Symbol	Meaning
i	Individual or case (Level-1)
j	Group or cluster (Level-2)
γ	Represents a fixed effect
γ_{00}	Grand mean
β_{0j}	Random intercept
β_{1j}	Random slope
ϵ_{ij}	Within-group variability (Level 1)
υ_{0j}	Between-group variability (Level 2)
τ_{00}	Res. var. of Level-2 random intercepts
σ^2	Res. var. of Level-1 random intercepts

Concept	Symbol	Level
Raw data	x_{ij}	Level 1 and 2
Grand mean centered	$\ddot{x}_{ij}$	Level 1 and 2
Group mean centered	$\dot{x}_{ij}$	Level 1
Group means	$\bar{x}_{.j}$	Level 2
Centered group means	$\bar{x'}_{.j}$	Level 2

IMPORTANT
Independent variables: x_1, x_2, $x_3 \ldots$
Dependent variable: Y

Level 1: **within-group** variation, represented by i
Level 2: **between-group** variation, represented by j

Note that Level 1 variables can carry both Level 1 and 2 variation. In x_{ij}, the subscript 0 represents the intercept and the 1 represents the slope. E.g., γ_{01} denotes the first predictor of the intercept and γ_{12} represents the second predictor of the slope.

The *fixed effects* of the model are constant for all cases in the population and do not carry i or j subscripts. Effects that vary across Level-2 units are denoted by γ and ϵ.

The *random effects* of the model vary across Level-1 and Level-2 units. Effects that vary across Level-2 units are denoted by υ and τ.

Intercept only Regression

$$\mathrm{Y_i} = \beta_0 + \epsilon_i$$

Simple linear regression

$$\mathrm{Y_i} = \beta_0 + \beta_1 x_i + \epsilon_i$$

Multiple linear regression

$$\mathrm{Y_i} = \beta_0 + \beta_1 x_i' + \epsilon_i$$

Moderation

$$\mathrm{Y_i} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$$

Moderation can be re-written as:

$$\mathrm{Y_i} = \beta_0 + \beta_1 x_1 + (\beta_2 + \beta_3 x_1) x_2 + \epsilon_i$$

Fixed effects model

$$\mathrm{Y_i} = \beta_1 x_{1i} + \beta_2 x_{2i} + \ldots + \beta_p x_{pi} + \epsilon_i$$

$$\epsilon_i \sim_{iid} N(0, \sigma^2)$$

Random effects ANOVA
Level-1:

$$\mathrm{Y_{ij}} = \beta_{0j} + \epsilon_{ij}$$

$$\epsilon_i \sim N(0, \sigma^2)$$

Level-2:

$$\beta_{\mathrm{0j}} = \gamma_{00} + \upsilon_{0j}$$

$$\upsilon_{0j} \sim_{iid} N(0, \tau_{00})$$

Reduced:

$$\mathrm{Y_{ij}} = \gamma_{00} + \upsilon_{0j} + \epsilon_{ij}$$

Random intercept regression model
Level-1:

$$\mathrm{Y_{ij}} = \beta_{0j} + \beta_{1j} x_{ij} + \epsilon_{ij}$$

$$\epsilon_i \sim N(0, \sigma^2)$$

Level-2:

$$\beta_{\mathrm{0j}} = \gamma_{00} + \upsilon_{0j}$$

$$\beta_{\mathrm{1j}} = \gamma_{10}$$

$$\upsilon_{0j} \sim_{iid} N(0, \tau_{00})$$

Reduced:

$$\mathrm{Y_{ij}} = \gamma_{00} + \gamma_{10} x_{ij} + \upsilon_{0j} + \epsilon_{ij}$$

Means as outcomes
Level-1:

$$\mathrm{Y_{ij}} = \beta_{0j} + \epsilon_{ij}$$

$$\epsilon_i \sim N(0, \sigma^2)$$

Level-2:

$$\beta_{\mathrm{0j}} = \gamma_{00} + \gamma_{01} w_j + \upsilon_{0j}$$

$$\upsilon_{0j} \sim_{iid} N(0, \tau_{00})$$

Reduced:

$$\mathrm{Y_{ij}} = \gamma_{00} + \gamma_{01} w_j + \upsilon_{0j} + \epsilon_{ij}$$

Intercepts as outcomes
Level-1:

$$\mathrm{Y_{ij}} = \beta_{0j} + \beta_{1j} x_{ij} + \epsilon_{ij}$$

$$\epsilon_i \sim N(0, \sigma^2)$$

Level-2:

$$\beta_{\mathrm{0j}} = \gamma_{00} + \gamma_{01} w_j + \upsilon_{0j}$$

$$\beta_{\mathrm{1j}} = \gamma_{10}$$

$$\upsilon_{0j} \sim_{iid} N(0, \tau_{00})$$

Reduced:

$$\mathrm{Y_{ij}} = \gamma_{00} + \gamma_{01} w_j + \gamma_{10} x_{ij} + \upsilon_{0j} + \epsilon_{ij}$$

Slopes as outcomes
Level-1:

$$\mathrm{Y_{ij}} = \beta_{0j} + \beta_{1j} x_{ij} + \epsilon_{ij}$$

Level-2:

$$\beta_{\mathrm{0j}} = \gamma_{00} + \gamma_{01} w_j + \upsilon_{0j}$$

$$\beta_{\mathrm{1j}} = \gamma_{10} + \gamma_{11} w_j + \upsilon_{1j}$$

$$\begin{pmatrix} \upsilon_{0j} \\ \upsilon_{1j} \end{pmatrix} \overset{iid}{\sim} \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & \tau_{10} \\ \tau_{10} & \tau_{11} \end{bmatrix} \right)$$

Reduced:

$$\mathrm{Y_{ij}} = \gamma_{00} + \gamma_{01} w_j + \gamma_{10} x_{ij} + \gamma_{11} w_j x_{ij} + \upsilon_{0j} + \upsilon_{1j} x_{ij} + \epsilon_{ij}$$

Simple Intercepts and Simple Slopes

$$\hat{\mathrm{y}} = \gamma_{00} + \gamma_{01} w + \gamma_{10} x + \gamma_{11} w x$$

And can be re-written as:

$$\hat{\mathrm{y}} = (\gamma_{00} + \gamma_{01} w) + (\gamma_{10} + \gamma_{11} w) x$$

Random Intercepts and Random Slopes Model
Level-1:

$$\mathrm{Y_{ij}} = \beta_{0j} + \beta_{1j} x_{ij} + \epsilon_{ij}$$

Level-2:

$$\beta_{\mathrm{0j}} = \gamma_{00} + \upsilon_{0j}$$

$$\beta_{\mathrm{1j}} = \gamma_{10} + \upsilon_{1j}$$

$$\begin{pmatrix} \upsilon_{0j} \\ \upsilon_{1j} \end{pmatrix} \overset{iid}{\sim} \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & \tau_{10} \\ \tau_{10} & \tau_{11} \end{bmatrix} \right)$$

Reduced:

$$\mathrm{Y_{ij}} = \gamma_{00} + \gamma_{10} x_{ij} + \upsilon_{0j} + \upsilon_{1j} x_{ij} + \epsilon_{ij}$$

Calculations
Correlation between random slopes and random intercepts:

$$r = \frac{\tau_{10}}{\sqrt{\tau_{00}}\sqrt{\tau_{11}}}$$

Intraclass correlation:

$$ICC = \frac{\tau_{00}}{\tau_{00} + \sigma^2}$$

Interpretation
Regression coefficient: β_i represents the predicted change in Y for a 1-unit increase in x_1, holding x_2 constant.

Centered regression coefficient: β_i is the influence if x_1' on Y when is x_2' is zero or for an individual at the mean of x_2.

Regression coefficient when an interaction is present: β_i is the change in Y for a 1-unit increase in x_1 when x_2 is zero.

Fixed effects
The *fixed effects* of the model are constant for all cases in the population and do not carry i or j subscripts. The Greek symbol gamma (γ) denotes fixed effects. E.g., γ_{00} has no subscript and does not vary.

$$\begin{aligned}\mathrm{Y_{ij}} = \gamma_{00} + \gamma_{10} Female_{ij} + \gamma_{20} Minority_{ij} + \\ \gamma_{30} Female_{ij} * Minority_{ij} + \upsilon_{0j} + \epsilon_{ij}\end{aligned}$$

The average language score for non-minority males is equal to γ_{00}. Female non-minorities scored γ_{10} points higher/lower on the language test compared to male non-minorities. Male minorities scored γ_{20} higher/lower on the language test compared to male non-minorities. Finally, female minorities scored γ_{30} points higher/lower compared to male minorities.

Covariance parameter estimates
τ_{00} can be interpreted as between-group variance or the covariance within cluster j, e.g., how much do students in the same class vary, or what percentage of the total variance is attributable to the differences between classrooms?

Confidence interval

β_1 = -0.11, 95% CI = [-0.22, -0.01]

The unknown population effect of mother's IQ on the number of children born to her (β_1) is expected to lie within 95% of similarly constructed intervals as [-0.22, -0.01].

Intraclass correlation
Within a cluster, how correlated are the Level-1 units? Of the total variation of your outcome, what percentage is between-cluster variation?

***p*-value and test-statistic**

β_1 = -0.11, t(8) = -2.48, p = .038.

There is a significant negative effect of mother's IQ on the number of children born to her. A 1-unit increase in mother's IQ is associated with a decrease of 1.1 children born to her.

Over repeated samples, the probability of observing a test statistic of -2.48 or more extreme, give that the null hypothesis of no effect of mother's IQ on children born to her is true, is 3.8%.