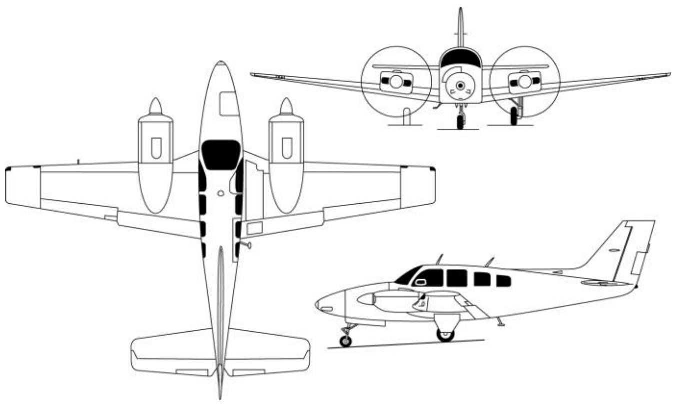
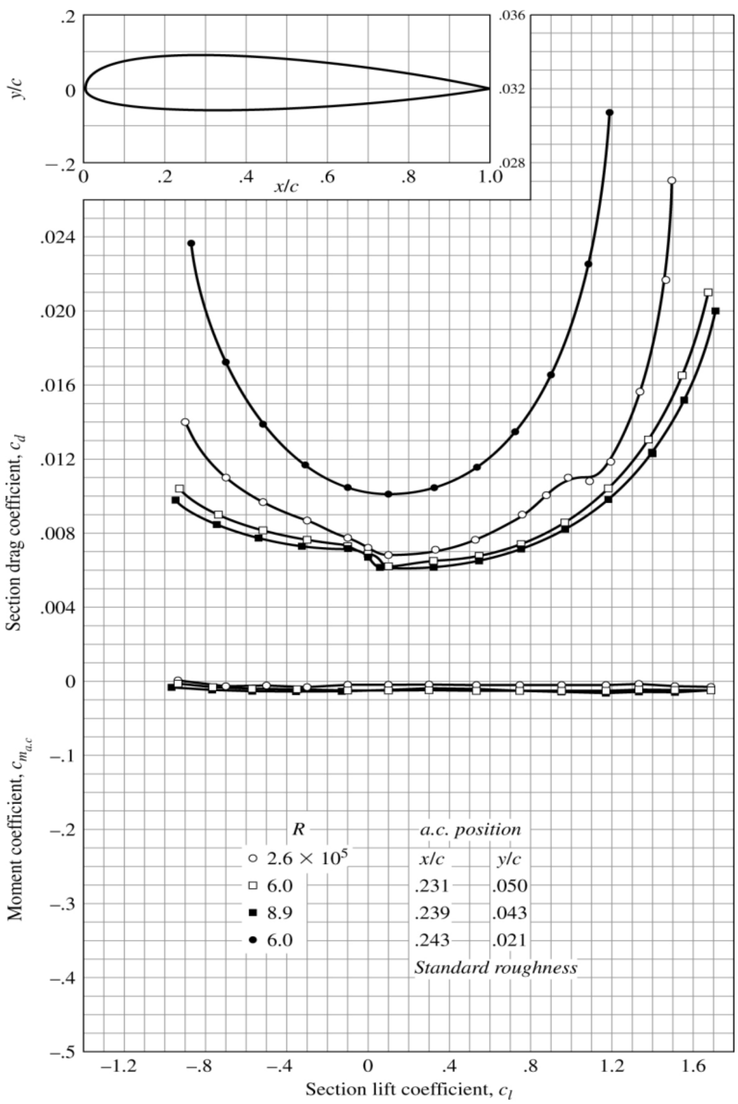
Section 7

Incompressible Flow

over Finite Wings (Chap A5)

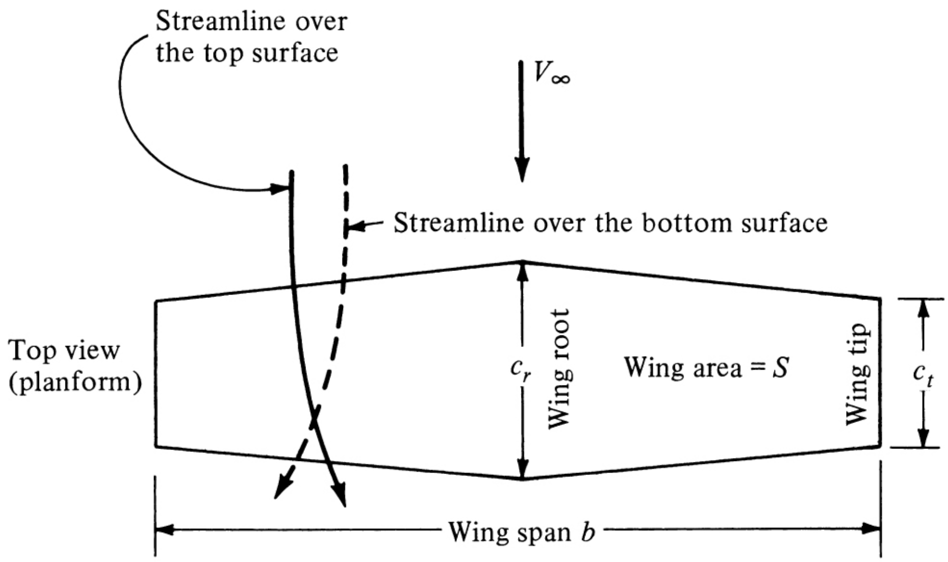
Motivation: 3D Effects

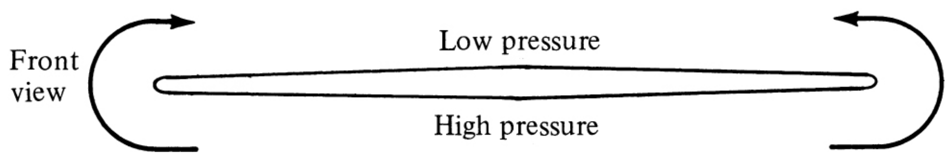


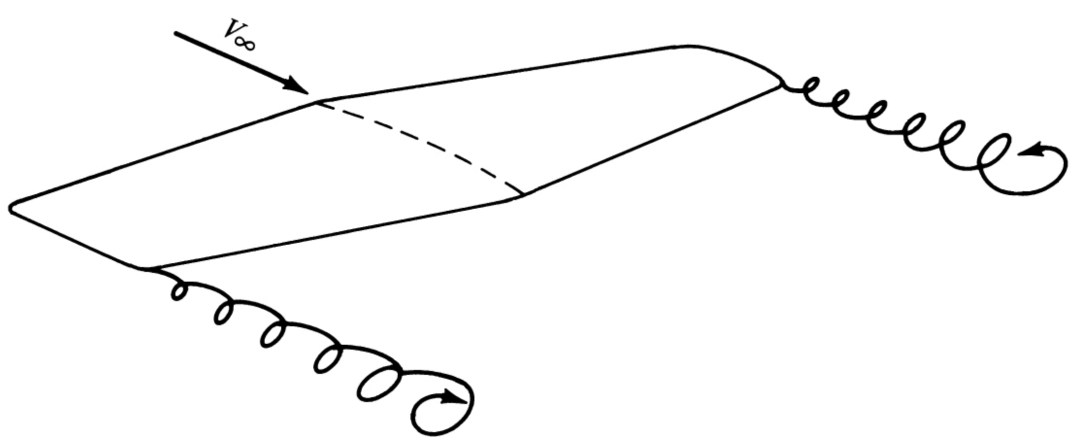


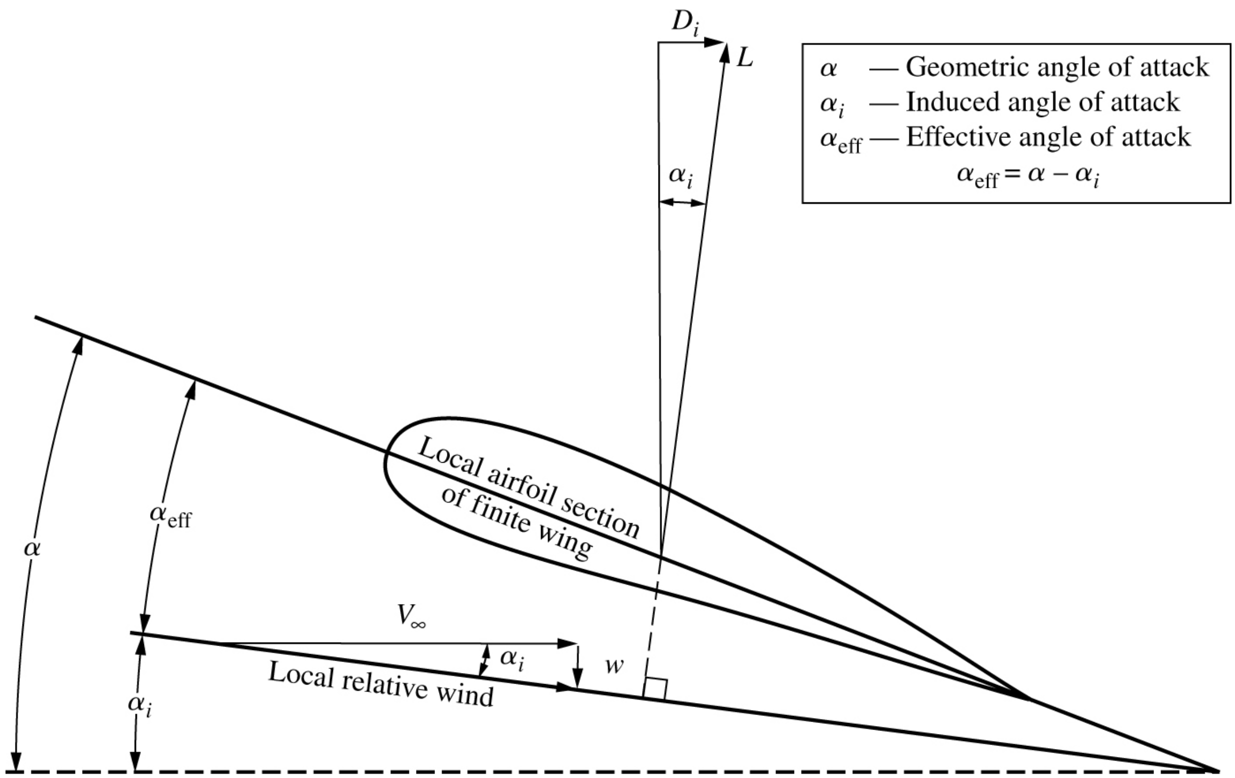
NACA 23015 Wing Section

Downwash and Induced Drag (A5.1)





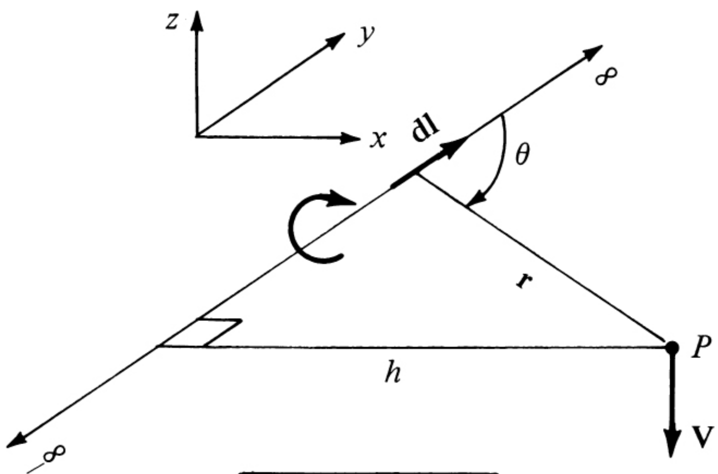



D_i
L
α — Geometric angle of attack
α_i — Induced angle of attack
α_{eff} — Effective angle of attack
$\alpha_{eff} = \alpha - \alpha_i$
α_i
Local airfoil section of finite wing
α
α_{eff}
V_∞
α_i
w
Local relative wind
α_i

Vortex Filament Aerodynamic Tools (A5.2)

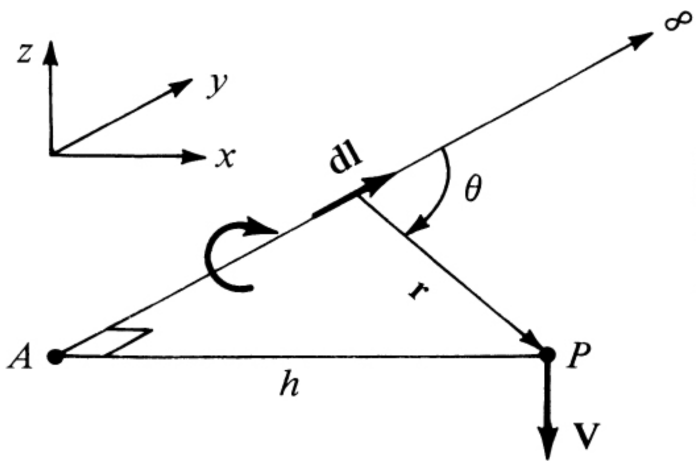
Biot-Savart Law

Biot-Savart applied to infinite straight vortex filament:



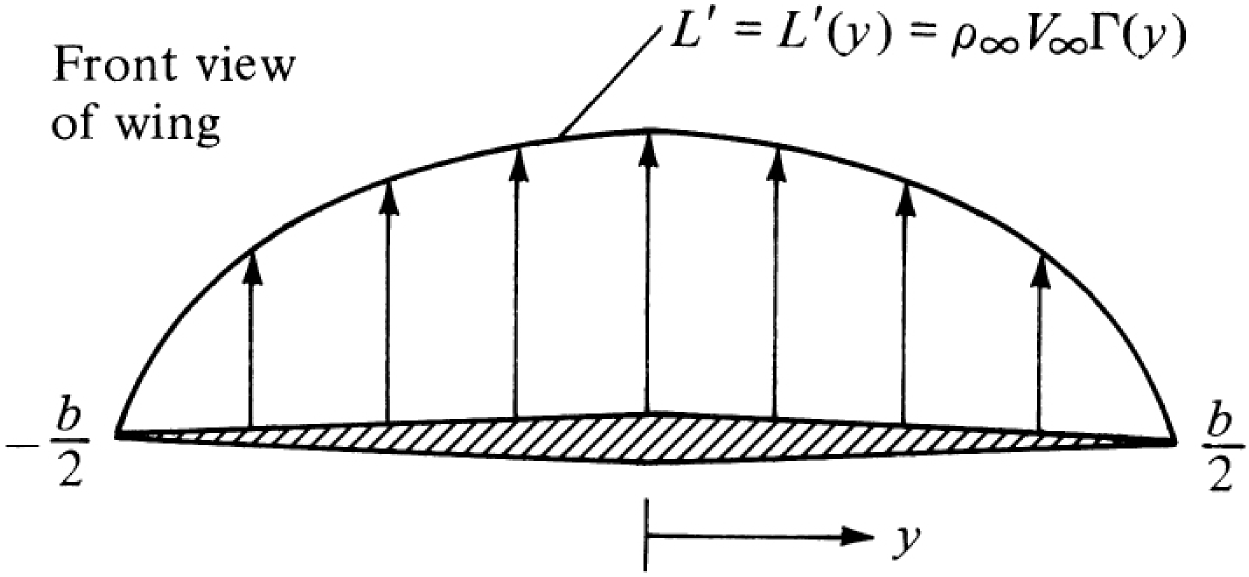
$$\boxed{V = \frac{\Gamma}{2\pi h}}$$

Biot-Savart applied to <u>semi-infinite</u> straight vortex filament:



$$\boxed{V = \frac{\Gamma}{4\pi h}}$$

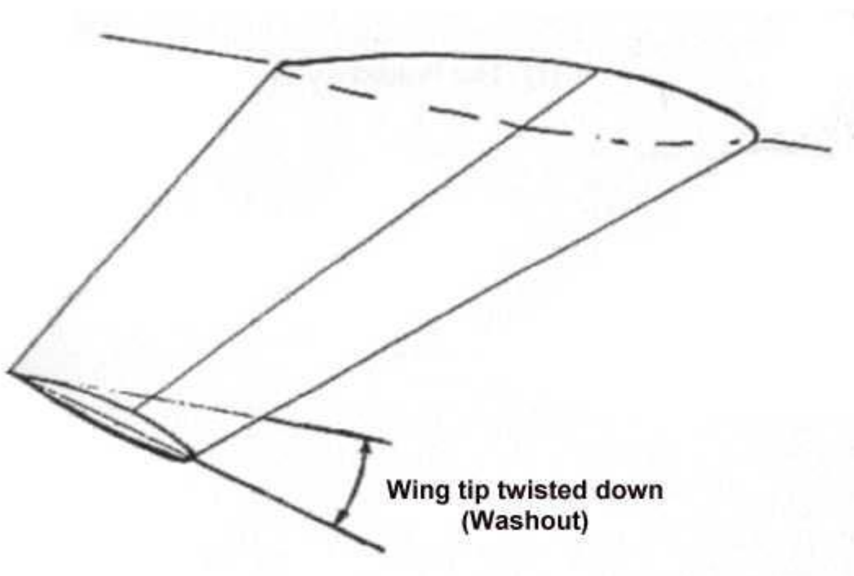
Lift Distribution over Wingspan

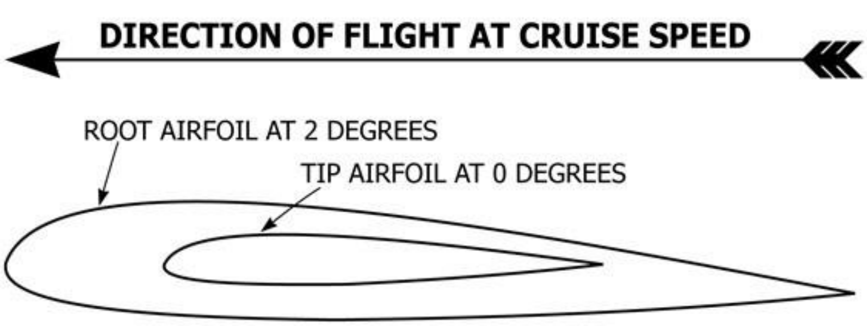


Real Wings: wingtip vortices

Varying chord:

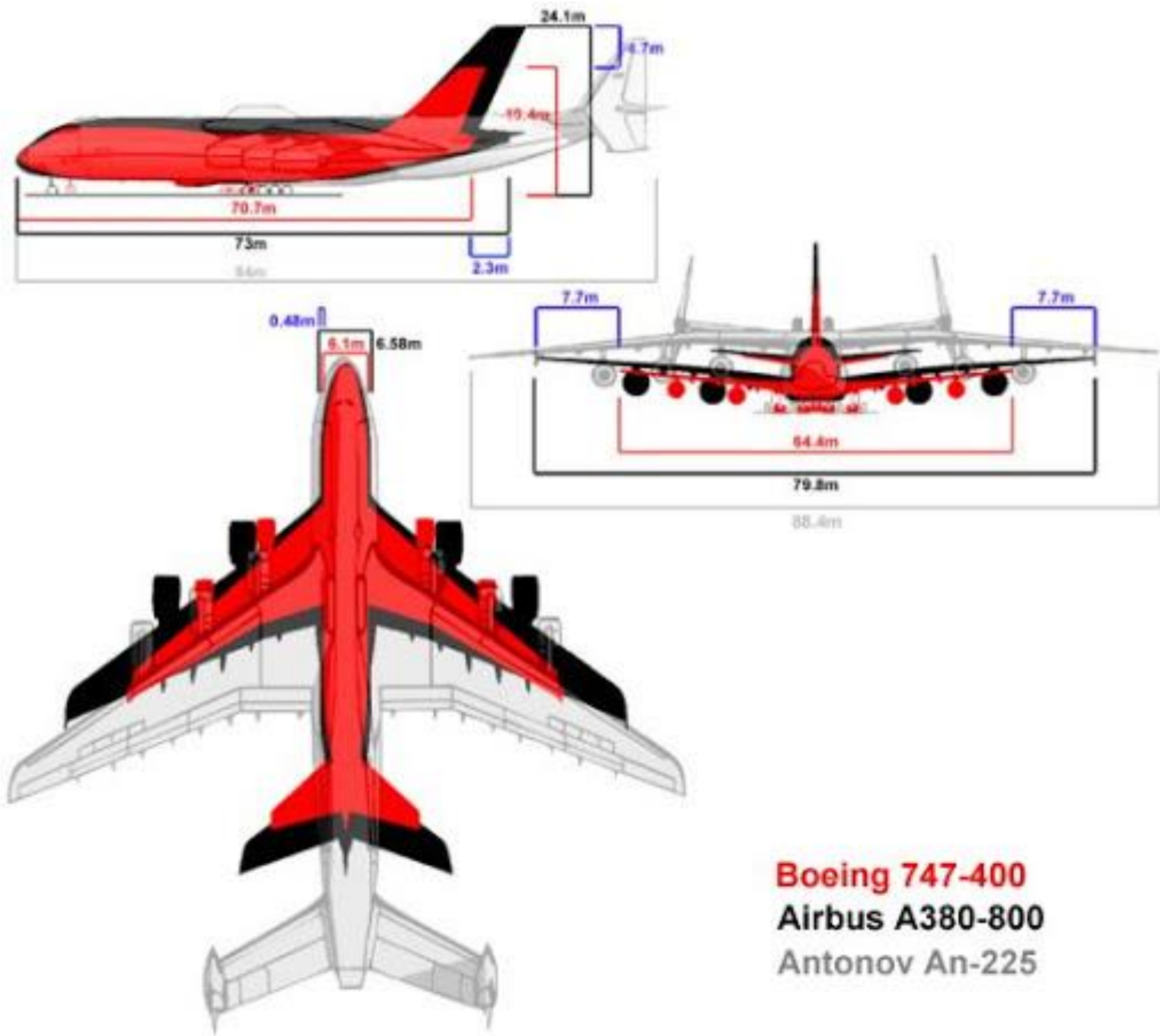
Washout:





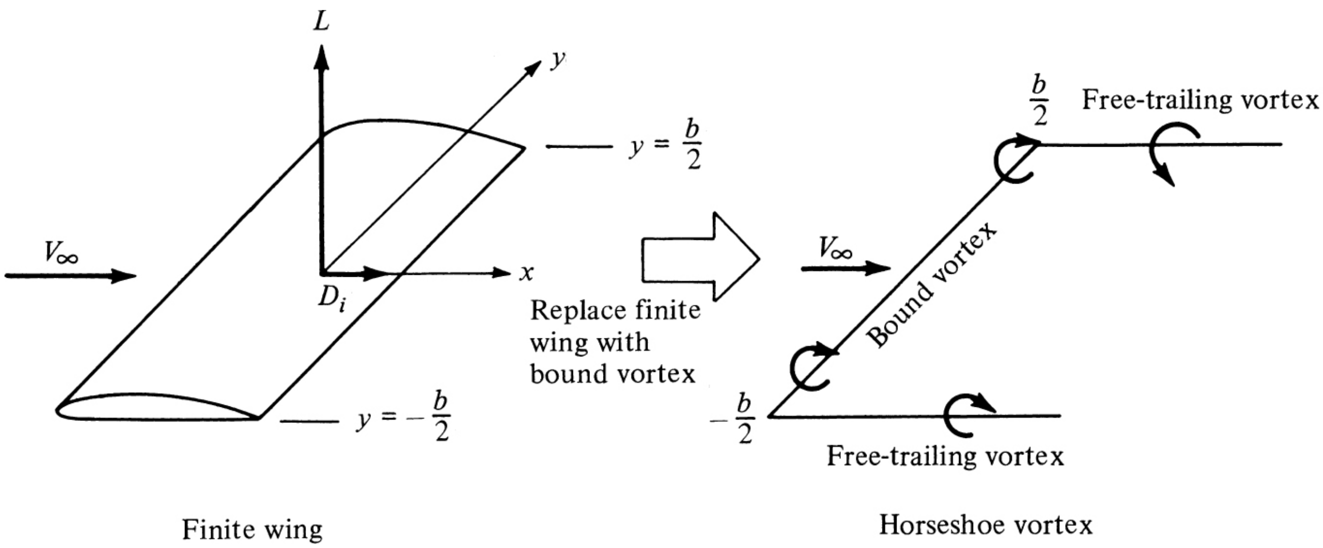
TYPICAL WASHOUT ANGLES FOR A **WW2** FIGHTER MODEL

Varying airfoil thickness:

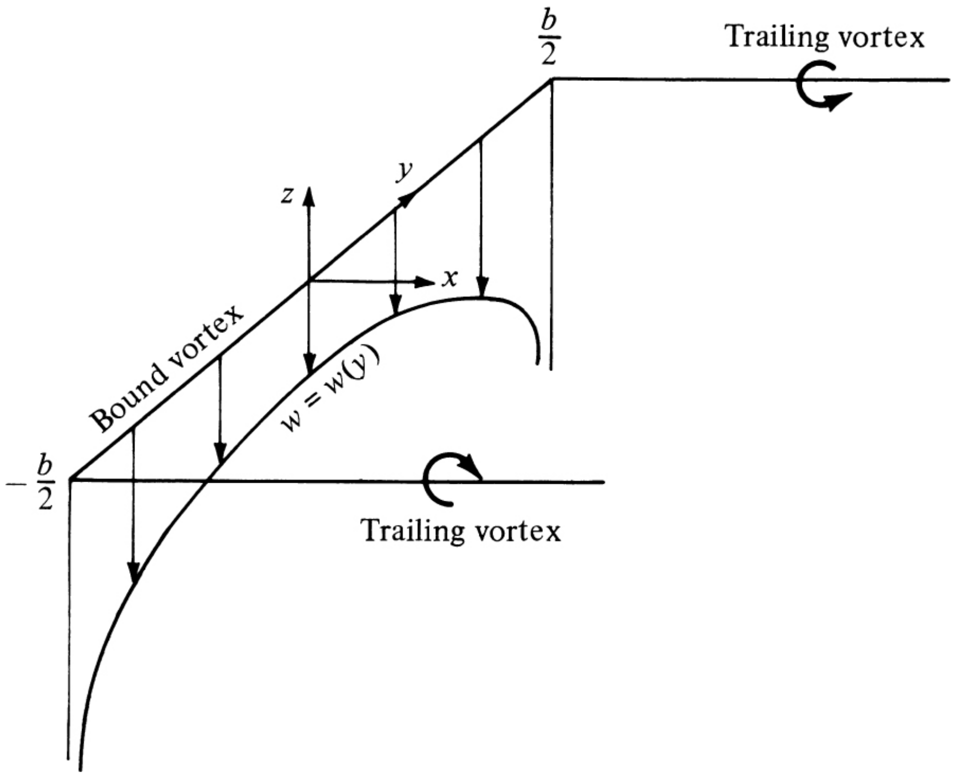


Agora digo eu: FANTASEADO POR MIM

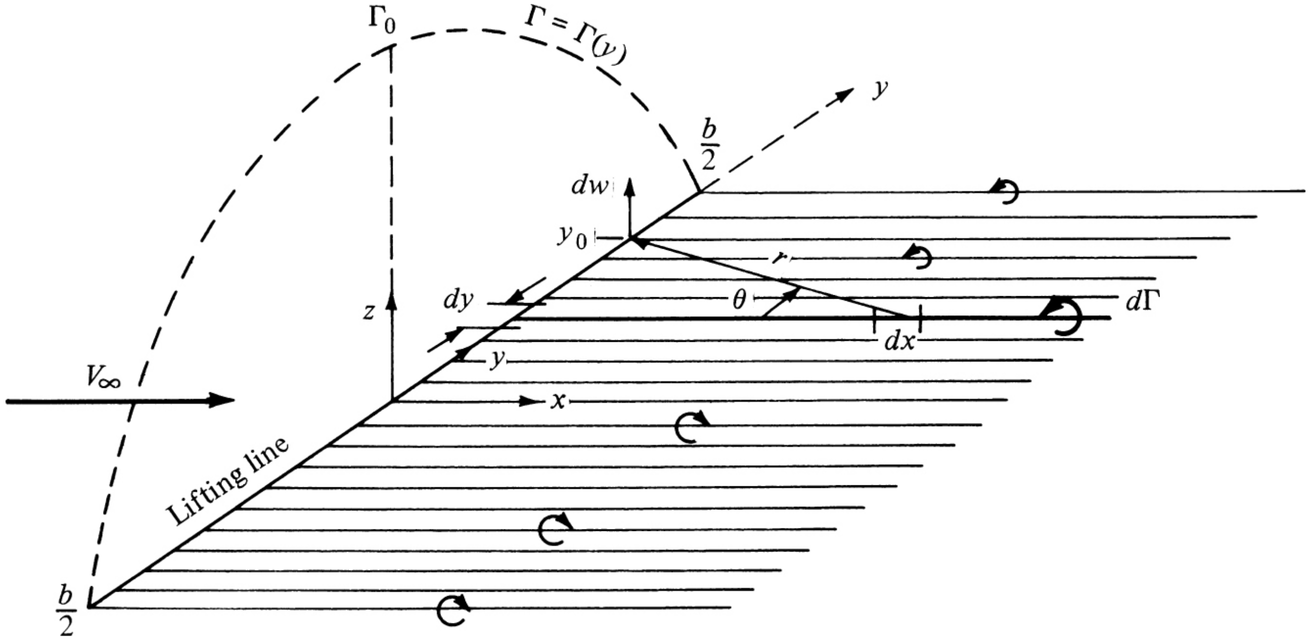
Prandtl's Lifting-Line Theory (A5.3)

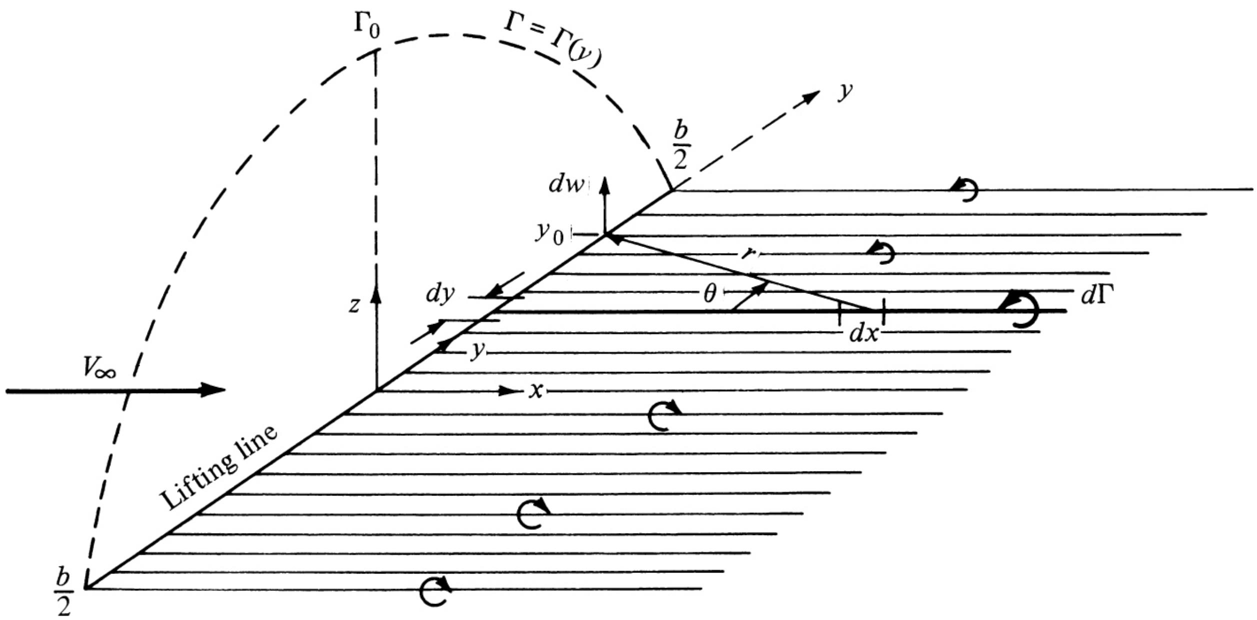


Spanwise Downwash Distribution



Superposition of "Horseshoe" Vortices




Γ_0
$\Gamma = \Gamma(y)$
y
$\frac{b}{2}$
dw
y_0
r
dy
θ
$d\Gamma$
z
dx
y
V_∞
x
Lifting line
$\frac{b}{2}$

Looking ahead: Computational strategy to compute Lift and induced Drag for a wing design involves estimating spanwise distribution of $\Gamma(y_0)$ from 2D airfoil data.(And remember: performance is reduced to due α_i, so we need to know it)

Recall:

Recall again:

Spanwise variation of: chord

AOA (geometric twist)

$\alpha_{L=0}$ (aerodynamic twist

Summary: Prandtl's Lifting Line Theory tells us that if we approximate an airplane wing with a collection of potential horseshoe vortices that superpose along a spanwise (tip-to-tip) "Lifting Line", then all performance parameters for the wing can be found from an assumed (or estimated) distribution of Γ along the Lifting Line.

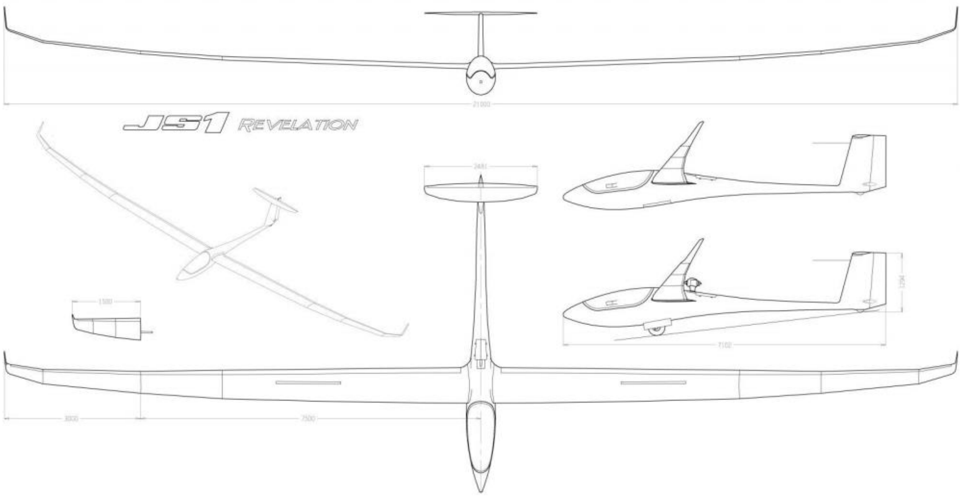
Elliptical Lift Distribution

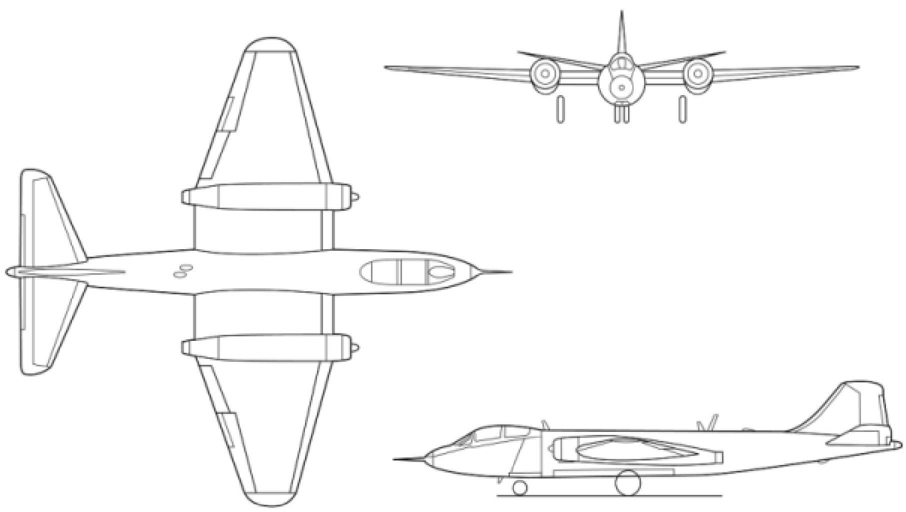
Continue with downwash at lifting line for elliptical distribution of Γ(y):

Recall: induced AOA:

Induced drag for elliptical dist:

Effect of Aspect Ratio on Induced Drag:



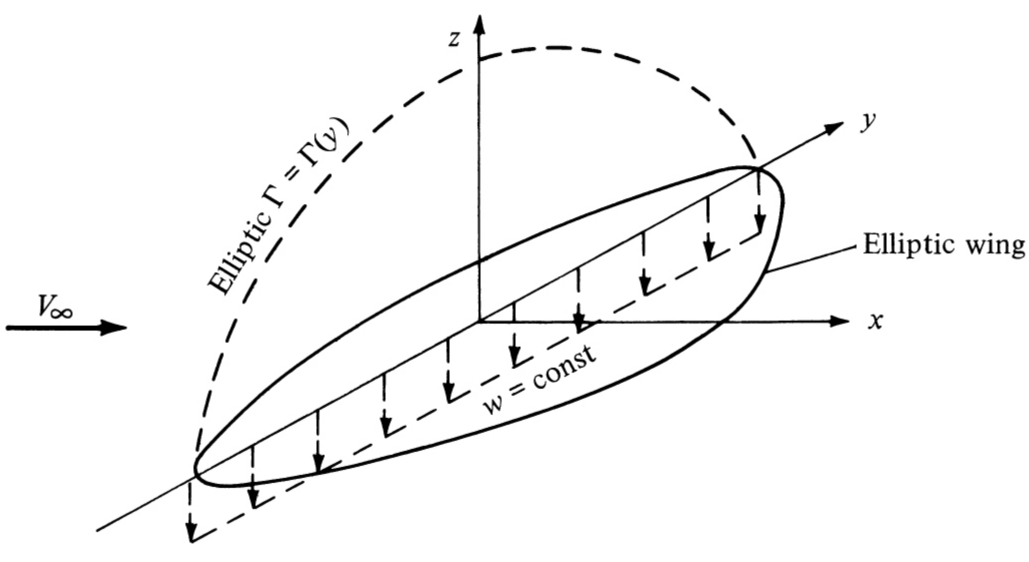


Dryden Flight Research Center February 1998
B-57B 3-view

Simple wing with elliptical Γ distribution,

no aerodynamic twist

no geometric twist



General (non-elliptical) Lift Distribution

Wait, why were we doing all this??

Ans: Prandtl's Lifting Line Theory for potential flow says that if you can determine spanwise circulation distribution Γ(y) for your wing design, then you can calculate the 3D coefficients of Lift (C_D) and Induced Drag (C_{Di}). So let's do that!

Math Wizardy: coordinate transformation

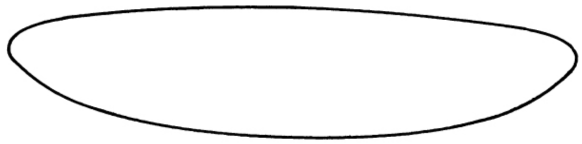
General Case: Circulation distribution over an arbitrary (non-elliptical) finite wing, expressed as Fourier Series in transformed variable θ:

How to determine $\Gamma(\theta)$ distribution for arbitrary wing:

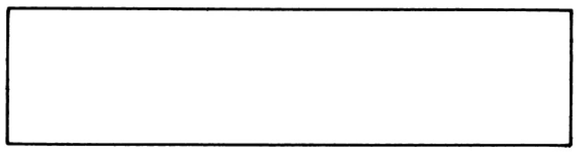
Now we know $\Gamma(y)$: Calculate C_L and C_{Di}

Lift:

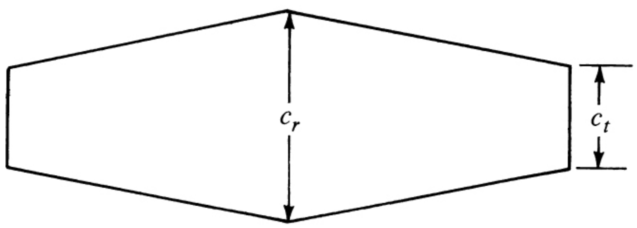
Drag:



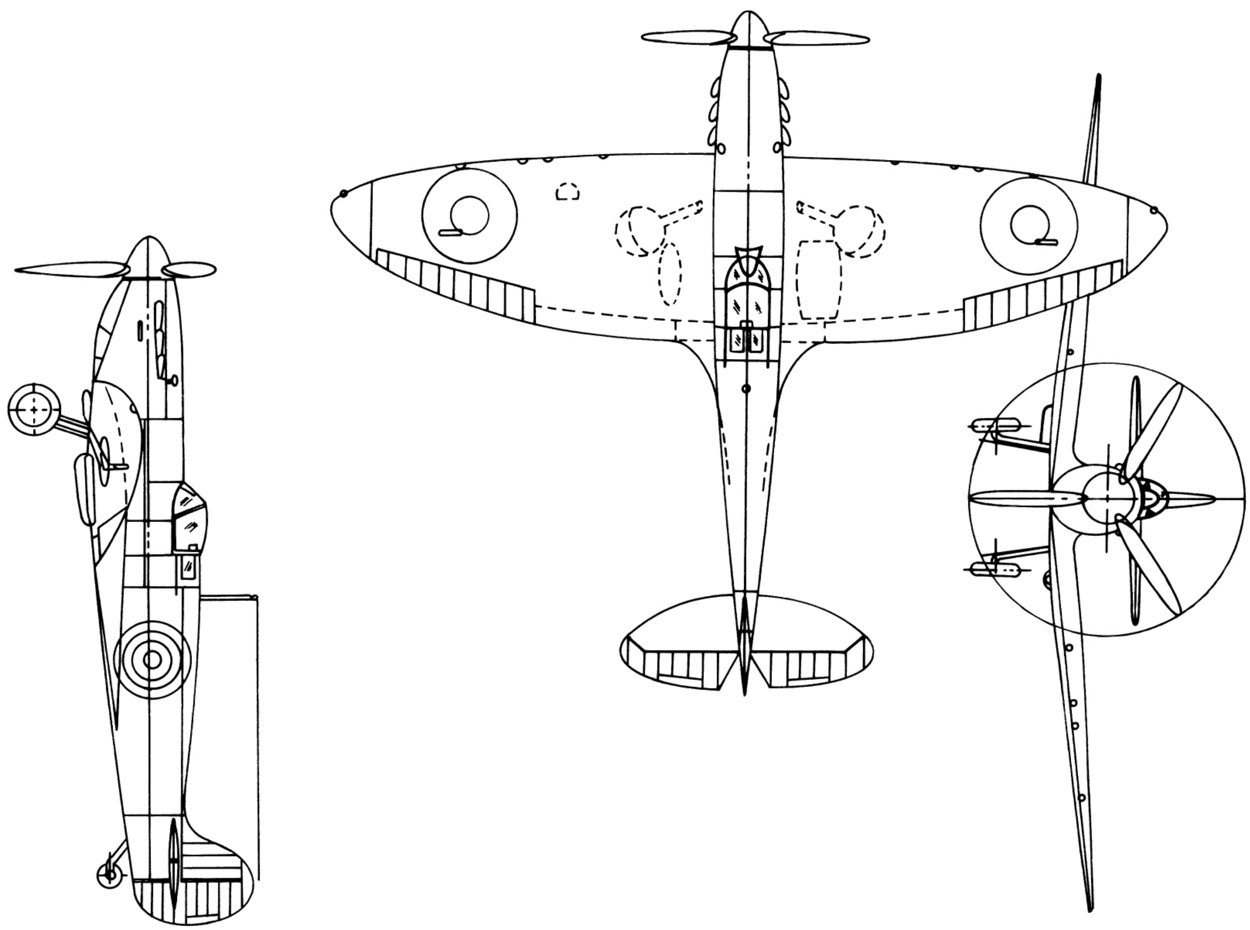
Elliptic wing



Rectangular wing

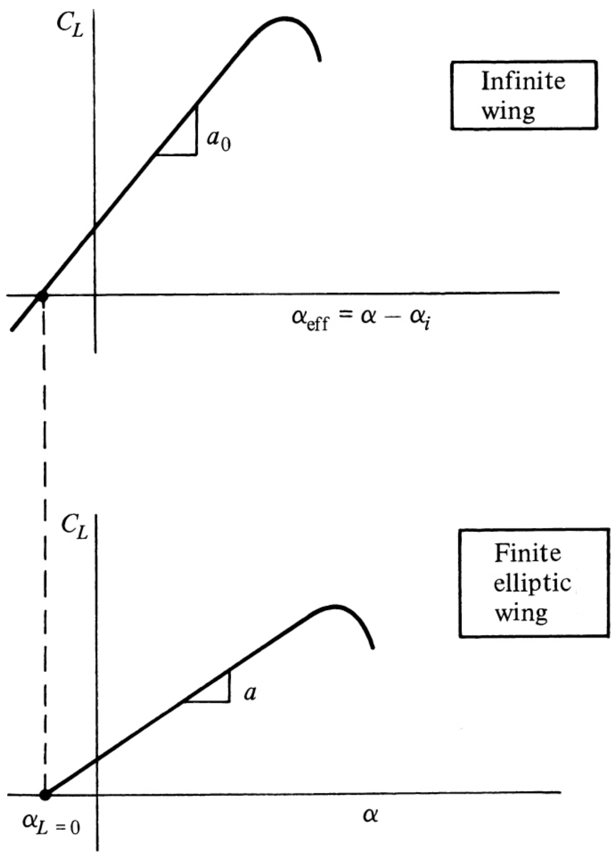


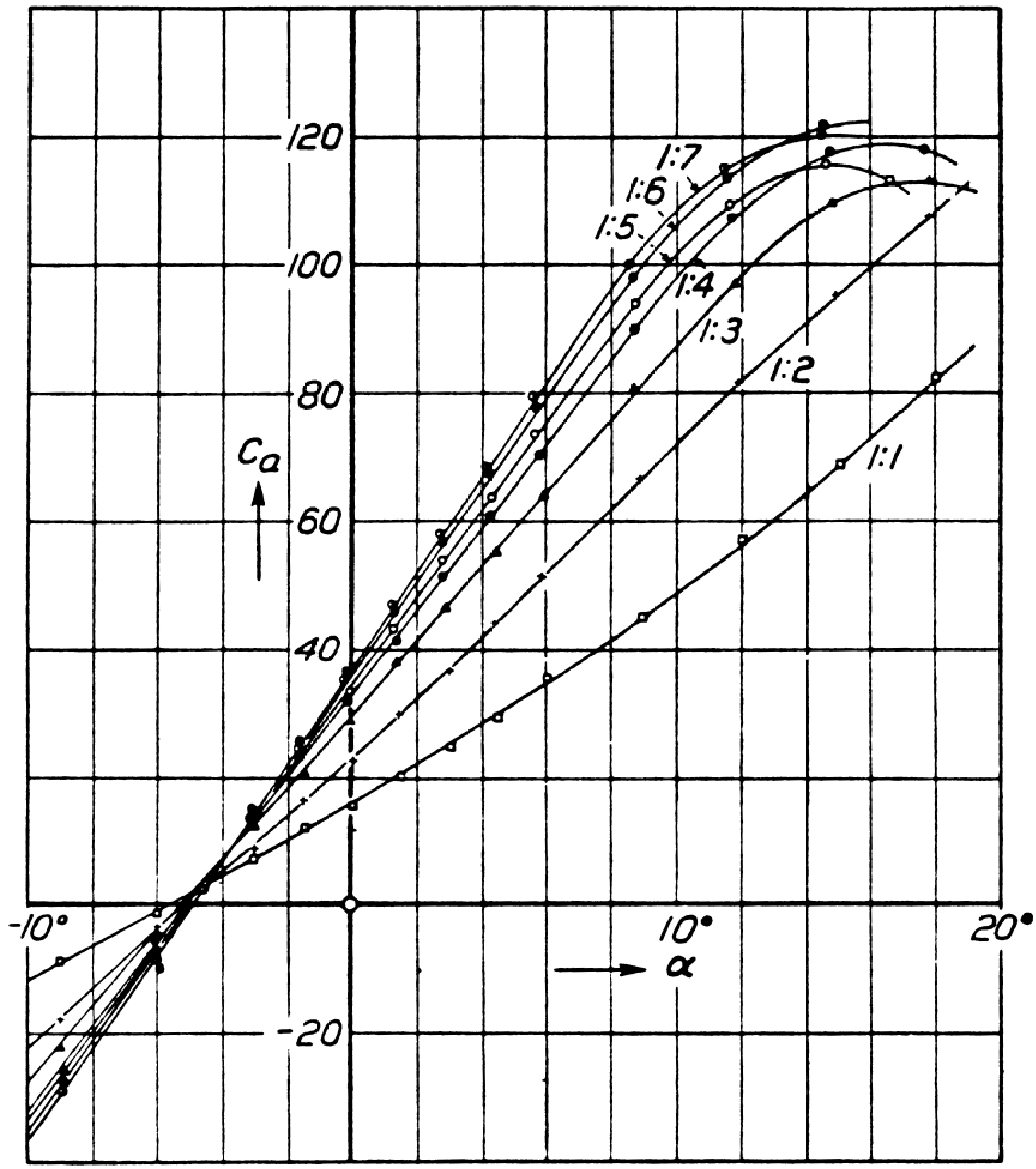
Tapered wing



Effect of Aspect Ratio on Total Drag

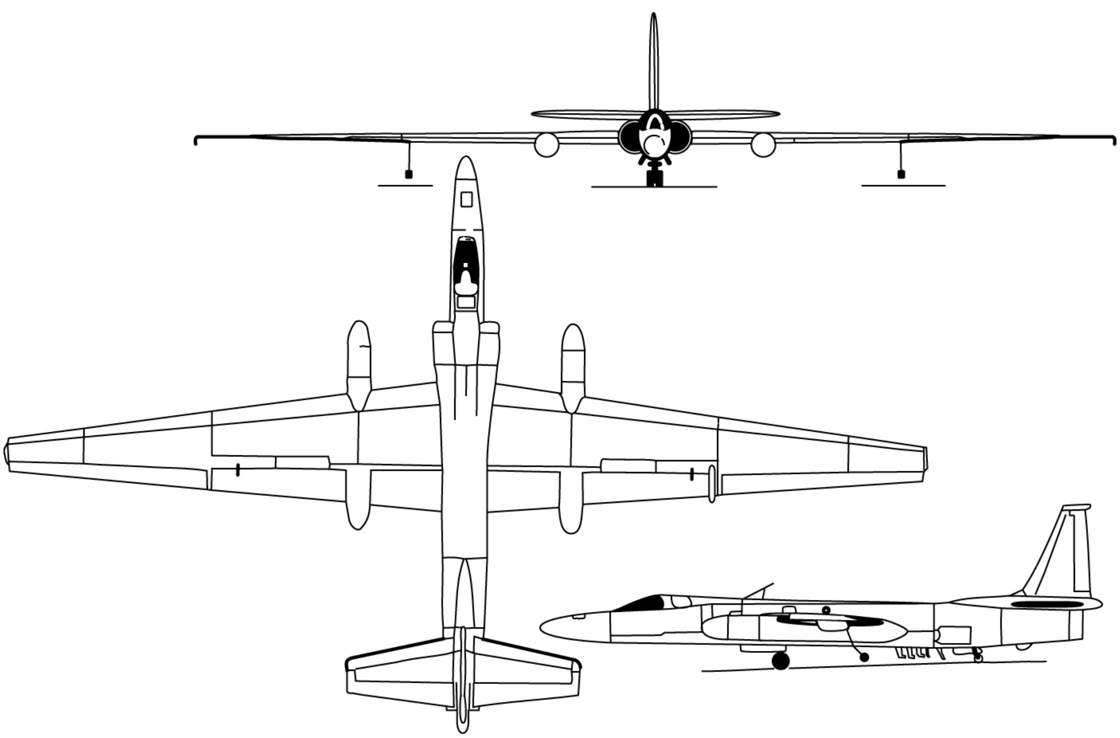
Effect of Aspect Ratio on Lift-Curve Slope



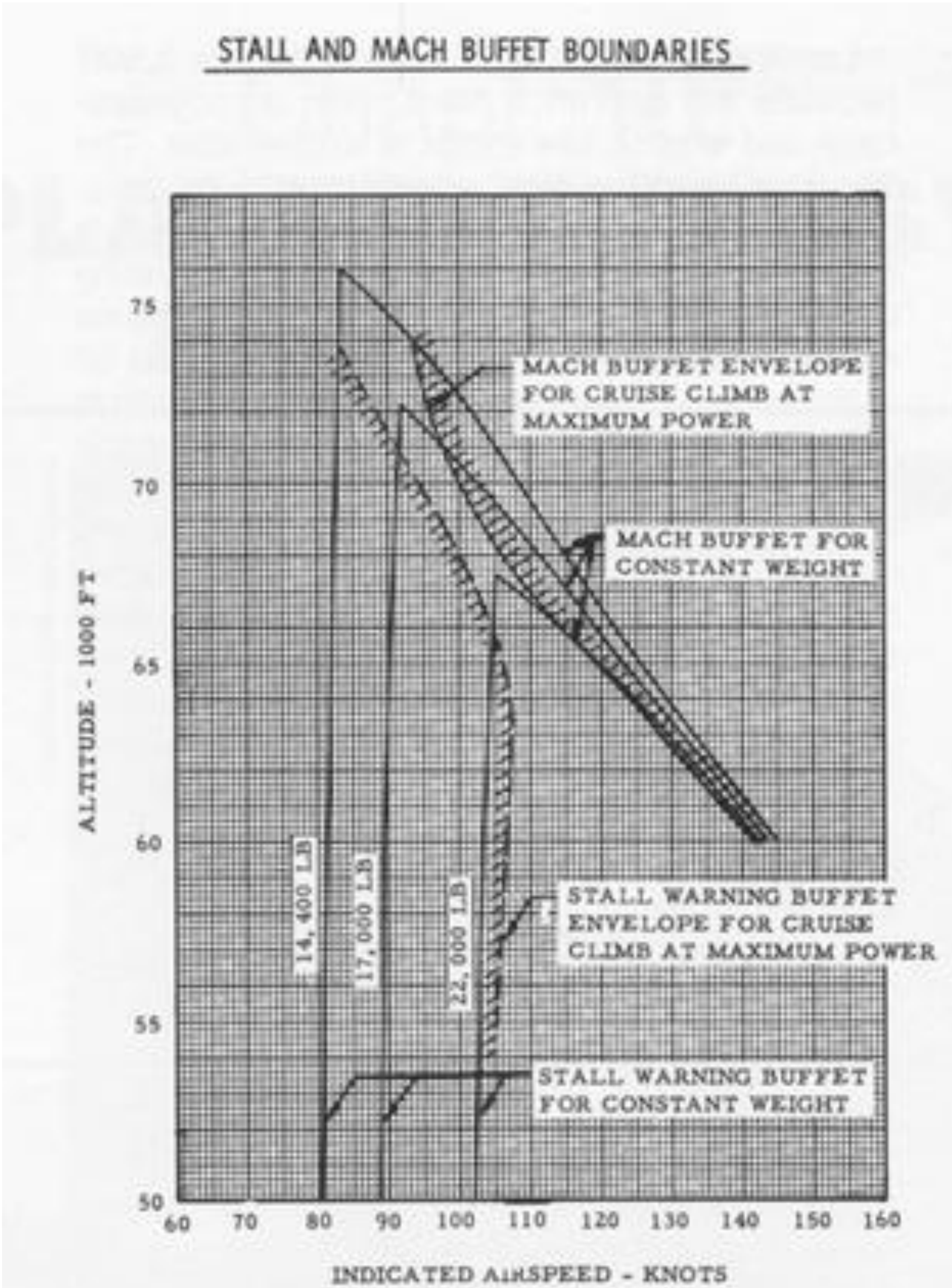


120
100
80
60
40
-20
C_a
-10°
10°
20°
α
1:7
1:6
1:5
1:4
1:3
1:2
1:1

Design Example: Lockheed U-2

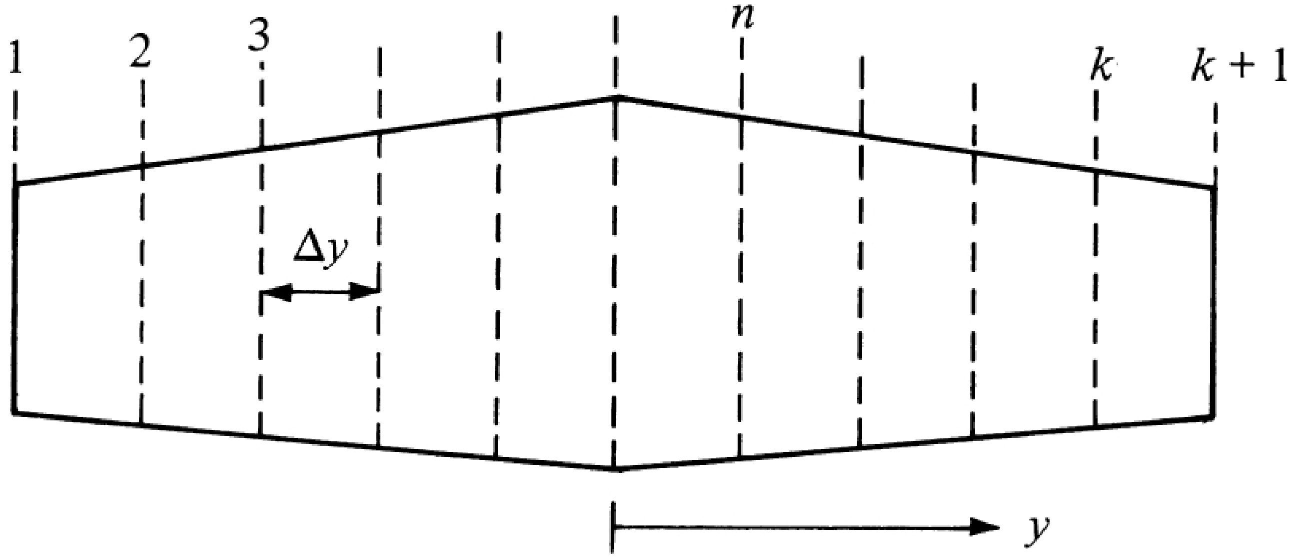


U2 "Coffin Corner"



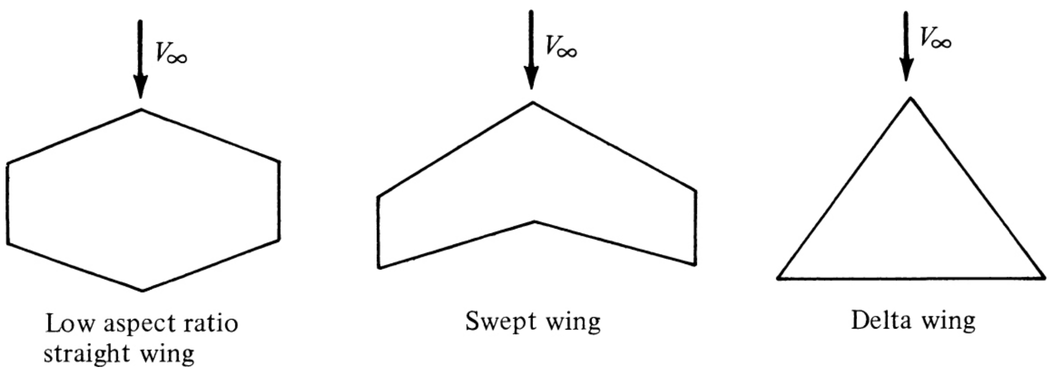
A Numerical Nonlinear Lifting-Line Method (A5.4)

(Project 8)



Vortex Lattice Numerical Method (A5.5)

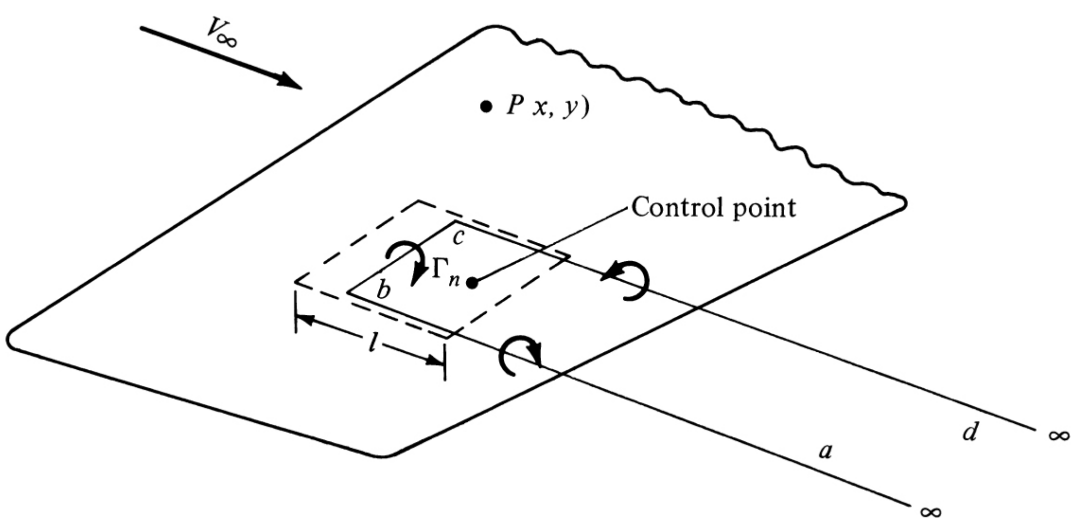
Lifting-Line theory good only for straight wings, not these:

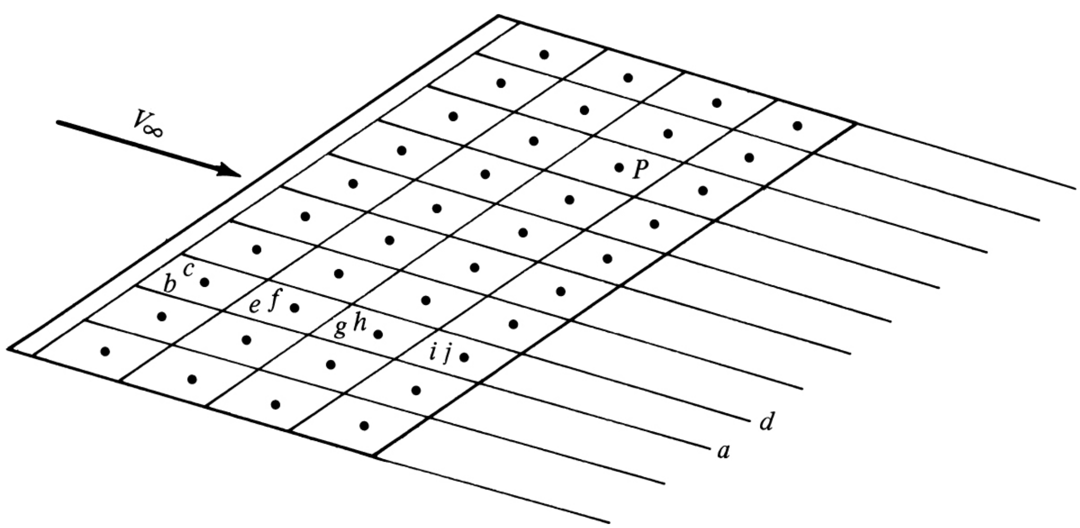


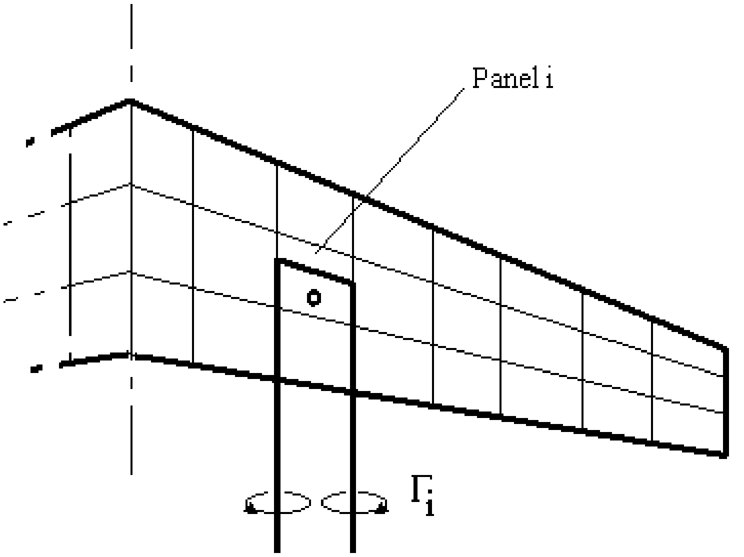
Example V-L code by Drela (MIT): AVL

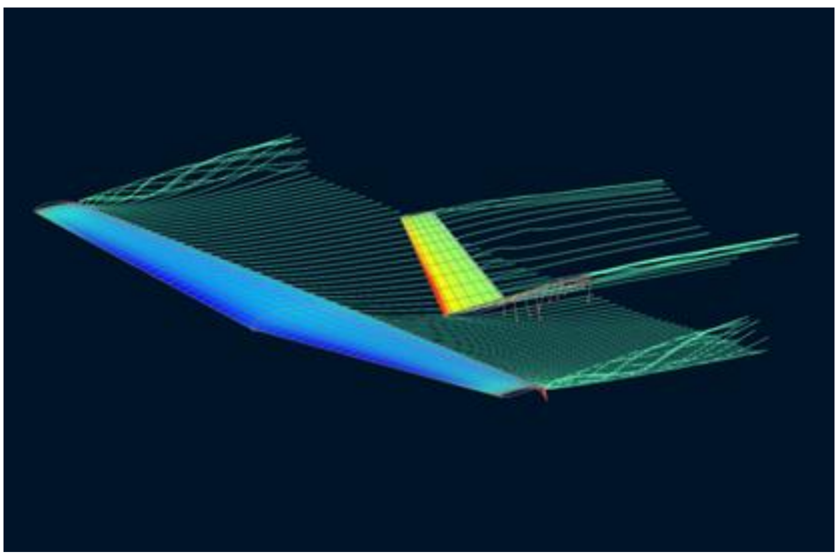
http://web.mit.edu/drela/Public/web/avl/

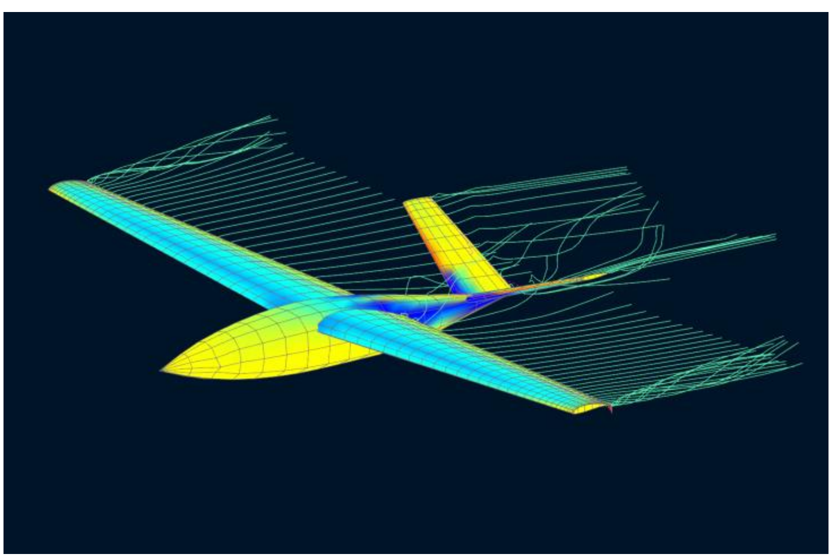
Vortex Lattice System



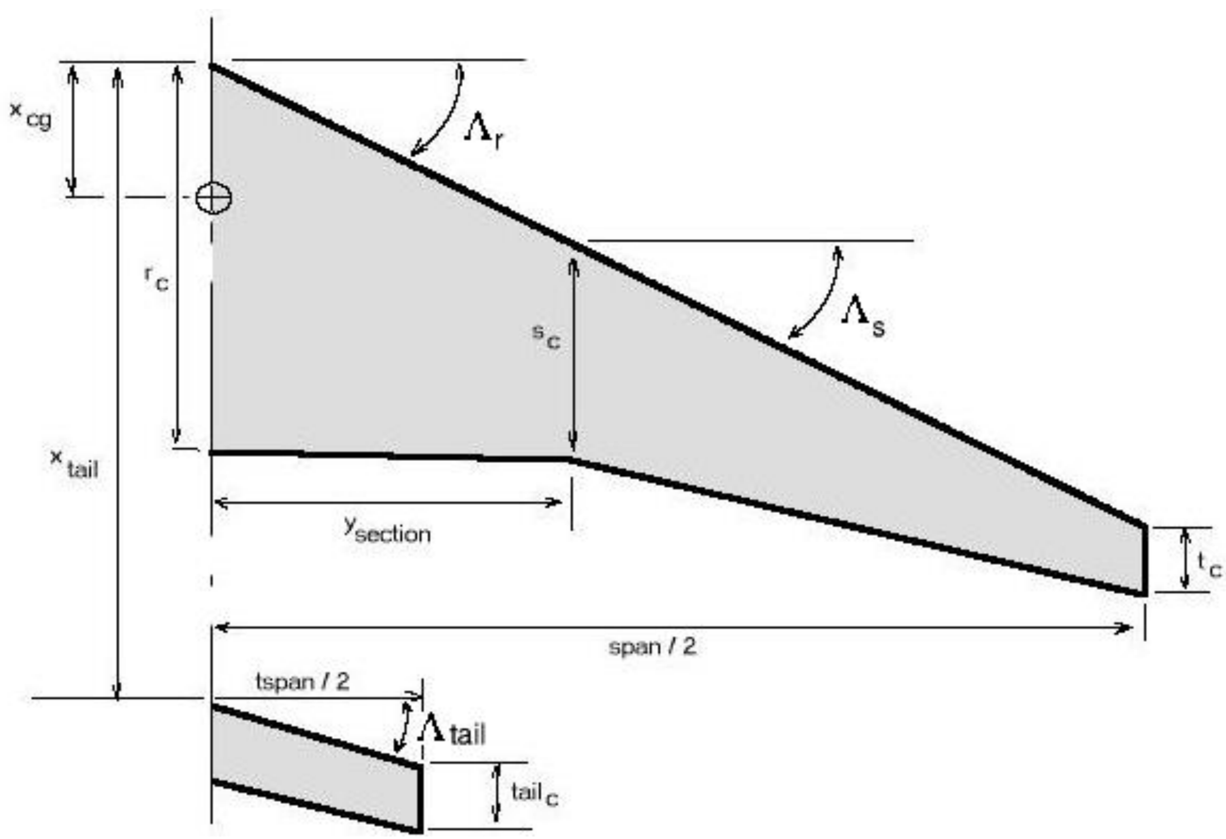



Panel i
Γ_i



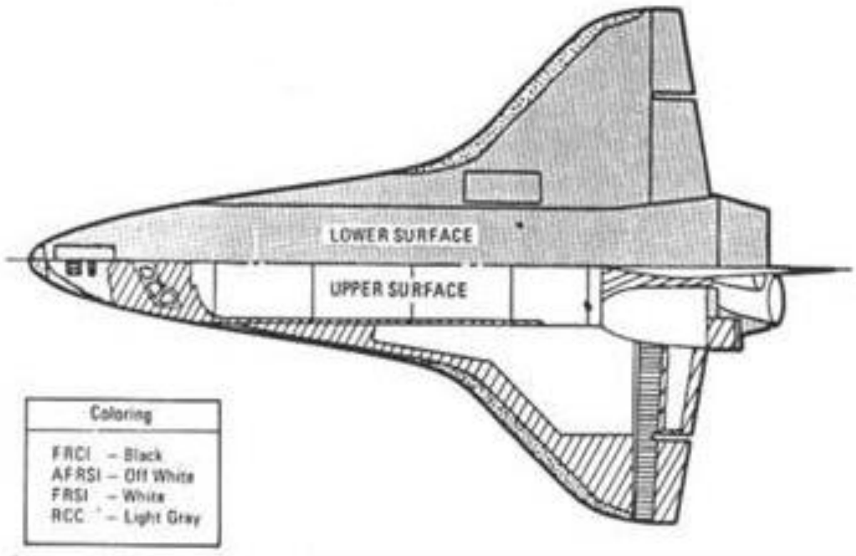


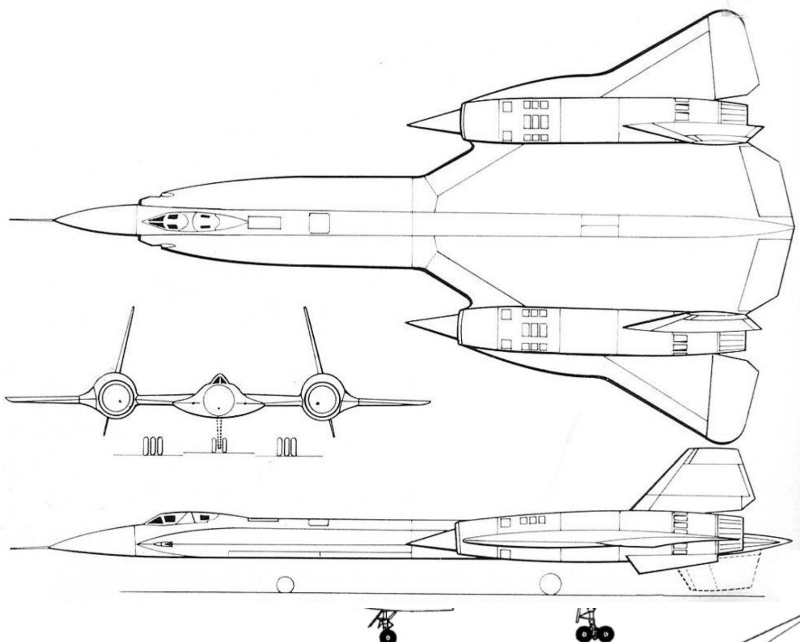
Vortex-Lattice Code: Example Wing Geometry Definition

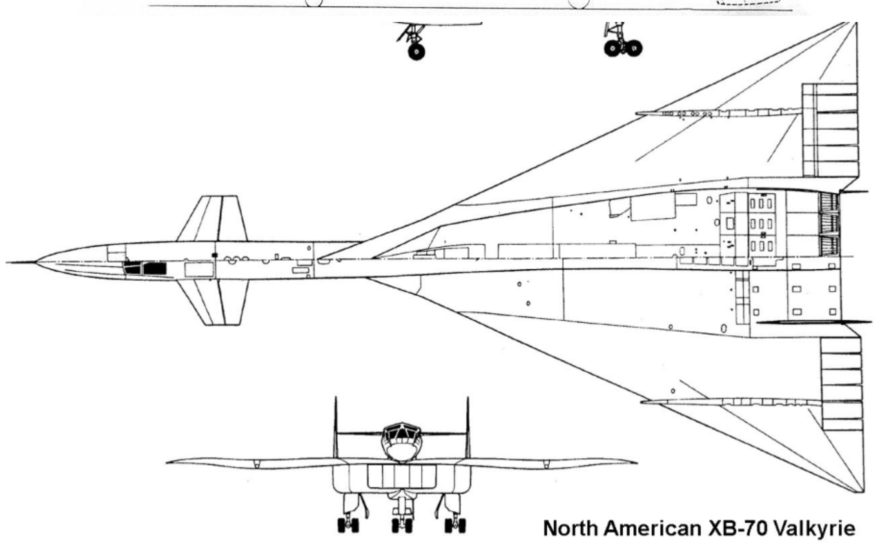


Panel methods gone wild:

The Delta Wing (A5.6)

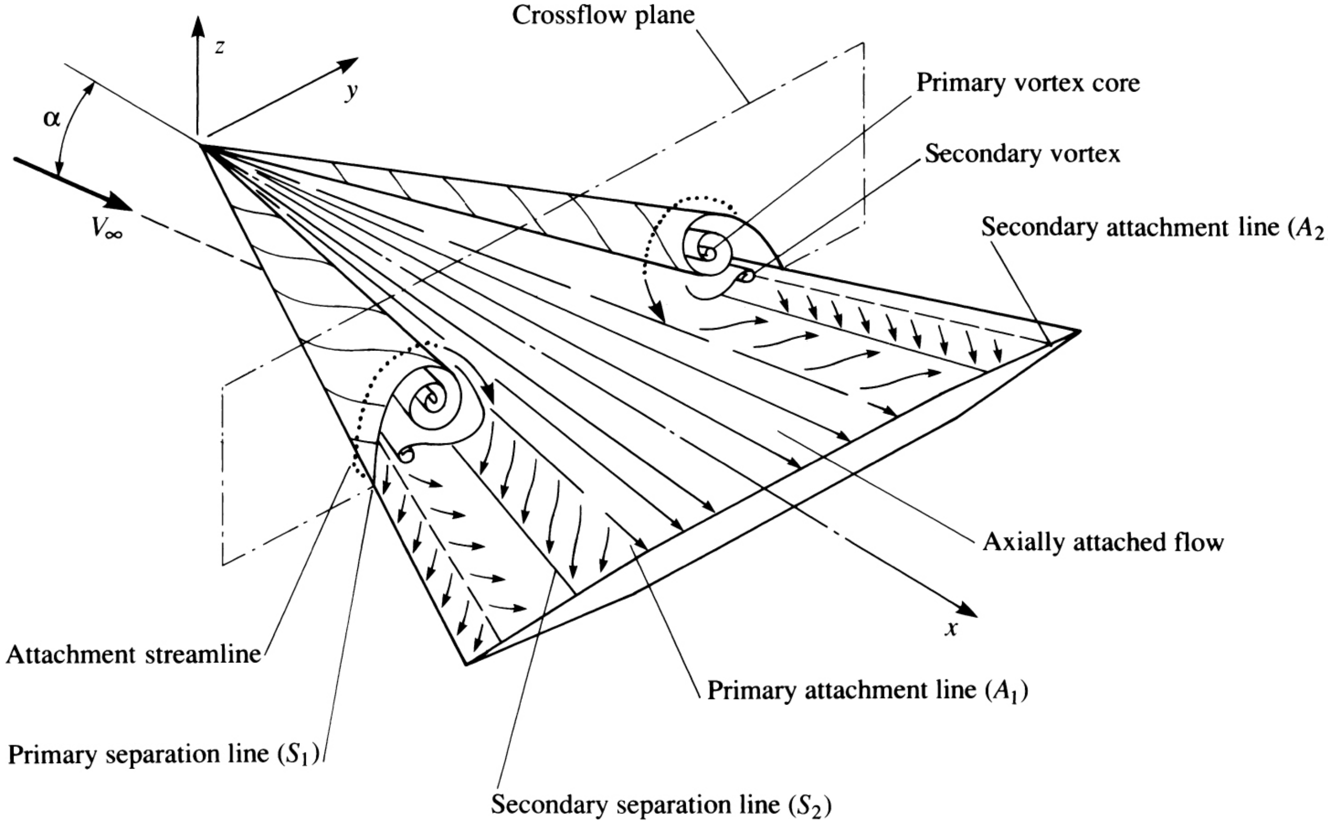


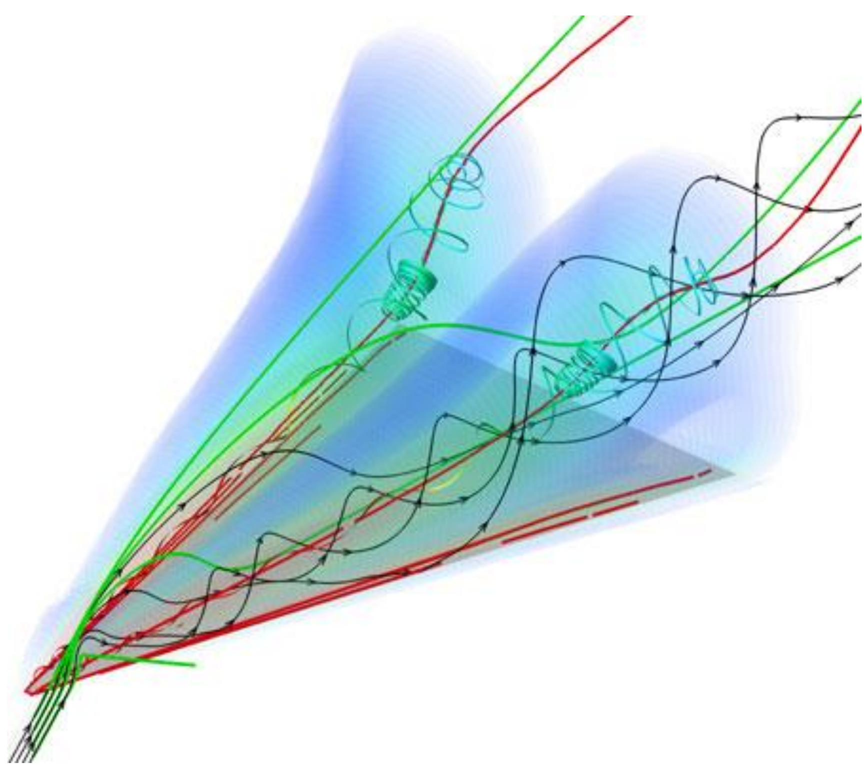


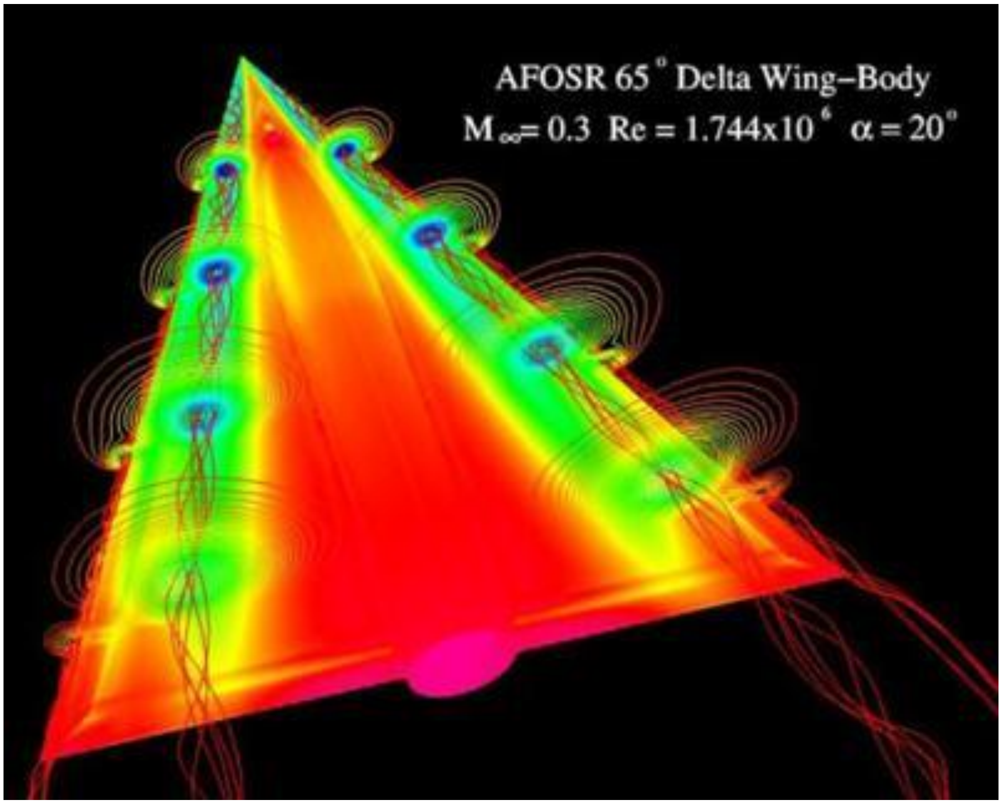


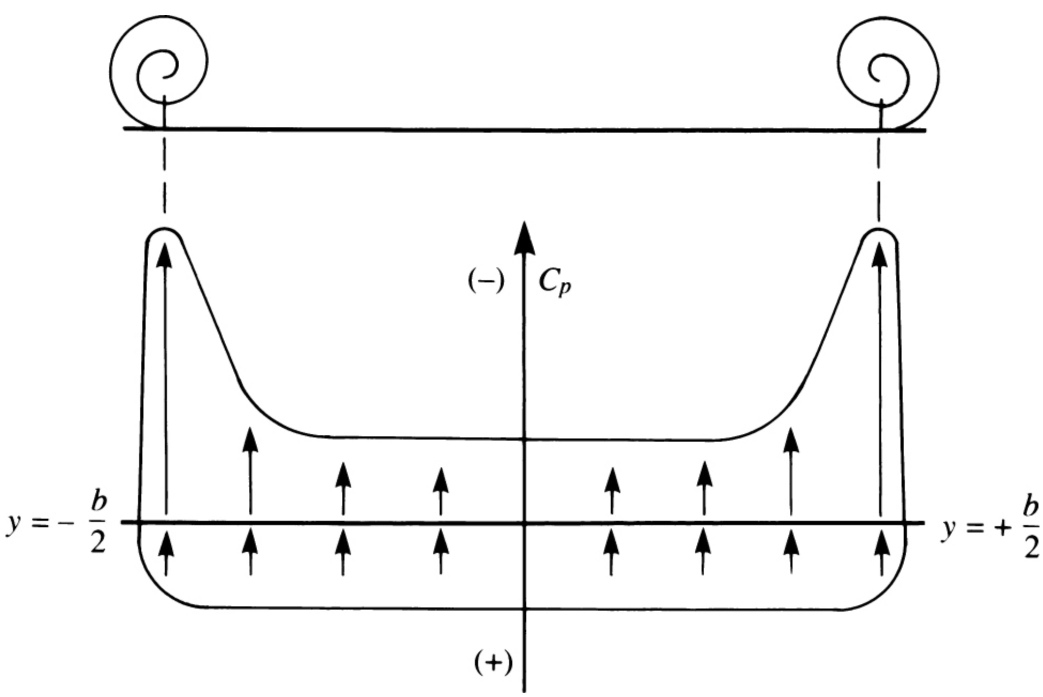
North American XB-70 Valkyrie

Subsonic Flowfield over Delta Wing:

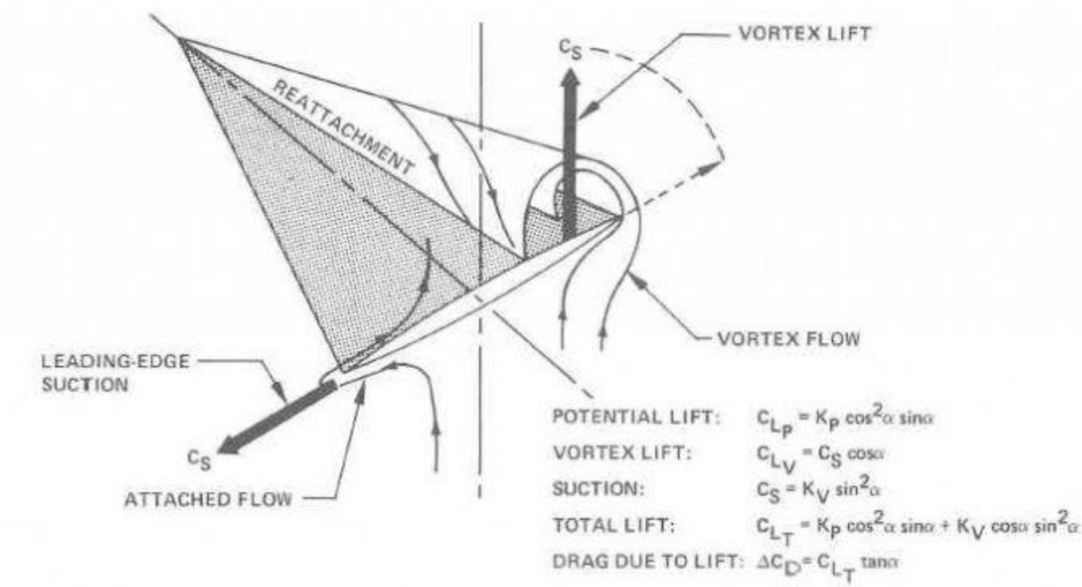




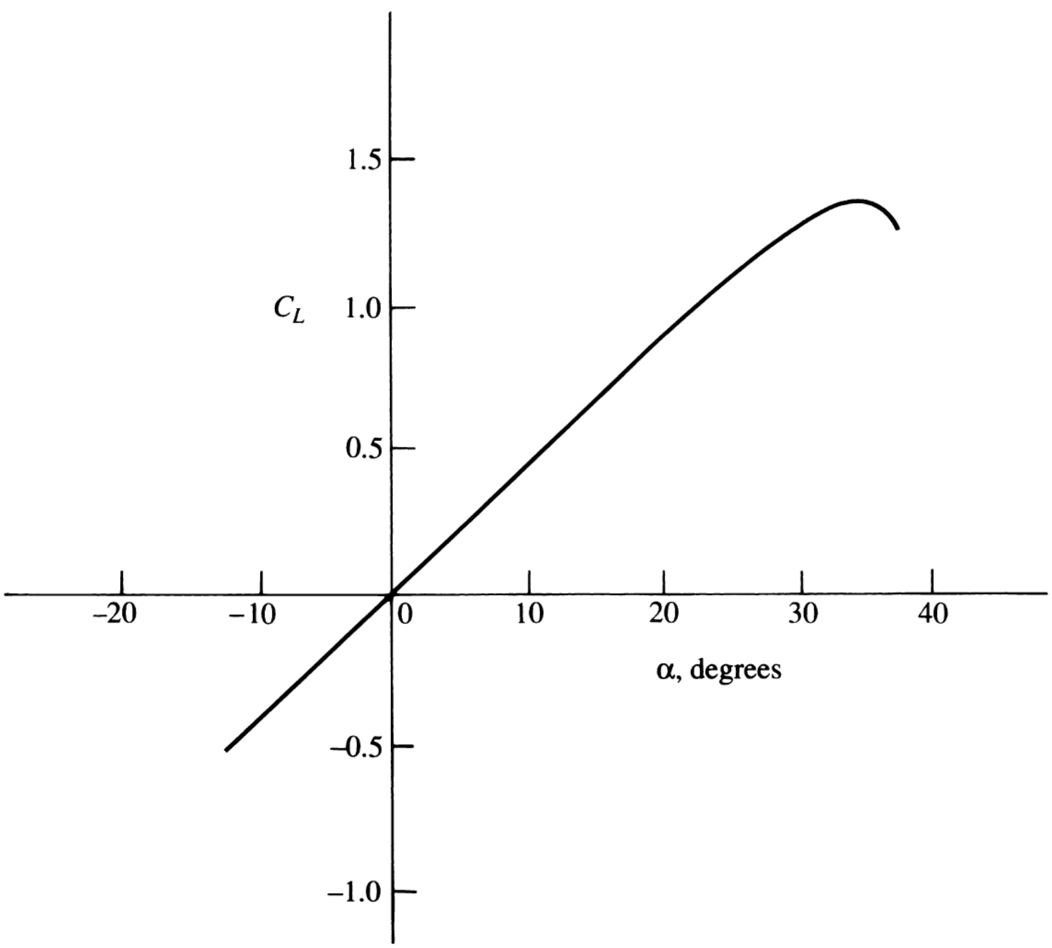

AFOSR 65° Delta Wing–Body
M∞= 0.3 Re = 1.744x10^6 α = 20°


(−) C_p
$y = -\frac{b}{2}$
$y = +\frac{b}{2}$
(+)

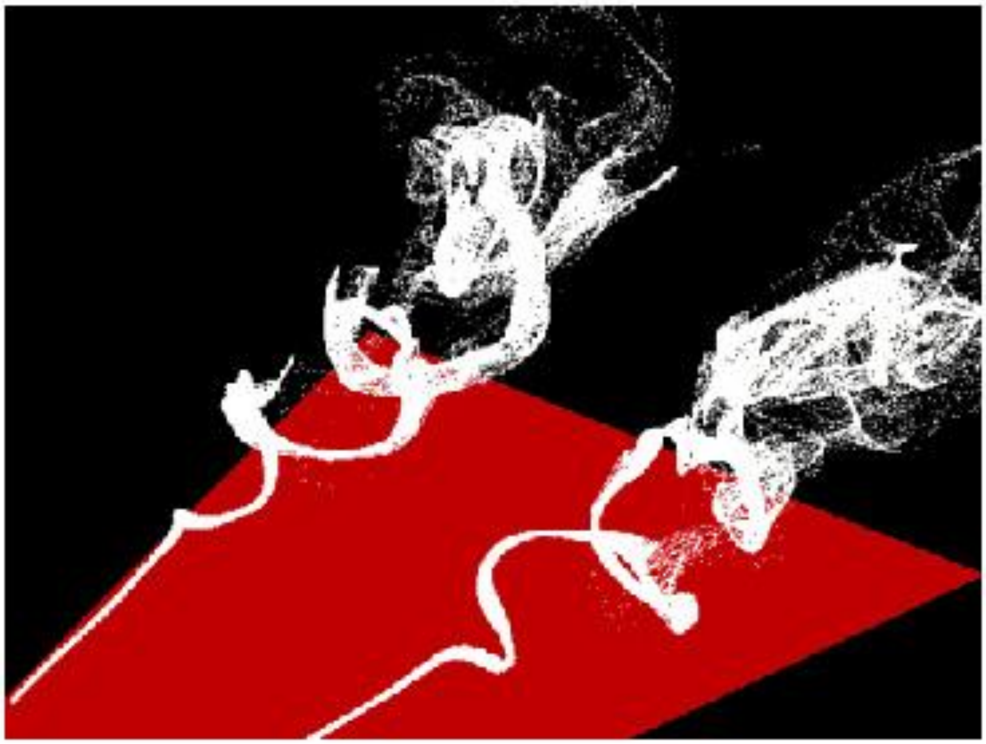
Delta Wing Lift:

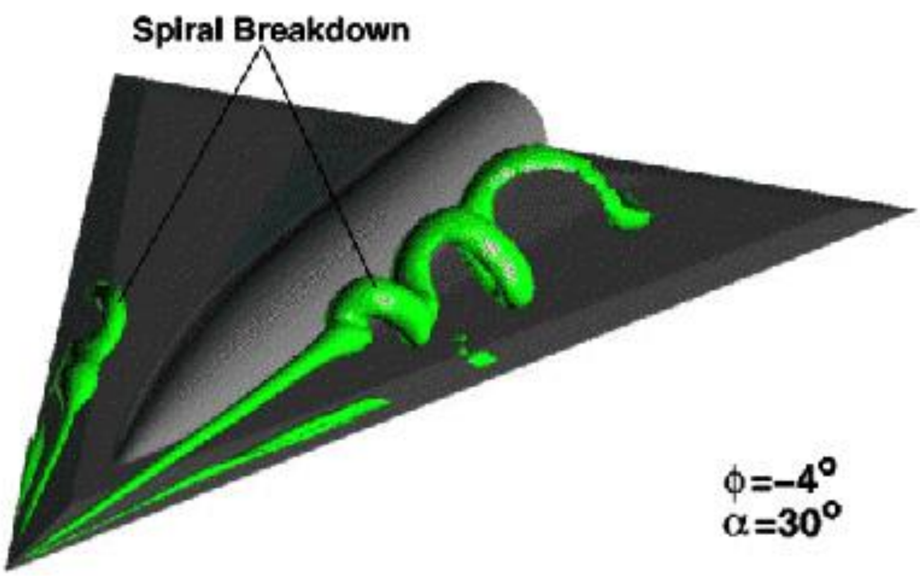


R.M. Kulfan, Wing Geometry Effects on Leading Edge Vortices, AIAA 79-1872

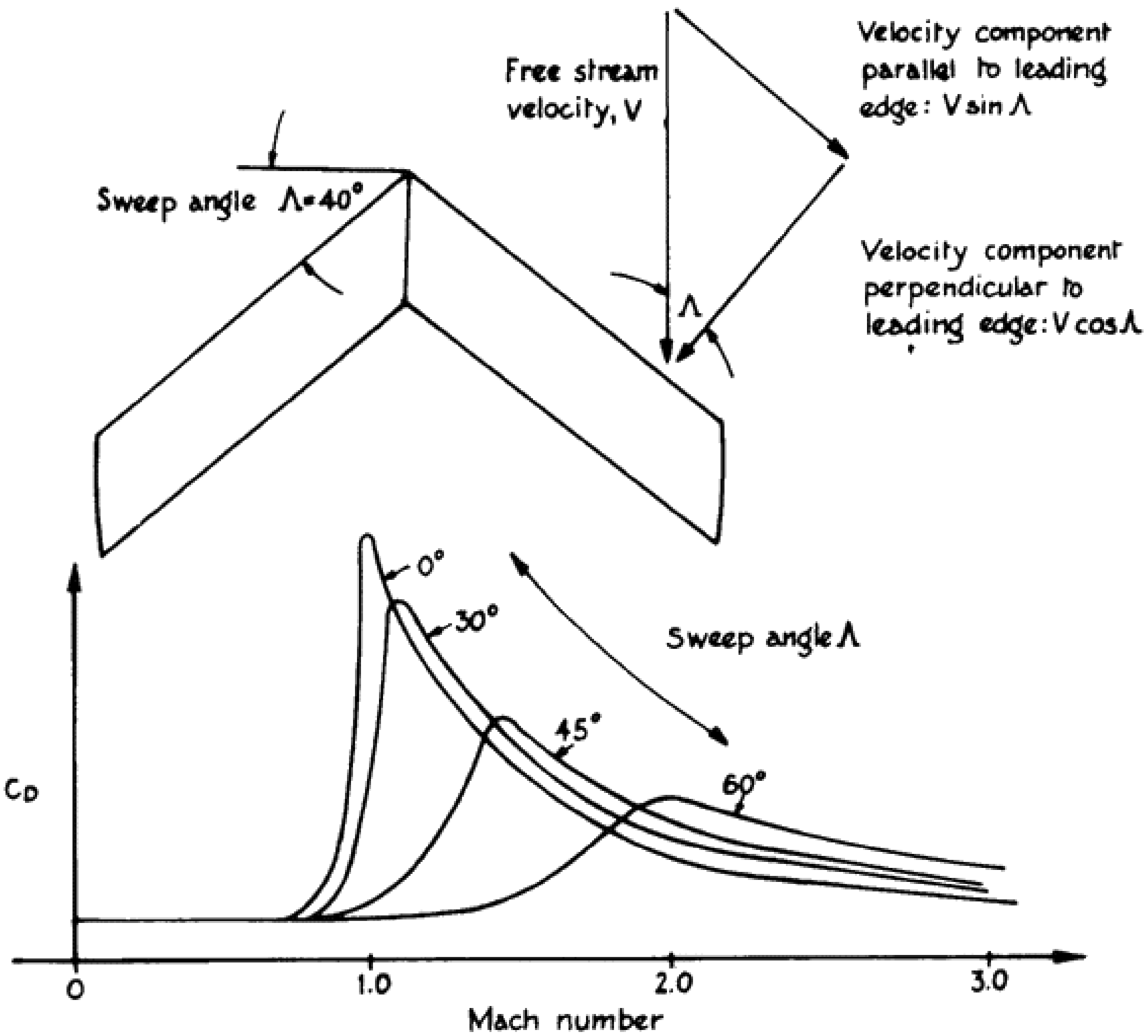

1.5
1.0
0.5
−0.5
−1.0
C_L
−20
−10
0
10
20
30
40
α, degrees

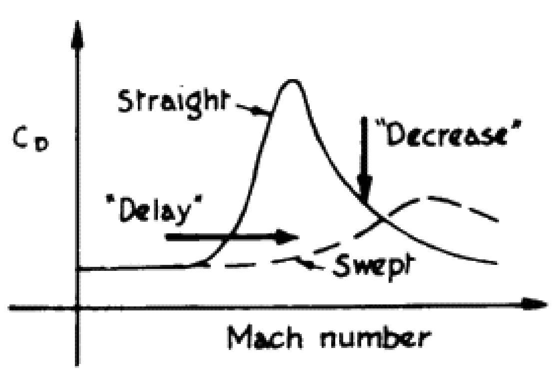
Delta Wing Vortex Breakdown:

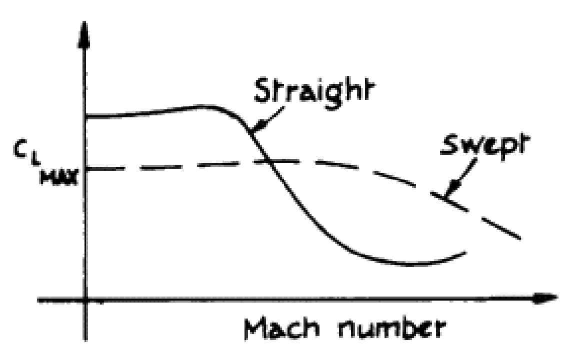




Drag Rise near Mach 1: Swept Wings







Variable-Sweep Wings: