1. Mixed Integer Linear Programming (MILP) for Flexible Job Shop Scheduling Problems (FJSSP)

Following is the description of the MILP modelling for the FJSSP to use as comparison benchmark on 402 commonly used benchmark instances [1]. The description is organised as follows: Section 1.1 defines the notation used to create the MILP model. Section 1.2 describes the objective used for the optimisation and Section 1.3 presents the constraints of the model.

1.1. ***Notation***

To describe the MILP formulation of the FJSSP the following notation is used:

- Subscripts:

$$i \ldots \text{subscript for machines where } 1 \leq i \leq m$$
$$j, j' \ldots \text{subscript for jobs where } 1 \leq j, j' \leq n$$
$$k, k' \ldots \text{subscript for operations where } 1 \leq k, k' \leq n_j$$

- Parameters:

$$m \ldots \text{number of machines}$$
$$n \ldots \text{number of jobs}$$
$$n_j \ldots \text{number of operations in job } j$$
$$O_{j,k} \ldots \text{operation } k \text{ in job } j$$
$$p_{j,k,i} \ldots \text{Processing time of operation } O_{j,k} \text{ on machine } i$$
$$L \ldots \text{a large number e.g. } L = \sum_{j=1}^{n} \sum_{k=1}^{n_j} \max_{i=1,\ldots,m} (p_{j,k,i})$$

- Decision variables:

$X_{j,k,j',k'}$ ...Binary: 1 if operation $O_{j,k}$ is processed after operation $O_{j',k'}$ on the same machine

$Y_{j,k,i}$ ...Binary: 1 if operation $O_{j,k}$ is processed on machine i

$C_{j,k}$ ...Continuous: Completion time of operation $O_{j,k}$

- Index sets:

For all operations $O_{j,k}$ for $1 \leq j \leq n, 1 \leq k \leq n_j$ we need

$$M_{j,k} = \{i : \text{ such that } O_{j,k} \text{ can be performed on } M_i\}$$

1.2. ***Objective***

$$C_{max} \rightarrow \text{ min : Makespan (maximum completion time)}$$

1.3. ***Constraints***

1.3.1. Assignment constraints

$$\sum\nolimits_{i \in M_{j,k}} Y_{j,k,i} = 1 \text{ for } 1 \leq j \leq n, 1 \leq k \leq n_j$$

This set of constraints guarantees that all operations $O_{j,k}$ are assigned to exactly one machine.

1.3.2. Sequencing constraints within jobs

A novel formulation using the completion times of the operations is used.

$$C_{j,1} \geq \sum\nolimits_{i \in M_{j,1}} Y_{j,1,i} p_{j,1,i} \text{ for } 1 \leq j \leq n$$

$$C_{j,k} \geq C_{j,k-1} + \sum\nolimits_{i \in M_{j,k}} Y_{j,k,i} p_{j,k,i} \text{ for } 1 \leq j \leq n, 1 < k \leq n_j$$

The completion time of the first operation $O_{j,1}$ of job j cannot be smaller than it's processing time. Furthermore the completion time of the k^{th} operation $O_{j,k}$ of job j can't be smaller then the completion time of its predecessor $O_{j,k-1}$ in job j plus it's processing time.

1.3.3. Sequencing constraints within machines

$$C_{j,k} \geq C_{j',k'} + p_{j,k,i} - L(3 - X_{j,k,j',k'} - Y_{j,k,i} - Y_{j',k',i})$$
$$C_{j',k'} \geq C_{j,k} + p_{j',k',i} - L(X_{j,k,j',k'} + 2 - Y_{j,k,i} - Y_{j',k',i})$$

Both constraints have to hold for operations $O_{j,k}$ with $1 \leq j < n$ and $1 \leq k \leq n_j$ and operation $O_{j',k'}$ with $j' > j$ and $1 \leq k' \leq n_{j'}$ and machines M_i with $i \in M_{j,k} \cap M_{j',k'}$.

The two sets of constraints are trivially fulfilled if the expression multiplied to the large number L is nonzero. If operation $O_{j,k}$ and $O_{j',k'}$ are processed on the same machine M_i ($Y_{j,k,i} = 1$ and $Y_{j',k',i} = 1$) the sequencing between these two operations is controlled by $X_{j,k,j',k'}$.

If $X_{j,k,j',k'} = 1$ the first constraint is active and guarantees that the processing time of the successor $O_{j,k}$ does not start before the predecessor $O_{j',k'}$ is finished. Otherwise ($X_{j,k,j',k'} = 0$ the second constraint is active and non overlapping processing times are implied by the second constraint.

1.3.4. Makespan constraint

$$C_{max} \geq C_{j,n_j} \text{ for } 1 \leq j \leq n$$

The makespan C_{max} can't be smaller than the completion time of the last operation O_{j,n_j} of all jobs.

1.3.5. Variable type constraints

$$\begin{aligned} &C_{j,k} \geq 0 \\ &Y_{j,k,i} \in \{0,1\} \\ &X_{j,k,j',k'} \in \{0,1\} \end{aligned}$$

The completion times $C_{j,k}$ are non-negative and $Y_{j,k,i}$, $X_{j,k,j',k'}$ are binary.

References

[1] D. Behnke, M. J. Geiger, Test Instances for the Flexible Job Shop Scheduling Problem with Work Centers 31.