**MAE 5930 Optimization**
**Homework 1**

Purpose: The problems assigned help develop your ability to

- recognize and formulate optimization problems.
- convert formulations into code using MATLAB's `linprog`, `quadprog`, and `fmincon`.
- find the minimum and infimum values of sets and functions.
- find the minimum values and locations of differentiable, unconstrained functions.

NOTE: You are welcome to use the equivalent functions in Python.

**Problem 1:** Consider the following table of nutritional data:

| Foods | Price (dol) per serving | Calories per serving | Fat (g) per serving | Protein (g) per serving | Carbs (g) per serving |
|---|---|---|---|---|---|
| Carrots | 0.14 | 23 | 0.1 | 0.6 | 6 |
| Potatoes | 0.12 | 171 | 0.2 | 3.7 | 30 |
| Bread | 0.20 | 65 | 0 | 2.2 | 13 |
| Cheese | 0.75 | 112 | 9.3 | 7 | 0 |
| Peanut Butter | 0.15 | 188 | 16 | 7.7 | 2 |

You must consume at least 2000 calories, 50g of fat, 100g of protein, and 250g of carbs. The goal is to meet these requirements and minimize the total cost of buying your food.

(a) Formulate this into an optimization problem by defining the variables, objective, and constraints.

(b) Write the problem in vector-matrix form suitable for MATLAB's `linprog`.

(c) Solve the problem using MATLAB's `linprog`. Provide your code and solution.

(d) Suppose only whole servings can be purchased. Solve the problem using MATLAB's `intlinprog`. Provide your code and solution. Explain how the integer solution is different than the continuous solution.

**Problem 2.** Think about an optimization problem that you "solve" in your daily life.

(a) Define the decision variables.

(b) Define the objective function.

(c) Define the constraints.

**Problem 3.** In MATLAB, create a noisy data set based on a quadratic function with coefficients all equal 1.

```
>> x  = linspace(0,1)';
>> y  = 1 + 1*x + 1*x.^2;
>> ym = y + 0.1*randn(100,1);
```

(a) Plot the quadratic function and the noisy data.

(b) Formulate the coefficient estimation problem as an optimization problem.

(c) Use MATLAB's `quadprog` to solve the problem.

(d) Why are the estimated coefficients not all one?

**Problem 4.** For each of the following functions, define the range. Using the range, determine the minimum value and infimum value. If they do not exist, explain why.

(a) $f(x) = 0$ with domain $\mathbb{R}$

(b) $f(x) = \cos(x)$ with domain $\mathbb{R}$

(c) $f(x) = x^3$ with domain $\mathbb{R}$

(d) $f(x) = -e^{-x}$ with domain $[0, \infty)$

(e) $f(x) = \sin(x)$ with domain $[0, 3\pi/2)$

**Problem 5.**

(a) Prove the following implication: *Let $f$ be real-valued differentiable function defined on an open interval of $\mathbb{R}$. If $x$ minimizes $f$, then $f'(x) = 0$.*

(b) Find a non-differentiable function where this condition fails.

(c) Find a differentiable function defined on a closed interval where this condition fails.

**Problem 6.** For each of the following functions defined on $\mathbb{R}$, use the first derivative condition to determine the minimum value (if it exists) and all locations where the minimum occurs.

(a) $f(x) = x^2$

(b) $f(x) = x^3$

(c) $f(x) = \cos(x)$

(d) $f(x) = (x - a)^2(x + a)^2$

**Problem 7.**

(a) Use the first derivative condition to find the minimum value and location of the function $f(x, y) = (\alpha - x)^2 + \beta(y - x^2)^2$.

(b) Use MATLAB's `fmincon` to solve the problem with $\alpha = 1$ and $\beta = 100$. Try different initial guesses. Provide your code and brief discussion of your observations.

**Problem 8.**

(a) Use the first derivative condition to find all local minima of $f(x) = x + 2\sin(x)$.

(b) How many local minima are there?

(c) Does a global minimum exist? If so, what is it?