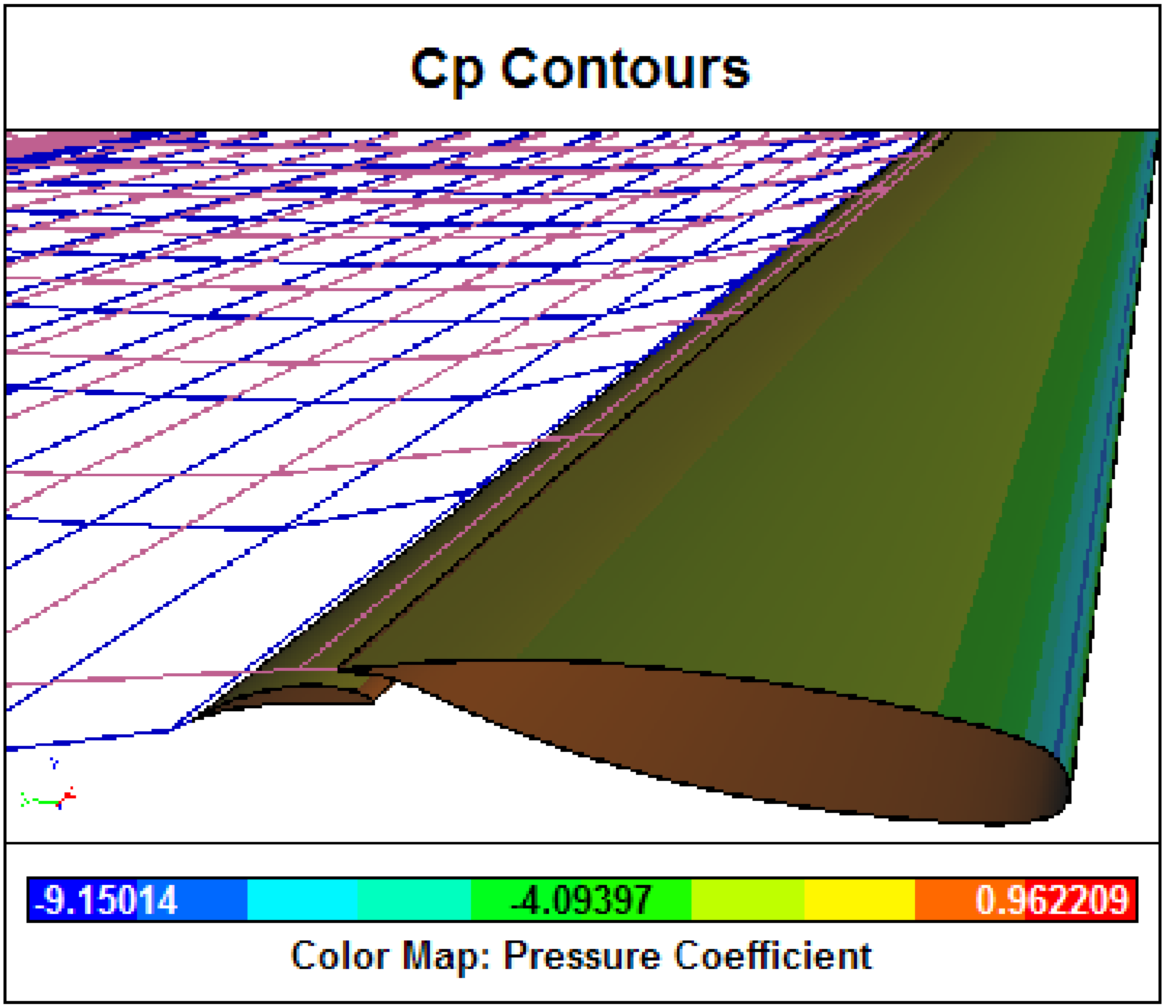


<u>Section 3</u>

<u>Potential Flows</u>

Motivation: Panel Methods to estimate flow over bodies

- No friction -> no boundary layers or wakes

Panel method treats the airfoil as a series of line segments

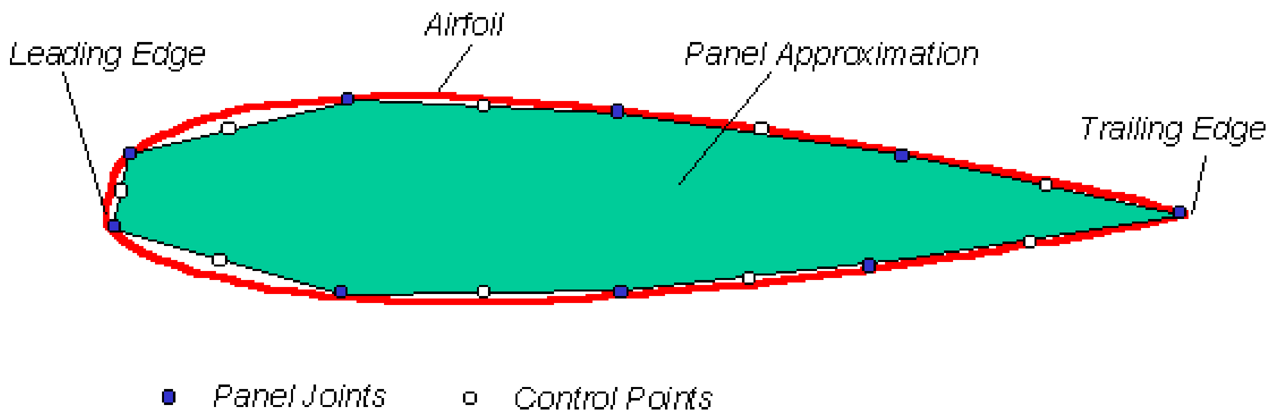


Figure 1. Vortex panel approximation to an airfoil.

On each panel, there is vortex sheet of strength $\Delta\Gamma = \gamma_0 \, ds_0$
Where ds_0 is the panel length.
Each panel is defined by its two end points (panel joints) and by the control point, located at the panel center, where we will
Apply the boundary condition ψ= Constant=C.
The more the number of panels, the more accurate the solution,
since we are representing a continuous curve by a series of broken straight lines

Section 5: Potential Flows (Chap A3) (3 lectures)

- Velocity Condition for Incompressible Flow (A3.6)
- Potential Flow – Laplace's Eqn (A3.7)
- Superposition of Elementary Flows: Concept (A3.9-12)
- Non-lifting Flow over a Circular Cylinder (A3.13)
- Potential Source Panel Method (A3.17)
- Potential Vortex (A3.14)
- Lifting Flow over a Circular Cylinder (A3.15)
- Kutta-Joukowski Lift (A3.16)

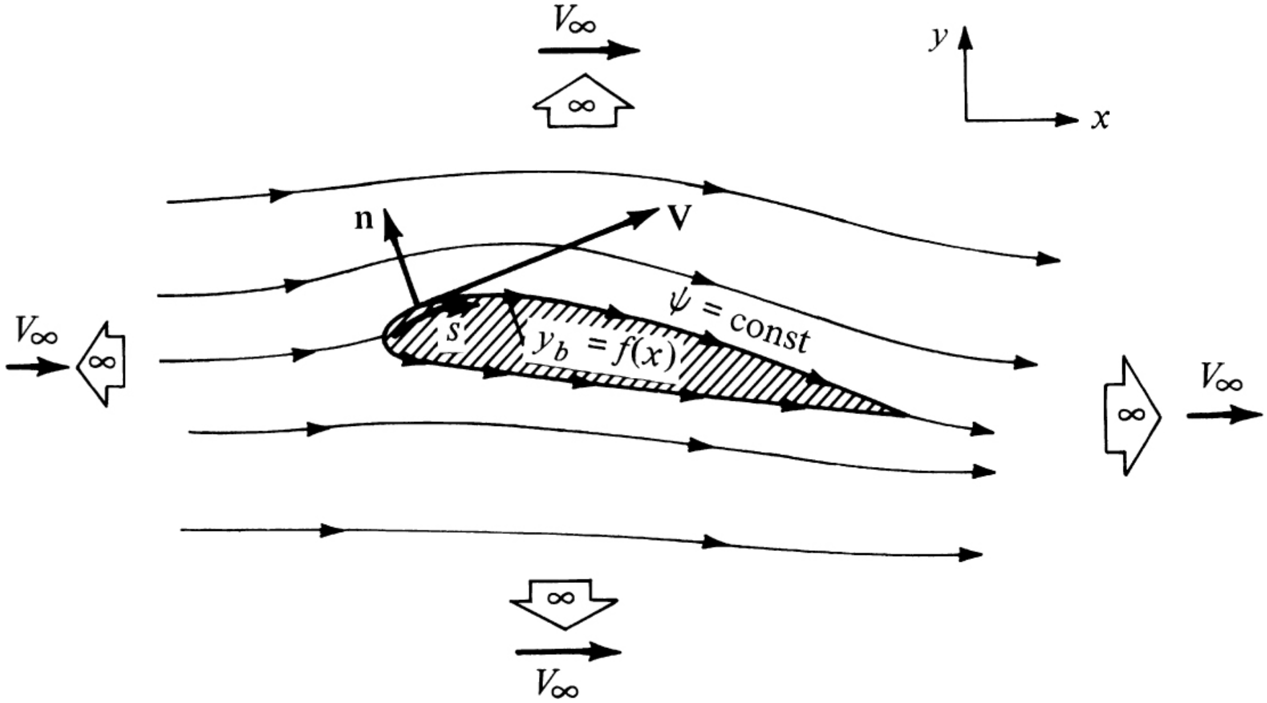
Condition on Velocity for Incompressible Flow (A3.6)

A Note About Vorticity

Potential Flow – LaPlace's Equation (A3.7)

LaPlace Equation in Different Ref Frames

Boundary Conditions for LaPlace Eqn



Two Sets of BC's for Any Body in Pot. Flow

Strategy to get surface forces on body in potential flow:

1) Solve with infinite and wall BC's appropriate for your particular flow (solutions usually a sum of elementary potential flows)

2) Obtain velocity field from potential function:

3) Obtain surface pressures via Bernoulli's along the surface (dividing) streamline:

4) Estimate boundary layer, calculate displace thickness , add to surface contour to get new surface

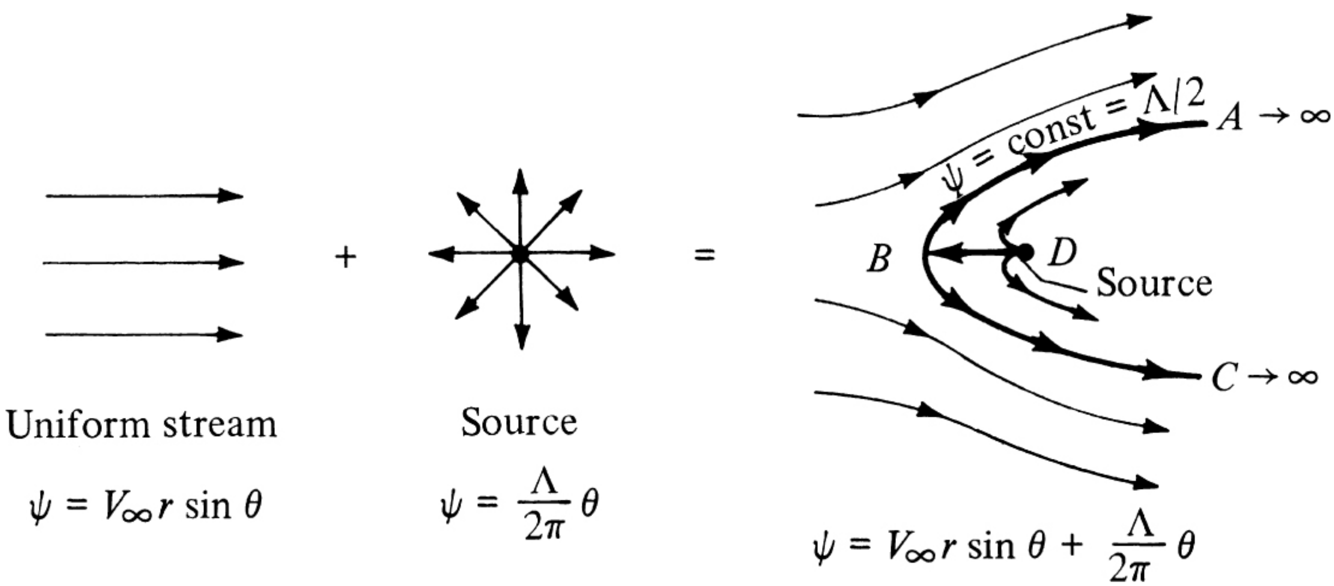
Elementary Potential Flows (Building Blocks)

Uniform Flow (A3.9):

Source Flow (A3.10):

Source Flow (con't):

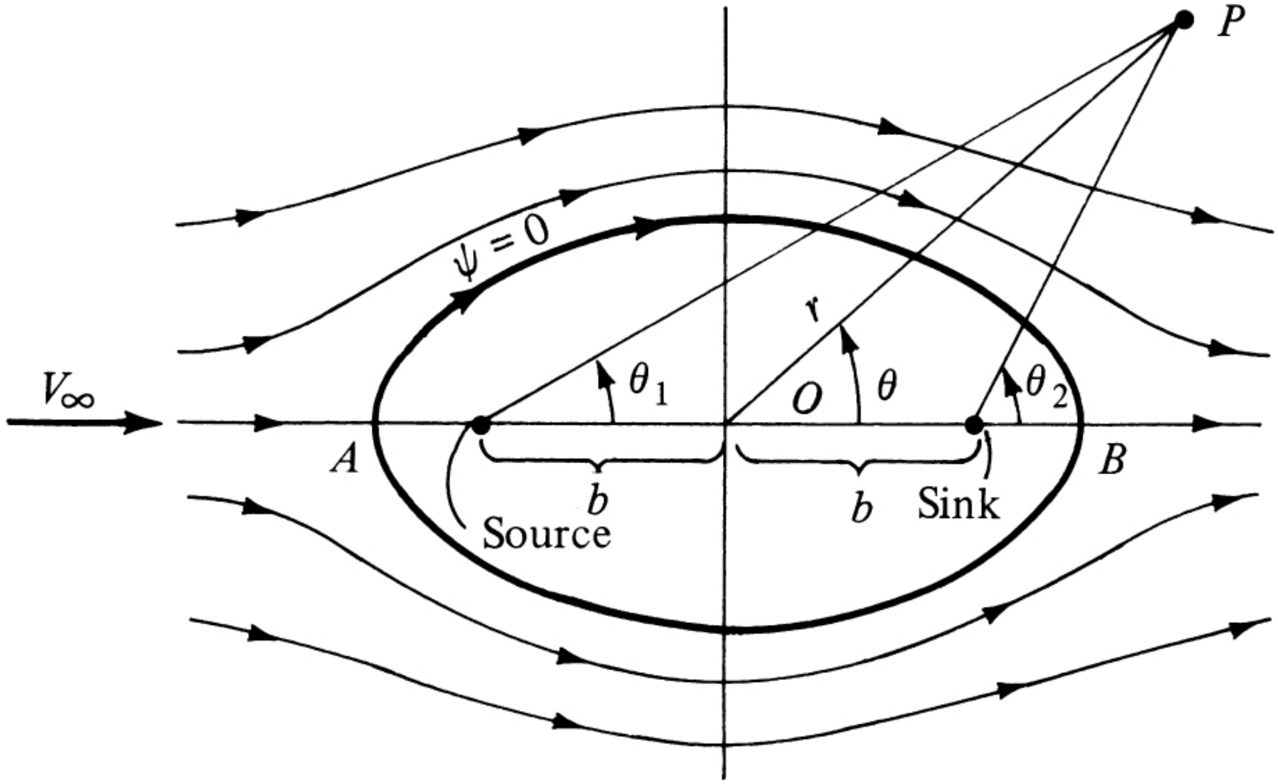
Uniform Flow + Source (A3.11):



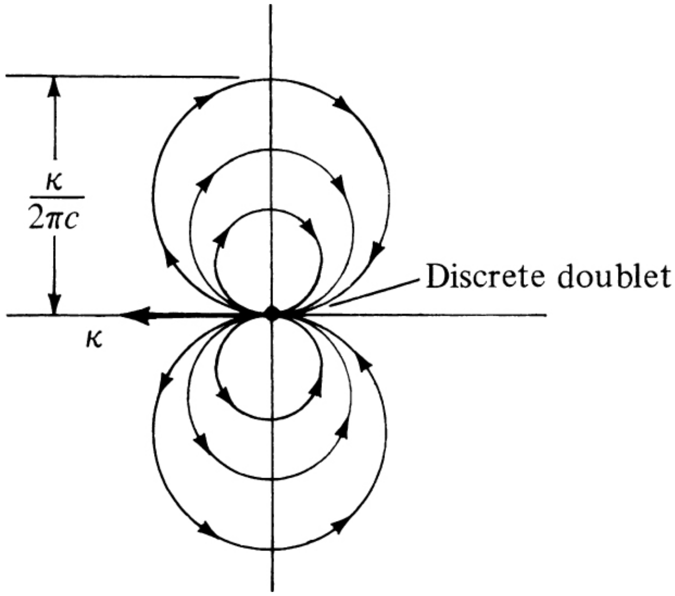
For this Combined Flow:

The whole point of potential-flow superposition:

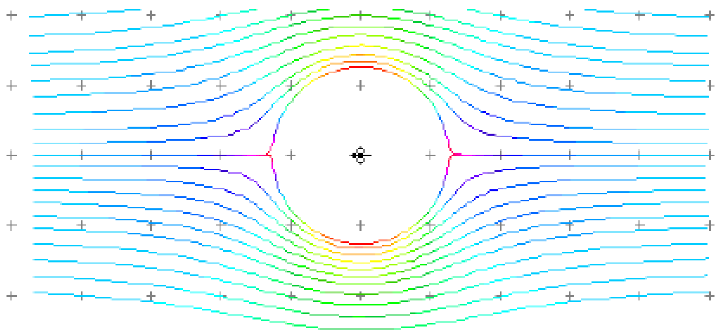
Rankine Oval:

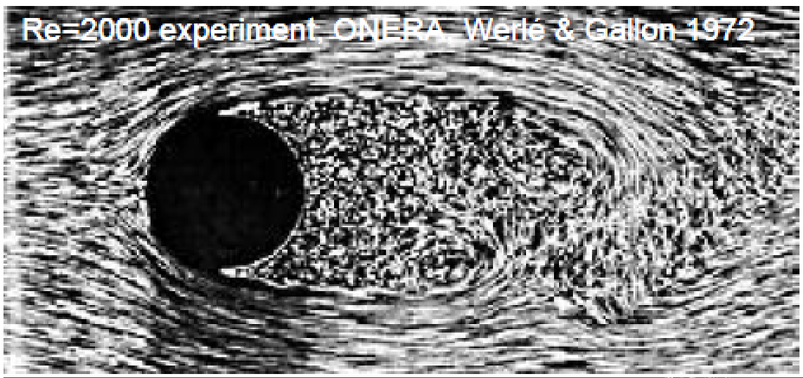


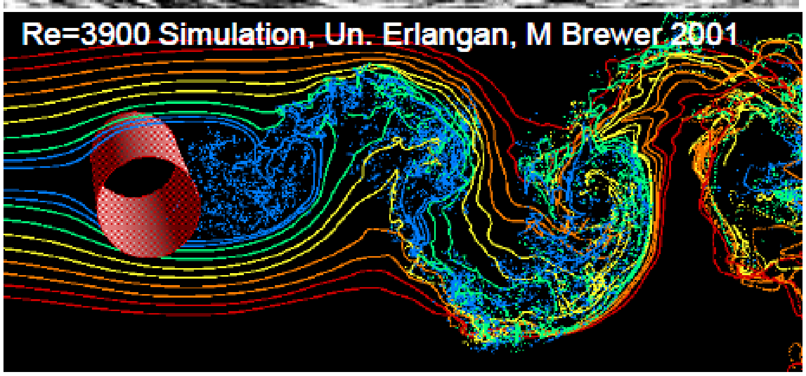
Source-Sink Combo: Doublet (A3.12):

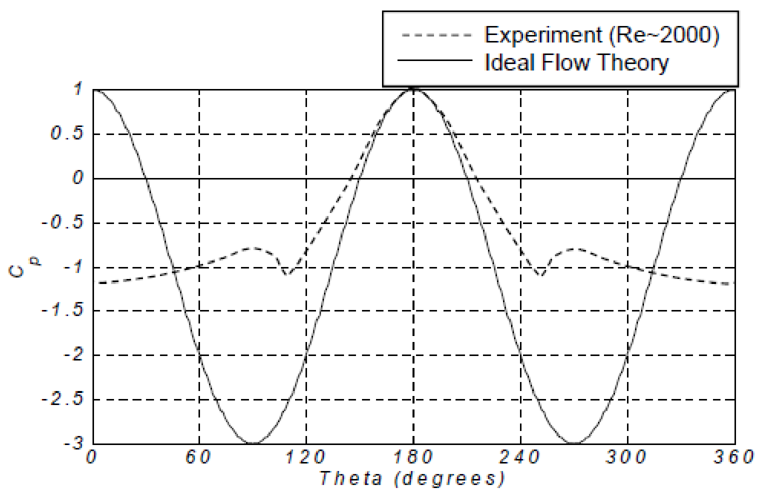


Cylinders in Crossflow:

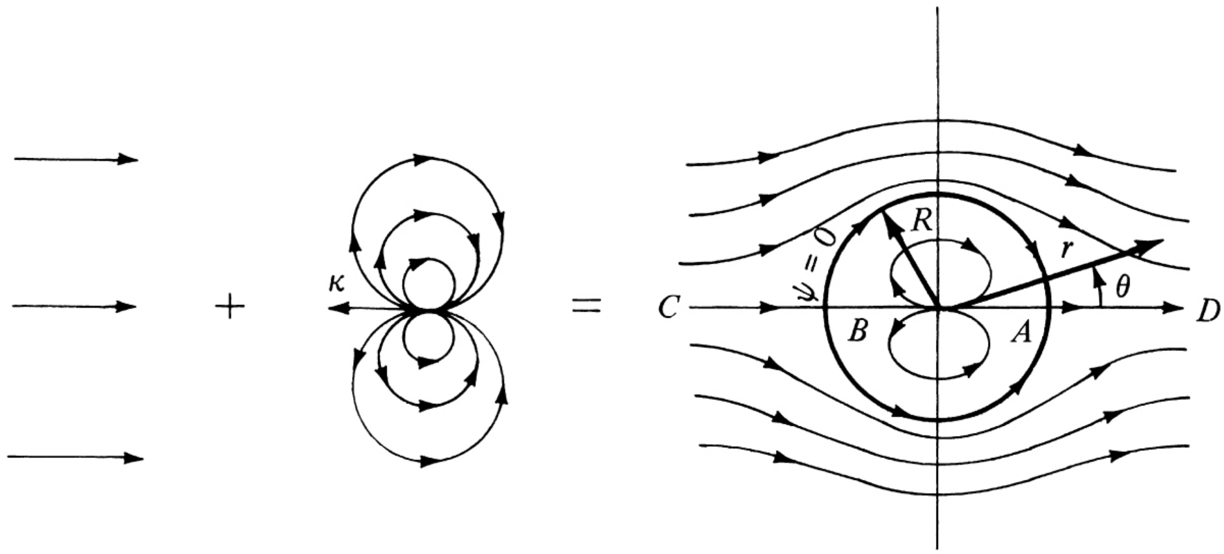








Non-Lifting Flow Over a Circular Cylinder (A3.13):



Uniform flow

$\psi = V_\infty r \sin\theta$

Doublet

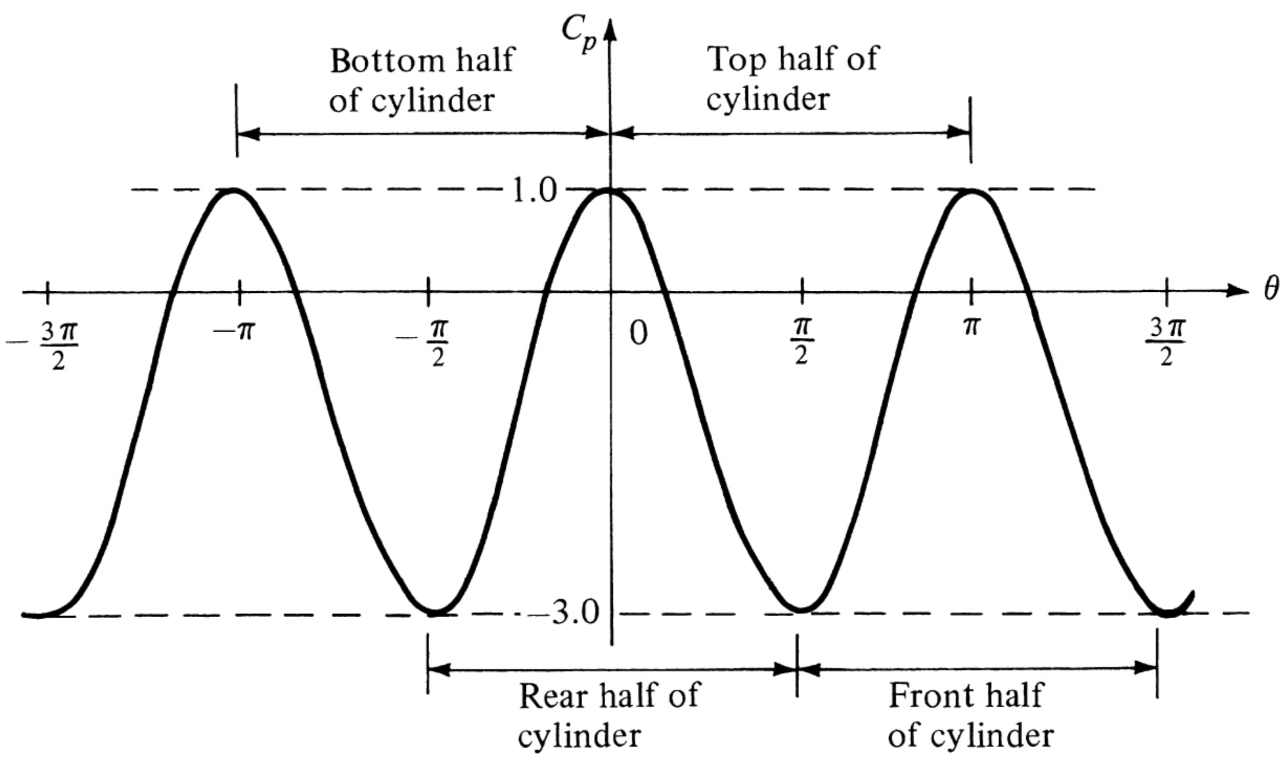
$\psi = \frac{-\kappa}{2\pi}\frac{\sin\theta}{r}$

