How do we use statistics

- We use statistics to confirm effects, estimate parameters, and predict outcomes
- It usually rains when I'm in Cape Town, but mostly on Sunday
  - *Confirmation:* In Cape Town, it rains more on Sundays than other days
  - *Estimation:* In Cape Town, the *odds* of rain on Sunday are 1.6–2.2 times higher than on other days
  - *Prediction:* I am confident that it will rain at least one Sunday the next time I go

Raining in Cape Town

- How we interpret data like this necessarily depends on assumptions:
  - Is it likely our observations occured by chance?
  - Is it likely they *didn't*?

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Vitamin A

- We measure the average heights of children raised with and without vitamin A supplements
  - *Estimate:* how much taller (or shorter) are the treated children on average?
  - *Confirmation:* are we sure that the supplements are helping (or hurting)?
  - *Range of estimates:* how much do we think the supplement is helping?

Outline

Estimation

- We use *P values* to say how sure we are that we have seen some effect
- We use *confidence intervals* to say what we think is going on (with a certain level of confidence)
- P values are *over-rated*
- *Never* use a high P value as evidence for anything, e.g.:
  - that an effect is small
  - that two quantities are similar

Vitamin A example

- We want to know if vitamin A supplements improve the health of village children
  - Is height is a good measure of general health?
  - How will we know height differences are due to our treatment?
    - We want the two groups to start from the same point – independent randomization of each individual
    - We may measure *changes* in height
    - Or *control for* other factors

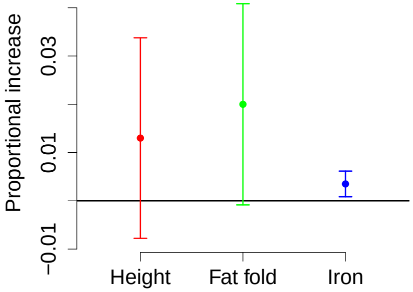
What do we hope to learn?

- Is vitamin A good for these children?
- How sure are we?
- How good do we think it is?
- How sure are we about that?

P values

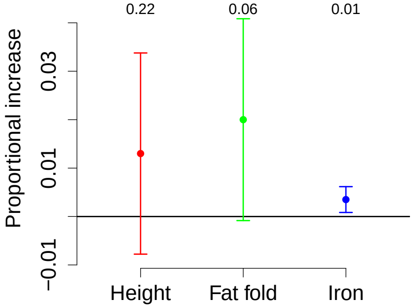
- What does it mean if I find a "significant P value" for some effect in this experiment?
- The difference is unlikely to be due to chance
  - So what! I already know vitamin A has strong effects on metabolism
- If I'm certain that the true answer isn't exactly zero, why do I want the P value anyway?

Confidence intervals



- What do these results mean?
- Which are significant?

Confidence intervals and P values



- A high P value means we can't see the sign of the effect clearly
- A low P value means we can

The meaning of P values

- More broadly, a P value measures whether we are seeing *something* clearly
  - It's usually the sign ($\pm$) of some quantity, but doesn't need to be

Types of Error

- Type I (*False positive:*) concluding there is an effect when there isn't one
  - This doesn't happen in biology. There is always an effect.
- Type II (*False negative:*) concluding there is no effect when there really is
  - This *should* never happen, because we should never conclude there is no effect

Experimental design

- Type I (*False positive:*) in the hypothetical case that the effect is exactly zero, what is the probability of falsely finding an effect
  - Should be less than or equal to my significance value
- Type II (*False negative:*) what is the probability of failing to find an effect that is there?
  - Useful, but can only be asked for a specific hypothetical effect *size*
- These are useful to analyze **power** and **validity** of a statistical design
  - You should do these analyses *before* you collect data, not after

A new view of error

- *Sign error:* if I think an effect is positive, when it's really negative (or vice versa)
- *Magnitude error:* if I think an effect is small, when it's really large (or vice versa)
- Confidence intervals clarify all of this

Low P values

- If I have a low P value I can see something clearly
- But it’s usually better to focus on what I see than the P value

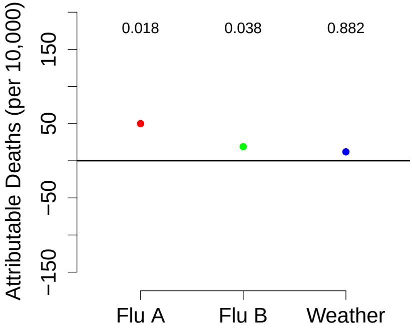
High P values

- If I have a high P value, there is something I *don't* see clearly
- It *may be* because this effect is small
- High P values should *not* be used to advance your conclusion

What causes high P values?

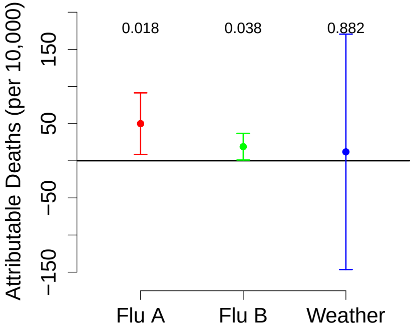
- Small differences
- Less data
- More noise
- An inappropriate model
- Less model resolution
- A lower P value means that your evidence for difference is better
- A higher P value means that your evidence for similarity is better – or worse!

Annualized flu deaths



- Why is weather not causing deaths at this time scale?

... with confidence intervals



- **Never** say: A is significant and B isn't, so $A > B$
- **Instead:** Construct a statistic for the hypothesis $A > B$
  - May be difficult

Syllogisms

- All men are mortal
- Jacob Zuma is mortal
- Therefore, Jacob Zuma is a man

Syllogisms

- All men are mortal
- Fanny the elephant is mortal
- Therefore, Fanny is a man

Bad logic

- A lot of statistical practice works this way:
  - bad logic in service of conclusions that are (usually) correct
- This sort of statistical practice leads in the aggregate to bad science
- The logic can be fixed:
  - Estimate a difference, or an interaction

Small effects

- We can’t build statistical confidence that something is small by failing to see it clearly
- We must instead see clearly that it is small
- This means we need a standard for what we mean by small

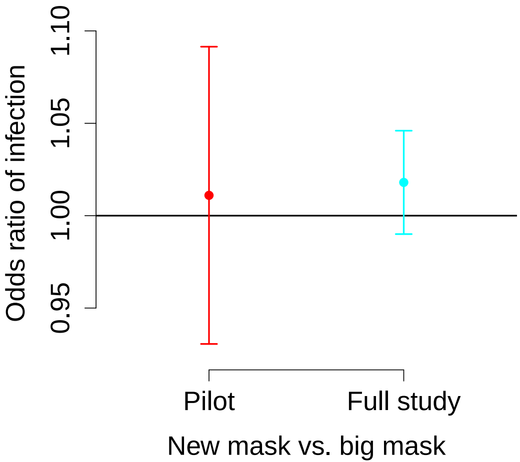
Flu masks


WARNING
This respirator helps protect
against certain particles.
Misuse may result in sickness
or death. For proper use, see
supervisor or box or call 3M
1-800-247-3941. NIOSH
3M 8210
N95

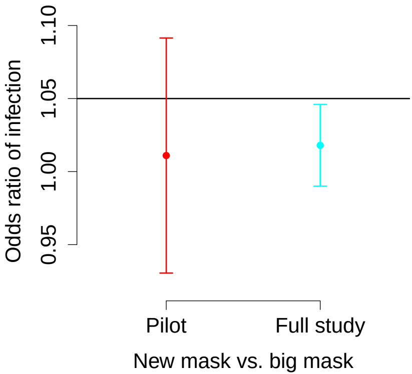
Flu mask example

- People who work in respiratory clinics sometimes have to wear bulky, uncomfortable, expensive masks
- They would like to switch to simpler masks, if those will do the job
- How can this be tested statistically? We don't want the masks to be "different".
  - Use a confidence interval
  - Decide how big a level is acceptable, and construct a P value for the hypothesis that this level is excluded!

Study results

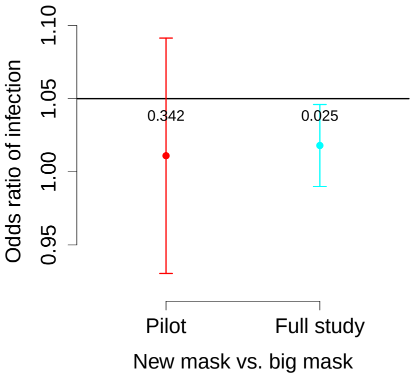


Non-inferiority trial



- Is the new mask "good enough"?
- What's our standard for that?

Non-inferiority trial



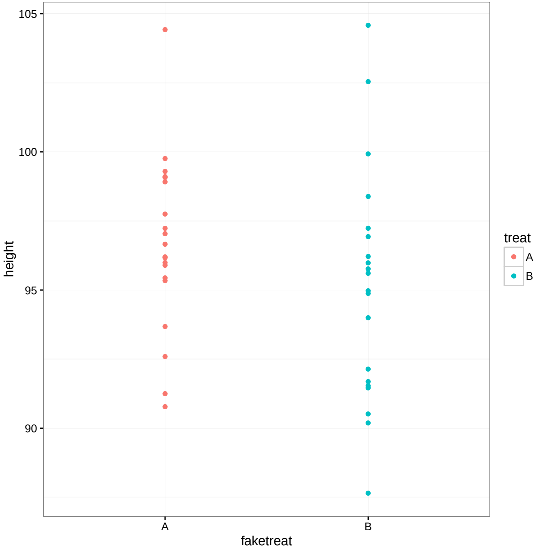
- We can even attach a P value by basing it on the “right" statistic.
- The right statistic is the thing whose sign we want to know:
  - The difference between the observed effect and the standard we chose

Outline

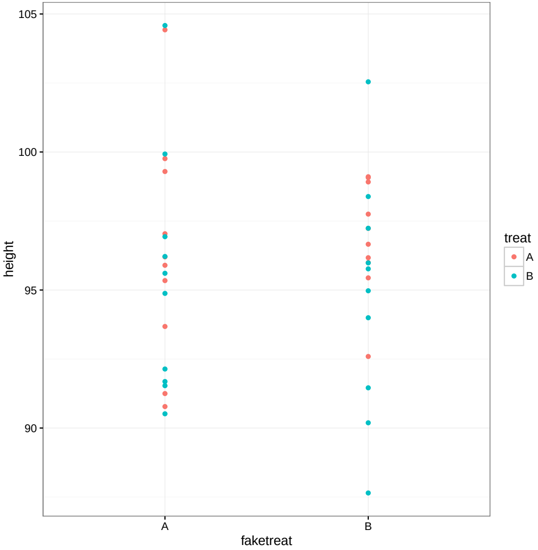
Frequentist paradigm

- Make a null model
- Test whether the effect you see could be due to chance
  - What is the probability of seeing exactly a 1.52 cm difference in average heights?
- Test whether the effect you see *or a larger effect* could be due to chance
  - This probability is the P value

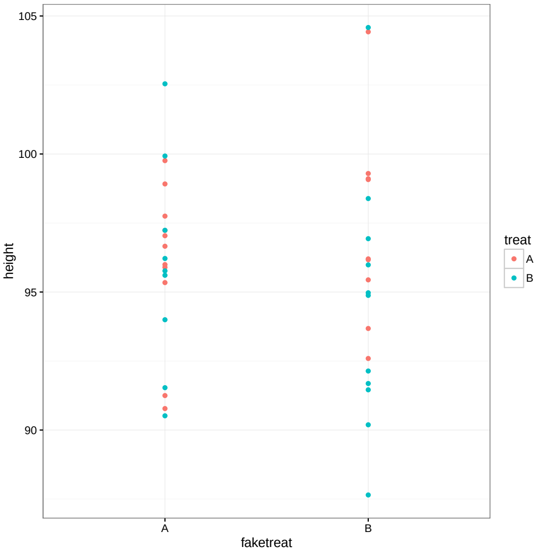
Height measurements



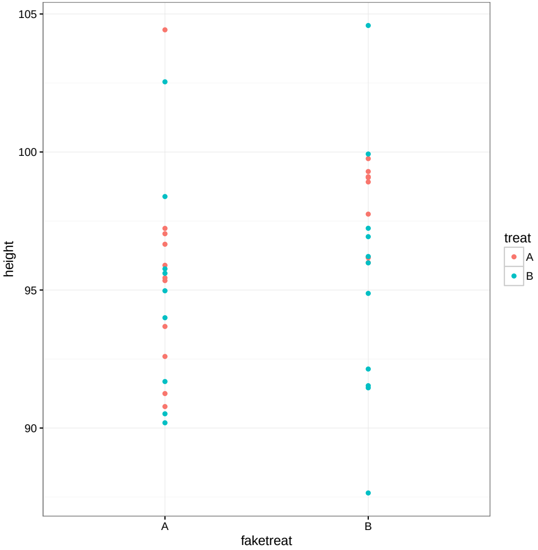
Scrambled measurements



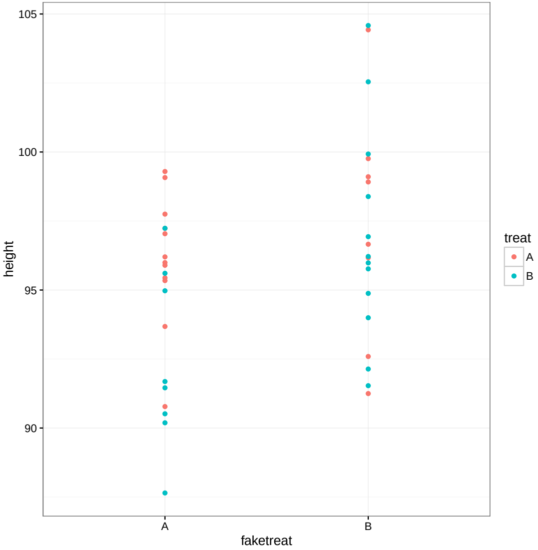
Scrambled measurements



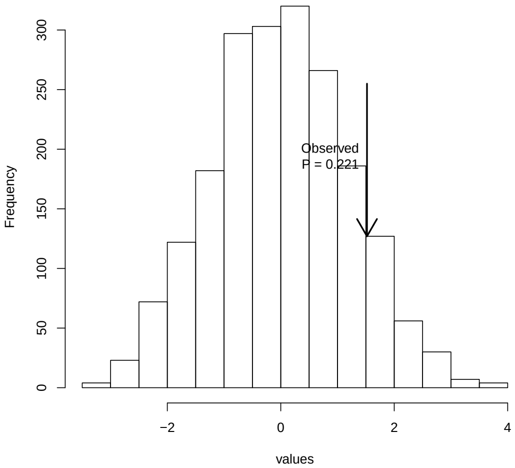
Scrambled measurements



Scrambled measurements



The null distribution



Bayesian paradigm

- Make a complete model world
- Use conditional probability to calculate the probability you want

A powerful framework

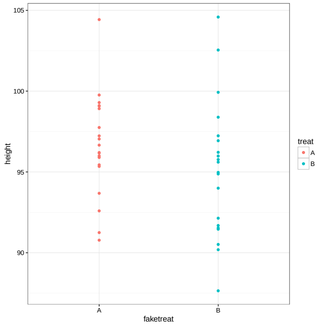
- More assumptions $\implies$ more power
- With great power comes great responsibility

Bayesian inference

- We want to go from a *statistical model* of how our data are generated, to a probability model of parameter values
  - Requires *prior* distributions describing the assumed likelihood of parameters before these observations are made
  - Use Bayes theorem to calculate posterior distribution – likelihood after taking data into account

Vitamin A study

- A frequentist can do a clear analysis right away
- A Bayesian needs a ton of assumptions – will try to make “uninformative” assumptions



Cape Town weather

- Frequentist: how unlikely is the observation, from a random perspective?
- Bayesian: what's my model world? What is my prior belief about weather-weekday interactions.

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Outline

Your philosophy

- Statistics are not a magic machine that gives you the right answer
- If you are to be a serious scientist in a noisy world, you should have your own philosophy of statistics
  - Be pragmatic: your goal is to do science, not get caught by theoretical considerations
  - Be honest: it's harder than it sounds.

Honesty

- You can always keep analyzing until you find a “significant” result
  - If you do this you will make a lot of mistakes
- You may also keep analyzing until you find a result that you already “know” is true.
  - This is confirmation bias; you’re probably right, but your project is not advancing science
- Good practice
  - Keep a data-analysis journal
  - Start *before* you look at the data