MAE 5930 Optimization
Homework 5

Purpose: The problems assigned help develop your ability to
- recognize and formulate integer programs.
- convert formulations into code using MATLAB's `intlinprog`.
- use piecewise linear approximations to solve nonconvex programs to global optimality.
- read a paper on optimization.

NOTE: Please write or type your formulations clearly so that a reader can understand what you are doing. You are welcome to use the equivalent functions in Python.

Problem 1: Consider the following nonlinear, nonconvex optimization problem.

$$\min f(x) = x^2 \quad \text{subject to} \quad g(x) = x^2 \sin(x) \leq 0 \quad \text{on} \quad x \in [1, 12]$$

(a) Plot the functions and visually confirm that the optimal point is $x \approx 3$.

(b) Solve the problem using MATLAB's `fmincon` with initial guesses $x = 1, 5, \text{and } 9$. Comment if the optimal solution is found for any of these initial guesses.

(c) Formulate the problem as a MILP by piecewise linearly approximating $f(x)$ and $g(x)$.

(d) Solve the MILP using 20 nodes, 50 nodes, and 100 nodes. Please provide your code and output showing the correct answer. Comment on any observations you have.

Problem 2: Formulate and solve the traveling salesman problem.

(a) Formulate the problem by defining the decision variables, objective, and constraints. As shown in class, the most obvious formulation does not avoid "sub-tours". Think of constraints you can add to avoid sub-tours. If you get tired of thinking, read the attached paper and implement their constraints.

(b) Solve the problem using MATLAB's `intlinprog` where the locations of the cities are $(0, 0)$, $(1, 1)$, $(2, 0.1)$, $(10, -.1)$, $(11, 1)$, $(12, 0)$.

(c) Solve the problem with 20 cities. Show the MATLAB output and plot your solution. Create x and y locations using `x = 20*rand(1,20); y = 20*rand(1,20);`

(d) Solve the problem with 40 cities. Show the MATLAB output and plot your solution. Create x and y locations using `x = 20*rand(1,40); y = 20*rand(1,40);`

(e) Solve the problem with 60 cities. Show the MATLAB output and plot your solution. Create x and y locations using `x = 20*rand(1,60); y = 20*rand(1,60);`

(f) If your computer did okay with 60 cities, keep increasing the number of cities until it doesn't.