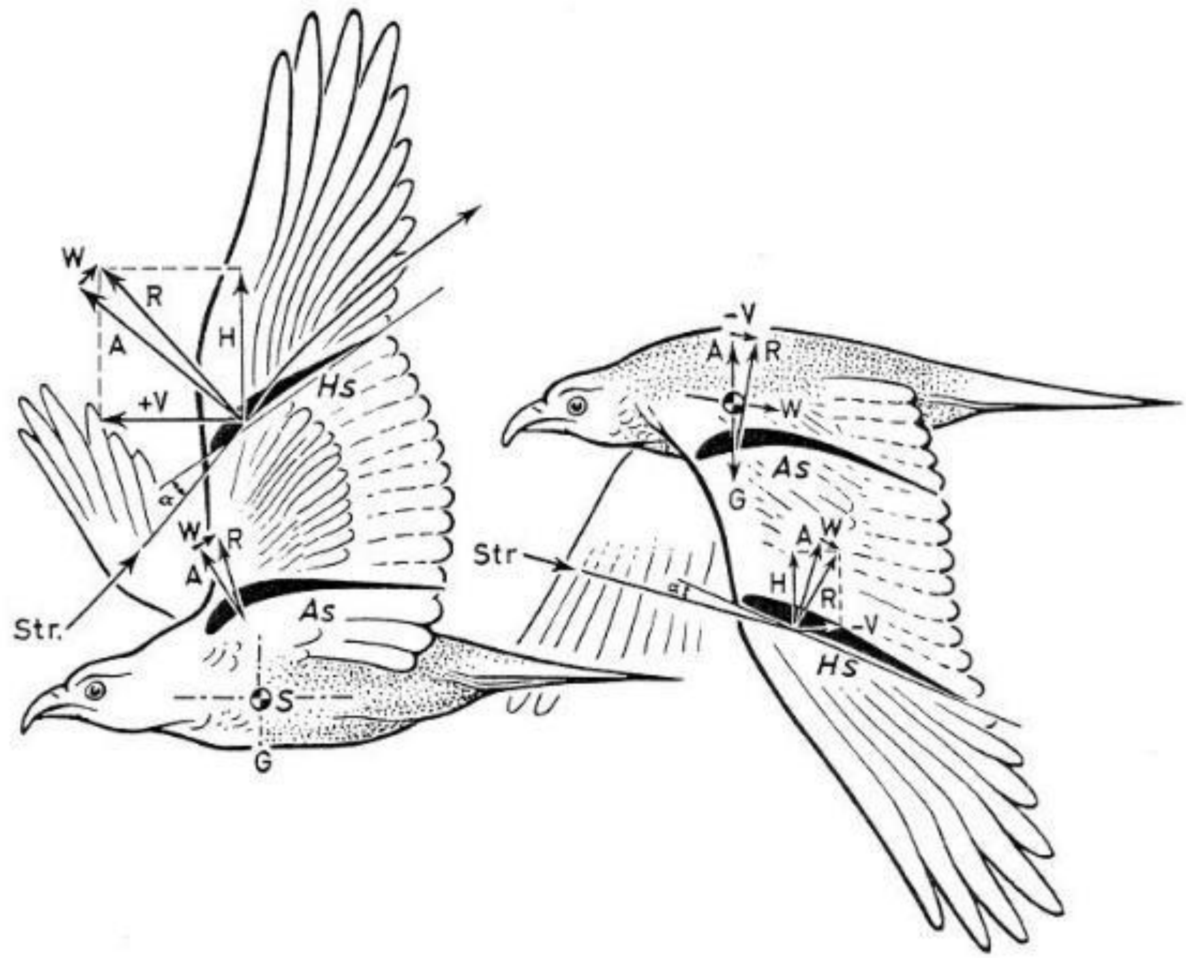
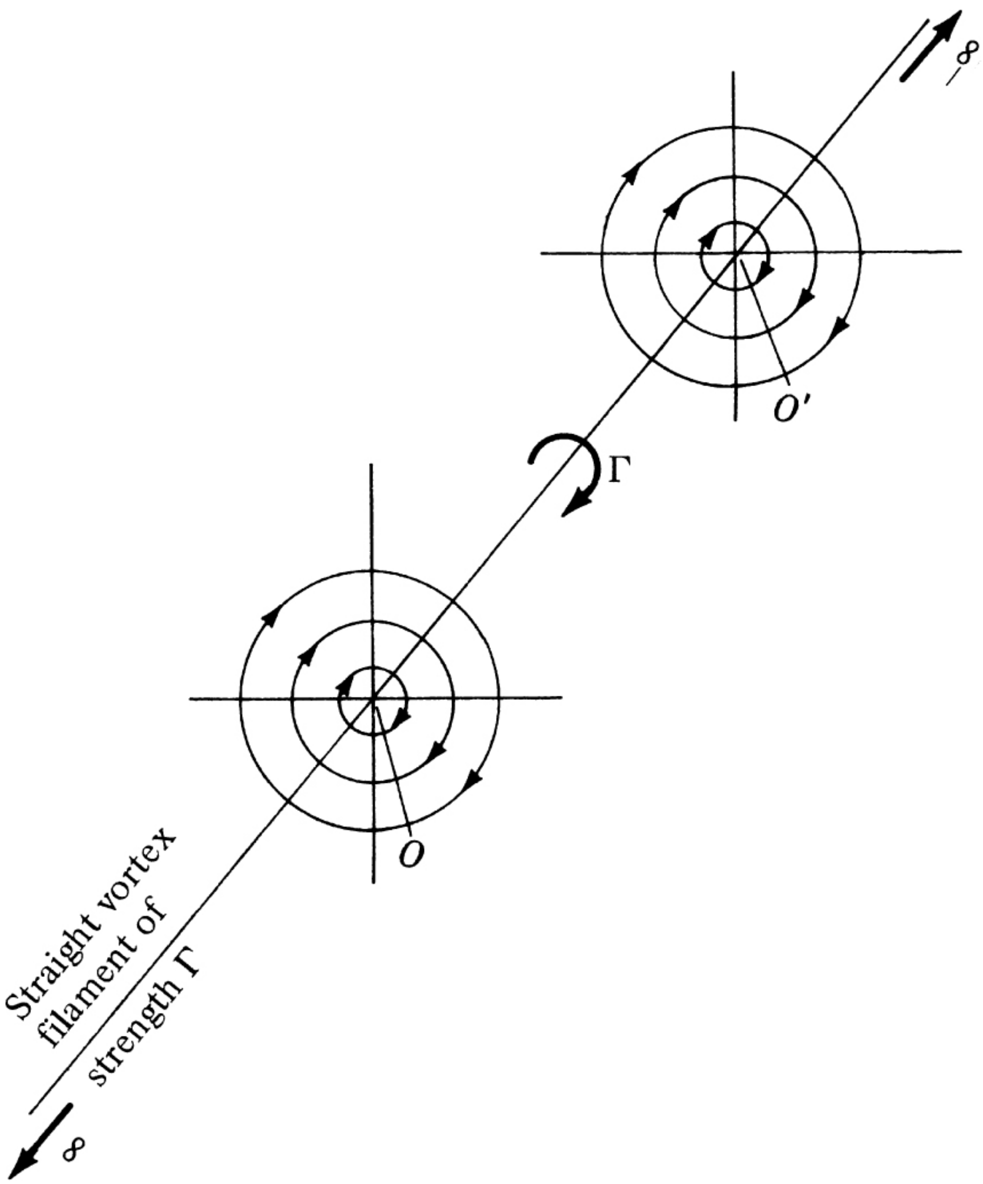
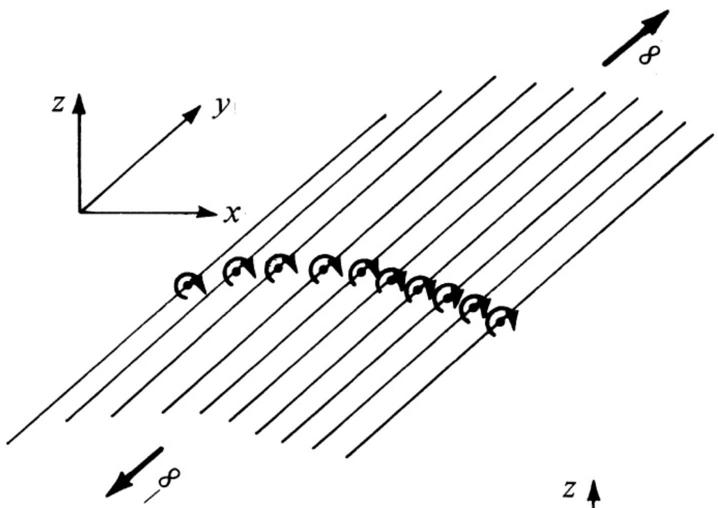


