_{min} \qquad \sigma_{ult} = ultimate\ \sigma$$

The above Gerber method shows 160.8 MPa corrected stress. This can be read off the S-N curve to find a corrected fatigue life partway between 1,000,000 and 10,000,000 cycles. This fits well with the simulated value of 1,262,000 cycles.

3.2.4. Other Metrics

Other simple checks were also carried out, for example:
- Centre of mass was in reasonable position
- Frame mass had a reasonable value
- Displacements of all nodes in static studies were small
- Highest fatigue damage was at areas of highest stress

3.3. Simulation Results

3.3.1. Aluminium Frame

First, the Frequency simulation was run on the aluminium frame. The simulation took over an hour and a half - but only provided the first 5 mode shapes. These were found to be inaccurate in the sanity check stage (see above), so the simulation was re-run to calculate the first 20 mode shapes.

The sanity checking revealed the final mode shape (#20) to be the first accurate representation of the frequency response of the frame:

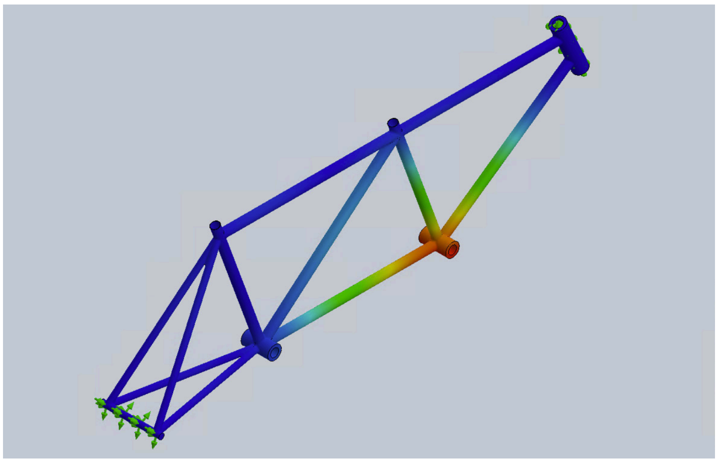


Figure 8: The frequency response of the first-iteration aluminium frame.

The natural frequency of this mode shape was 50.87 Hz, which is well above the specified minimum of 30 Hz.

The fatigue simulation was run for the same frame with the same boundary conditions. The maximum stresses in the frame were not enough to cause any fatigue damage at all, and the damage percentage at 1,000,000 cycles was, therefore, 0.

Total mass of this frame was 2.97 kg.

3.3.2. Titanium Frame

The frequency simulation was run for the titanium version for the same frame. Sanity checking revealed that the first accurate mode shape was the 15^{th}.

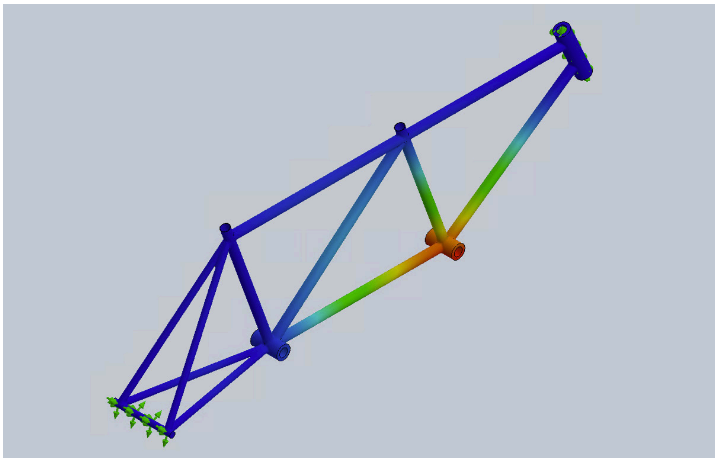


Figure 9: The 15th (and first correct) mode shape of the titanium frame.

The natural frequency was found to be 49.01 Hz. The similarity of this to that of the aluminium frame (50.87 Hz) is a good sign: fundamental frequency is proportional to E/ρ, and this ratio is similar for 7075-T6 aluminium (72/2810 = 0.0255) and Ti-6Al-4V titanium (104.8/4429 = 0.0260).

As with the aluminium version of this frame, the fatigue simulation was run next. As titanium Ti-6Al-4V is more durable than 7075-T6 aluminium, no damage was expected. The simulation showed this prediction to be correct: again, damage percentage at 1,000,000 loading cycles was 0.

Total mass of this frame was 4.68 kg.

3.4. Second Iteration

3.4.1. Refinement Rationale

The brief asks for a 'lightweight tandem frame,' and the refinement of the frame should be aimed at increasing natural frequency.

To decrease weight, the back down tube was removed as it was found to be unnecessary: it was under very little stress (under 10 MPa) or strain (in the order of 10^{-4}). This would, however, *decrease* natural frequency, as:

$$\omega = \sqrt{k/m}$$

Removing the tube will have a larger effect on k, the stiffness of the frame, than on mass m, so, in order to increase fundamental frequency ω, other changes need to be made too.
The changes were analysed by analogy to the fundamental frequency of a cantilever beam:

$$\omega_n = a_n{}^2 \sqrt{\frac{E}{\rho}\frac{I}{L^4 A}} \qquad \frac{I}{A} = \frac{1}{4} \times \left(R_o{}^2 + R_i{}^2\right)$$

Fundamental frequency increases as the $I/A = 1/4\,(R_o{}^2 + R_i{}^2)$ term in the above equation increases. This means that thinner pipe walls and / or larger pipes will increase ω. Wall thickness cannot be further reduced as it can only be changed in 1mm increments, so pipe diameter will be increased by 2 mm everywhere, with walls kept at 1 mm.

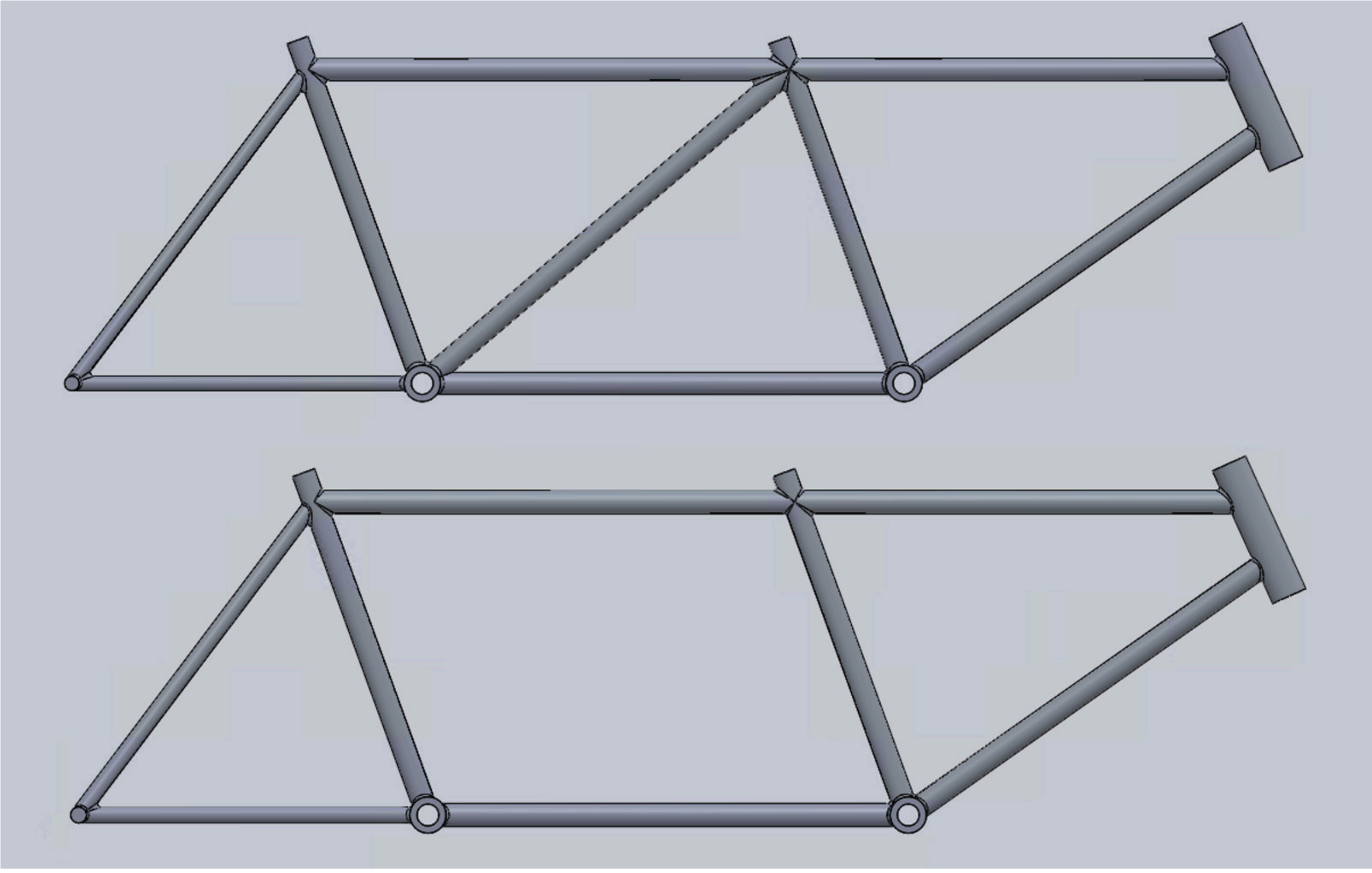


Figure 10: The old (above) and new frames. New frame is thicker but missing a tube.

3.4.2. Aluminium Frame

The new frame was analysed in the same way as the old. The new aluminium frame was found to have a mass of 2.94 kg, 1% lighter than the previous iteration. Its fundamental frequency was 57.2 Hz, representing a 2.12% improvement despite the removal of a member.

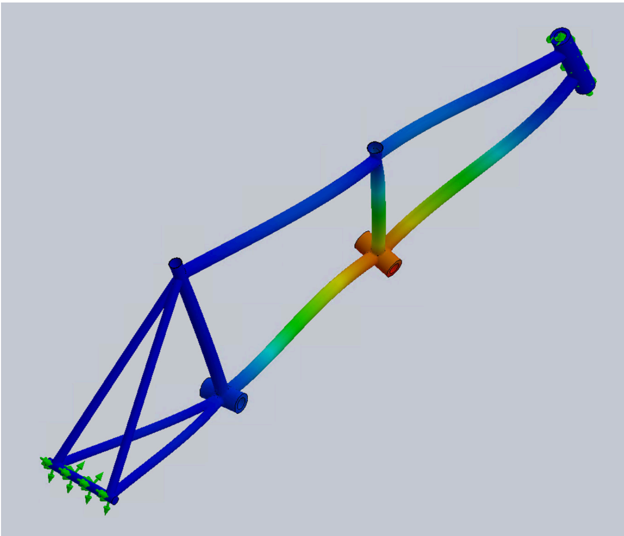


Figure 11: The mode shape of the fundamental frequency.

The fatigue life of this frame was not infinite under the defined loading conditions: the maximum damage percentage on the frame after 1,000,000 loading cycles was 79.2%: overall life of the frame was 1.26 million cycles. This decrease was because the various loads affect the frame in different ways, and with different pipe size they contribute differently to overall stress - meaning the range of the stress is higher.

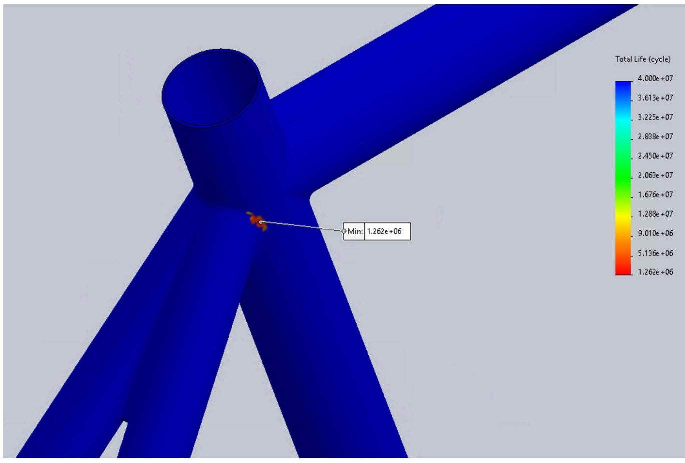


Figure 12: Plot showing fatigue concentrator in same location as stress concentrators.

3.4.3. Titanium Frame

The new titanium frame had a mass of 4.63 kg, again 1% lighter than the first. Its fundamental frequency was 50.21 Hz - similarly, 2.02% better than before.

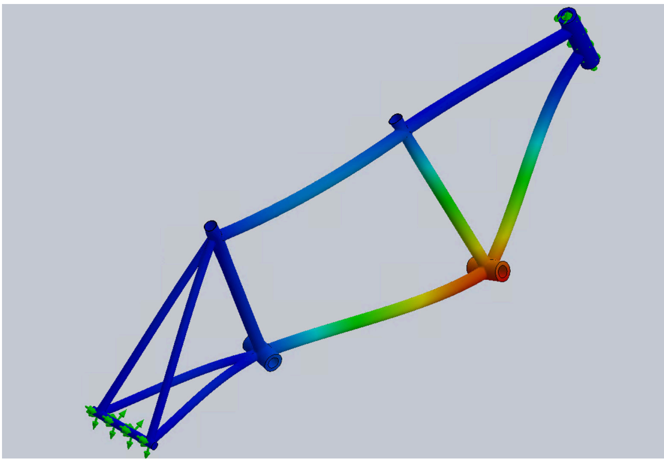


Figure 13: The mode shape of the fundamental frequency of the second titanium frame.

After the 1,000,000 loading cycles, this iteration again suffered no damage at all as alternating stresses were under the S-N curve for titanium.

3.5. Study Results

Table 2: Simulation results.

Simulation Results					
Frame	**Mass /kg**	**Max. Stress Intensity /MPa**	**Fundamental Frequency /Hz**	**Fatigue Life /cycles**	**Damage Percentage @ 1E6 Cycles**
Aluminium 1	2.97	350.8	50.87	N/A	0
Titanium 1	4.68	350.4	49.01	N/A	0
Aluminium 2	2.94	342.6	57.15	1.26E+06	79.22%
Titanium 2	4.63	343.0	50.21	N/A	0

4. DISCUSSION

There are three main sources of error in FEA: modelling error, discretisation error and numerical error. Numerical errors are caused by loss of precision in computation, so are hard to control without hugely increasing computing time. The other two sources of error, however, have been actively controlled throughout the project.

First, care was taken to make sure the model was accurate, with no self-intersecting geometry or overlapping parts - this helps to reduce modelling error. Next, a manual mesh convergence study was carried out to minimise discretisation error - again, this was a fight against computing time and power, so some compromises had to be made. Better convergence was achieved through use of local mesh refinement. Finally, returning to modelling error, the initial conditions and study parameters were designed with care so as to reduce errors introduced before simulations were run. Despite this, studies can only ever be an idealised version of the problem, so some errors were inevitable - for example, pedalling forces were applied only to one side of the frame, as a worst-case scenario - in reality, riders pedal on both sides of the frame.

After the running of the simulations, sanity checking was used as described to make sure all calculated values were reasonable.

One source of error that it was not possible to check or minimise, however, was the S-N curve for titanium: the raw data were not available, so they were measured off a graph. Measurement was done using an automated tool to reduce human error.

All four frames passed the necessary criteria of a natural frequency greater than 30 Hz and ability to withstand over 1,000,000 loading cycles. The most successful frame was the second aluminium one, as it was the lightest to fit the brief, weighing 2.94 kg. This may seem very little for an entire tandem bike frame, however, frame mass only accounts for 15-20% of bike mass[4] - and high-end aluminium single bicycle frames weighing around 1.5 kg are available, so this value is plausible.

5. REFERENCES

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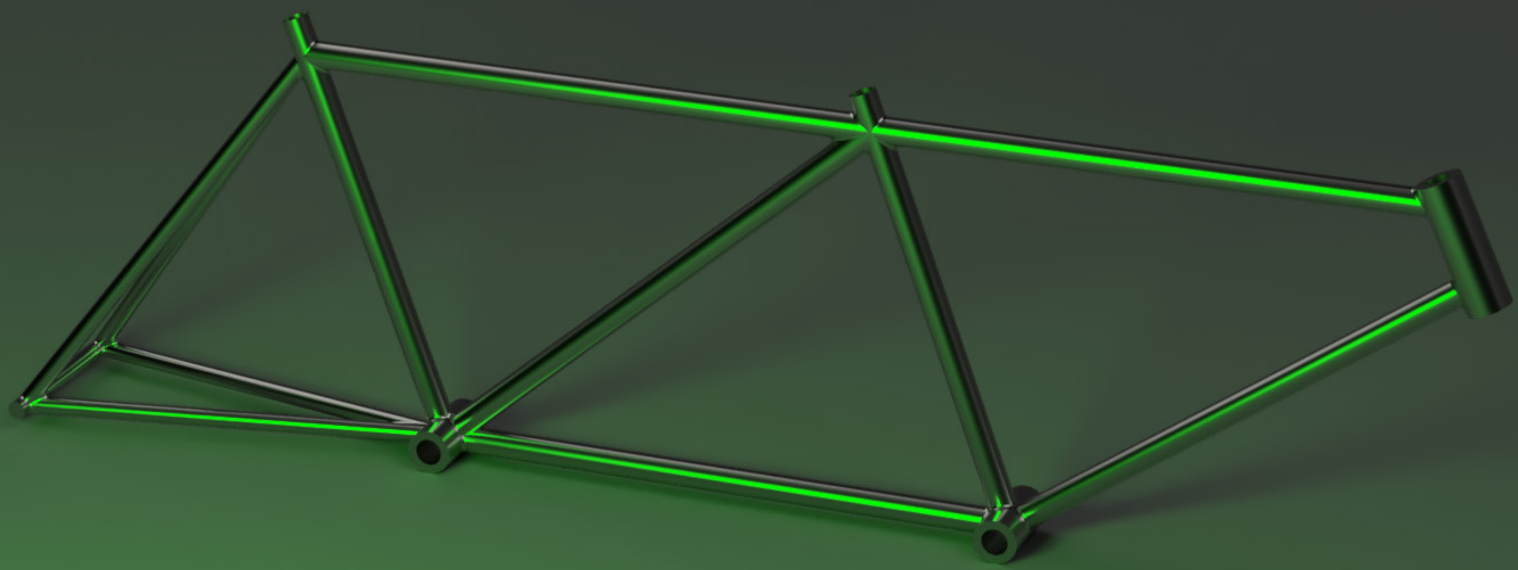

Modelled on SolidWorks 2021 • Rendered on Fusion 360

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