A Two Field Model of Inflation with a Transverse Instability

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Abstract

A two field model of inflation is explored in which, while inflation is driven by the longitudinal field, the transverse field experiences a temporary instability.The evolution of the system during inflation is calculated by making use of a lattice simulation. The change in ζ as sourced by the gradient terms in the transverse field generated during its instability is calculated.

1 Introduction

Inflation, referring to a period of accelerating expansion in the early universe, has been found to have a great deal of power in explaining some peculiarities in the universe we observe today. Inflation was first proposed as a mechanism to explain away the so called horizon and flatness problems [1]. The horizon problem is that regions of the universe which would seemingly, without a period of inflation, be causally disconnected are observed to have a high degree of homogeneity, this homogeneity is evidenced by the near uniformity of the cosmic microwave background. Inflation alleviates the horizon problem by allowing for regions of the universe which at early times were causally connected to gain a degree of homogeneity through causal mechanisms before the accelerating expansion of the universe ends the causal contact between these regions. The flatness problem is that, in a situation where the expansion of the universe is not accelerating, a solution to a homogeneous and isotropic universe is predicted to be driven away from flat when the energy density is different from a critical value. To avoid an appeal to fine tuning parameters in order to create a flat universe, some mechanism capable of having made the universe close enough to being flat at some time in the past that it has not deviated significantly from flat at the present time is required. Such a mechanism is provided by inflation when we consider that accelerated expansion is equivalent to changing the characteristic scale over which curvature of the universe is measured, inflation then sets conditions in the past when the universe is close to flat by changing the characteristic scale over which curvature is measured.

Since its original proposal, inflation has also provided an explanation for the origin of structure in the universe in terms of curvature perturbations. Due to the expansion of the universe different regions of space become causally disconnected during inflation and curvature perturbations remain between these causally disconnected regions. After the end of inflation when regions, once causally disconnected, come back into causal connection the curvature perturbations act to seed the formation of structure.[2]

Given the fruitfulness of inflation in explaining the universe as we see it today, investigations into inflation are of particular interest. In the present work we examine the effects of an instability during inflation acting perpendicular to the inflaton field. The remainder of this report is divided as follows: in section 2 an overview of the relevant theory is presented, in section 3 general features of the computational techniques are discussed, section 4 presents the results of lattice simulation of a two field inflation model including an instability, and concluding remarks are given in section 5.

2 Theory

To reduce clutter in the equations we will make use of units in which $c = \hbar = 1$ and will take as our mass scale the reduced Planck mass, $M_{Pl} \equiv (8\pi G)^{-1/2}$.

The simplest model that can be considered for the universe is one in which the universe is spatially homogeneous and isotropic. A universe model which satisfies the conditions of spatial homogeneity and isotropy is referred to as a Friedmann-Robertson-Walker (FRW) universe. The spacetime of a FRW universe is characterized by a FRW metric of which there are three varieties: open, closed, and flat, categorized by the amount of curvature. We will restrict ourselves to the special case of a flat FRW universe, the spacetime of which can be described by a metric with corresponding line element

$$\mathrm{d}^2 s = -\,\mathrm{d}^2 t + a^2(t)(\,\mathrm{d}x^2 + \,\mathrm{d}y^2 + \,\mathrm{d}z^2). \tag{1}$$

In (1) x, y, z are known as comoving coordinates while $a(t)$ is referred to as the scale factor. The physical significance of these quantities is that a test particle initially at rest at one position (as measured in comoving coordinates) will remain at rest at that position (again, as measured in comoving coordinates) while the physical distance between any two such stationary test particles is proportional to the scale factor and will evolve in time.

The time evolution of a universe described by a FRW metric is determined by substituting the metric associated with (1) into the Einstein equation, the result is the Friedmann equation

$$H^2 = \frac{1}{3M_{Pl}^2}\rho. \tag{2}$$

We have introduced $H \equiv \dot{a}/a$ which is known as the Hubble parameter and measures the rate of expansion of the universe. An important property of the Hubble parameter is its inverse, H^{-1}, provides the relevant length scale for regions which are causally connected.

Another relation which proves useful and takes a simple form in a FRW universe is the fluid equation. In the adiabatic case the fluid equation is arrived at by equating the rate of change of energy in a unit comoving volume to the rate at which work is done by pressure in expanding that volume

$$\frac{\mathrm{d}}{\mathrm{d}t}(\rho a^3) = -p\frac{\mathrm{d}}{\mathrm{d}t}(a^3), \tag{3}$$

$$\dot{\rho} + 3H(\rho + P) = 0. \tag{4}$$

We can write down the action for a set of scalar fields minimally coupled to gravity as

$$S = \int \left\{\frac{R}{16\pi G} - \frac{1}{2}g^{\mu\nu}\phi_{i,\mu}\phi_{i,\nu} - V(\phi_i)\right\}\sqrt{-g}\,\mathrm{d}^4x, \tag{5}$$

where R is the Ricci scalar, $g^{\mu\nu}$ is the metric tensor, g is the determinant of the metric tensor, uppercase Roman indices run over the set of scalar fields, sums over repeated indices being implied.

In this report we will be mainly with concerned with the case of two scalar fields minimally coupled to gravity. It is assumed that these fields are sufficiently close to being uniform and isotropic that the metric tensor is well approximated a FRW type metric with a line element of the form $\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2)$. With these assumptions the action can be rewritten as

$$S = \int \Big\{ \frac{R}{16\pi G} + \frac{1}{2a^2}(a^2\dot{\phi}_i^2 - (\nabla\phi_i)^2) - V(\phi_i) \Big\} a^3 \,\mathrm{d}t\,\mathrm{d}^3x. \tag{6}$$

Applying the Euler-Lagrange condition to extremize the above action results in the following equations of motion for the scalar fields:

$$0 = \ddot{\phi}_i + 3H\dot{\phi}_i + (-\frac{\Delta\phi_i}{a^2} + V_{,\phi_i}). \tag{7}$$

Two notable differences between this scalar field equation of motion and its Minkowski space counterpart are the addition of a drag term proportional to the Hubble parameter, and the scaling of the Laplacian by a factor of a^{-2}. The first of these differences is dubbed the Hubble drag term and results from the time dependence of a. The second of these differences can be intuitively understood as the need to differentiate with respect to physical distances instead of coordinate distances.

With fields in units of the reduced Planck mass, the pressure and energy density of the scalar fields is given by

$$\rho = \sum_i \Big(\frac{1}{2}\dot{\phi}_i^2 + \frac{1}{2a^2}(\nabla\phi_i)^2\Big) + V(\phi_i), \tag{8}$$

$$P = \sum_i \Big(\frac{1}{2}\dot{\phi}_i^2 - \frac{1}{6a^2}(\nabla\phi_i)^2\Big) - V(\phi_i). \tag{9}$$

2.1 Inflation in Brief

A number of equivalent definitions of inflation can be given, each highlight different aspects of the physics [2]. We start by taking as the defining feature of inflation to be an accelerating expansion, written in terms of the scale factor this condition reads

$$\ddot{a} > 0. \tag{10}$$

Recalling the definition of H it is easily verified that this condition of accelerating expansion can be expressed equivalently as

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{H^{-1}}{a}\right) < 0. \tag{11}$$

The physical significance of inflation can be extracted from this second definition by recognizing H^{-1} as the relevant distance scale over which points can be causally connect, known as the cosmic horizon, while a is the relevant distance scale between a pair of comoving points. As this ratio of scales decreases the cosmic horizon effectively shrinks with respect to comoving coordinates so that two comoving points initially causally connected may cease to be causally connected at some later time during inflation. Once inflation has terminated and (11) no longer holds, a pair of points that were removed from causal contact during inflation will, at later time, reestablish causal contact in what is referred to as reentering the horizon.

So far we have described inflation in terms of the scale factor and Hubble parameter, but have not described a scenario that would lead to inflation actually taking place and would like to relate such a scenario back to scalar fields. To relate inflation back to scalar fields we first derive a condition for inflation to proceed. By differentiating both sides of the Friedmann equation (2) and substituting using the continuity equation (4) and Friedman equation (2) it is possible to give conditions on when inflation will occur.

$$\frac{\mathrm{d}}{\mathrm{d}t}(H^2) = \frac{1}{3M_{Pl}^2}\frac{\mathrm{d}}{\mathrm{d}t}(\rho) \tag{12}$$

$$\frac{\ddot{a}}{a}M_{Pl}^2 = \frac{\dot{\rho}}{6H} + H^2 \tag{13}$$

$$\frac{\ddot{a}}{a}M_{Pl}^2 = -\frac{1}{6}(\rho + 3P) \tag{14}$$

The above calculation immediately gives as a necessary and sufficient condition for inflation to occur in a FRW universe that $p + \rho/3 < 0$. For the case of inflation driven by single, spatially homogeneous, scalar field this condition can be rewritten using (8) and (9) to relate ρ and p to quantities relating directly to the field.

$$\dot{\phi}^2 - V(\phi) < 0. \tag{15}$$

For definiteness we will reserve ϕ to refer to the inflaton field, that is the field driving inflation.