Section 6

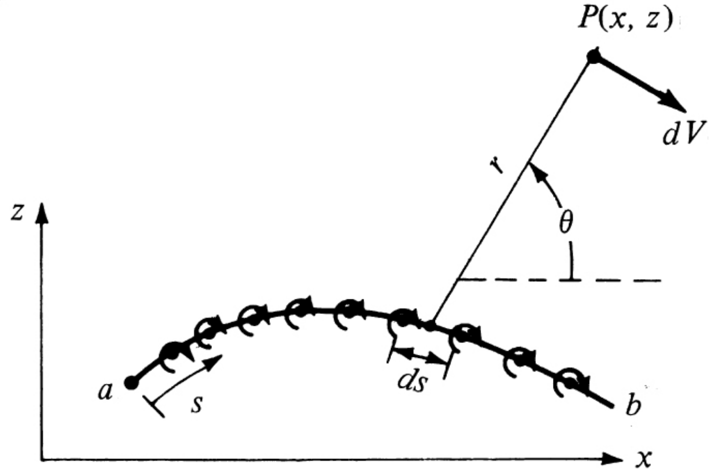
Incompressible Flow over Airfoils (Chap A4)

Vortex Sheet Concept (A4.4)





Vortex sheet in perspective

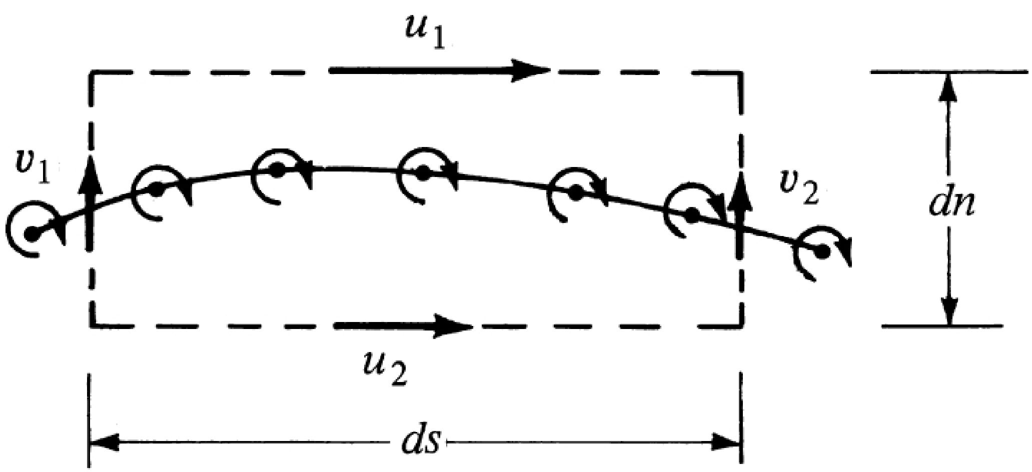


Edge view of sheet

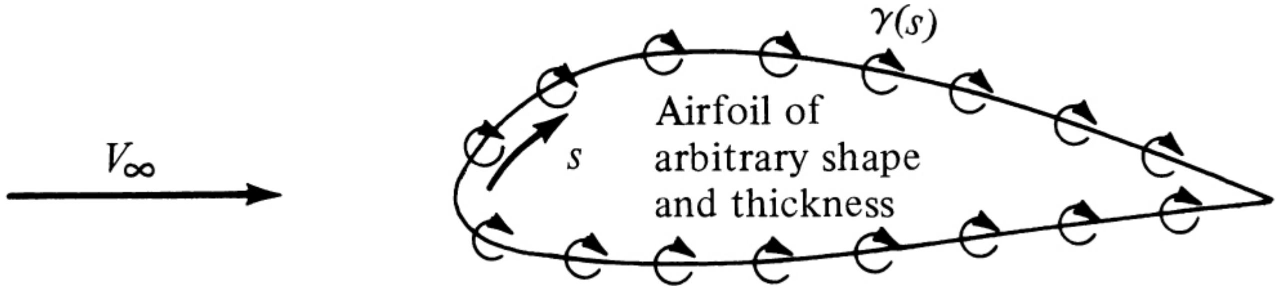
Discontinuous change in velocity across singularity sheets:

Source Sheet:

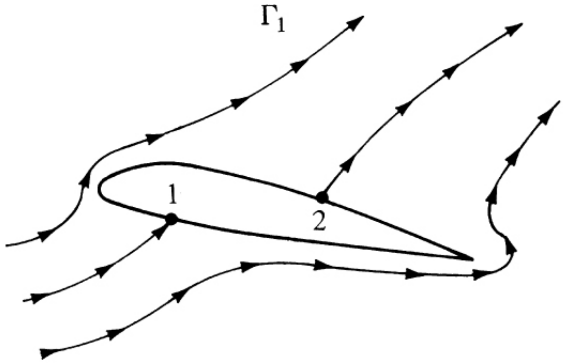
Vortex Sheet:

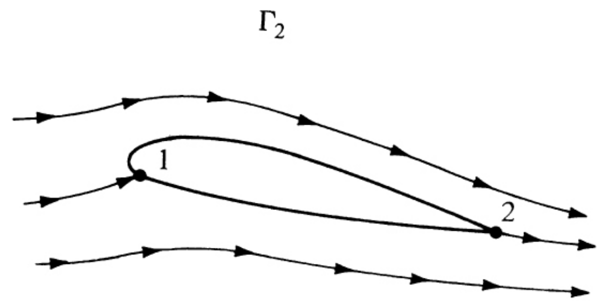


Vortex sheet over an airfoil surface:

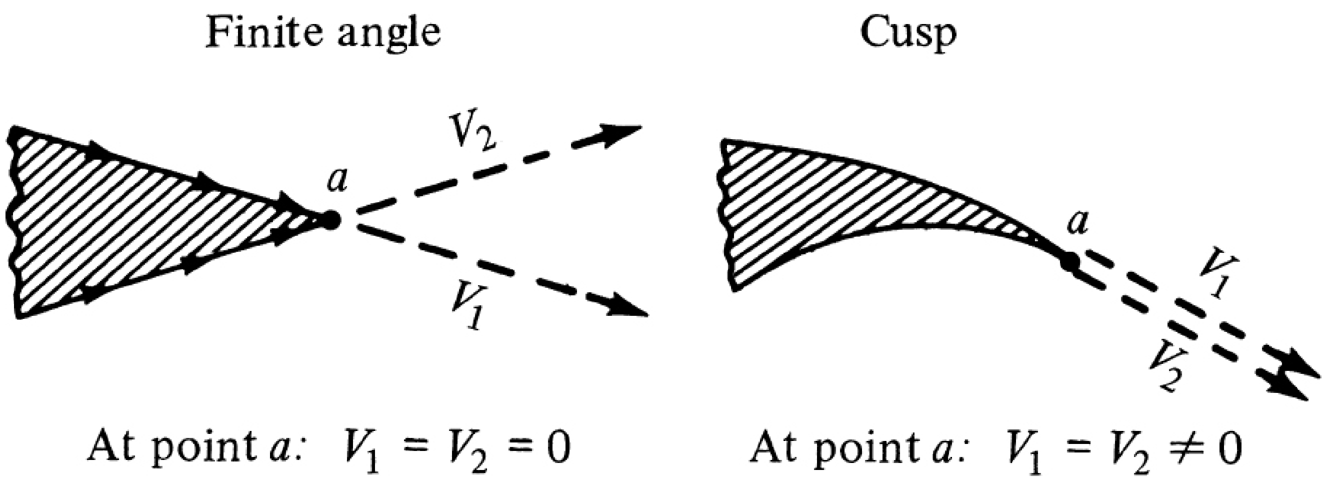


The Kutta Condition (A4.5)



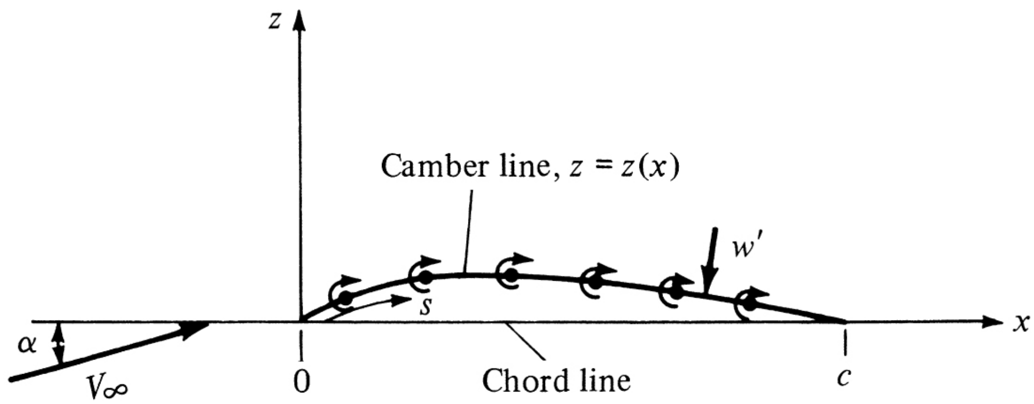


Flow at the Trailing Edge:

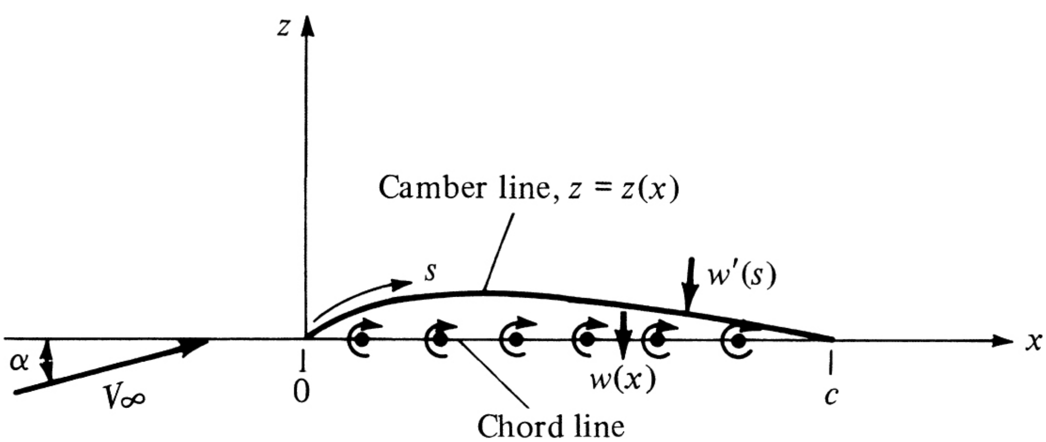


Without friction would we have lift?:

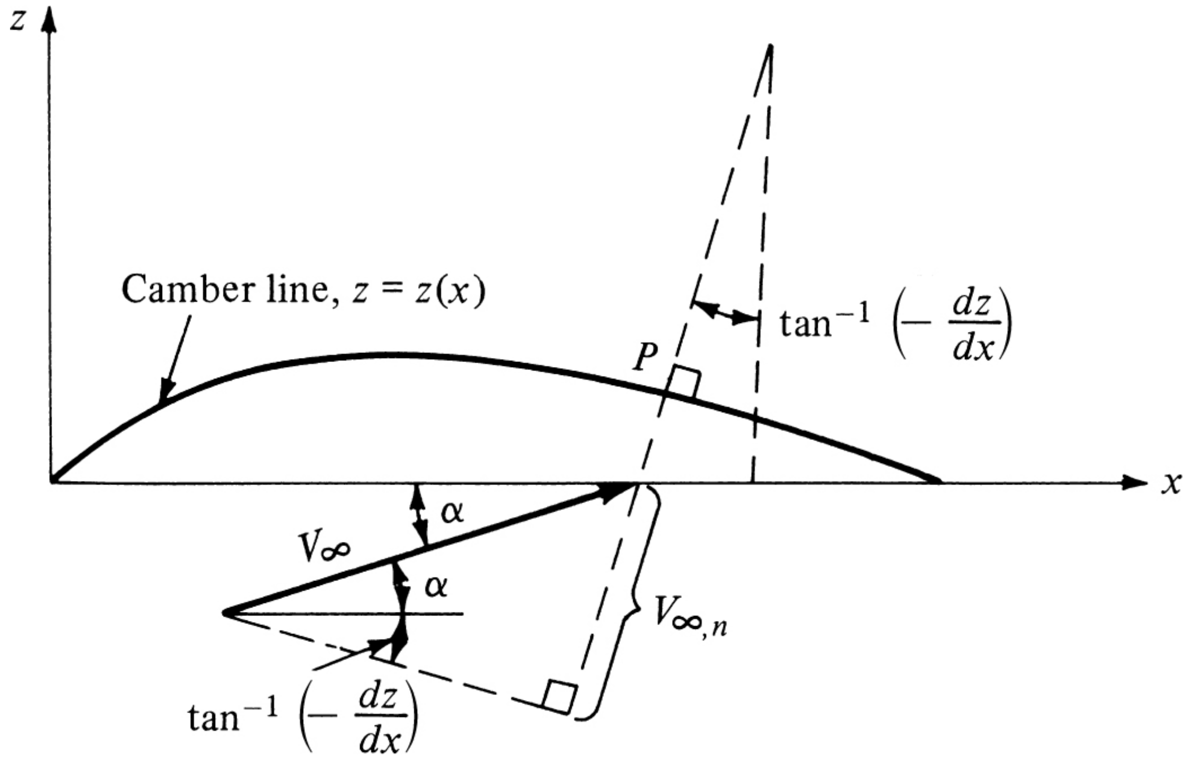
Classical Thin Airfoil Theory: Symmetric Airfoil (A4.7)