Flow over a cylinder

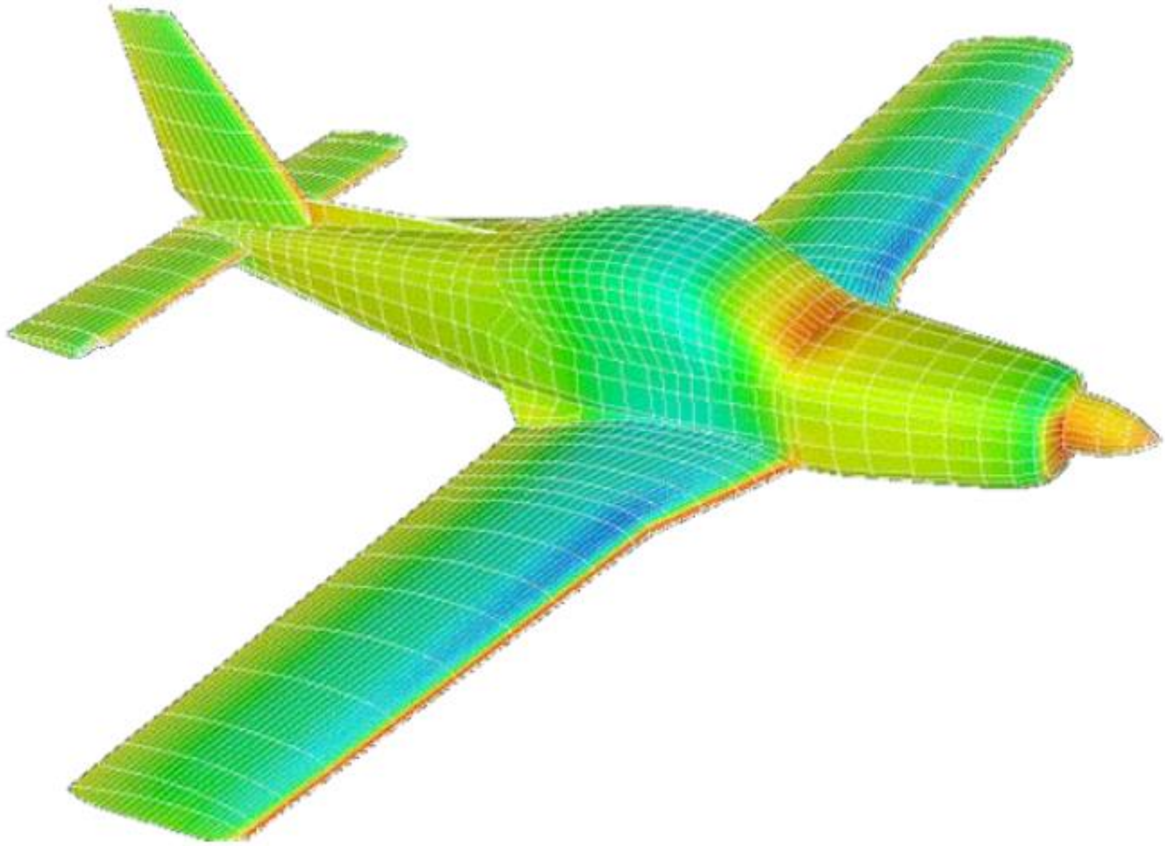
$\psi = V_\infty r \sin\theta - \frac{\kappa}{2\pi}\frac{\sin\theta}{r}$

Find Dividing SL (thru stagnation points)

Find "ideal" pressure dist over cylinder



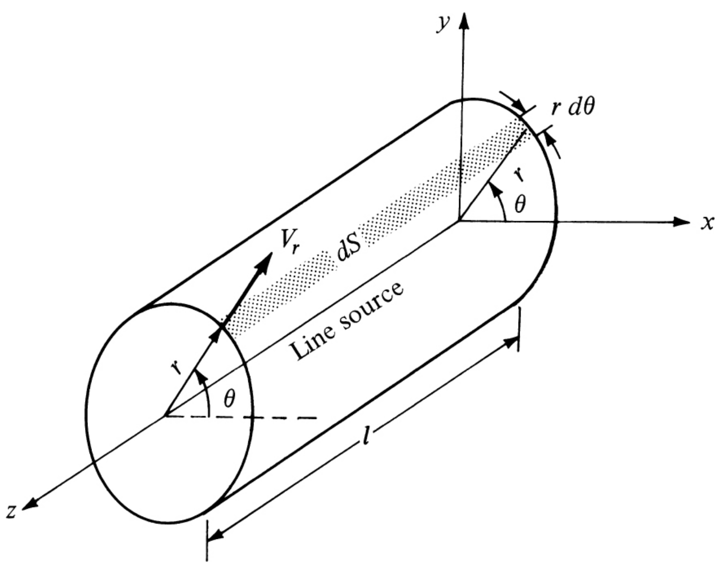
Panel Method Overview



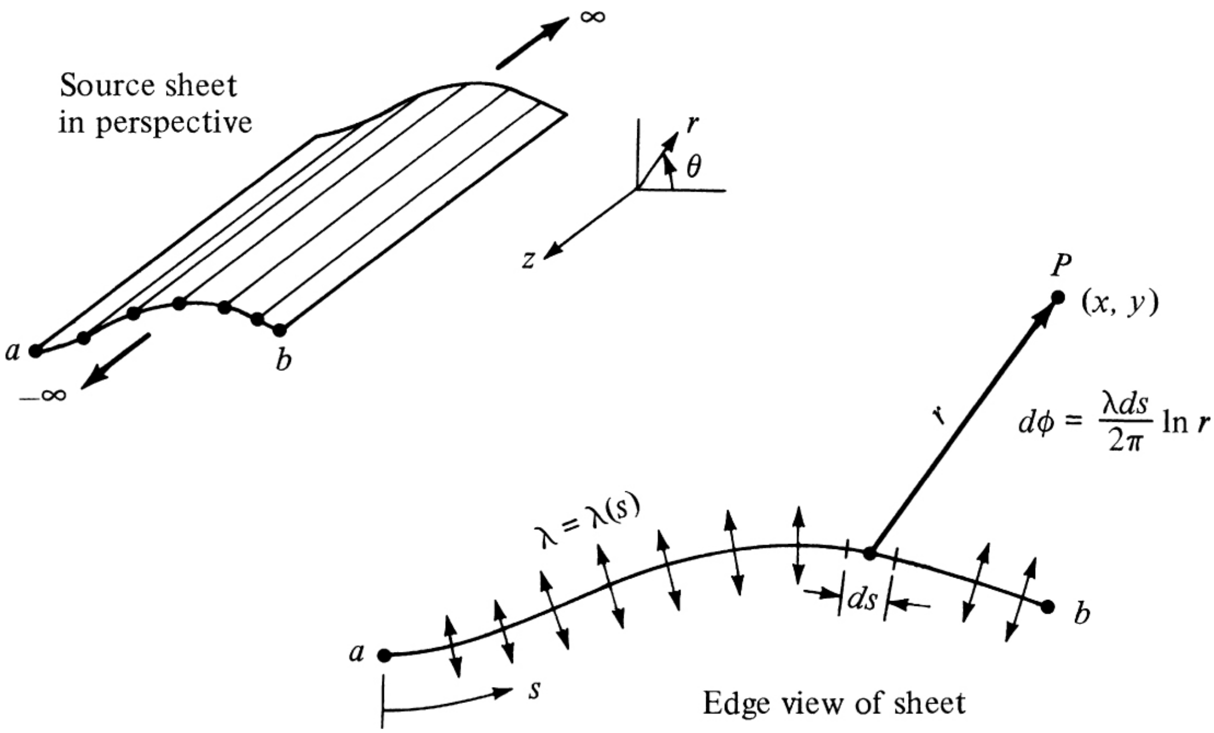
- So far, we have superposed elementary potential flows and then discovered what shape is defined by the "dividing streamline".
- But as aerodynamicists, we would rather define the shape first, then solve for the distribution of summed potential flow elements that give the required shape as the dividing SL.
- Then, we have the potential flowfield over our shape, which means we have Φ, V, and P anywhere in the flow, including on the surface of our shape.