In standard inflationary scenarios inflation is driven by a single scalar field which is initially far from the minimum of its potential and homogeneous apart from small fluctuations. For an appropriate form of potential the scalar field evolving according to the equation of motion (7) will reach a situation analogous to terminal velocity with $\ddot{\phi} \approx 0$ due to the Hubble drag term. When this situation occurs, the order of the equation of motion is effectively reduced and it becomes possible to make a set of simplifying assumptions, known as the slow-roll approximation, that will allow for the qualitative behaviour of the system to be more easily extracted. The standard slow-roll approximation makes additional assumptions about the form of the potential in order to approximate the Friedmann equation (2) by a simpler form [3]. As our interest in the slow-roll approximation will be to explain features in the evolution of the scalar fields and not the calculation of the Hubble parameter we will take the slow-roll approximation to refer to $\ddot{\phi} \approx 0$, or equivalently

$$3H\dot{\phi} \approx -V_{,\phi}. \tag{16}$$

The validity of (16) of course depends on the accuracy required and must be verified with a more detailed calculation, but in general (16) remains a useful tool for analyzing the qualitative behaviour of the inflaton field during inflation and is valid in this context until $\ddot{\phi}$ becomes significant as ϕ nears the end of inflation.

After the end of inflation as the inflaton approaches the minimum of its potential and (7) becomes a damped oscillator equation (albeit with a drag term which has its own equation of motion given by (2)). So, after inflation the inflaton will oscillate around the minimum of its potential. A calculation of the evolution of such an inflaton field during inflation and shortly after the end of inflation is shown in Figure 1.

For definiteness we will take $\phi > 0$ at initial conditions, then noting that ϕ decreases monotonically during inflation (see Figure 1) ϕ can be thought of as a decreasing time-like variable during inflation.

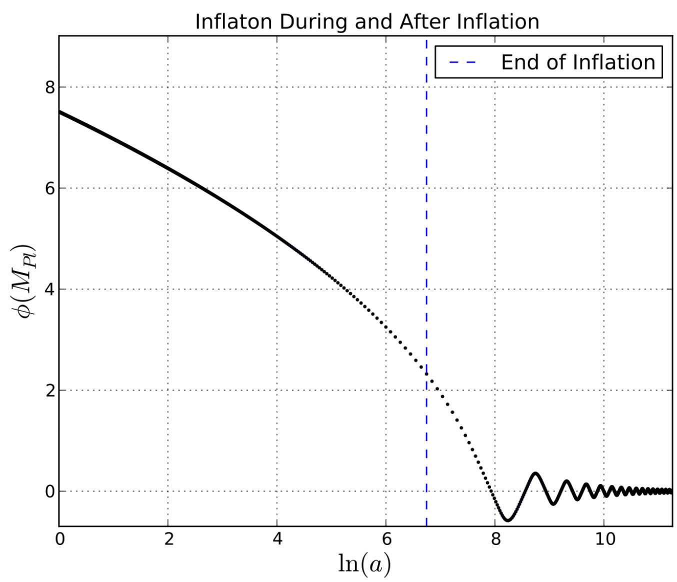

Figure 1: Value of the inflaton field (in units of M_{Pl}) plotted against the logarithm of the scale factor in an inflationary model with a potential $V \sim \phi^4$, the dashed line marks the end of inflation. During inflation ϕ changes monotonically, sometime after the end of inflation the behaviour of ϕ becomes oscillatory.

2.2 The ζ Parameter

In this section the ζ parameter is introduced as a way of tracking when the production of entropy occurs. To identify the production on entropy consider

$$\mathrm{d}(\rho V) + P\,\mathrm{d}V = T\,\mathrm{d}S \tag{17}$$

$$\mathrm{d}\rho + (\rho + P)\,\mathrm{d}(\mathrm{ln}V) = \frac{T\,\mathrm{d}S}{V}. \tag{18}$$

The time derivative of entropy within a comoving volume can now be calculated by rewriting the physical volume in terms of a constant comoving volume and the scale factor, $V = V_{comoving}a^3$,

$$\frac{T}{V}\frac{\mathrm{d}S}{\mathrm{d}t} = \dot{\rho} + 3(\rho + P)H. \tag{19}$$

An immediate consequence of the above equation, (19), is that entropy within a comoving volume will be conserved so long as the fluid equation (4) holds. The situation in which (4) fails to hold is one in which there is is energy transfer between comoving volumes, such a situation requires some amount of inhomogeneity. This requirement for inhomogeneity links the non-conservation of ζ to the production of spatial gradients in the scalar fields.

With the intention of identifying when a change in entropy occurs we now define the ζ parameter [4] by its differential as

$$\mathrm{d}\zeta \equiv \frac{\mathrm{d}\mathrm{ln}\,\rho}{3(1 + P/\rho)} + \mathrm{d}\,\mathrm{ln}a. \tag{20}$$

The time derivative of ζ is then given by

$$\frac{\mathrm{d}\zeta}{\mathrm{d}t} = \frac{1}{3(\rho + P)}\frac{\mathrm{d}\rho}{\mathrm{d}t} + H. \tag{21}$$

Equivalently in the case of scalar fields, using the energy density, pressure, and equation of motion for a scalar field, (8), (9), and (7) respectively, allows (21) to be rewritten as

$$\frac{\mathrm{d}\zeta}{\mathrm{d}t} = \sum_i \frac{\nabla\dot{\phi}_i \cdot \nabla\phi_i + \dot{\phi}_i\nabla^2\phi_i}{3a^2(\rho + P)}. \tag{22}$$

The condition for the right-hand side of (21) to be nonzero is exactly the condition that the right-hand side of (19) also be nonzero, so ζ is conserved except during periods of entropy production.

If the evolution of the scalar fields and scale factor is known then (22) can be integrated to find ζ as a function of time, by convention $\zeta = 0$ is chosen as the initial condition.

3 Computation Techniques

The equations of motion for a scalar field in an expanding universe are partial differential equations and are non-linear due to the dependence of the Hubble parameter H on the scalar field and its derivatives and, in general, non-linear terms from $V_{,\phi_i}$. In this work the equations of motion are solved numerically using an as of yet unnamed lattice simulation code developed by Jonathan Braden [5]. Using a lattice simulation to solve the equations of motion has the advantage that nonlinear effects are included in the calculation.

In order to solve the equations of motion (7) the system is recast into a Hamiltonian framework within the lattice simulation and space is evenly discretizes within a fixed comoving volume. A lattice site refers to a discrete point within the comoving volume being simulated, while the set of all lattice sites is referred to simply as the lattice. Values of scalar fields, ϕ_i and their momenta are stored for each lattice site with periodic boundary conditions imposed on the lattice. While the scalar fields are allowed to vary over the lattice, a flat FRW universe is assumed for the calculation of the scale factor, a, and the Hubble parameter, H, which are calculated uniformly over the lattice. The assumption that a FRW universe provides a good estimate for the evolution of spacetime can be physically motivated by the near homogeneity of the inflaton field during inflation.

The lattice simulation makes use of a pseudo-spectral method to calculate spacial derivatives, whereby a fast Fourier transform (FFT) is taken of the value to be differentiated over the lattice and the derivative calculated in Fourier space and finally the inverse FFT is used to return the derivatives to real space.

Initial conditions for the inflaton field and its momentum are chosen to have a mean value consistent with the slow-roll approximation $\ddot{\phi} \approx 0$. Fluctuations to the fields are added as a sampling of a Gaussian random field, the physical motivation for these field fluctuations is to provide one realization of the quantum fluctuations of the fields.

To the benefit of this work in investigating the effects of an instability during inflation, the lattice simulation code used is easily configurable to change the potential being considered. By running the lattice simulation twice from identical initial conditions and fluctuations, once with an instability in the potential and once without, the effects of the instability can be isolated by comparing between the two runs.

4 Transverse Instability in the Two Field Model

In what follows, we will consider an extension to the single field inflationary scenario described above by including a second scalar field, χ, which is initialized with small fluctuations around the minimum of its potential. Inflation is still driven by the field ϕ while additional dynamics are associated with the field χ, the terminology is that ϕ is the longitudinal field while χ is the transverse field. The specific scenario considered is one in which as inflation proceeds in the longitudinal direction the potential changes shape so as to create an instability in the transverse direction, this instability is transient and as inflation continues in the longitudinal direction the transverse instability is removed and the potential returns to its original form.

4.1 The Two Field Potential

The specific form of potential chosen to explore the above scenario is

$$V(\phi,\chi) = \frac{\lambda_\phi}{4}\phi^4 + \frac{\lambda_\chi}{4}\chi^4 + \Delta V(\phi,\chi), \tag{23}$$

where ΔV provides the transient instability in the transverse direction. The form chosen for ΔV is given by

$$\Delta V = -\frac{A^2\sqrt{e}}{b}(\phi-\phi_p)\mathrm{exp}\left[-\frac{(\phi-\phi_p)^2}{2b^2}\right]\chi^2. \tag{24}$$

The parameters A^2, b, and ϕ_p respectively control the magnitude, width, and placement along the longitudinal direction of ΔV, the overall sign convention for ΔV is a result of having chosen the convention that $\phi > 0$ during inflation. It can be noted that this form of ΔV is an effective mass term for the χ field. When $|\phi - \phi_p| \gg, b$ $\Delta V \approx 0$ and has negligible effect. As ϕ approaches ϕ_p from above the χ field acquires a negative effective mass squared which reaches a minimum at $\phi = \phi_p + b$

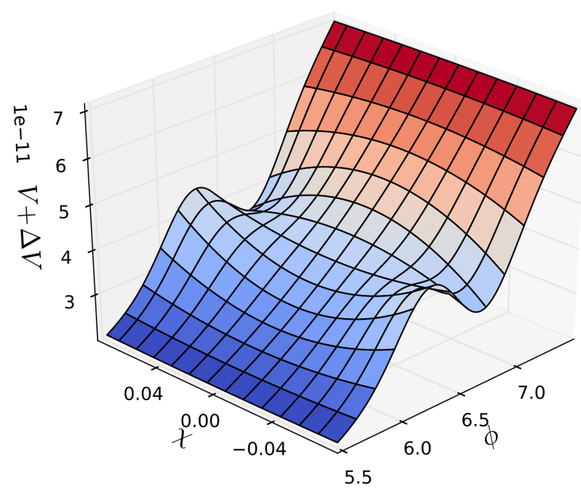

Figure 2: An example potential surface in the form of (23). The transverse instability is visible as the portion of the potential surface which is concave down along the χ direction, $\chi = 0$ becomes stable for the portion of the potential surface that is concave up along the χ direction. The overall downwards slope from larger to lower values of ϕ is due to the ϕ^4 term in the potential and is mainly responsible for driving inflation in the longitudinal direction. Fields are in units of M_{Pl}.

(the potential remaining bounded from below by the χ^4 term in (23)). Likewise, as ϕ approaches ϕ_p from below the χ field acquires a positive effective mass squared which reaches a maximum at $\phi = \phi_p - b$. The surface of this potential is shown in Figure 2 while the effective mass of χ is shown in Figure 3.

With the inclusion of a nonzero ΔV in (23) there is, associated with the instability in the transverse direction, a change in $V_{,\phi}$ which becomes more significant as χ is driven away from zero

$$V_{,\phi} = \lambda_\phi \phi^3 - \frac{A^2\sqrt{e}}{b}\left(1 - \frac{(\phi-\phi_p)^2}{b^2}\right)\exp\left[-\frac{(\phi-\phi_p)^2}{2b^2}\right]\chi^2. \tag{25}$$

There is then the possibility that applying a transverse instability of the form (24) will significantly alter the evolution of the longitudinal field, potentially causing an

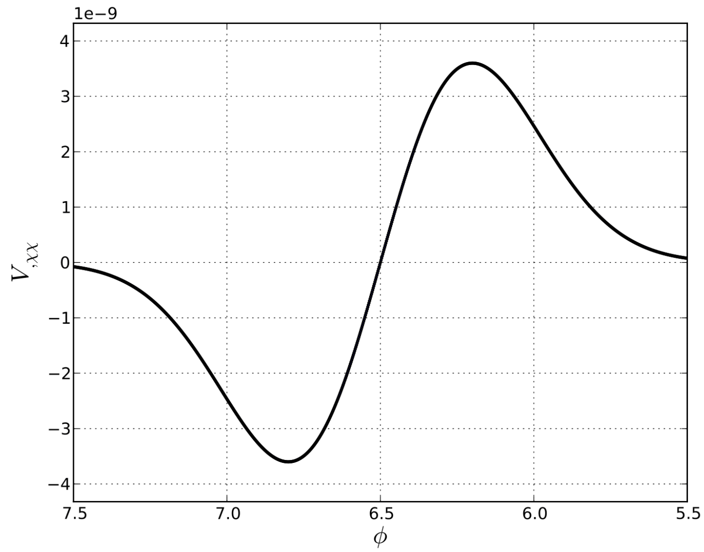

Figure 3: A visualization of the transverse instability, the effective mass squared of χ (evaluated along $\chi = 0$ with parameters $b = 0.3M_{Pl}$, $\phi_p = 6.5M_{Pl}$). For $\phi > \phi_p$ the transverse field is unstable around $\chi = 0$, for $\phi < \phi_p$ the transverse field becomes stable around $\chi = 0$. This plot relates to the potential surface in Figure 2 by being a measure of its curvature in the χ direction.

early end to inflation. As the target of investigation for the current work is one in which inflation continues along the longitudinal direction it will be necessary to place appropriate bounds on the potential considered (the case where $V_{,\phi}$ changes sufficiently to stop ϕ from proceeding towards zero is a realization of the trpping mechanism, but not the subject of this report [6]).

If we wish to consider the case that inflation continues during the transverse instability we should require $\dot{\phi} < 0$ during inflation, that is ϕ continues to roll down its potential without reversing direction. In order to remain consistent with this requirement for inflation to continue we impose a set of bounds on the parameters of ΔV. To estimate this set of bounds, note that in the slow-roll approximation the requirement $\dot{\phi} < 0$ is equivalent to $V_{,\phi} > 0$. In the slow-roll approximation it is sufficient to require that over the range of values of ϕ and χ covered during the evolution of the system $V(\phi, \chi)$ does not attain a local maximum. It is in principle possible to solve for this condition numerically, however by making several approximations a sufficient condition can be derived.

The domain of interest, where inflation along the longitudinal direction could be stopped by the addition of ΔV to the potential is the region where $\Delta V_{,\phi} < 0$, from (25) this is $|\phi - \phi_p| < b$. The inclusion of the χ^4 term in the potential insures that the range of χ remains bounded, let $|\chi|_{max}$ be a bounding value on χ so that $|\chi| \leq |\chi|_{max}$. Within the range $|\phi - \phi_p| < b$ and $|\chi| < |\chi|_{max}$ a lower bound can be placed on $\Delta V_{,\phi}$, comparing to (25) we have

$$\Delta V_{,\phi}(\phi, \chi) \geq \Delta V_{,\phi}(\phi = \phi_p, \chi = |\chi|_{max}) \tag{26}$$

$$= -\frac{A^2\sqrt{e}}{b}|\chi|^2_{max}. \tag{27}$$

The above bound on $\Delta V_{,\phi}$ leads to a lower bound on $V_{,\phi}$ for values of ϕ and χ in the stated range,

$$V_{,\phi}(|\phi - \phi_p| < b, |\chi| < |\chi|_{max}) \geq \lambda_\phi(\phi_p - b)^3 - \frac{A^2\sqrt{e}}{b}|\chi|^2_{max}. \tag{28}$$

Then in the same range and in the slow-roll approximation a sufficient condition to insure that inflation along the longitudinal direction is not stopped by including the ΔV term in the potential is given by

$$\frac{\lambda_\phi(\phi_p - b)^3 b}{\sqrt{e}A^2} \geq |\chi|^2_{max}. \tag{29}$$

It should be noted that $|\chi|_{max}$ is also determined by the potential parameters and the initial magnitude of χ fluctuations, so the bound on potential parameters provided by (29) is incomplete. In principle $|\chi|_{max}$ can be estimated from the parameters of the potentials, however even without performing such an estimate (29) can still be used a scaling relation for the maximum effect size the transverse instability can provide while still allowing inflation to continue along the longitudinal direction.

4.2 Effects of Nonzero ΔV

The effect of introducing a nonzero ΔV term in the form of (24) to the potential is that as inflation proceeds in the longitudinal direction and ϕ decrease, χ will experience a transient tachyonic instability for values of $\phi \approx \phi_p + b \pm b$. As the transverse field, χ, initially has a mean of zero the fluctuations of χ will take on both positive and negative values. During the instability positive fluctuations of χ will grow exponentially more positive, while negative fluctuations will grow exponentially more negative. The net effect of this instability is to cause a separation between trajectories of initially similar fluctuations in χ, increasing the importance of gradient terms in the equations of motion. Once ϕ has decreased so that $\phi \leq \phi_p$ the transverse instability terminates and the ΔV term in the potential instead bestows χ with an effective mass, causing the field to oscillate with time for $\phi \approx \phi_p - b \pm b$. These oscillations are damped by the Hubble drag term in the equations of motion. This separation of trajectories is illustrated in Figure 4 which compares the evolution of the χ field sampled at different lattice sites as calculated with the inclusion of a ΔV term in the potential.

To examine, in more detail, the behaviour of χ after the transverse instability has terminated we can write down its equation of motion from (7), (23) and (24)

$$0 = \ddot{\chi} + 3H\dot{\chi} + (-\frac{\Delta\chi}{a^2} + \lambda_\chi \chi^3 + \Delta V_{,\chi}) \tag{30}$$

$$0 = \ddot{\chi} + 3H\dot{\chi} + (-\frac{\Delta}{a^2} + \Delta V_{,\chi\chi})\chi + \lambda_\chi \chi^3. \tag{31}$$

For $\Delta V_{,\chi\chi} > 0$ this equation resembles a damped oscillator equation with time dependent parameters, and anharmonicities arising due to the $\lambda_\chi \chi^3$ term and the indirect χ dependence of $\Delta V_{,\chi\chi}$ from coupling between ϕ and χ . The qualitative

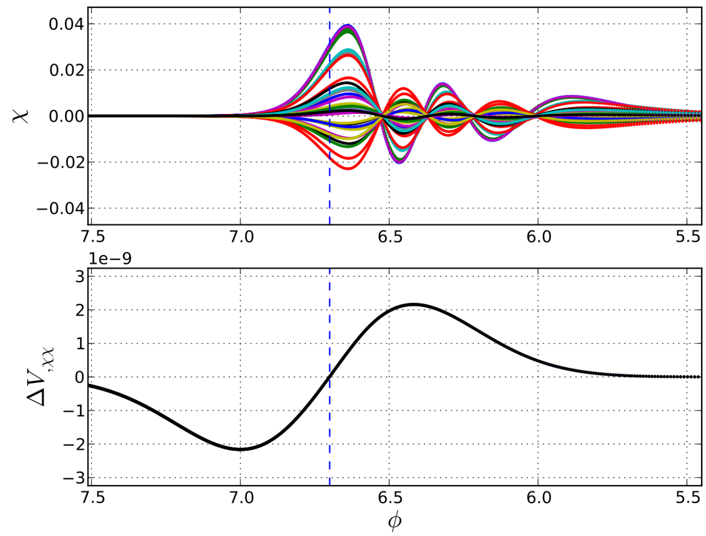

Figure 4: *Top:* The evolution of χ at sampled lattice points showing the effects of ΔV. *Bottom:* $\Delta V_{,\chi\chi}$ is a measure of the transverse instability, it is plotted here for the parameters of ΔV used in is the calculation of χ shown in the top plot.
The vertical dashed line is located at $\phi = \phi_p$, where the transverse direction transitions from unstable to stable. For $\phi > \phi_p$ trajectories of χ separate due to the transverse instability causing growth of fluctuations, initial fluctuations in the χ field are smaller than the scale visible on this plot. For $\phi < \phi_p$ the transverse direction is stable and the χ field oscillates. Fields are units of M_{Pl}, $\Delta V_{,\chi\chi}$ is in units of M_{Pl}^2.

behaviour of χ after the transverse instability can be explained assuming that the time dependence of the parameters in (31) vary slowly enough that treating (31) as a damped oscillator equation is a good approximation over short enough periods of time. If this is the case, then in analogy with a damped oscillator an effective restoring force is supplied by $\Delta V_{,\chi\chi}$ and Laplacian terms of χ, with damping being provided by the Hubble drag term.

With both sources of the effective restoring force becoming exponentially suppressed ($\Delta V_{,\chi\chi}$ due to the Gaussian tail for $\phi < \phi_p - b$ and $-\Delta/a^2$ due to the approximately exponential growth of a during inflation) and the drag term H only

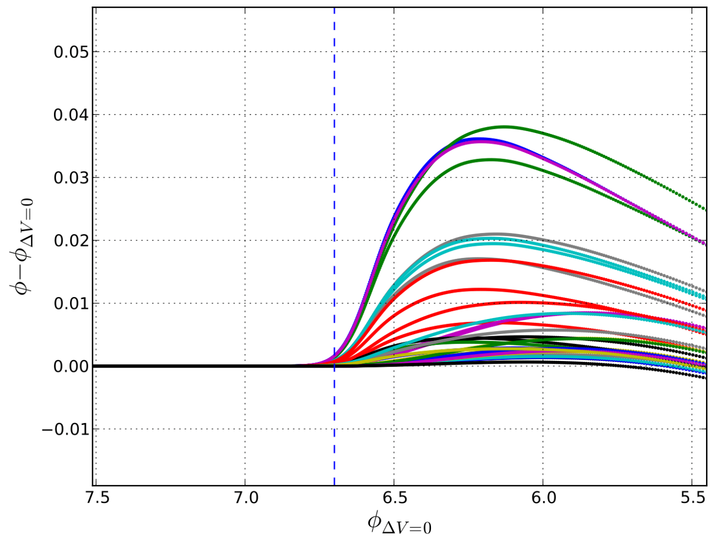

Figure 5: The difference between ϕ as calculated with and without a ΔV term in the potential at sampled points on the lattice plotted on the vertical axis, with ϕ as calculated without a ΔV term in the potential along the horizontal axis, the vertical dashed line is located at $\phi = \phi_p$. The separation of trajectories of ϕ is due to a secondary effect of the transverse instability causing growth in χ fluctuations, as χ fluctuations also cause growth in fluctuations of $V_{,\phi}$ (see (25)). Trajectories of ϕ separate at a delay to those of χ show in Figure 4. Fields are in units of M_{Pl}.

slowly varying during inflation the behaviour of the χ field will at some point change over from an oscillatory to an overdamped state. At some point after χ begins exhibiting overdamped behaviour the effective restoring force will become negligible and the field χ will approach a constant comoving configuration for the duration of inflation, this can be seen in Figure 4.

There is, associated with the transverse instability, a separation of trajectories in the ϕ field. As fluctuations of the χ field grow during the transverse instability $\Delta V_{,\phi}$ becomes more spatially inhomogeneous as well as growing in magnitude. The result is $V_{,\phi}$ becomes less homogeneous and there is a growth of the nonzero Fourier modes of ϕ. From the slow-roll approximation (16) and the form of $V_{,\phi}$, it should be noted that for $\phi < \phi_p$ the larger the fluctuation of χ at a particular lattice site the more slowly ϕ will roll down its potential. This means the effect of ΔV on the ϕ field is to slow its rate of change, but to do so in a way which is inhomogeneous. The separation of ϕ trajectories is shown in Figure 5.

The separation of field values at different comoving positions (lattice sites) shown in Figure 4 and Figure 5 allows for energy flow to occur between comoving volumes as long as they remain causally connected. The flow of energy between comoving volumes violates the adiabatic assumption used for the derivation of the fluid equation (4), as previously mentioned in Section 2.2, this leads to the non-conservation of the ζ parameter.

Fluctuations of the fields at initial conditions will also lead to non-conservation of ζ. In order to isolate the change in ζ that is due to the transverse instability two calculations for the evolution of the system are performed from identical initial conditions, one including the ΔV term in the potential and one setting $\Delta V = 0$. Figures 6 and 7 compare the production of ζ with and without the transverse instability at sampled lattice sites and averaged over the lattice respectively.

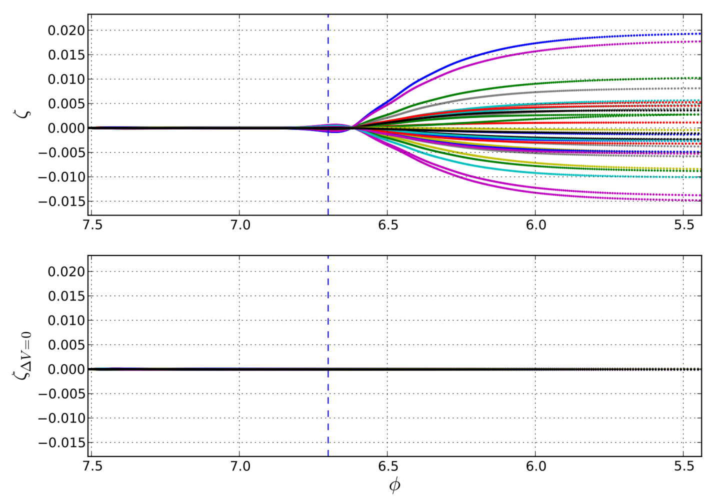

Figure 6: *Top:* ζ as calculated at sampled lattice sites with a nonzero ΔV term included in the potential. *Bottom:* ζ as calculated at sampled lattice sites with the ΔV term excluded from the potential.
For the top plot the parameters of ΔV are the same as those used in Figures 4 and 5, the vertical dashed line is located at $\phi = \phi_p$. The scales of both the top and bottom plots are the same, demonstrating that a transverse instability during inflation is capable of producing a change in ζ that is large compared to the change in ζ caused by the initial fluctuations of the fields, while still allowing inflation to continue in the transverse direction.

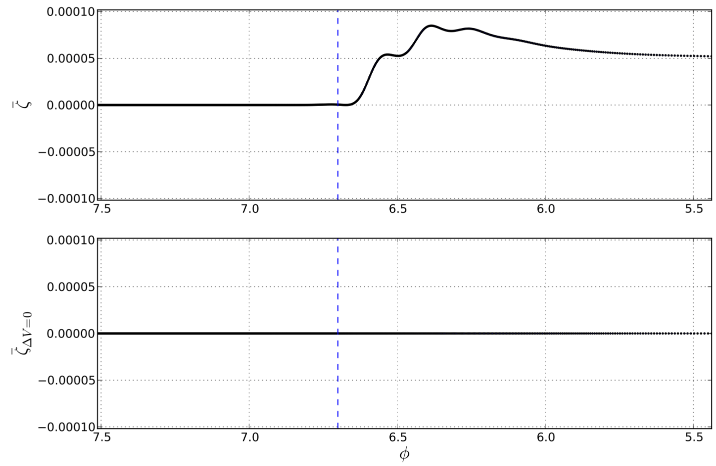

Figure 7: *Top:* ζ calculated as an average over the lattice with a nonzero ΔV term in the potential. *Bottom:* ζ calculated as an over the lattice with the ΔV term excluded from the potential.

5 Conclusion

We have explored the effects of a temporary transverse instability in a model of two field inflation using a lattice simulation. Using parameters which were selected to allow inflation to continue along the longitudinal direction, the effects of including a ΔV term in the potential were analyzed for both the transverse and longitudinal field. The change in the ζ parameter was calculated for the scenario of including a transverse instability, with a baseline established in the absence of such an instability. It was found that the variation in ζ between lattice sites was greatly enhanced by the inclusion of the ΔV term to the potential. It remains for future work to determine the post-inflation effects associated with the large production of ζ varying across the lattice.

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