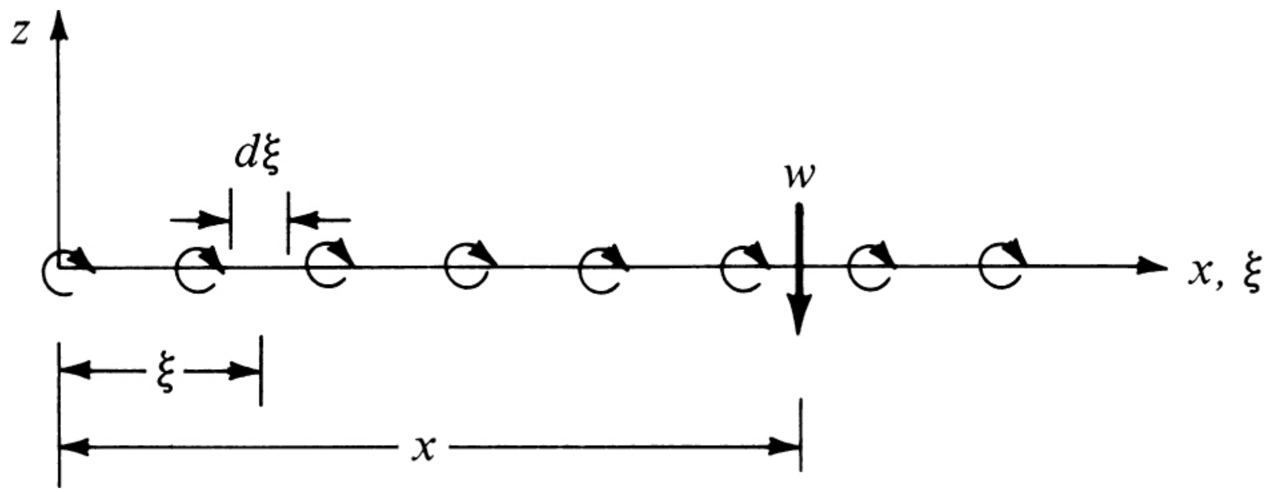


(a) Vortex sheet on the camber line



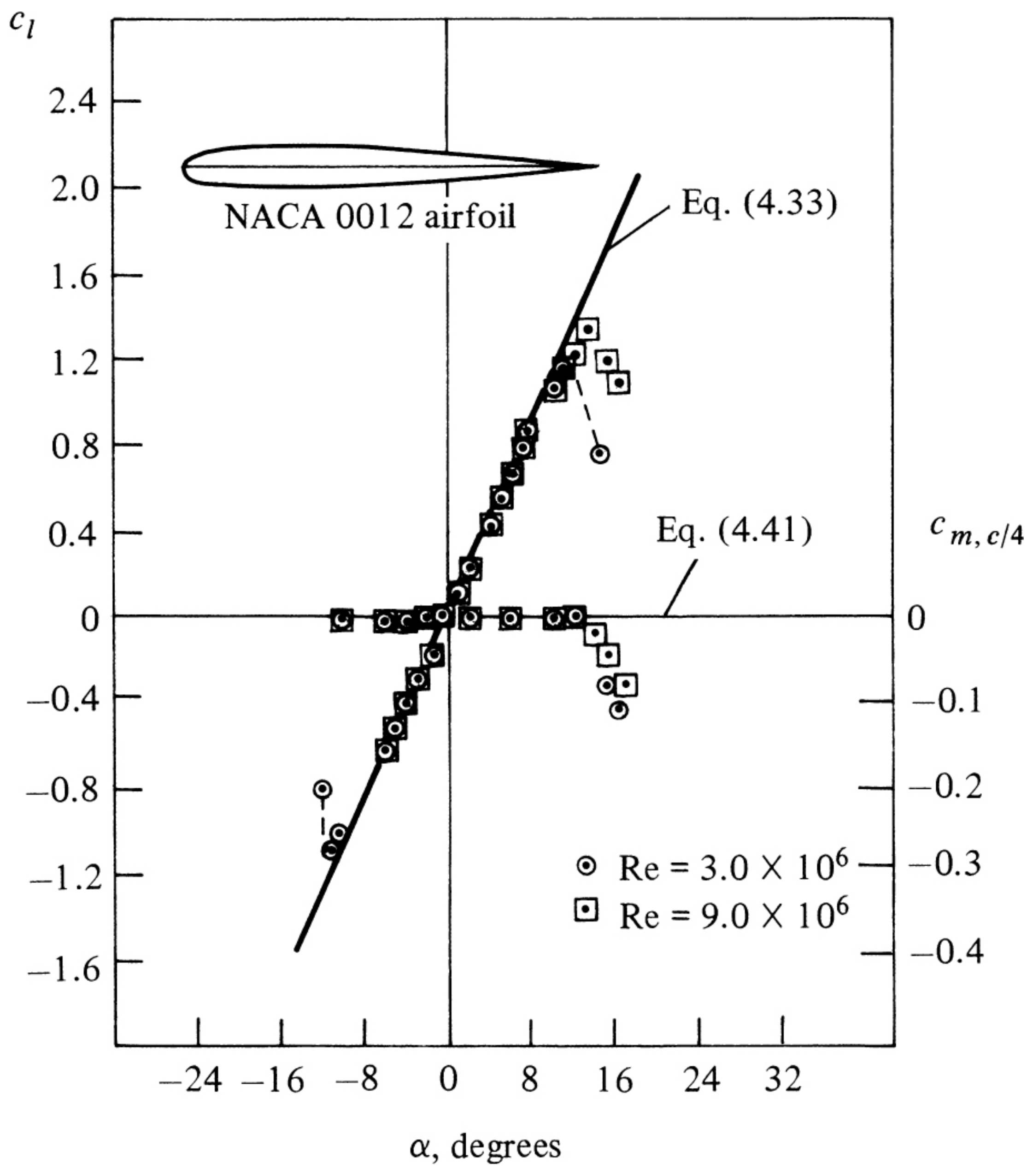
(b) Vortex sheet on the chord line


z
x
Camber line, $z = z(x)$
P
$\tan^{-1}\left(-\frac{dz}{dx}\right)$
α
α
V_∞
$V_{\infty,n}$
$\tan^{-1}\left(-\frac{dz}{dx}\right)$

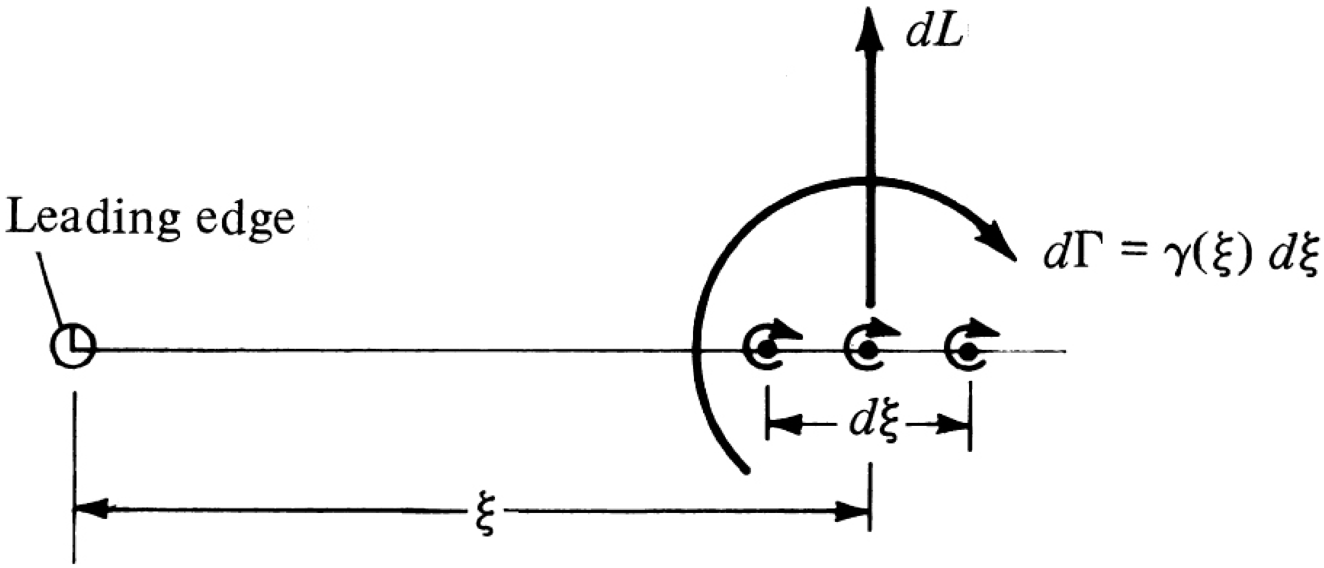

z
dξ
w
x, ξ
ξ
x

Apply Thin Airfoil Theory to Symmetric, Lifting Airfoil

Comparison with Experiment:



Moments about Leading Edge:



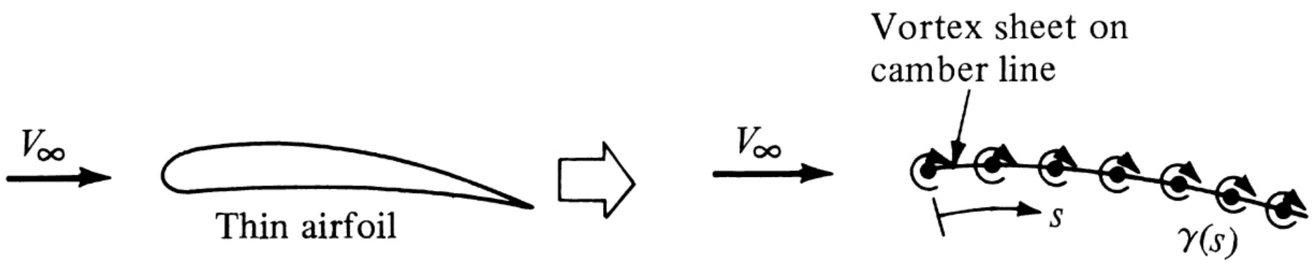
Transform variables as before:

Thin Airfoil Theory: Cambered Airfoil (A4.8)

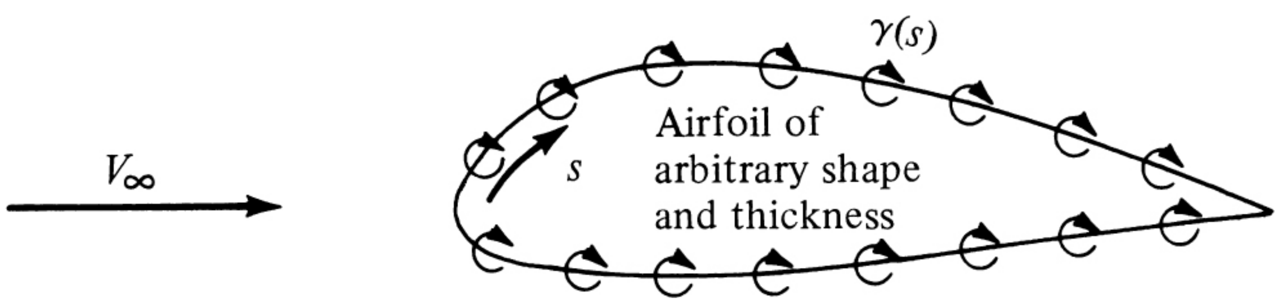
Solution obtained by calculus:

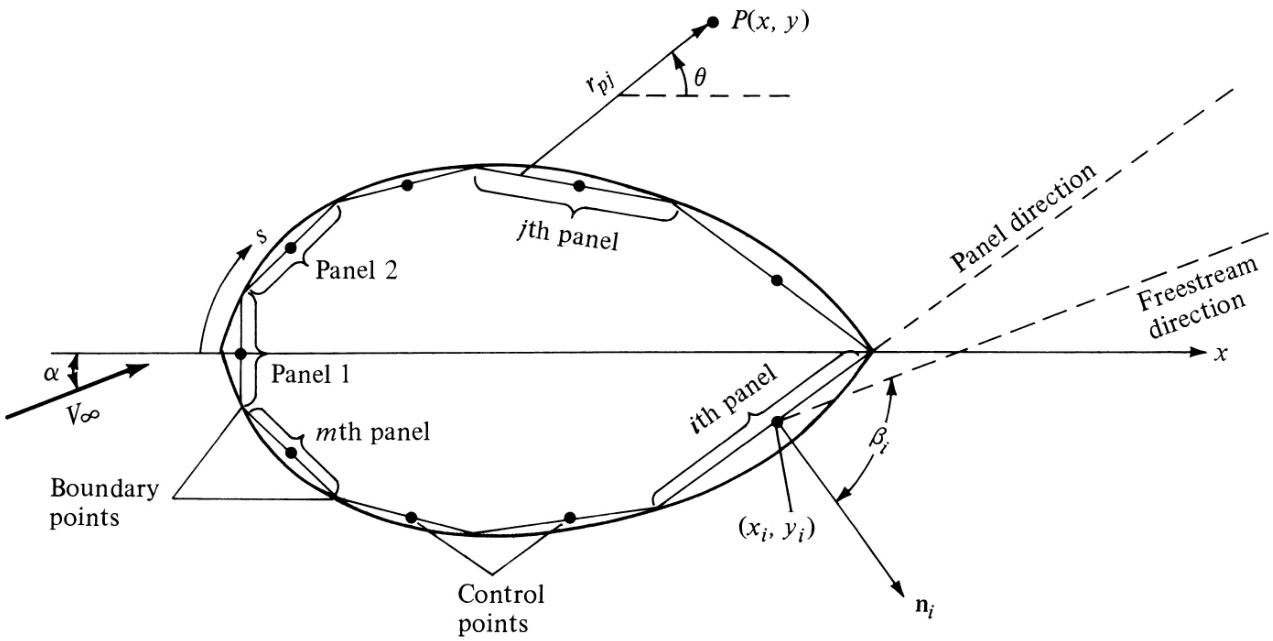
Potential Vortex Panel Method (A4.10)

Thin Airfoil Approach:



Vortex Sheet on Surface Approach:





- $\gamma(s)$ = vortex strength/unit length (unknown) = constant for each panel
- Set control points at mid-panel
- Apply wall BC (normal velocity = 0) at Control Points
- Follow same steps as for source panels:

1) Integrate over jth panel to get expression for potential at P due to panel j
2) Sum individual panel contributions to get potential at P due to all panels
3) Place P at Control Point of ith panel to get potential there
4) Compute normal velocity (due to FS and induced) at CP and require it to be zero

Self-Induced Velocity at Panel

How to satisfy Kutta Condition? (Extra Equation!)

Several Methods:

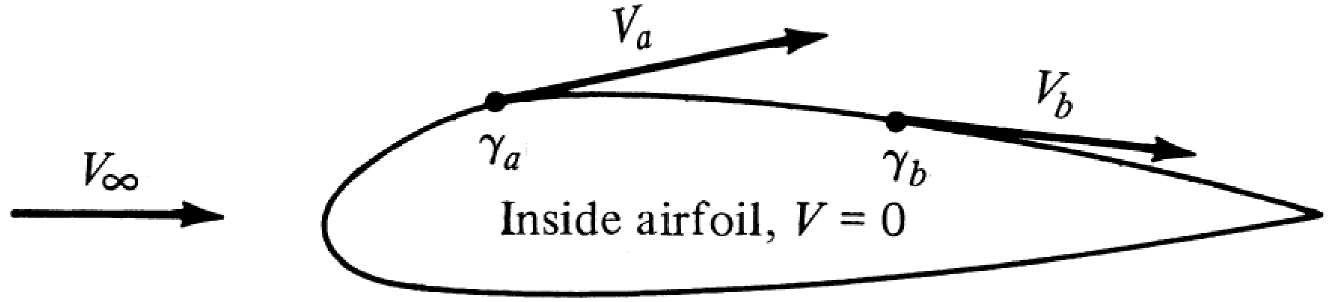
Combined Source/Vortex Panels:

- Use source-sheet panels for airfoil shape/thickness, and vortex panels for circulation.
- Give all vortex panels same strength – thus only one additional unknown
- So, n source eqns plus Kutta Condition
- N unknown source strengths, plus one unknown vortex strength
- n+1 eqns, n+1 unknowns

Vortex Panels only (text):

- Vortex panels only, so n unknowns (γ_i) for n panels.
- Kutta condition γ(TE) = 0 imposed by:

- Ignore one vortex panel to get n eqns, n unknowns


V_a
V_b
V_∞
γ_a
γ_b
Inside airfoil, $V = 0$

Second Order Panel Method: