MAE 5930 Optimization
Homework 3

Purpose: The problems assigned help develop your ability to • formulate linear programs (LPs). • solve LPs analytically using the necessary and sufficient optimality conditions. • solve LPs numerically using MATLAB's `linprog`. NOTE: You are welcome to program in MATLAB or Python.

Problem 1: You are the project manager for a house building company. Building a house requires the following steps (and completion time):

- B: Building the foundation (takes 3 weeks)
- F: Framing (2 weeks)
- E: Electrical (3 weeks)
- P: Plumbing (4 weeks)
- D: Dry Wall (1 weeks)
- L: Landscaping (2 weeks)

Some tasks can start only after another task is complete. The following sequencing rules must be followed: F after B, L after B, E after F, P after F, D after E, D after P. You are allowed to do tasks in parallel. Your job as project manager is to build the schedule to complete the project as quickly as possible.

(a) Formulate this as a linear programming problem by defining the variables, objective, and constraints.

(b) Solve the linear program using MATLAB's `linprog`.

(c) Comment on any observations you have.

(We discussed this problem in class in the "Introduction to Linear Programming" lecture.)

Problem 2. You are a facilities engineer at a manufacturing plant. A byproduct of the manufacturing processes is a hazardous chemical. Only 1,000 liters (L) of this chemical are allowed on site at any given time, and none can be stored over night.

Fortunately, your facility is connected by pipeline to a reprocessing plant, which can take the byproduct at a cost. The table below shows your production and the cost of piping to the reprocessing plant on an hourly basis.

	9-10AM	10-11AM	11-12PM	12-1PM	1-2PM	2-3PM
Production (L)	300	240	600	200	300	900
Cost ($/L)	30	40	35	45	38	50

The work days is from 9AM to 3PM. As facility engineer, your job is to determine how much chemical byproduct should be sent each hour to the reprocessing plant to minimize cost and meet environmental regulations.

(a) Formulate this as a linear programming problem by defining the variables, objective, and constraints.

(b) Solve the linear program using MATLAB's `linprog`.

(c) Comment on any observations you have.

(We discussed this problem in class in the "Introduction to Linear Programming" lecture.)

Problem 3. Consider the linear programming (LP) problem with objective

$$\begin{aligned} \text{minimize} \quad & x_1 - x_2 \\ \text{subj. to} \quad & x_1 + x_2 \leq 1 \\ & -x_1 + 2x_2 \leq 2 \\ & x_1 \geq -1 \\ & -x_1 + 3x_2 \geq -3 \end{aligned}$$

You solved the problem graphically in Homework 2.

(a) Write the linear program in inequality form by defining c, A, and b.

(b) Use the optimality conditions (see Theorem 1 below) to find the minimum point.

(c) Is this point a local minimum, global minimum, both, or neither?

Problem 4. Consider the following objective function:

$$\text{minimize} \qquad x_1 + x_2$$

For each of the constraints below, solve the problem graphically by plotting lines of constant cost and the constraints. Then, use the optimality conditions to solve the problem. To do this part, write each problem in inequality form by defining c, A, and b and apply Theorem 1. Your graphical and analytical answers should agree.

(a) $x_1 \geq 0$ and $x_2 \geq 0$.

(b) $x_1 \geq 0$.

(c) $-x_1 - x_2 \leq 0$.

Theorem 1 (Optimality Conditions for LP in Inequality Form)**.** *The linear program in inequality form has a minimum at x if and only if the following system is solvable:*

$$Ax \leq b \qquad \lambda \geq 0, \qquad \lambda^T(Ax - b) = 0, \qquad A^T\lambda + c = 0.$$