- First, is the problem convex? No. $\nabla_x f = \begin{bmatrix} x_2 \\ x_1 \end{bmatrix} \rightarrow \nabla_x^2 f = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ which has eigvals $= -1, 1$ which aren't both $\geq 0 \Rightarrow \nabla_x^2 f$ not PSD $\rightarrow f$ not convex $\Rightarrow$ this is not a convex program.
- Let's build some additional intuition before jumping in to the math. In general, we'll minimize $x_1 x_2$ if one of $\{x_1, x_2\} > 0$ and the other of $\{x_1, x_2\} < 0$, with $|x_1|$ or $|x_2|$ a very large value, e.g. a positive times a negative is a negative, and the larger in magnitude the values x_1 & x_2 the larger in (absolute) magnitude their product, which will be negative. We know $x_2 \geq x_1 \Rightarrow x_2$ will be the positive and x_1 will be the negative. Then we just need $x_1 + x_2 \geq 2 \Rightarrow |x_2| \geq |x_1| + 2$.
- For example: Let $x_2 = 12$ and $x_1 = -10$. This satisfies the constraints and gives $f(x) = (-10)(12) = -120$, a pretty low (minimal) objective!
- But what if $x_2 = 102$ and $x_1 = -100$. This also satisfies the constraints but gives $f(x) = (-100)(102) = -10{,}200$ an even lower (more min.) objective.

- We can see this trend can continue interminably. Therefore we know a min of $f(x) = x_1 x_2$ DNE (it continually approaches $-\infty$ as x_1 and x_2 grow in magnitude.)

- Now let's use more formal math to substantiate our intuition and line of reasoning that min $f(x) = x_1 x_2$ DNE.
- $L(x) = \lambda_0 x_1 x_2 + \lambda_1(-x_1 - x_2 + 2) + \lambda_2(x_1 - x_2)$

$$\partial L / \partial x_1 = \lambda_0 x_2 - \lambda_1 + \lambda_2 \stackrel{\text{set}}{=} 0 \longleftarrow \boxed{\text{eq } 1}$$

$$\partial L / \partial x_2 = \lambda_0 x_1 - \lambda_1 - \lambda_2 \stackrel{\text{set}}{=} 0 \longleftarrow \boxed{\text{eq } 2}$$

- Complimentarity: $\lambda_1(-x_1-x_2+2)=0 \leftarrow \boxed{\text{comp 1}}$, $\boxed{\text{comp2}} \rightarrow \lambda_2(x_1-x_2)=0$

- Case 1: $\lambda_0=0$
  - Eq1 becomes: $-\lambda_1+\lambda_2=0 \Rightarrow \lambda_1=\lambda_2$, Eq2 is now: $-\lambda_1=\lambda_2$. For both to hold, we must have $\lambda_1=\lambda_2=0$, BUT $\lambda_0=0$ and $\lambda^T=[\lambda_1=0, \lambda_2=0]$ violates the non-triviality condition. Thus, λ_0 must $=1$ for any potential solutions.

- Case 2: $\lambda_0=1$, $\lambda_1=0$, $\lambda_2=0$
  - Eq1 becomes: $x_2=0$, Eq2 becomes $x_1=0$. Is $(0,0)$ feasible? No, it's not. Consider $x_1+x_2\geq 2 \rightarrow 0+0=0 \not\geq 2 \Rightarrow$ this case isn't feasible.

- Case 3: $\lambda_0=1$, $\lambda_1=0$, $(x_1-x_2=0)$
  - If $x_1-x_2=0 \Rightarrow x_1=x_2$. Eq1 becomes: $x_2+\lambda_2=0$, Eq2 becomes: $x_1-\lambda_2=0$.
  - Now we have $x_2=-\lambda_2$ and $x_2=\lambda_2$ which only works if $\lambda_2=0$ which then makes $x_2=0 \Rightarrow x_1=0$ and we just showed $(0,0)$ to be infeasible in Case 2. Thus this case, case 3, is also infeasible.

- Case 4: $\lambda_0=1$, $\lambda_2=0$, $(-x_1-x_2+2=0)$
  - Eq1 becomes: $x_2-\lambda_1=0$, Eq2 becomes: $x_1-\lambda_1=0 \Rightarrow x_1=x_2=\lambda_1$
  - Rewriting $(x_1+x_2=2) \rightarrow x_1+x_1=2 \rightarrow x_1=1 \Rightarrow x_2=1 \Rightarrow \lambda_1=1$.
  - The candidate $x=[x_1=1, x_2=1]$ is feasible and satisfies optimality conditions with $\lambda_0=1$, $\lambda^T=[\lambda_1=1, \lambda_2=0]$. But we've already shown the pt $(-100, 102)$ has a much more minimal objective value, thus $(1,1)$ is not a minimizer.

- Case 5: $\lambda_0=1$, $(x_1-x_2=0)$, $(-x_1-x_2+2=0)$
  - $x_1-x_2=0 \Rightarrow x_1=x_2 \rightarrow$ into other eqn: $-x_1-x_1=-2 \Rightarrow x_1=1$, $x_2=1$, same as case 4.

- Thus, $\nexists$ a minimizer of $f(x)=x_1x_2$ by using FJ optimality conditions, and our exploration at the beginning is upheld: $\nexists$ a minimizer for this problem.