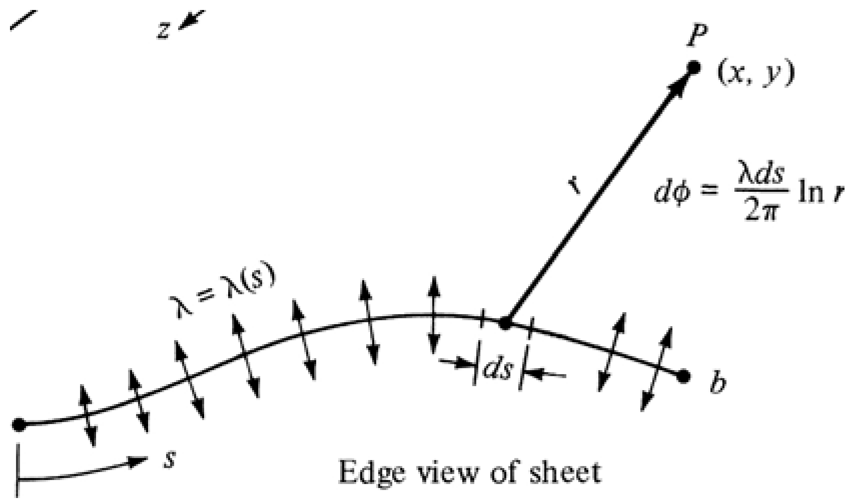
Review of Point Source Flow

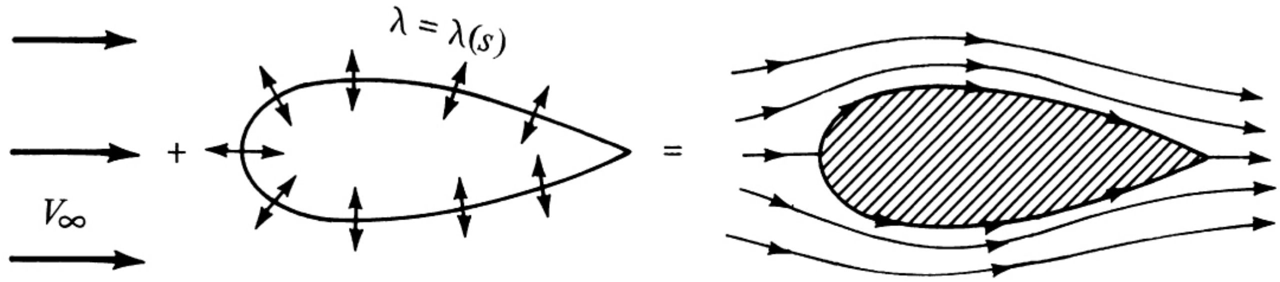
Potential Source Panel Method for Non-Lifting Bodies (A3.17):



Many Line-Sources = <u>Source Sheet</u>




z
P
(x, y)
r
$d\phi = \frac{\lambda ds}{2\pi} \ln r$
$\lambda = \lambda(s)$
ds
b
s
Edge view of sheet



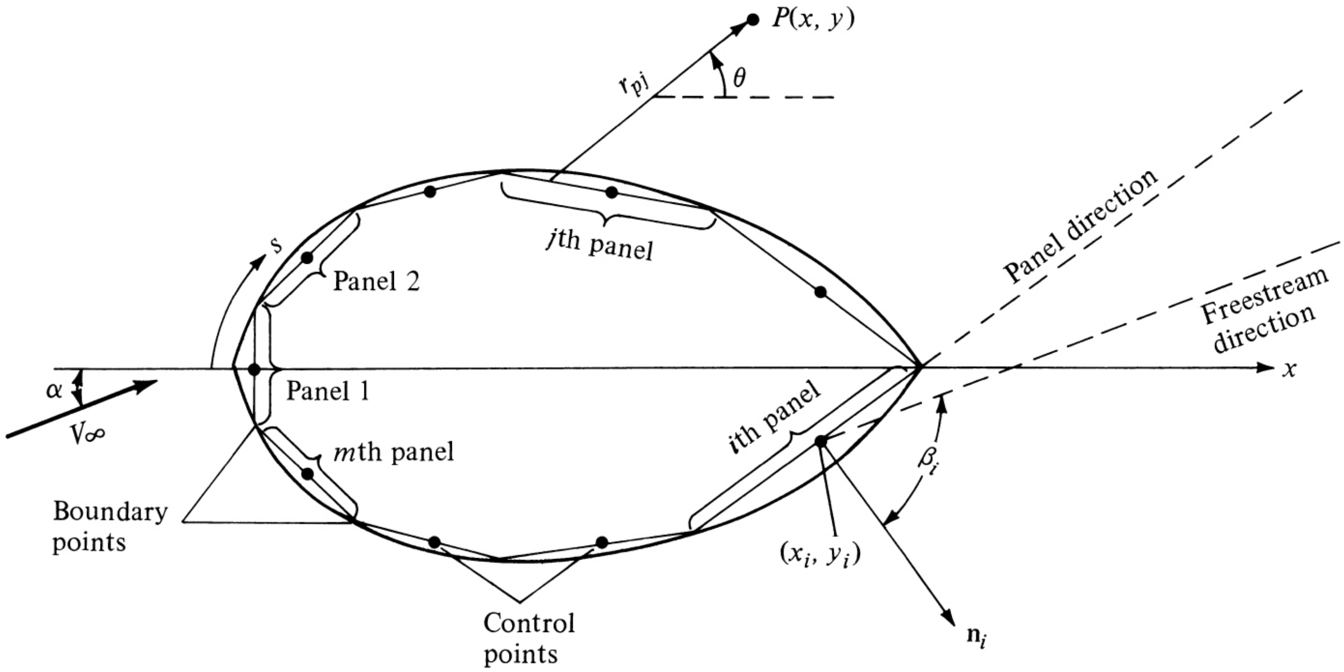
Uniform flow

Source sheet on surface of body, with $\lambda(s)$ calculated to make the body surface a streamline

Flow over the body of given shape

Panel Method for Flow Over a Non-Lifting Body: Recipe

- **Start with chosen symmetrical shape**
- **Mix of Cartesian and Cylindical coords**



Recipe for finding flowfield:

1) Approximate source sheet over surface by straight panels

2) Let λ = constant over each panel, but a different value for each panel

3) Goal: solve for all λ_j so that dividing SL is the body surface

4) Enforce wall BC: flow is tangent to body, zero flow-through

5) Enforce this wall BC only at the midpoint of each panel – called the panel's "Control Point"

6) Do this by varying λ_i's until wall BC is met at the Control Point of every panel

Formal Method

Recall: Midpoint of each panel is called the "Control Point", and the wall BC is imposed by requiring that the wall-normal velocity at each CP must be zero.

How do we impose our wall BC's at each Control Point?

