Human Perception of Two Dimensional Symmetry

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Abstract

This paper investigates human ability to innately perceive multiple different features of symmetry. While most work has focused on simple bilateral reflection symmetry, these experiments make a case that humans can perceive much more than that. By using a set of images that each conform to one of the seventeen wallpaper groups, we investigate which wallpapers groups are easiest and most difficult to tell apart. This allows us to provide evidence that rotation and reflection are not sufficient in explaining perception of symmetry, challenging some prevailing notions.

Introduction

The concept of *symmetry* refers to a common property of everyday objects and images. Symmetry is present when, for instance, one half of the object appears mirrored along an axis. Faces are a good example: they are approximately symmetrical along a vertical axis. Symmetry has received much attention as a feature of visual perception and attention (see (?, ?) for a review). However, symmetry as a geometric concept is much more general: symmetry refers to any transformation of an object or image that leaves it with the exact same appearance. Mathematically, exactly four types of symmetry are distinguished in two dimensional images.

Prior work has focused largely on reflection. It has even been suggested that this is the only type of symmetry humans can process at once (?, ?). In this study, we present empirical data suggesting that this is not the case. The experiments are designed to test whether people perceive differences in symmetry, and whether they can identify and distinguish all groups of objects that are formed by two dimensional symmetries.

We will use *wallpapers* to create images that use combinations of these symmetry types. Wallpapers are two-dimensional images made from an infinitely repeating symmetric *tile*. These tiles are characterized by a specific set of symmetries. There are exactly seventeen wallpaper groups, as has been well-known for over a hundred years (?, ?). In other words, every two-dimensional repeating image has one of seventeen sets of symmetries. These wallpapers can be arranged in a hierarchy, which we argue relates to how they are perceived.

Reflection

Figure : Characterized by the number and placement of its axes

Rotation

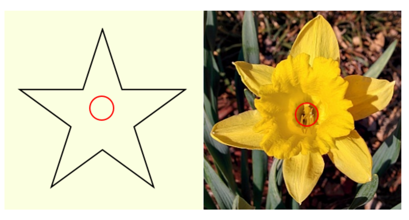

Figure : Characterized by the number of rotation angles

Translation

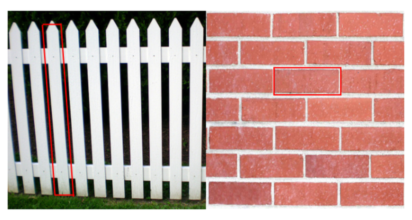

Figure : Characterized by the repeating shape

Glide Reflection

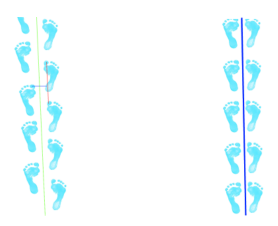

Figure : Characterized by the number and location of the axes

Wallpaper Groups

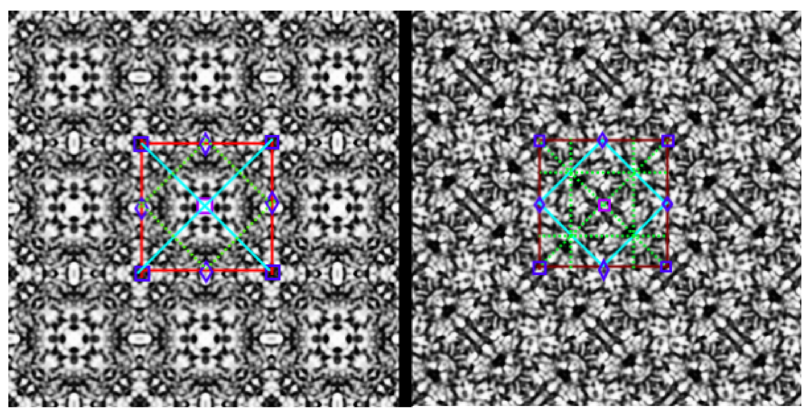

Figure : Different Groups: On the left is P4M, on the right is P4G.

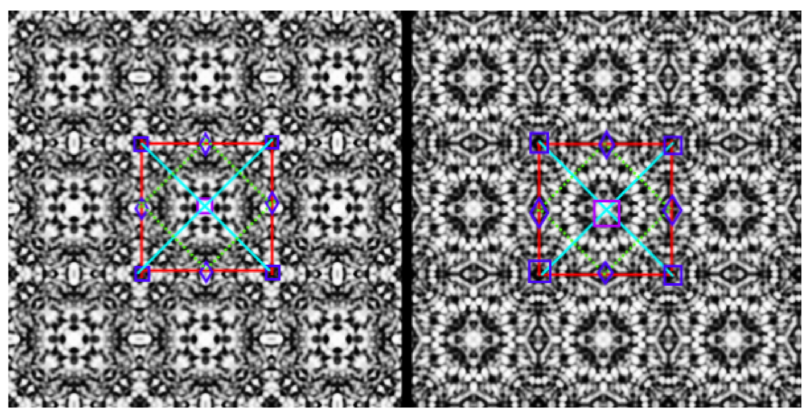

Figure : Same Groups: Both P4M.

Task

We recruited 106 subjects from the Amazon Mechanical Turk platform to complete the task. We pruned twelve subjects for not taking the task seriously. All subjects were compensated for their time. The task compared participants' ability to distinguish among wallpaper groups

