MAE 5930 Optimization
Homework 4

Purpose: The problems assigned help develop your ability to
- recognize and formulate integer programs.
- convert formulations into code using MATLAB's `intlinprog`.

NOTE: Please write or type your formulations clearly so that a reader can understand what you are doing. You are welcome to use the equivalent functions in Python.

Problem 1: You are the supervisor for ACME Security, and you are responsible for assigning security officers a position. In the table below, seven officers' availability and cost are shown. That is, the table shows the times each officer comes to work, leaves work, and how much you must pay them to work that time. Your goal as the supervisor is to ensure that at least one person is always on duty between 1pm and 9pm while minimizing cost. You cannot pay for a partial shift.

Officers	Jenna	Doug	Tyler	Cindy	Brad	Ian	Ellie
Hours	1-5	1-3	4-7	4-9	6-9	5-8	8-9
Cost ($)	30	18	21	38	20	22	9

As an example, you could choose Jenna, Tyler, and Brad, and this would cost you $30 + $21 + $20 = $71.

(a) Formulate this into an optimization problem by defining the variables, objective, and constraints. Be sure to specify if the variables are integers or reals.

(b) Solve the program using MATLAB's `intlinprog`. Please provide your code and output showing the correct answer.

Problem 2: You are the logistics engineer for an air transportation company. Your goal is to find out how to pack crates on a plane in an optimal way. The crates are of five possible types denoted A, B, C, D, and E. The table below shows the mass, volume, and value of each crate type.

Type	A	B	C	D	E
Mass (kg)	500	1500	2100	600	400
Volume (m^3)	25	15	13	20	16
Value ($)	50,000	60,000	90,000	40,000	30,000

The plane is divided into three segments – front, middle, and back. Each segment has a limited mass and volume capacity. These capacities are summarized in the table below.

Segment	Front	Middle	Back
Volume Capacity (m^3)	200	500	300
Mass Capacity (kg)	8,000	20,000	6,000

For stability reasons during flight, the plane must also be balanced correctly. The balancing requirements are:

mass of middle cargo $\geq$ mass of front cargo + mass of back cargo
mass of middle cargo $\leq 2 \times$ (mass of front cargo + mass of back cargo).

Additionally, there are only 12 crates of type A, 8 of B, 22 of C, 15 of D, and 11 of E. You need to decide how many crates of each type are going in what segment of the plane to maximize the value delivered.

(a) Formulate this into an optimization problem by defining the variables, objective, and constraints. Be sure to specify if the variables are integers or reals.

(b) Solve the program using MATLAB's `intlinprog`. Please provide your code and output showing the correct answer.

Problem 3: THIS PROBLEM IS VOLUNTARY. YOU DO NOT HAVE TO SOLVE IT.

Formulate the Sudoku game as an integer program. Solve the program in MATLAB and use the "Problem Based" modeling tools to help. For more information on those tools, see `https://www.mathworks.com/help/optim/problem-based-approach.html`.

Depending on the initial values provided in the game, you may find that the problem is easy to solve or quite difficult. You can also try installing two commercial solvers – CPLEX and Gurobi – to see how much more powerful they are than the built-in MATLAB solver.