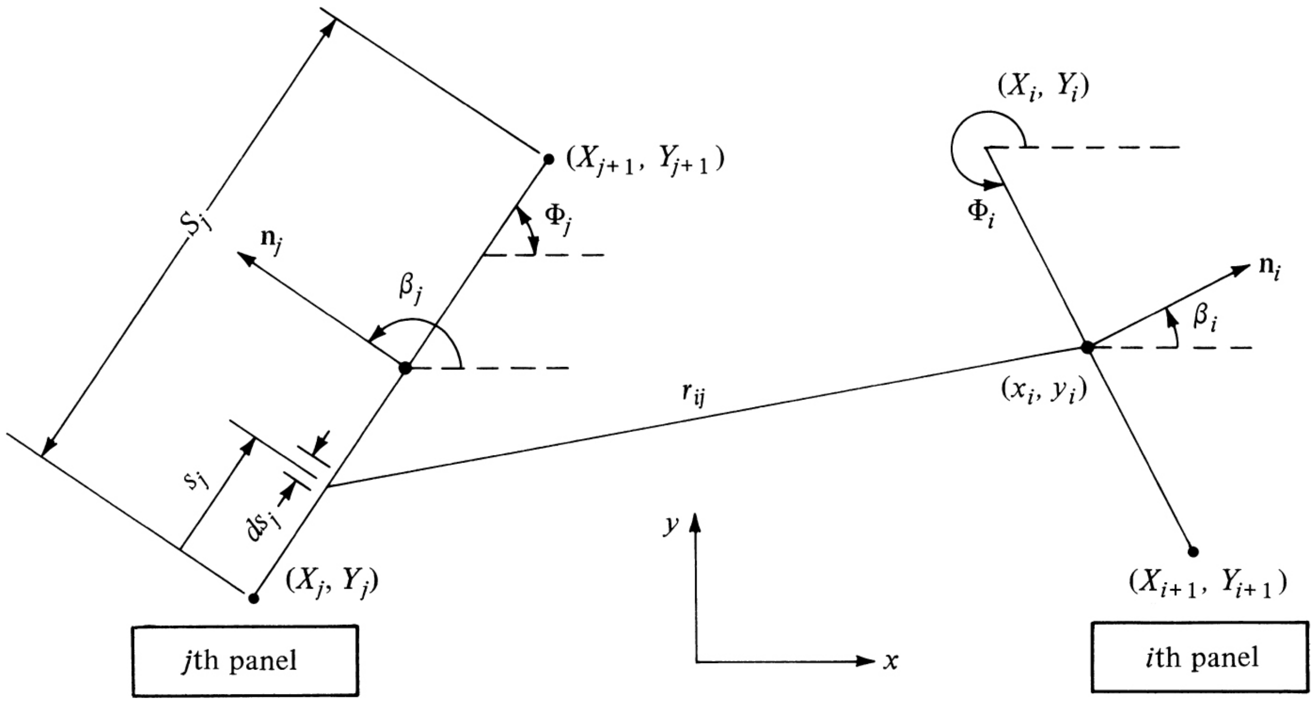
Oh-Oh! Trouble with Self-Induced Normal Flow

<u>Note</u> that the integral term in 3.152 is not a flow property – depends only on panel-to-panel geometry:

Let's Find the Pressure on Each Panel

Panel Method Consistency Check

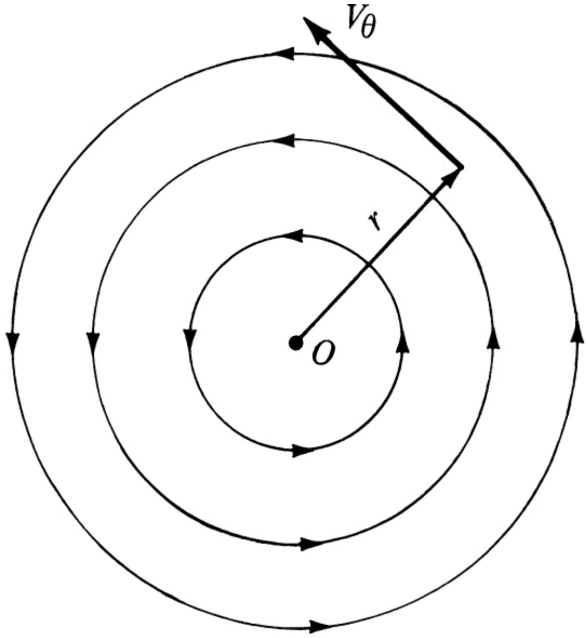
Coding: Co-Influence Between any two Panels



Co-Influence Between any two Panels (con't)

Potential Vortex (A3.14):

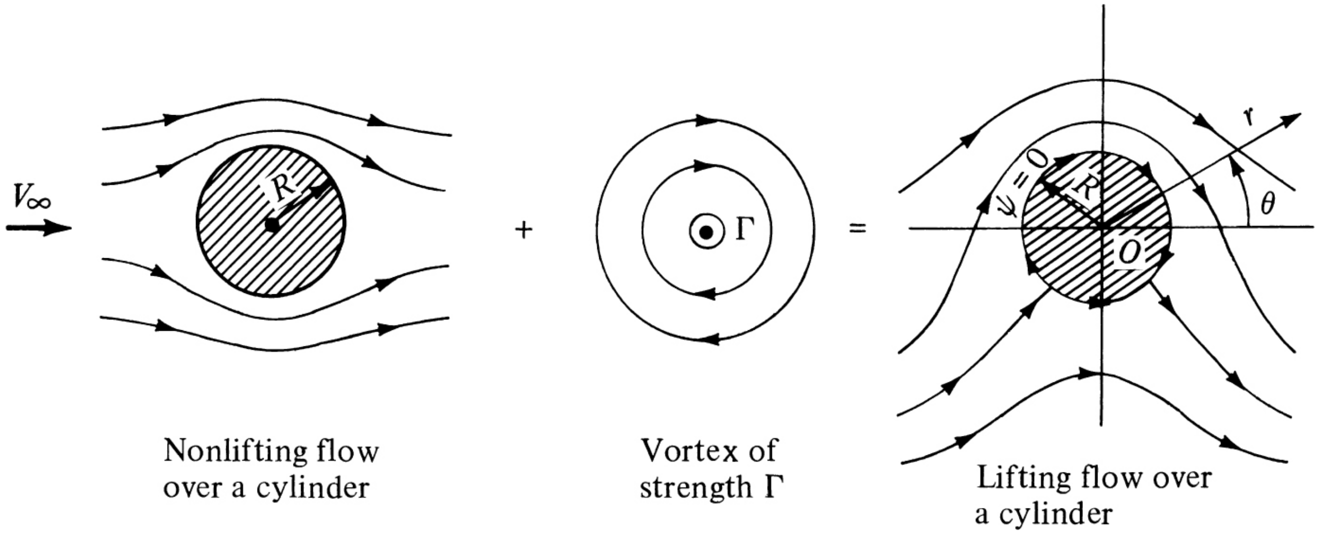
A vortex in a flow without vorticity???



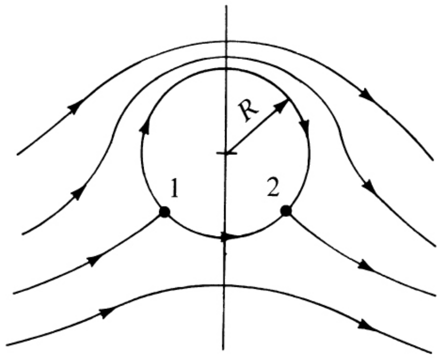
Summary of Potential Elemental Flows

Type of flow	Velocity	ϕ	ψ
Uniform flow in x direction	$u = V_\infty$	$V_\infty x$	$V_\infty y$
Source	$V_r = \dfrac{\Lambda}{2\pi r}$	$\dfrac{\Lambda}{2\pi} \ln r$	$\dfrac{\Lambda}{2\pi} \theta$
Vortex	$V_\theta = -\dfrac{\Gamma}{2\pi r}$	$-\dfrac{\Gamma}{2\pi} \theta$	$\dfrac{\Gamma}{2\pi} \ln r$
Doublet	$V_r = -\dfrac{\kappa}{2\pi} \dfrac{\cos\theta}{r^2}$	$\dfrac{\kappa}{2\pi} \dfrac{\cos\theta}{r}$	$-\dfrac{\kappa}{2\pi} \dfrac{\sin\theta}{r}$
	$V_\theta = -\dfrac{\kappa}{2\pi} \dfrac{\sin\theta}{r^2}$		

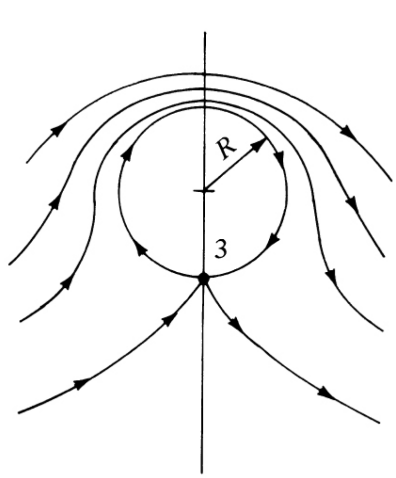
Lifting Flow Over a Cylinder (A3.15)



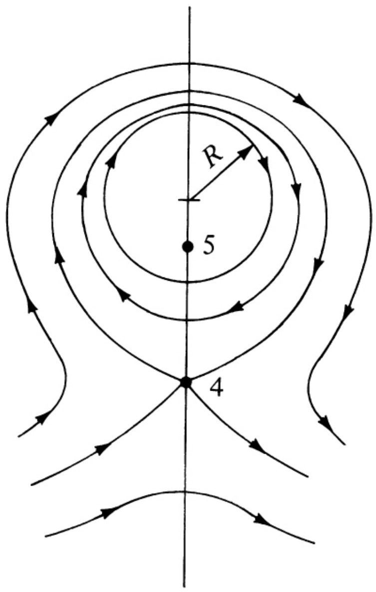
Velocity Field for a Spinning Cylinder in Crossflow



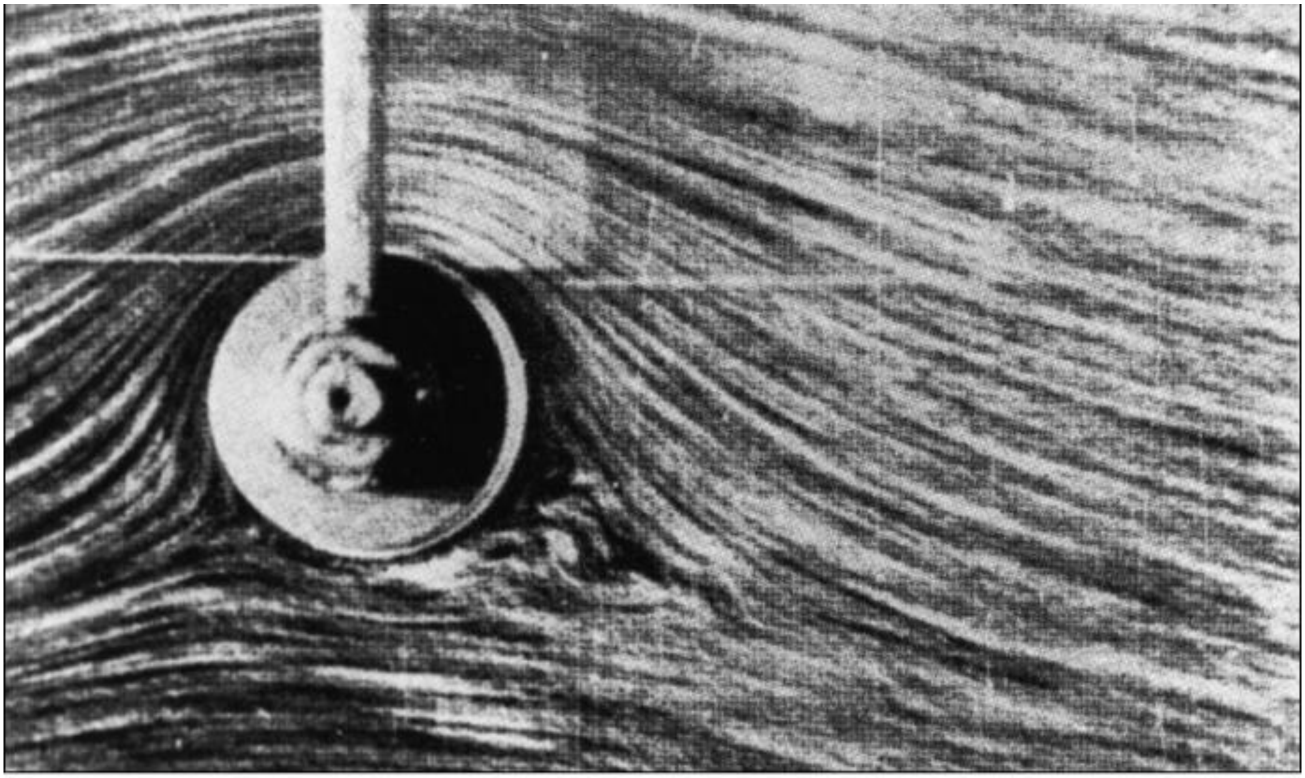
(a) $\Gamma < 4\pi V_\infty R$



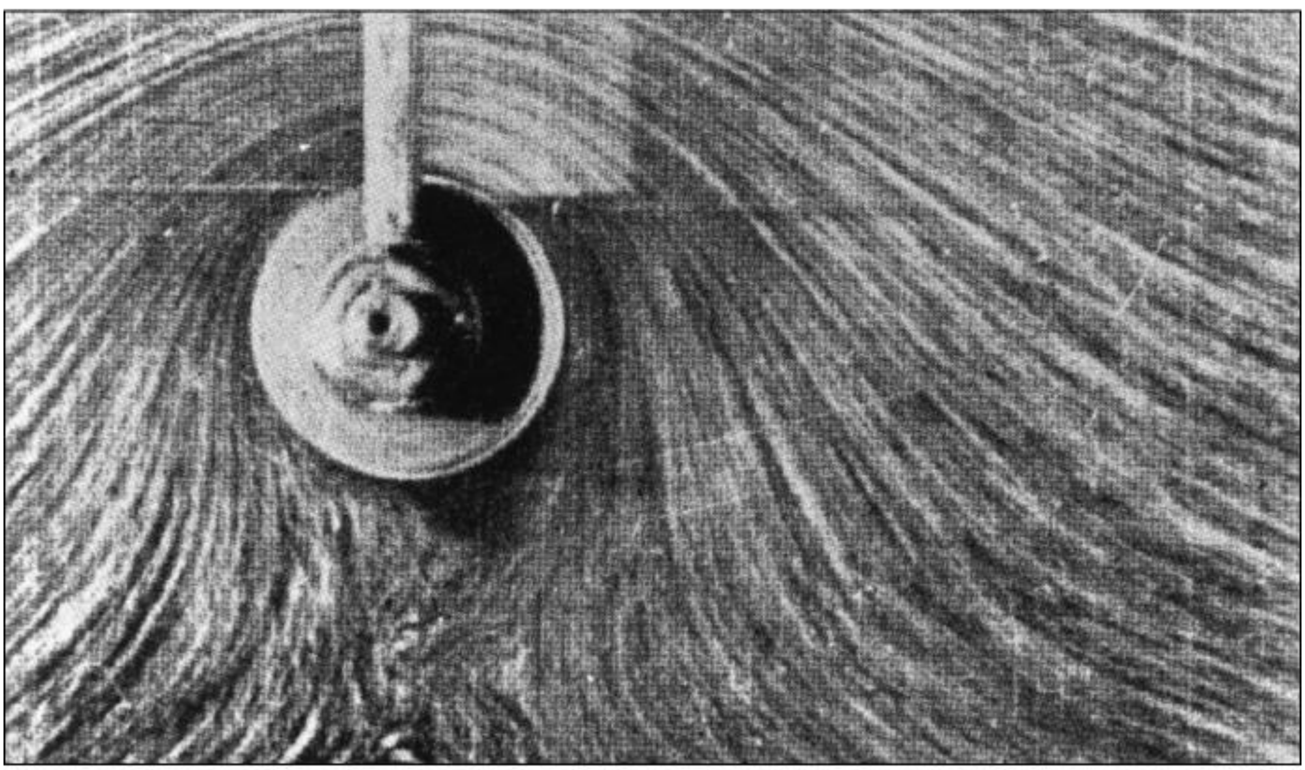
(b) $\Gamma = 4\pi V_\infty R$



(c) $\Gamma > 4\pi V_\infty R$



(b)



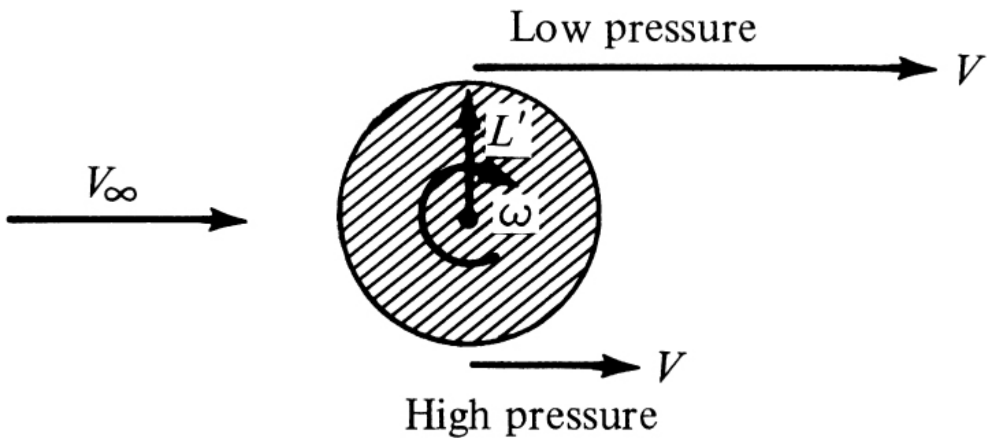
(c)

Source: Prandtl and Tietjens, Reference 8

Calculate Potential Lift on a Circular Cylinder with Circulation

Coefficient of Lift (per unit span)

Relate Γ to ω for a Spinning Cylinder



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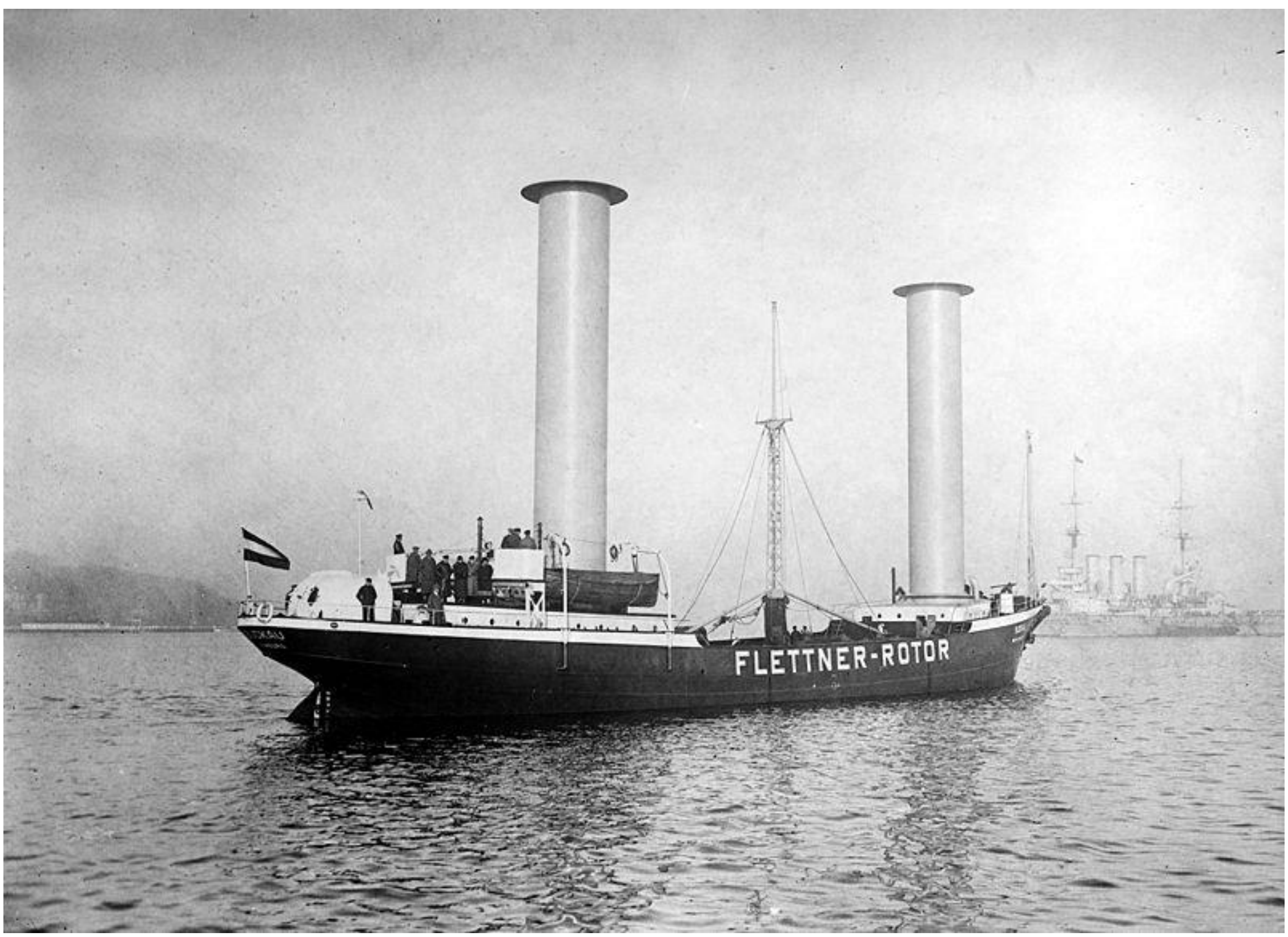

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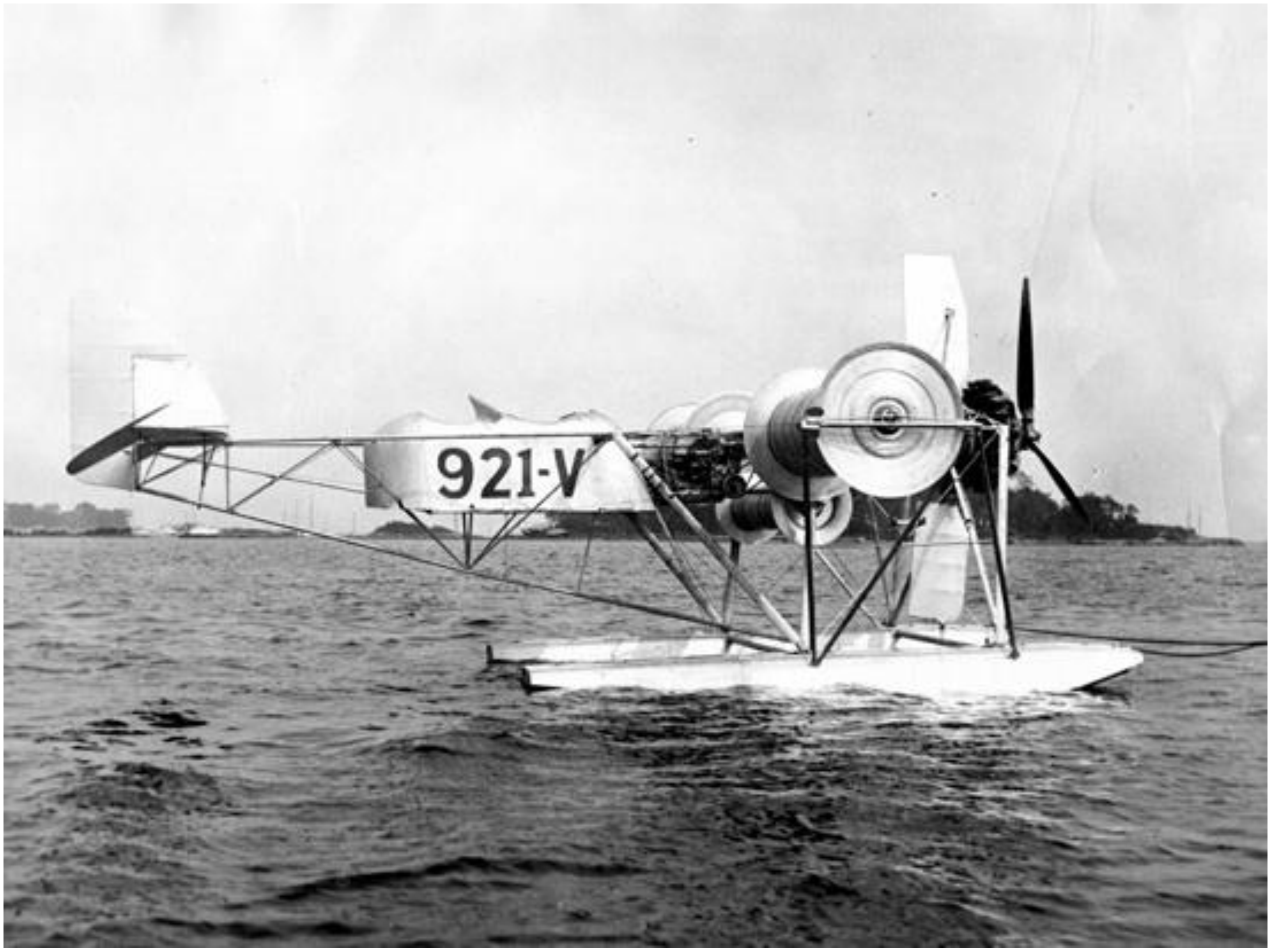

2012

http://www.youtube.com/watch?v=Ra8y6gGotwY

Review Def’n of Circulation and its relation to Vorticity

Kutta-Joukowski Lift (A3.16)