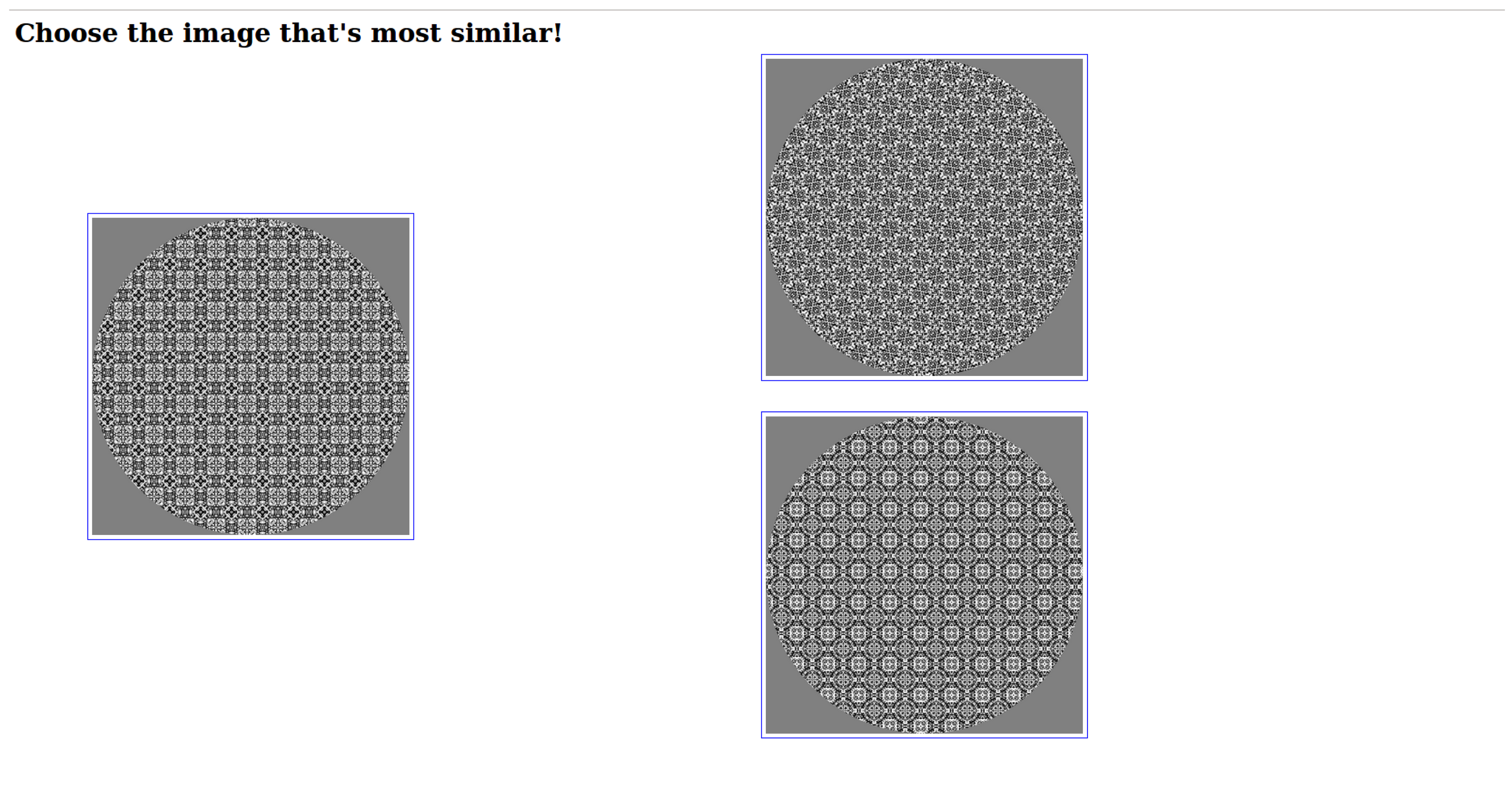

Figure : A screenshot of the experimental task

Reflection

Group	2-fold	3-fold	4-fold	6-fold	T_1	T_2	D_1	D_2	tile
P1	F	F	F	F	None	None	None	None	O
P2	T	F	F	F	None	None	None	None	O
PM	F	F	F	F	Refl	None	None	None	Re
PG	F	F	F	F	Glide	None	None	None	Re
CM	F	F	F	F	None	None	Refl	None	Rh
PMM	T	F	F	F	Glide	Refl	None	None	Re
PMG	T	F	F	F	Glide	Refl	None	None	Re
PGG	T	F	F	F	Glide	Glide	None	None	Re
CMM	T	F	F	F	None	None	Refl	Refl	Rh
P4	T	F	T	F	None	None	None	None	S
P4M	T	F	T	F	Refl	Refl	Refl	Refl	S
P4G	T	F	T	F	Glide	Glide	Refl	Refl	S
P3	F	T	F	F	None	None	None	None	H
P3M1	F	T	F	F	None	None	Refl	None	H
P31M	F	T	F	F	Refl	Refl	Refl	None	H
P6	T	T	F	T	Refl	None	None	None	H
P6M	T	T	F	T	Refl	Refl	Refl	Refl	H

Table : Wallpaper groups represented as their symmetries. The first four columns are whether the group has that type of rotation symmetry. The second four columns refer to the four main axes on the tile. (Refl=Reflection, Glide=Glide Reflection, Re=Rectangular, Rh=Rhombic, O=Oblique, S=Square, H=Hexagonal)

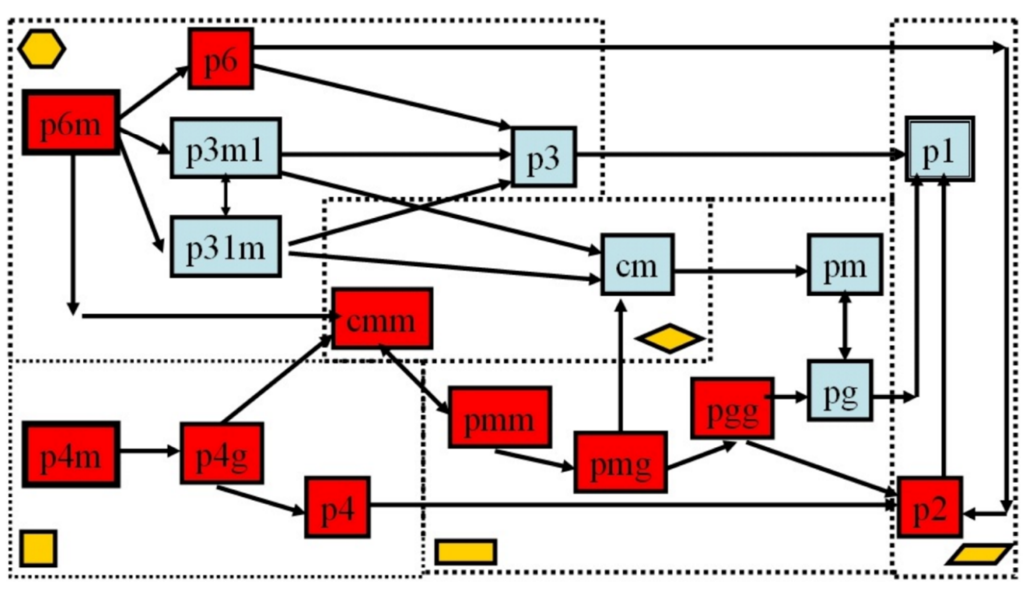

Figure : The subgroup relation graph. If an arrow points from any given box, ***A***, toward any given box ***B***, that means that ***B***'s symmetries are subset of ***A***'s symmetries.

Rotation

Translation

Results

The best model was identified according to Aikake Information Criterion (AIC), among the models including the pool of features described earlier. This best model included distance, but additionally included the T_1 axis, which would correspond to bilateral reflection symmetry, the D_1 axis, which is the positive diagonal, 4-fold rotation, and 3-fold rotation.