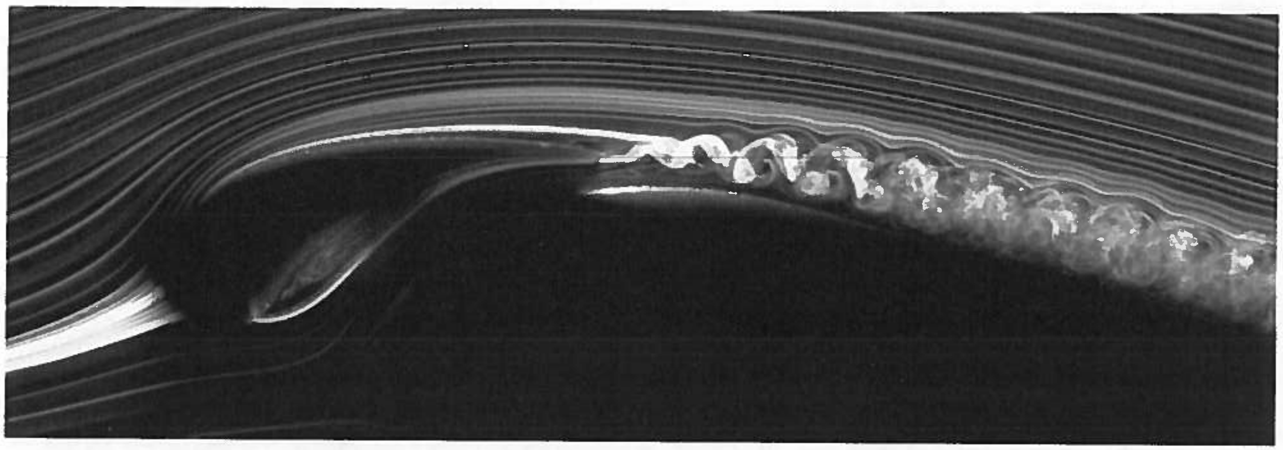
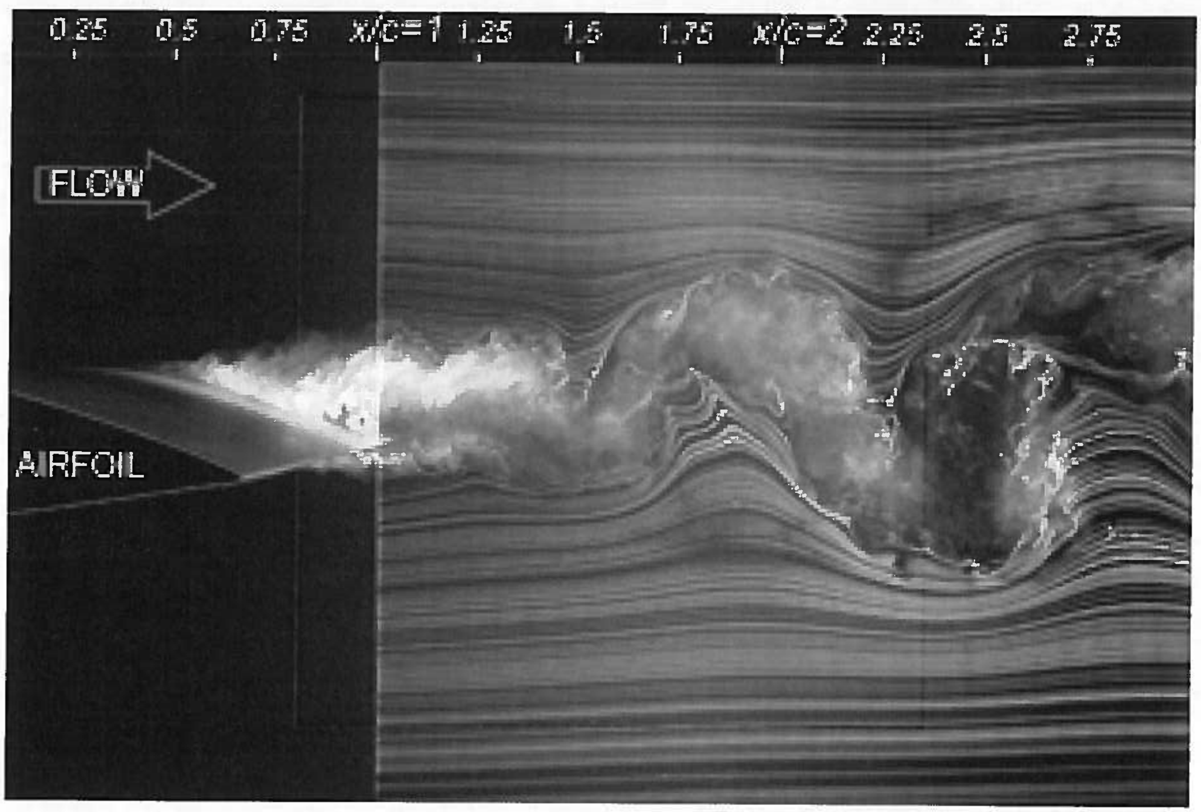
Section 2

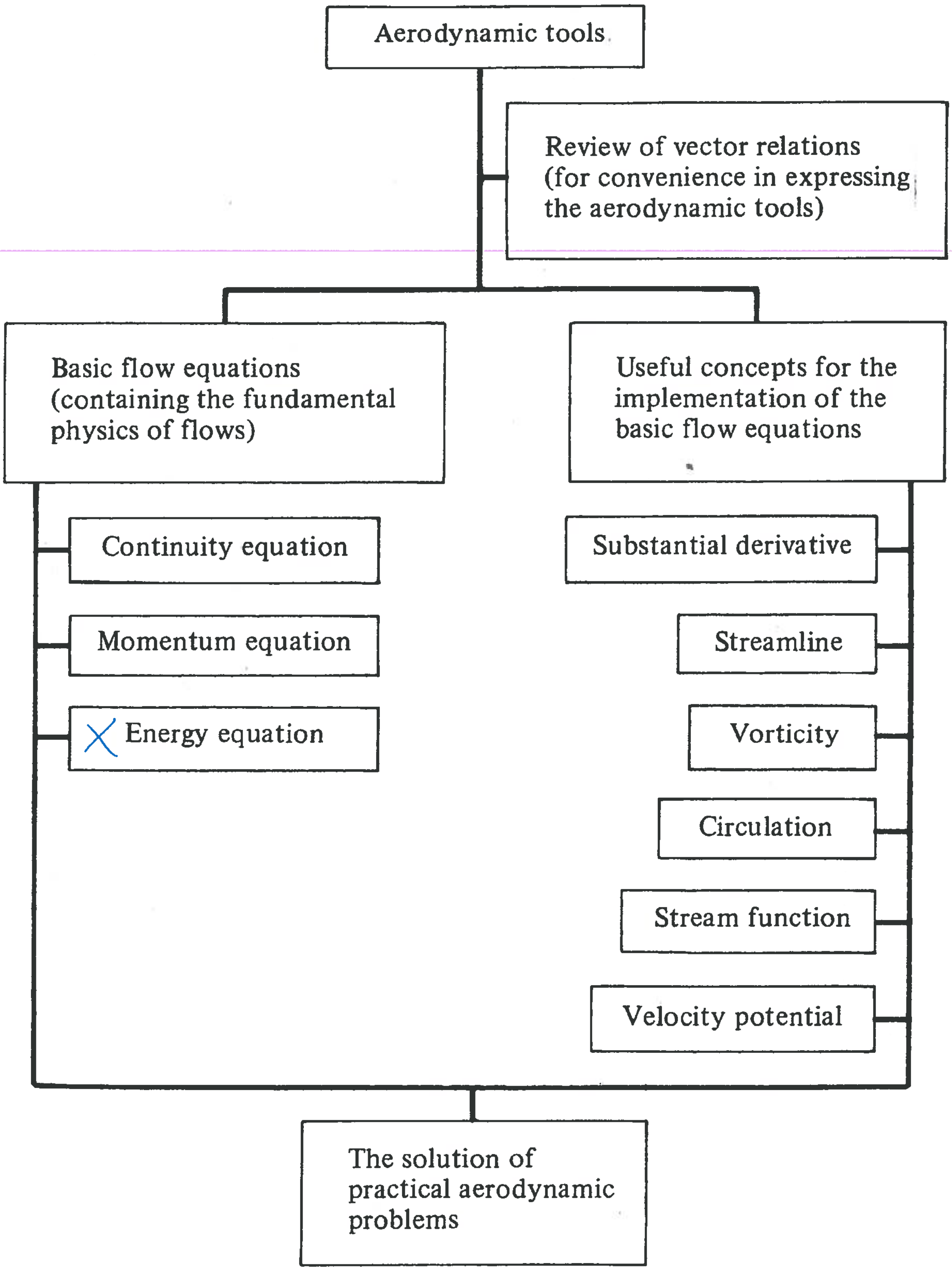
Fluids Review (Chap A2)





(self-study lectures)

Roadmap (A2.1):


Aerodynamic tools
Review of vector relations (for convenience in expressing the aerodynamic tools)
Basic flow equations (containing the fundamental physics of flows)
Useful concepts for the implementation of the basic flow equations
Continuity equation
Momentum equation
Energy equation
Substantial derivative
Streamline
Vorticity
Circulation
Stream function
Velocity potential
The solution of practical aerodynamic problems

Vector Math (A2.2):

Gradient of a Scalar Field (A2.2.5)

scalar $P = p(x, y, z)$

gradient of P : $\vec{\nabla} P$ magnitude = max spatial rate of change

direction = direction of max rate of change

$$\vec{\nabla} p = \frac{\partial p}{\partial x}\vec{i} + \frac{\partial p}{\partial y}\vec{j} + \frac{\partial p}{\partial z}\vec{k}$$

Divergence of a Vector Field (A2.2.6)

$$\vec{V} = \vec{V}(x, y, t) = V_x \vec{i} + V_y \vec{j} + V_z \vec{k}$$

$$\vec{\nabla} \cdot \vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} \quad \text{(scalar)}$$

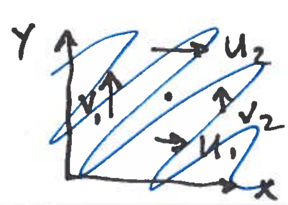
Curl of a Vector Field (A 2.2.7)

$$\vec{\nabla} \times \vec{V} = \left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right)\vec{i} + \left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right)\vec{j} + \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)\vec{k}$$

Vorticity: $2\vec{\omega} = \vec{\nabla} \times \vec{V}$

~ 0 for many ~~[illegible]~~ flows near surfaces

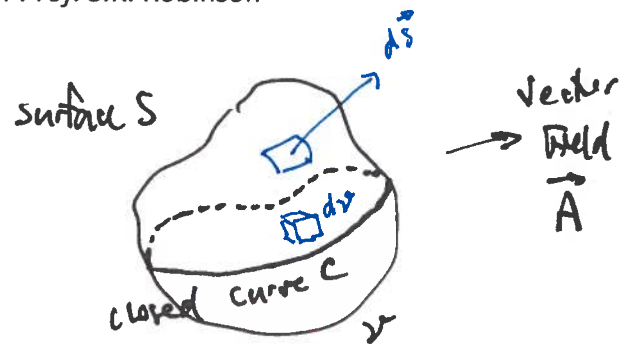
2-D flow: $\vec{\omega} = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$



$\boxed{\text{Vorticity} \neq \text{vortex}}$

Just shear layer / velocity grad → vorticity!

Useful Theorems



Stokes :

$$\oint_C \vec{A}\cdot d\vec{s} = \iint_S \underbrace{(\vec{\nabla}\times\vec{A})}_{\text{curl of } \vec{A}}\cdot d\vec{s}$$

Divergence :

$$\oiint_S \overset{\text{vector field}}{\vec{A}}\cdot d\vec{s} = \iiint_v (\vec{\nabla}\cdot\vec{A})\, dv$$

Gradient :

$$\iint_S \overset{\text{scalar field}}{p}\, d\vec{s} = \iiint_v \vec{\nabla} p\, dv$$

Continuity (A2.4)

- Mass-Flow Rate = area x density x flow velocity normal to area

$$\dot{m} = \rho V_n A \qquad \left(\frac{M}{L^3} \frac{L}{T} L^2 = \frac{M}{T} \right)$$

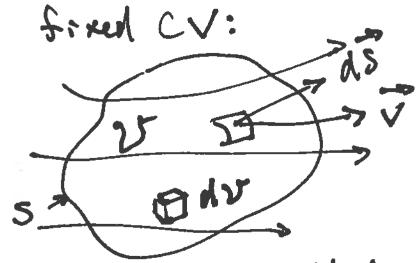
- Mass Flux $\frac{\dot{m}}{A} = \rho V_n$

- Notation: $\vec{V}(x,y,t) = V_x \vec{i} + V_y \vec{j} + V_z \vec{k}$

$$= u\vec{i} + v\vec{j} + w\vec{k}$$

Principal: Mass can be neither created nor destroyed

fixed CV:



$$\Downarrow$$

volume $\mathcal{V}$
surface S

Net mass-flow out of CV through bounding surface S (LHS) = Time rate of decrease of mass inside CV (RHS)

mass-flow across elemental surface dS $= \rho V_n \, dS = \underbrace{\rho \vec{V} \cdot d\vec{S}}_{\text{outflow if } > 0}$

$d\vec{S}$: always positive by defin

Integrate mass-flow over entire surface S

$$\iint_S \rho \vec{V} \cdot d\vec{S} = \text{LHS}$$

- RHS

$$\underbrace{-}_{\text{negative for decrease}} \overbrace{\frac{\partial}{\partial t}}^{\text{time rate of change}} \underbrace{\iiint_{\mathcal{V}} \underbrace{\rho \, d\mathcal{V}}_{\text{elem volume mass}}}_{\text{mass in entire volume } \mathcal{V}}$$

- LHS = RHS

$$\boxed{\frac{\partial}{\partial t} \iiint_{\mathcal{V}} \rho \, d\mathcal{V} + \iint_{S} \rho \vec{V} \cdot \vec{dS} = 0}$$

Continuity in Integral Form

- Continuity in Differential Form:

$$\frac{\partial}{\partial t}\iiint_{\mathcal{V}} \rho \, d\mathcal{V} + \iint_{S} \rho \vec{V} \cdot d\vec{S} = 0$$

fixed cv

use Divergence Theorem to convert to volume integral

$$\iiint_{\mathcal{V}} \frac{\partial \rho}{\partial t} d\mathcal{V} + \iiint_{\mathcal{V}} \vec{\nabla} \cdot (\rho \vec{V}) \, d\mathcal{V}$$

or $$\iiint_{\mathcal{V}} \underbrace{\left[\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{V})\right]}_{\text{only way is for integrand to equal zero!}} d\mathcal{V} = 0$$

volume is arbitrary - must always be true

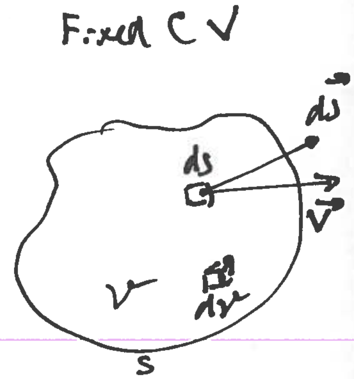
$$\therefore \quad \boxed{\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{V}) = 0}$$ Continuity in Differential Form

- No assumptions at all except Continuum - totally general!

- Incompressible Steady : $\vec{\nabla} \cdot \vec{V} = 0$ "Divergence-Free"

Momentum (A2.5)

- Principle: $\Sigma \vec{F} = \frac{d}{dt}(m\vec{V})$



for fixed collection of mass : externally applied forces results in rate of change of momentum

- Externally applied forces :

Body (gravity, etc. acting on $\mathcal{V}$)
Surface (pressure and shear acting on S surface)

$$\Sigma \vec{F} = \underbrace{\iiint_{\mathcal{V}} \rho \, \vec{f}_{body} \, d\mathcal{V}}_{\text{body}} \overbrace{- \underbrace{\iint_{S} p \, d\vec{s}}_{\text{pressure}} + \underbrace{\vec{F}_{visc.}}_{\text{shear}}}^{\text{surface}}$$

force per unit mass

pressure (negative because force in direction opposite $d\vec{s}$)

- Rate of change of Momentum =

net outflow rate of momentum across surface S + time rate of change of momentum inside $\mathcal{V}$

(Term 1) (term 2)

Momentum (Con't)

~~Drag of a 2-D Body (2.6)~~

Term 1

- time rate of change of momentum (fixed CV):

recall $\dot{m} = \rho \vec{V} \cdot d\vec{S}$ $\left(\frac{M}{T}\right)$ mass-flow rate through area $d\vec{S}$

$\therefore$ flow rate of momentum across $d\vec{S}$ $= \underbrace{\underbrace{(\rho \vec{V} \cdot d\vec{S})}_{\frac{\text{mass}}{\text{unit time}}} \vec{V}}_{\frac{\text{momentum}}{\text{unit time}}}$

- Net out flow rate of momentum through entire surface S $= \boxed{\iint_S \underbrace{(\rho \vec{V} \cdot d\vec{S})}_{\text{positive if outflow}} \vec{V}}$ Term 1

Term 2

Momentum of fluid in elemental volume $d\mathcal{V}$ $= \underbrace{\overbrace{(\rho d\mathcal{V})}^{dm} \vec{V}}$

rearrange

so time rate of change of momentum inside entire CV $\mathcal{V}$ $= \boxed{\frac{\partial}{\partial t} \iiint \rho \vec{V} \, d\mathcal{V}}$ Term 2

- Rate of change of momentum inside CV = Term 1 + Term 2

Bring all terms back together:

$$\frac{d}{dt}(m\vec{V}) = \Sigma \vec{F}$$

$$\Downarrow$$

$$\frac{\partial}{\partial t}\iiint_{\mathcal{V}} \rho \vec{V}\, d\mathcal{V} + \iint_{S} (\rho \vec{V} \cdot d\vec{S})\vec{V} = -\iint_{S} p\, d\vec{S} + \iiint_{\mathcal{V}} \rho \vec{f}\, d\mathcal{V} + \vec{F}_{visc.}$$

time rate of change of momentum inside CV	net outflow rate of momentum through CV's surface S	pressure on surface of CV, S	body forces	viscous force

(2.64)

General Integral form of Momentum Equation

- Follow text (p133-134) to get Differential Form of Mom. Eqn:

$$x: \quad \frac{\partial(\rho u)}{\partial t} + \vec{\nabla}\cdot(\rho u \vec{V}) = -\frac{\partial p}{\partial x} + \rho f_x + F_{x_{visc}}$$

$$y: \quad \frac{\partial(\rho v)}{\partial t} + \vec{\nabla}\cdot(\rho v \vec{V}) = -\frac{\partial p}{\partial y} + \rho f_y + F_{y\,visc} \qquad (2.70)$$

$$z: \quad \frac{\partial(\rho w)}{\partial t} + \vec{\nabla}\cdot(\rho w \vec{V}) = -\frac{\partial p}{\partial z} + \rho f_z + F_{z_{visc}}$$

- Simplified flow case: steady
  inviscid
  no body forces

$$\boxed{\begin{array}{l}
\underline{\text{Momentum Equation}} \text{ becomes:} \\[1em]
\displaystyle \iint_S (\rho \vec{V} \cdot d\vec{S})\, \vec{V} = -\iint_S p\, d\vec{S} \\[1em]
\text{or} \quad \vec{\nabla} \cdot (\rho u \vec{V}) = -\dfrac{\partial p}{\partial x} \\[0.5em]
\qquad\ \ \vec{\nabla} \cdot (\rho v \vec{V}) = -\dfrac{\partial p}{\partial y} \\[0.5em]
\qquad\ \ \vec{\nabla} \cdot (\rho w \vec{V}) = -\dfrac{\partial p}{\partial z}
\end{array}}$$

(2.71)

Steady
Inviscid
No Body Force

(2.72)

- Notation reminder : expansion of terms

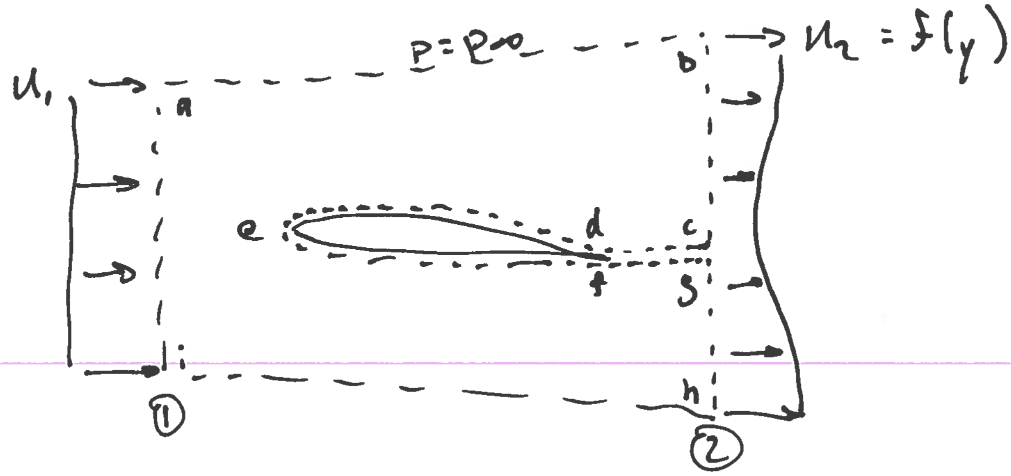
$$\vec{\nabla} \cdot (\rho u \vec{V})$$

$$= \left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k} \right) \cdot \left(\rho u^2 \vec{i} + \rho u v \vec{j} + \rho u w \vec{k} \right)$$

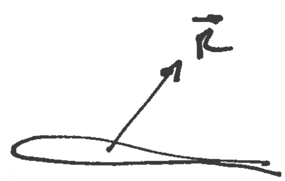
$$= \frac{\partial}{\partial x}(\rho u^2)\vec{i} + \frac{\partial}{\partial y}(\rho u v)\vec{j} + \frac{\partial}{\partial z}(\rho u w)\vec{k}$$

spatial gradients of momentum terms

Drag of a 2-D Body (2.6)

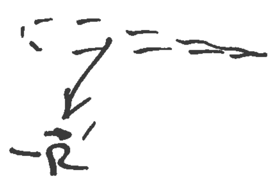


Force on airfoil due to flow



(resultant of P and τ)

Force on CV due to airfoil



$$\Sigma F_{CV} = \text{force due to pressure dist over CV} + \text{force on CV due to airfoil}$$

$$= -\iint_{CV} p\, d\vec{S} - \vec{R}'$$

- Steady flow: Momentum Eqn:

$$\iint_S (\rho \vec{V} \cdot d\vec{S})\vec{V} = -\iint_{CV} p\, d\vec{S} - \vec{R}'$$

• x-component :

$$D' = -\iint_S (\rho \vec{V} \cdot d\vec{S})\, u \; - \iint_{CV} (p\, dS)_x$$

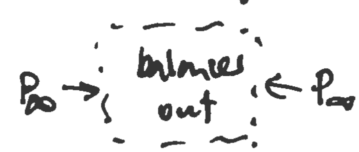
drag per unit span

component of pressure force in x-dir

CV boundaries are far enough out that all static pressure = p_∞ (although dyn pressure changes in wake)
and Total

$$\cancel{\iint_{CV} (p\, dS)_x} \rightarrow 0$$

external press on CV



$$\therefore \quad D' = -\iint_S (\rho \vec{V} \cdot d\vec{S})\, u \quad \text{(x-dir only)}$$

Top and bottom of CV : SL, so $V_n = 0$

$$\therefore \quad D' = \int_i^a \rho_1 u_1^2\, dy - \int_h^b \rho_2 u_2^2\, dy \quad \ldots$$

Drag in Continuity ...

$$\boxed{D' = \int_a^b \rho_2 u_2 \underbrace{(u_1 - u_2)}_{\text{"wake deficit"}} \, dy}$$

$u_1 = \text{const}$

$u_2 = f(y)$

drag = momentum deficit

$\frac{M}{L^3} \; \frac{L}{T} \; \frac{L}{T} \; L$

$\frac{M}{T^2} = \frac{F}{L} \cancel{T^2} \frac{1}{\cancel{T^2}}$

Drag per unit span

Substantial Derivative Forms (A2.9-10)

- Time rate of change, following a moving fluid element:

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

$$= \underbrace{\frac{\partial}{\partial t}}_{\text{local derivative}} + \underbrace{\vec{V}\cdot\vec{\nabla}(\quad)}_{\text{convective derivative}}$$

- Continuity:

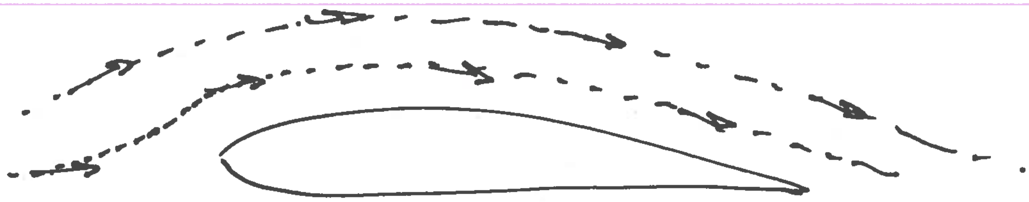
$$\frac{D\rho}{Dt} + \rho\vec{\nabla}\cdot\vec{V} = 0$$

- X-Momentum:

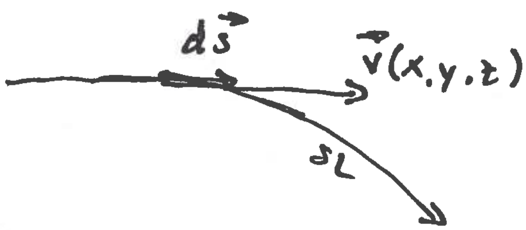
$$\underbrace{\rho\frac{Du}{Dt}}_{\text{rate of chg of momentum}} = \underbrace{-\frac{\partial p}{\partial x}}_{\text{pressure}} + \underbrace{\rho f_x}_{\text{body}} + \underbrace{F_{x_{visc}}}_{\text{viscous shear}}$$

Streamlines (A2.11)

- Defin: a curve whose tangent at every point is parallel to velocity vector at that point.



- Equation:



$d\vec{s}$ locally $\parallel$ to $\vec{V}$

or $d\vec{s} \times \vec{V} = 0$

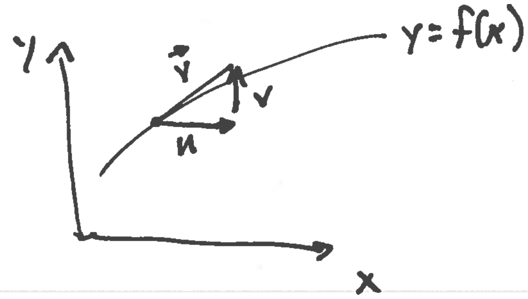
$$(dx\,\vec{i} + dy\,\vec{j} + dz\,\vec{k}) \times (u\,\vec{i} + v\,\vec{j} + w\,\vec{k}) = 0$$

$$\underbrace{(w\,dy - v\,dz)}_{=0}\vec{i} + \underbrace{(u\,dz - w\,dx)}_{=0}\vec{j} + \underbrace{(v\,dx - u\,dy)}_{=0}\vec{k} = 0$$

If we know $\vec{V}(x,y,z)$ then integrate to get SL eqn $f(x,y,z)$

• 2D flow : $v\,dx - u\,dy = 0 \Rightarrow \dfrac{dy}{dx} = \dfrac{v}{u}$

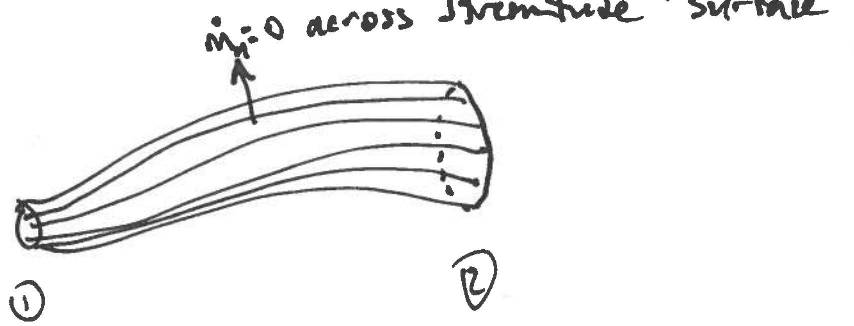
Slope of 2D streamline



• Streamline : mass does not cross SL

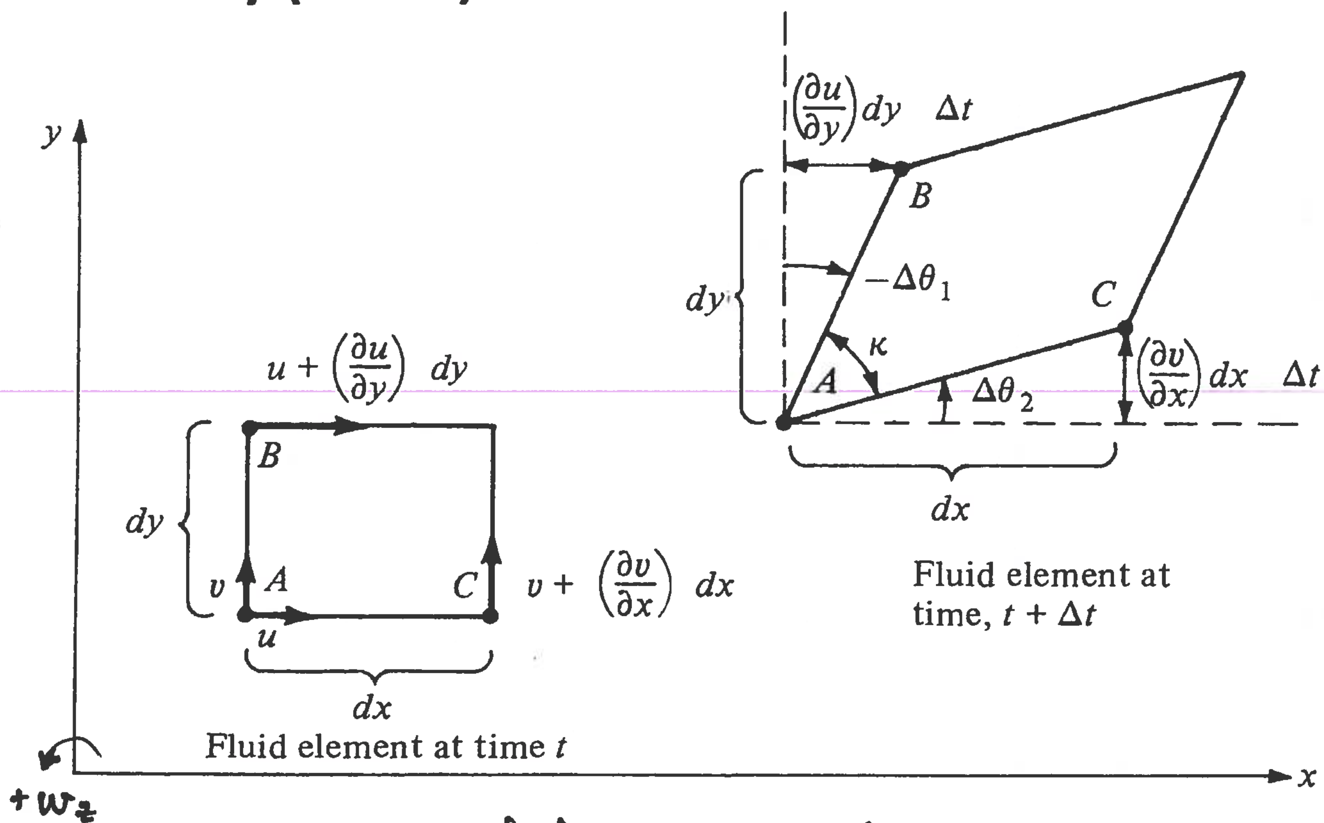
(since velocity always ∥ to it)

• StreamTube : bundle of SL's



$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 \quad \text{(cont)}$$

Vorticity (A2.12)



- Angular Velocity of fluid element

=

Avg angular velocity of AB and AC lines

or, $\omega_z = \frac{1}{2}\left(\frac{d\theta_1}{dt} + \frac{d\theta_2}{dt}\right)$ (+)

find these in terms of velocity field

- From figure:

AC: $\tan\Delta\theta_2 = \dfrac{\left(\frac{\partial v}{\partial x}\right) dx\,\Delta t}{dx} = \frac{\partial v}{\partial x}\Delta t \cong \Delta\theta_2$

$\frac{d\theta_2}{dt} = \lim_{\Delta t \to 0} \frac{\Delta\theta_2}{\Delta t} = \frac{\partial v}{\partial x}$; $\frac{d\theta_1}{dt} = -\frac{\partial u}{\partial y}$

avg rotational velocity → call it ω_z

$$\therefore \quad \omega_z = \frac{1}{2}\left(\frac{d\theta_1}{dt} + \frac{d\theta_2}{dt}\right)$$

$$= \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

- In general,

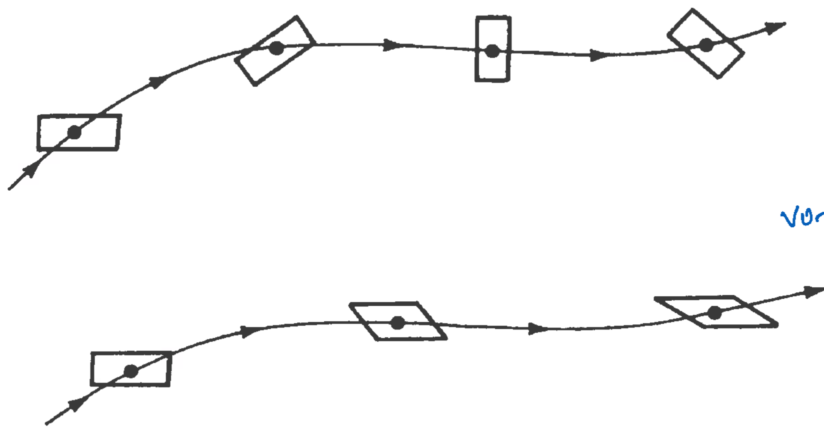
$$\vec{\omega} = \frac{1}{2}\left[\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\vec{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\vec{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}\right]$$

angular velocity of fluid element

x_i

- Or, define Vorticity $\xi = 2\omega$

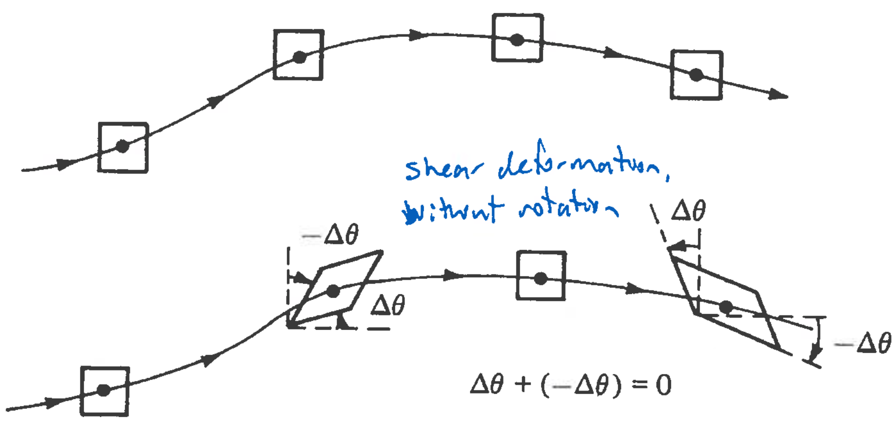
$$\boxed{\text{Vorticity } \vec{\xi} = 2\vec{\omega} = \vec{\nabla} \times \vec{V}}$$



Rotational Flow

vorticity $\vec{\xi} \neq 0$

(viscous flows are rotational!)



Irrotational Flow

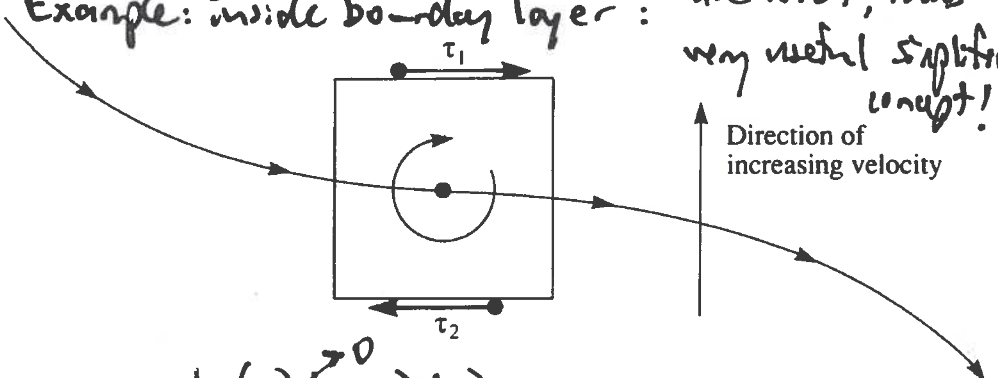
$\vec{\xi} = 0$

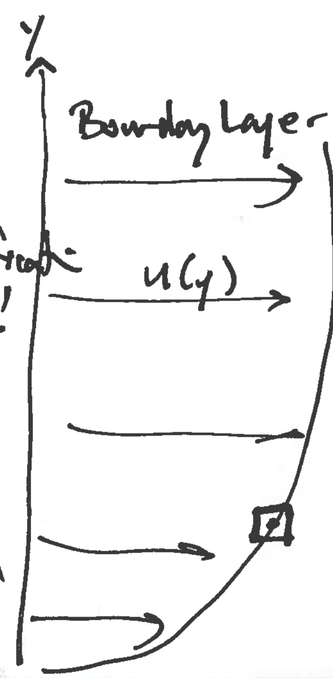
2D: $\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$

$\boxed{\text{Viscous} = \text{Rotational!}}$

Many (not all) inviscid flows are irrot, thus very useful simplification concept!

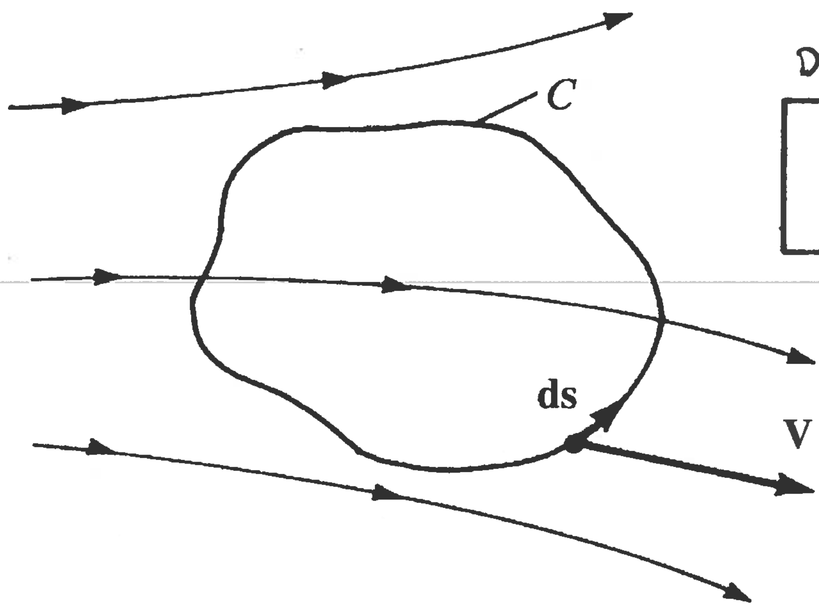
Example: inside boundary layer (or shear):





$$\omega_z = \frac{1}{2}\left(\cancelto{0}{\frac{\partial v}{\partial x}} - \frac{\partial u}{\partial y}\right)$$

Circulation (A2.13) - Fundamental Concept for Computing Lift



Defn of Circulation

$$\boxed{\Gamma = -\oint_C \mathbf{V} \cdot d\mathbf{s}}$$

- No flow rotation implied, term refers to path of integration
- $+\Gamma$ vs. + integral path ∴ neg. sign

- But circulation is related to vorticity:

use Stokes Thm

$$\boxed{\Gamma \equiv -\oint_C \vec{V} \cdot d\vec{s} = -\iint_S \underbrace{(\vec{\nabla} \times \vec{V})}_{\text{vorticity}} \cdot d\vec{s}}$$

Circulation = Surface Integral of Vorticity

- If flow w/in C is irrotational, then $\Gamma = 0$

Stream and Potential Functions (A2.14-16)

Stream Function – 2D Steady Flow Only!

- Defined by its gradient in a flowfield:

$$\boxed{u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x}}$$

incompressible defin

Derived from the definition of a streamline

- ψ = constant along streamlines
- Defined for both rotational + irrot flows
- Defined for 2D flows only

Velocity Potential ϕ

- Irrot. Flow: $\vec{\nabla} \times \vec{V} = 0$

- Vector Identity: if ϕ is a scalar,

  then $\vec{\nabla} \times (\vec{\nabla}\phi) = 0$

  (curl of a gradient of a scalar $= 0$)

- So for irrot. flow we can define a scalar ϕ

  such that $\boxed{\vec{V} = \vec{\nabla}\phi}$ definition of velocity potential function

  ① irrot flows only ($\vec{\omega} = 0$)

  ② defined in 3D (unlike stream function - 2D only)

- $u = \dfrac{\partial\phi}{\partial x}$, $v = \dfrac{\partial\phi}{\partial y}$, $w = \dfrac{\partial\phi}{\partial z}$

- Irrotational Flows = "<u>Potential Flows</u>"

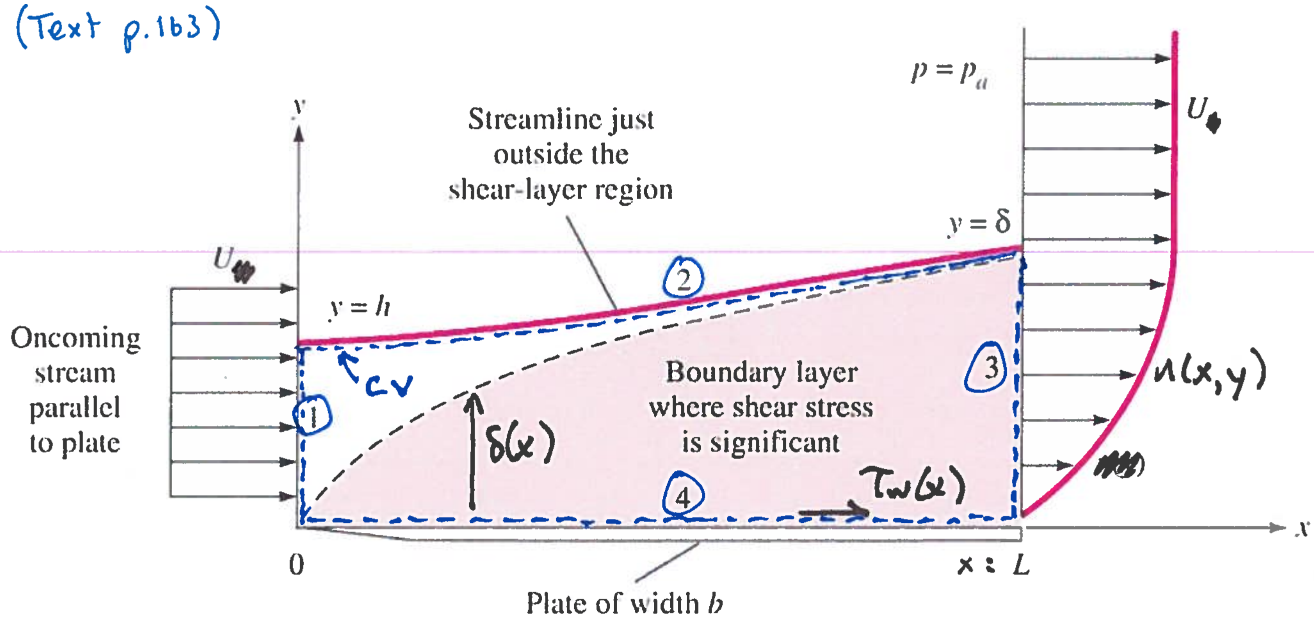
☆ • Much simpler, since solve for one unknown ϕ, instead of 3 (u, v, w)

- Lines of constant ψ and ϕ (where both defined) are $\perp$ (2D irrot flows only)

Text 7.2: Momentum Integral Estimates for Turbulent Boundary Layers

Example 3.11 (Sec 3.4 Linear Momentum Integral Equation)

(Text p.163)



- In Chapter 3, we used the Reynolds Transport Theorem (RTT) to derive a form of the linear momentum principle that can be used for flow-through control volumes (CV's). Let's apply that to a boundary layer (laminar or turbulent) to find the drag force D on a flat plate.
- Carefully choose your CV (this takes practice!):

~~side 1 1-D inlet~~ , → not yet

Text 7.2: Momentum Integral Estimates for Turbulent Boundary Layers

review from 3.4 :

Linear Momentum for fixed CV

- $\Sigma \vec{F} = \frac{d}{dt}(m\vec{V})$

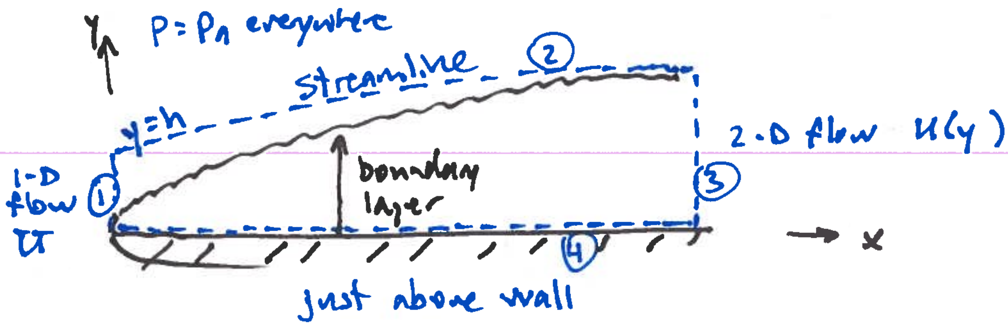
$$\Big\Downarrow \text{RTT}$$

$$\underbrace{\Sigma \vec{F}}_{\substack{\text{vector sum}\\ \text{of all forces}\\ \text{acting on CV}}} = \underbrace{\frac{d}{dt}\left(\int_{CV} \vec{V}\rho \, d\mathcal{V}\right)}_{\substack{\text{rate of change of}\\ \text{momentum within CV}}} + \underbrace{\int_{CS} \vec{V}\rho(\vec{V}\cdot\vec{n})\, dA}_{\substack{\text{rate of momentum}\\ \text{outflow - inflow}\\ \text{through CV}}} \qquad (3.37)$$

= surface forces (P and τ)
+
body forces (gravity)

Text 7.2: Momentum Integral Estimates for Turbulent Boundary Layers

- Choose CV carefully:



Side ① : at start of BL, so velocity is constant (1-D flow)

$$\vec{V} \cdot \vec{n} = -U$$

② : along SL, so no mass flows through it (by defn of SL)

$$\vec{V} \cdot \vec{n} = 0 \quad \text{(smart choice!)}$$

③ : cuts through BL, so velocity depends on y (2D flow)

$$\vec{V} \cdot \vec{n} = u(y)$$

④ : along wall, just above; shear force acts on wall

$$\vec{V} \cdot \vec{n} = 0 \quad \text{(why?)}$$

☆ Note that wall applies equivalent drag force to CV of $-D\,\vec{i}$ (watch those signs!)

Text 7.2: Momentum Integral Estimates for Turbulent Boundary Layers

Boundary Layer

- Apply x-momentum equation to our BL Control Volume:

$$\Sigma F_x = \cancel{\frac{d}{dt}\left(\int_{CV} u \rho \, dV\right)}^{\,0 \text{ steady}} + \int_{CS} u \, \overbrace{\rho (\vec{V} \cdot \vec{n}) \, dA}^{\dot{m}}$$

$$-D = \rho \int_1 u(0,y)(\vec{V}\cdot\vec{n})\, dA + \rho \int_3 u(L,y)(\vec{V}\cdot\vec{n})\, dA$$

$$= \rho \int_0^h U(-U)\, b\, dy + \rho \int_0^\delta u(L,y)[u(L,y)]\, b\, dy$$

$$\underbrace{\boxed{D = \rho U^2 b h - \rho b \int_0^\delta u^2 \, dy \Big|_{x=L}}}_{\text{rate of change in x-momentum flux through CV inlet and outlet}}$$

Drag on entire plate from $x = 0$ to $x = L$

but wait, we don't know h!

Next tool in the kit is Continuity →

Text 7.2: Momentum Integral Estimates for Turbulent Boundary Layers

- Conservation of Mass :

Note that the CV is a streamtube → flow only through ends

$$\rho \int_{CS} (\vec{V} \cdot \vec{n})\, dA = 0 \quad \text{((Cons. Mass))}$$

$$= \rho \int_0^h (-U)\, b\, dy + \rho \int_0^\delta u\, b\, dy \Big|_{x=L}$$

$$\therefore \; U h = \int_0^\delta u\, dy \Big|_{x=L}$$

- Plug this into drag equation :

(7.2)

Drag of plate for span $x = 0 \to L$

$$\boxed{D = \rho b \int_0^{\delta(x)} u (U - u)\, dy \Big|_{x=L}}$$

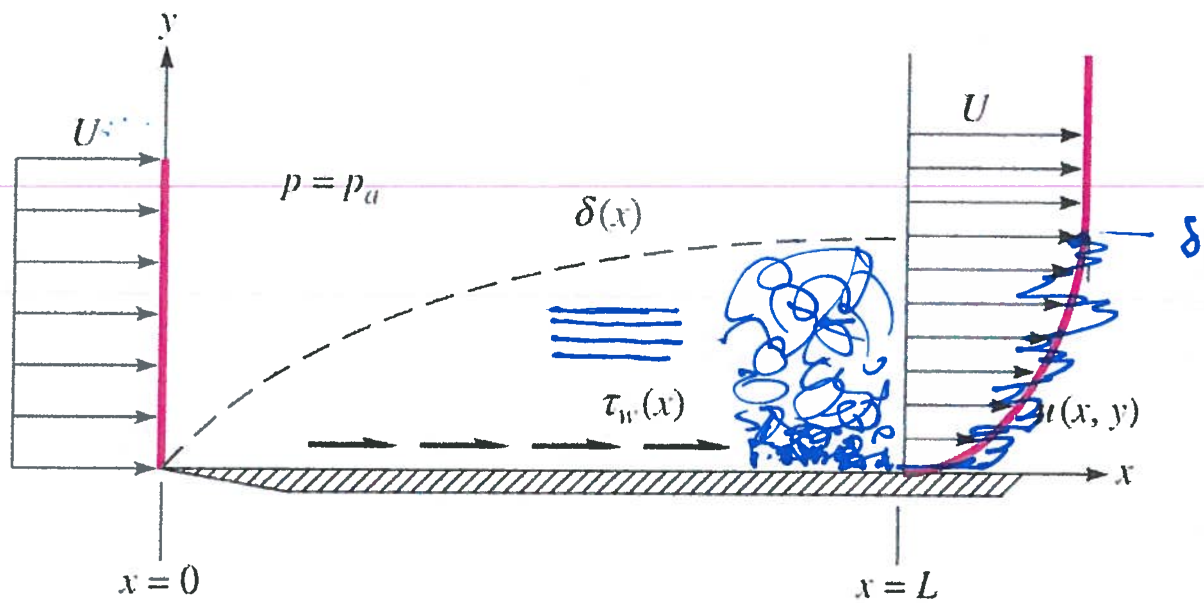
note only need info at end of plate !

where $\rho u (U - u)$ = "momentum deficit" in BL

Von Karman

Text 7.2: Momentum Integral Estimates for Turbulent Boundary Layers

TBL "Momentum Thickness": θ



- We just found that drag on the plate is

$$D(x) = \rho b \int_0^{\delta(x)} u(U-u)\,dy \qquad \left(\begin{array}{l}\text{Total} \\ \text{Drag for plate} \\ \text{between 0 and } x\end{array}\right)$$

or, $$= \rho b U^2 \theta$$

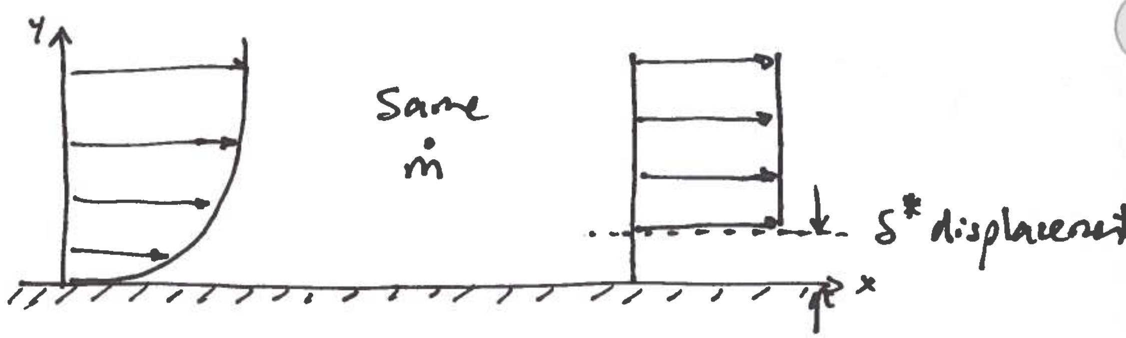
where $$\boxed{\theta \equiv \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \quad \text{Momentum Thickness}}$$ 7.3

∴ Momentum Thickness relates shape of the velocity profile $u(y)$ in a boundary layer to its drag. (Total Plate Drag)

Review of Displacement Thickness δ^*

(using flat-plate boundary layer)

- Idea is to estimate "displaced" boundary that would give same mass-flow rate past body that a frictionless flow would.



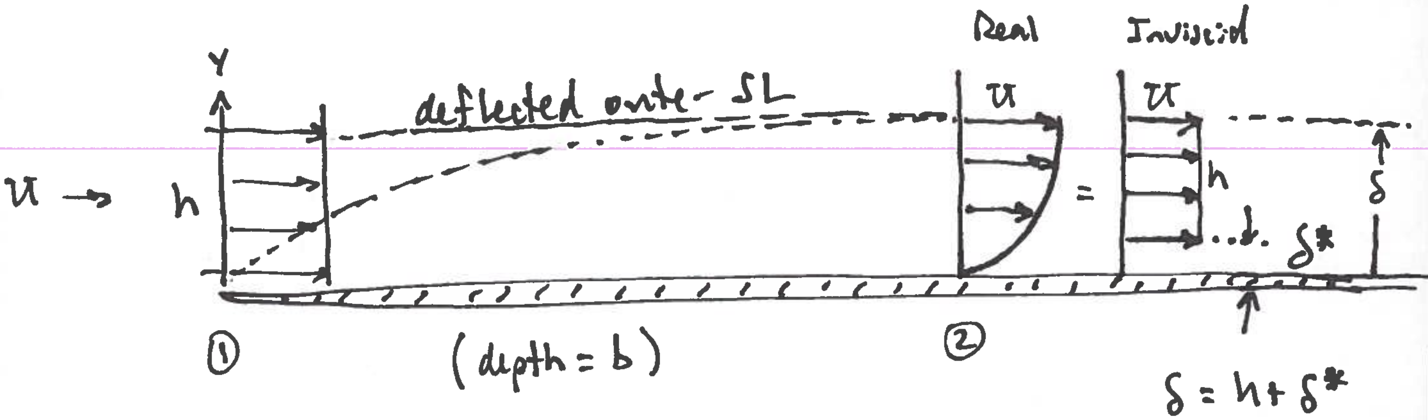
actual viscous boundary layer

hypothetical frictionless flow, displaced by δ^*

- How do we derive an expression for this δ^* ?

Use continuity, since considering $\dot{m}$

- A boundary layer deflects outer streamline (@ BL edge) outward by distance δ^*:



- Continuity between ① and ②: $\dot{m}_1 = \dot{m}_2$

$$\int_0^h \rho U b \, dy = \int_0^\delta \rho u b \, dy$$

const @ inlet

$$U h = \int_0^\delta u \, dy = \int_0^\delta \left[U + (u - U)\right] dy$$

$$= U\delta + \int_0^\delta (u - U)\, dy$$

$$= U(h + \delta^*) + \int_0^\delta (u - U)\, dy$$

- Solve for δ^*:

$$\boxed{\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy}$$

displacement of outer SL due to presence of BL

Note nice integration characteristic: $1 - \frac{u}{U} \to 0$ as $y \to \delta$ unlike δ_{99}

Predicting Flow Patterns, Velocity, and Pressure over a Body - Solving the Equations of Fluid Motion (A2.~~12~~ 17)

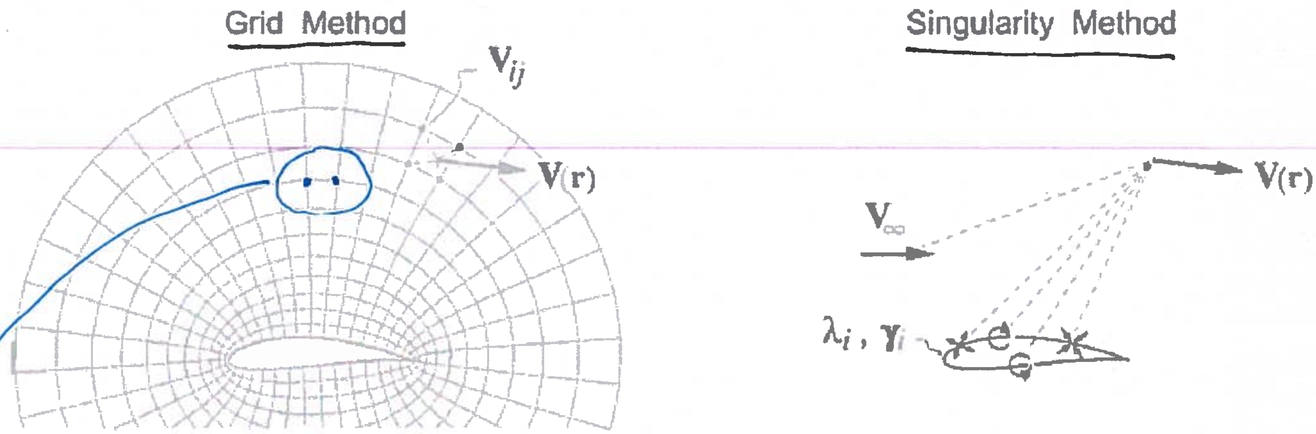
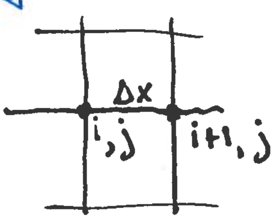


Figure 2.1: Grid and singularity methods used to represent a velocity vector field V(r).

- Grid Method: governing diff eqns are discretized to solve for velocity field at each grid node



Taylor Series Expansion: example velocity gradient:

$$u_{i+1,j} = u_{i,j} + \left(\frac{\partial u}{\partial x}\right)_{i,j} + \text{H.O.T.'s}$$

$$\left(\frac{\partial u}{\partial x}\right)_{i,j} = \frac{u_{i+1,j} - u_{i,j}}{\Delta x} - \text{truncation error} \left(\begin{matrix}\text{Fwd}\\ \text{Diff}\end{matrix}\right)$$

- Singularity Method: Point flow-inducers (source, sink, vortex) distributed on bounding surface. Strengths + BC manipulated to approximate natural flow patterns

Iterative Approach to Solving Potential Flows over Bodies with Thin Viscous Layers:

Potential (inviscid, Irrotational) Flow

↓

Distribute flow-inducing point singularities (source, sink, vortex) on boundaries; adjust singularity strengths to match BC's

↓

Superpose solutions to linear potential flow (Laplace) equations to generate potential flowfield over body

↓

Use boundary layer analysis in potential flowfield to estimate displacement thickness over body

↓

Add displacement thickness distribution to original body shape to get new shape – start over with potential flow analysis

↓

Converged potential flowfield with viscous effects accounted for