- First, is the problem convex? No. $\nabla_x^2 f(x) = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \Rightarrow$ eigvals $= -1, 2$, since $-1 \ngeq 0 \Rightarrow$ not all positive eigvals $\Rightarrow \nabla_x^2 f(x)$ not PSD $\Rightarrow f(x)$ objective is not convex $\Rightarrow$ this is not a convex program.

- Let's see if we can solve the problem without using the FJ conditions. This will help us know if candidates are good, correct, lend intuition.

- Intuition, to minimize $2x_1^2 - x_2^2$ we should make x_1^2 as small as possible (occurs when $x_1 = 0$) and make $|x_2|$ as large as possible. Thus we'll have $0 -$ (some big number).

- If $x_1 = 0 \Rightarrow h(x) = x_1^2 x_2 - x_2^3 = 0 \rightarrow 0 x_2 - x_2^3 = 0 \rightarrow x_2^3 = 0 \Rightarrow x_2 = 0$.

Is $(x_1 = x_2 = 0)$ the best we can do? (obj value $= 0$)

- From $h(x)$ we see $(x_1^2)(\cancel{x_2}) = (\cancel{x_2})(x_2^2)$. $\Rightarrow \pm x_1 = \pm x_2$. But since both x_1 and x_2 are squared in $f(x)$, let's just pick $[x_1 = x_2] > 0$ for ease.

Say $x_1 = x_2 = 2 \longrightarrow$ obj: $f(x) = 2(2^2) - (2^2) = 8 - 4 = 4 \nleq 0$ (best obj. so far).

- Intuitively, since x_1^2 must $= x_2^2$, in the objective function the first term will always be double the second term. Since the terms are always non-negative, the best objective value is when the $x_1^2 = x_2^2 = 0$, which occurs when $x_1 = x_2 = 0$.

- Thus, our best objective value is $f(x) = 0$ at $[x_1 = 0, x_2 = 0]$. We didn't arrive here through rigorous proofs, but rather through intuition, logical deduction, and some simple math. (Thus, can't declare global optimality.) Let's see if use of the FJ conditions substantiates this finding.

- $L(x) = \lambda_0 (2x_1^2 - x_2^2) + \lambda_1 (x_1^2 x_2 - x_2^3)$

  $\partial L / \partial x_1 = 4\lambda_0 x_1 + 2\lambda_1 x_1 \overset{set}{=} 0 \rightarrow x_1 (2\lambda_0 + \lambda_1) = 0 : \boxed{eq\,1}$

  $\partial L / \partial x_2 = -2\lambda_0 x_2 + \lambda_1 x_1^2 - 3\lambda_1 x_2^2 \overset{set}{=} 0 : \boxed{eq\,2}$

- Complimentarity:

$\boxed{c1}$: $\lambda_1 (x_1^2 x_2 - x_2^3) = 0 \Rightarrow$ either $\lambda_1 = 0$ or $(x_1^2 x_2 - x_2^3 = 0)$

- CASE 1: $\lambda_0 = 0, \lambda_1 \neq 0$

- $\boxed{Eq1}$ becomes: $x_1(\lambda_1) = 0 \Rightarrow x_1 = 0$ since we specified $\lambda_1 \neq 0$ in case.
- From $\boxed{C1}$, $\lambda_1 \neq 0 \Rightarrow x_1^2 x_2 - x_2^3 = 0$ with $x_1 = 0 \rightarrow x_2^3 = 0 \Rightarrow x_2 = 0$
- From $\boxed{eq2}$: $-0 + \lambda_1 (0^2) - 3\lambda_1 (0^2) = 0 \rightarrow 0 + 0 - 0 = 0$ holds $\forall \lambda_1 \in \mathbb{R}$.

> - Thus, the candidate $[x_1 = 0, x_2 = 0]$ with $\lambda_0 = 0$ and $\lambda_1 > 0$ (to avoid violation of non-triviality condition) is viable per the optimality conditions. From our logic at the beginning of the problem, we know this is the best solution we can achieve. Let's be thorough and check the remaining cases.

- CASE 2: $\lambda_0 = 1, \lambda_1 = 0$

- $\boxed{c1}$ satisfied since $\lambda_1 = 0$. $\boxed{Eq1}$ becomes $x_1(2\lambda_0) = 0 \rightarrow x_1(2) = 0 \Rightarrow x_1 = 0$
- $\boxed{Eq2}$ becomes $-2\lambda_0 x_2 + 0 - 0 = 0 \rightarrow -2x_2 = 0 \Rightarrow x_2 = 0$

> - Once again, $(x_1 = 0, x_2 = 0)$ is a viable candidate, this time with $\lambda_0 = 1$ and $\lambda_1 = 0$.

- CASE 3: $\lambda_0 = 1$, $\lambda_1 > 0$

- To satisfy $\boxed{c1}$ we have $x_1^2 x_2 - x_2^3 = 0 \longrightarrow (x_1^2)(\cancel{x_2}) = (\cancel{x_2})(x_2^2) \Rightarrow x_1^2 = x_2^2$

$\Rightarrow$ either $x_1 = x_2$ or $-x_1 = x_2$ (equivalent to $x_1 = -x_2$).

- When $x_1 = x_2$: $\boxed{eq1}$ is now $x_1(2+\lambda_1) = 0 \Rightarrow x_1$ must $= 0 \Rightarrow x_2 = 0$
  - Does $\boxed{eq2}$ hold? $-2x_2 + \lambda_1 x_2^2 - 3\lambda_1 x_2^2 = 0 \longrightarrow -2x_2 - 2\lambda_1 x_2^2 = 0$

  $\rightarrow -2(x_2 - \lambda_1 x_2^2) = 0 \longrightarrow x_2(1 - \lambda_1 x_2) = 0$, which works when $x_2 = 0$, which we're explored. So let's explore $1 - \lambda_1 x_2 = 0 \rightarrow x_2 = 1/\lambda_1$ and $\lambda_1 = 1/x_2$

  - In this case: objective $= 2\left(\frac{1}{\lambda_1}\right)^2 - \left(\frac{1}{\lambda_1}\right)^2 = \frac{1}{\lambda_1}$. Since $\lambda_1 > 0$ we can take $\lim_{\lambda_1 \to \infty} \frac{1}{\lambda_1} = 0$.

  - Thus, the candidate $(x_1 = 1/\lambda_1,\ x_2 = 1/\lambda_1)$, with $\lambda_0 = 1$ and $\lambda_1 \to \infty$ is a viable candidate, but never quite achieves $f(x) = 0$, just $f(x) \to 0$ as $\lambda_1 \to \infty$.

- When $-x_1 = x_2$: $\boxed{eq2}$ becomes $-2x_2 + \lambda_1(-x_1)^2 - 3\lambda_1(x_2^2) = 0$

$\rightarrow -2x_2 + \lambda_1 x_2^2 - 3\lambda_1(x_2^2) = 0 \longrightarrow -2x_2 - 2\lambda_1 x_2^2$, which we run into above.

Thus, there are no new λ_1 values to explore.

- Conclusion: we have explored all possible non-trivial cases, and found candidates $(x_1 = 0, x_2 = 0)$ and $(x_1 = x_2 = 1/\lambda_1$ as $\lambda_1 \to \infty)$. The best objective value from these is $f(x) = 0$. Since this isn't a CP, we can't guarantee $(0,0)$ to be a/the global optimizer. However, we know it's a viable candidate according to optimality conditions (both when $\lambda_0 = 0, \lambda_1 > 0$ and when $\lambda_0 = 1$ and $\lambda_1 = 0$). Also, based on our reasoning at the beginning of the problem, it is very likely a global optimizer (though rigorous proof would need to hold to guarantee this formally.)