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MODELLING AND REAL-TIME CONTROL OF TRAFFIC FLOW ON THE SOUTHERN PART OF BOULEVARD PERIPHERIQUE IN PARIS: PART II: COORDINATED ON-RAMP METERING

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Abstract—A coordinated, feedback ramp metering strategy is derived for the southern part of Boulevard Périphérique in Paris. The derivation is based on well-known methods of Automatic Control theory (Linear Quadratic Optimisation) using a linearized version of a nonlinear macroscopic traffic flow model presented in a preceding paper. Efficiency of the presented strategy is tested and compared to no control, fixed-time control, and local feedback control on the basis of simulation investigations.

1. INTRODUCTION

The present paper describes development and simulation testing of coordinated on-ramp metering strategies for the southern part of the Boulevard Périphérique (BP) in Paris. The paper is understood as the continuation of Papageorgiou *et al.* (1990) which will be referred to as "Part I" throughout this paper.

Congestions on freeways have become a common phenomenon leading to delays, reduced traffic security, increased fuel consumption, and severe air pollution. Freeway traffic flow can be influenced by on-ramp metering, variable speed limitation signs, variable route recommendations, information provided to the drivers, and further variable traffic signs. A substantial amelioration may be achieved by application of adequate tools provided by Automatic Control and Computer Technology aiming at transforming traffic flow on freeways into a controllable, optimally operating system. This general approach has been successfully applied to optimal operation of a broad spectrum of technological and semi-technological processes with limited capacity (see e.g. Singh, 1987).

A convenient way of deriving coordinated feedback control strategies for ramp metering is by application of the well-known Linear-Quadratic Methodology. This was already done in the past by Isaksen and Payne (1973), Knapp (1972), Athans *et al.* (1975), Papageorgiou (1984), Goldstein and Kumar (1982).

It is the scope of the present study to develop a coordinated control strategy for the southern part of BP on the basis of the Linear-Quadratic (LQ) optimization theory. A number of traffic occupancy measurements from the mainstream being available, a LQ-regulator is a simple linear connection (via a gain matrix) between these measurements and the on-ramp volumes required in order to achieve a desired traffic state. Since the elements of the gain matrix are constant, the LQ-regulator appears particularly simple and easy to implement as compared to other coordinated strategies. Furthermore, the LQ-regulator is fairly insensitive with respect to stochastic measurement errors due to its closed-loop mode of control (see Papageorgiou, 1986 for a review of coordinated on-ramp metering strategies).

In distinction to previous applications of the LQ-methodology to ramp metering, the present paper presents a multivariable regulator with integral parts which appears to have some advantages as compared to the classical LQ-approach. Moreover, the possibility of using occupancy (instead of density) measurements is envisaged. Control theoretic developments are briefly outlined. More details may be found in Papageorgiou (1988) and in the aforementioned applications of the LQ-methodology to ramp metering. The present paper, which is an extract of the second part of a report by Papageorgiou (1988), includes some simulation tests of the developed strategies.

2. PRELIMINARY CONSIDERATIONS

Potential control benefit

Measured volume flow diagrams depicted in figures 4 and 5 (Part I) indicate underutilisation of the Boulevard Périphérique during the peak hour congestion. The fact that the existing infrastructure is underutilized just at the time it is most urgently needed, is a shocking paradox which provides an ultimate motivation for introduction of traffic control measures.

In order to estimate the amount of underutilisation, let us consider a traffic situation characterized by the traffic volume flow diagram of Fig. 1. Since mainstream capacity is not exceeded, we may assume corresponding traffic densities to be undercritical, in which case the spatially average speed along the BP-stretch amounts to 55 km/h. Exit rates correspond to real life averages.

Table 1 gives the spatially averaged speed and the total served traffic volume (sum of all exit volumes) for the traffic situations of Fig. 1 (desired conditions), figure 4 of Part I (congestion), and figure 5 of Part I (severe congestion). It is seen that under desired conditions, the total served volume increases by 11%, respectively 36%, and the average passing time through the BP-stretch decreases by 36%, respectively 64% as compared to congested respective severely congested conditions. Loosely speaking, if there is a means to reach and preserve the desired traffic conditions, one may be able to double the average speed and serve 20% more cars as compared to actual traffic conditions during peak hours. This defines the potential benefit of ramp metering strategies.

Controllability aspects

The desired traffic conditions depicted in Fig. 1 represent just one example out of a high number of possible entry volume combinations all leading to more or less similar results with respect to average speed and total served volume, provided congestion is avoided and capacity flow is preserved. The differences between these combinations concern the distribution of individual on-ramp volumes to be served under mainstream capacity constraints.

Generally, mainstream traffic volumes at all sites increase if congestion is avoided. As a consequence, the surplus volume may be distributed among the individual on-ramps and eventually a net benefit may be attributed to each on-ramp as compared to the noncontrolled case. This implies, however, that all important entry volumes are controllable or that they do not considerably deviate from their attributed values. In fact, if any entry volume (e.g. mainstream entry of Fig. 1) exceeds its designed value, subsequent entry volumes (e.g. on-ramp Italie) will have to be reduced in order to avoid congestion, and if this happens permanently, congestion avoidance may be possible, if at all, only if controllable on-ramps are disadvantaged as compared to the noncontrollable ones.

We have thus addressed the item of system controllability. We may generally postulate that a desired traffic situation (including desired individual on-ramp volumes) may be reached more or less accurately according to the actual degree of controllability. Eventually, in case of low controllability, the aforementioned potential benefit of control may not be met fully but up to a corresponding degree.

The actual degree of controllability depends upon:

(i) The number and importance of controllable entry volumes.
(ii) The degree of restrictions applying to the controllable entry volumes.

For the present application, the two most important entry volumes, namely mainstream entry and A6, are not controllable. This diminishes substantially the controllability of the traffic system, particularly in presence of several on-ramp volume restrictions.

Control constraints

Controllable on-ramp volumes are subject to a minimum green phase constraint in order to avoid ramp closure.

If existing storage space at some on-ramps is not sufficient, excessive queues may interfere with surface street traffic and cause undesired delays. This may be avoided by increasing on-ramp volumes whenever excessive on-ramp queues are detected (control override). The override tactics applied in the present study are described in full detail in Papageorgiou (1988).

Desired traffic condition

The principal aim of the control strategies to be considered in the present study is to achieve and preserve on the southern part of BP desired traffic conditions. The desired traffic conditions may be specified as the solution of an optimisation problem (see e.g. Wattleworth, 1965). However, in view of the low controllability of the present application, the set of feasible desired conditions is rather limited and hence formulation of an optimisation problem for desired condition specification makes little sense. Hence an

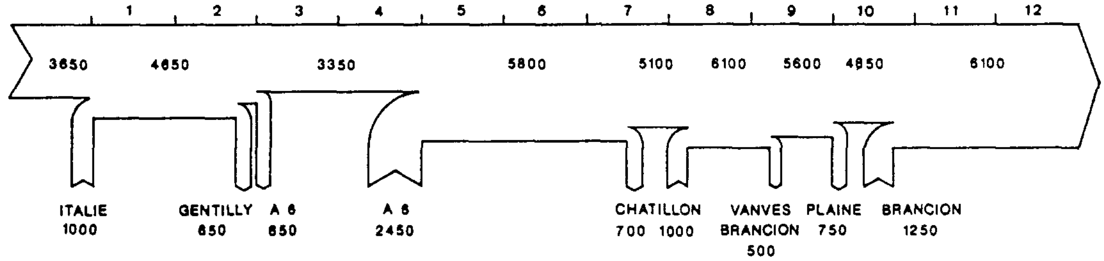

Fig. 1. Desired traffic conditions.

Table 1. Comparison of traffic situations

Traffic situation	Speed (km/h)	Total volume (veh/h)	Car count on mainstream (veh)
desired state	55	9,350	600
congestion	35	8,400	1,000
severe congestion	20	6,850	1,250

adhoc specification of the desired traffic conditions of Fig. 1 has been effectuated taking into consideration the particular needs of the present application.

Diversion aspects

An important issue to be addressed in the context of ramp metering is diversion: the additional delay which may be caused due to on-ramp queues may motivate a portion of drivers to modify their route choice and use surface streets instead of the Boulevard Périphérique. Diverting cars represent an additional load of surface streets which may also be saturated during peak hours. Thus, under certain conditions it may seem preferable from a corridor point of view to tolerate a certain degree of congestion on the BP in order to keep drivers away from the surface streets (Fargier *et al*, 1979).

In order to identify the importance of this issue for the present application, let us refer once more to figures 4 and 5 (Part I), and Fig. 1. The approximate number of cars included in the BP-stretch at a given time instant for each situation is given in Table 1. It may be seen that the additional amount of cars included in the BP-stretch due to congestion, is not more than the surplus amount of cars served under desired conditions during some 30 minutes. In other words, ramp metering may cause diversion during the first 30 minutes of the peak hour period but afterwards it will facilitate absorption of an even greater number of cars from the surface streets due to the higher service level resulting from congestion avoidance. In fact, delay due to congestion on BP motivate drivers to divert (i.e. either to exit from BP before having reached their final destination or to change completely their route choice).

In conclusion, ramp metering may redistribute traffic diversion in space and time but the final load of surface streets during peak hours may be reduced as compared to the noncontrolled situation. This phenomenon was already observed in real-life experiments at the on-ramp Brancion (Haj-Salem *et al.*, 1988). Loosely speaking, if some 500 cars are kept away from the BP during the initial phase of the peak hour period (in order to avoid congestion), an even higher number of cars may be absorbed from the surface streets later on. Clearly, a condition for a net benefit is that the initial diversion of 500 cars does not create lasting congestion phenomena on the surface network.

On the other hand, amelioration of traffic conditions on the Boulevard Périphérique may attract more drivers to use it rather than the surface streets. In view of the reduced delay on BP, these drivers may be willing to accept longer delays at the on-ramp queues and may thus activate control override due to excessive queue lengths thus rendering some on-ramps uncontrollable. We conclude that even if all on-ramps are controllable, lack of storage space may reduce the degree of controllability due to excessive queue override.

Fixed-time control

In principle, desired traffic conditions may be reached by fixing the corresponding on-ramp metering values. However, limited controllability, small disturbances, model inaccuracies, etc. may lead to partial underutilisation, respectively oversaturation of traffic flow if measurements of mainstream traffic are not utilized.

Under low controllability conditions of the present application, fixed-time control may at best reduce congestion up to a certain degree. At the same time, partial underutilisation of BP capacity is very likely to occur.

Local feedback control

Local feedback control aims at maintaining a desired traffic density in the section of each controllable on-ramp by using occupancy respective density measurements from this section. A local control law (ALINEA) derived in Papageorgiou *et al.* (in preparation) is utilised in the simulation tests in the following form:

$$r_i(k) = r_i(k-1) - K\,[\rho_i(k) - \rho_i^d] \qquad (1)$$

where ρ_i^d is the desired density, k is the sample time index, and $K = 16$ is the constant gain. This control law has excellent robustness properties and sensitivity of results with respect to the value of the gain K is low. In Automatic Control theory, the control law (1) is called an I-(Integral) feedback regulator.

The desired densities corresponding to the traffic volumes of Fig. 1 read $\rho_1 = 112$ veh/km (Italie), $\rho_8 = 125$ veh/km (Chatillon), and $\rho_{10} = 125$ veh/km (Brancion). The control law (1) becomes active every 40 s, the duration of a traffic cycle.

Feedback control reacts to actual traffic conditions in order to avoid or eliminate congestion. At the same time, underutilisation of the mainstream capacity is avoided if there is sufficient demand in the corresponding on-ramp. The main disadvantage of feedback law (1) is due to its local character: reaction to congestion does not occur before congestion has reached the corresponding section. In some situations, however, this may be insufficient to dissolve congestion, and for this reason there is an interest in developing multivariable feedback control strategies.

It should be noted that for the local control, maintenance of the desired traffic conditions corresponds to maintenance of the desired traffic densities in the on-ramp sections. In fact, eqn (1) makes it apparent that, if actual densities deviate from their desired values (e.g. due to uncontrolled volumes further upstream), local control will modify the on-ramp volumes as much as necessary for eliminating the density deviations. In other words, feedback control (1) tolerates on-ramp volume deviations in order to avoid density deviations from the desired values.

3. COORDINATED FEEDBACK CONTROL

Unfortunately, Automatic Control Theory does not provide any theoretical tools for derivation of multivariable feedback control laws for large-scale nonlinear systems like the freeway traffic system at hand. On the other hand, a powerful tool for design of multivariable feedback control laws for linear systems is the linear quadratic (LQ) optimisation theory. This technique is also applicable to nonlinear systems which are linearized around a desired steady-state.

A quadratic optimization criterion is used to penalize deviations of the problem variables from their steady-state. The usual way of specifying the weighting factors for the particular deviations in the optimization criterion is a trial-and-error procedure. This methodology for deriving coordinated ramp metering strategies was applied in the past by various researchers (see e.g. Isaksen and Payne, 1973; Athans *et al.*, 1975; Goldstein and Kumar, 1982).

Derivation of coordinated feedback control for the considered freeway system on the basis of the linear quadratic (LQ) optimization theory is presented in full detail in Papageorgiou (1988). Two alternative types of multivariable control are derived: a classical LQ-control law

$$\underline{r}(k) = \underline{r}^d - K_{LQ}[\underline{\varrho}(k) - \underline{\varrho}^d] \tag{2}$$

and a linear quadratic integral (LQI) control law

$$\begin{aligned}\underline{r}(k) = \underline{r}(k-1) - K^1_{LQI}[\underline{\varrho}(k) - \underline{\varrho}(k-1)] \\ - K^2_{LQI}[\hat{\underline{\varrho}}(k) - \hat{\underline{\varrho}}_d]\end{aligned} \tag{3}$$

where

$\underline{r} = [r_1\ r_8\ r_{10}]^T$ is the vector of controllable on-ramp volumes.

$\underline{\varrho} = [\rho_1\ \rho_2\ .\ .\ .\ \rho_{12}]^T$ is the vector of densities.

$\hat{\underline{\varrho}} = [\rho_2\ \rho_8\ \rho_{10}]^T$ is the vector of some selected bottleneck densities.

The gain matrices have the dimensions 3×12 (K_{LQ}, K^1_{LQI}) and 3×3 (K^2_{LQI}). The desired density values are

$$\rho_j^d = 112 \text{ veh/km}, j = 1, 2, 6, 7, 9, 11, 12$$
$$\rho_j^d = 125 \text{ veh/km}, j = 5, 8, 10$$
$$\rho_j^d = 75 \text{ veh/km}, j = 3, 4$$

that is, they are equal to the critical densities of the corresponding sections according to figure 14 (Part I). For LQI-Control, the maximum number of bottleneck densities corresponds to the number of controllable on-ramps, see Papageorgiou (1988). Thus, it appears reasonable to assign one bottleneck density to each controllable on-ramp. The corresponding bottleneck sections are those for which a congestion is most probable to appear first (e.g. on-ramp sections, lane-drop sections, etc). For the present application the densities of sections 2, 8, 10 are considered as bottleneck densities due to the lane drop (section 2: this is the section where congestion is first observed in real life) or due to the existing on-ramp (sections 8 and 10).

Both control laws have been derived on the basis of the modelling equations of META (see Part I) lineralized around the desired steady-state of Fig. 1 and have been given the name METALINE. Strictly speaking, the values of the gain matrices depend on the desired traffic conditions. However, nonlinearities in traffic behaviour are not very much pronounced for noncongested traffic. Since the specified desired traffic conditions are always noncongested, the sensitivity of gain matrix values with respect to modified desired conditions is low. Thus, the same gain matrices can be used regardless the choice of the desired conditions.

Obviously, the control laws (2) and (3) are some kind of generalisation of the local control (1): twelve density measurements included in the vector $\underline{\varrho}(k)$ are linearly related to three on-ramp volumes $\underline{r}(k)$ by use of some gain matrices K_{LQ}, respectively K^1_{LQI}, K^2_{LQI} with appropriate dimensions. The control laws become active every 40 s (traffic cycle). In Automatic Control theory, control law (2) is called a multivariable state regulator and the control law (3) is called a multivariable state regulator with integral parts. To the best of our knowledge, application of LQI-Control to coordinated ramp metering is proposed in this paper for the first time. For this reason, a brief presentation of the control theoretic background is provided in Appendix 2.

The precise values of the gain matrices are given in Appendix 1. Figure 2 depicts the spatial distribution of the gain matrix elements K_{ij} of LQ control. It becomes apparent that the highest gain value of each on-ramp volume occurs at the section it runs into. The gain values reduce rapidly for distant sections. A similar spatial distribution is present in the matrix K^1_{LQI} but the matrix elements corresponding to bottleneck sections are higher (see Appendix 1). The quadratic matrix K^2_{LQI} is found to be roughly diagonal. Due to this overlapping structure of the gain matrices an overlapping LQ-methodology was proposed in the past by Isaksen and Payne (1973), and by Goldstein and Kumar (1982).

Following the LQ methodology, the gain matrices are calculated as solution of the well-known Ricati equation. However, application of the multivariable feedback laws (2) and (3) to other freeway systems

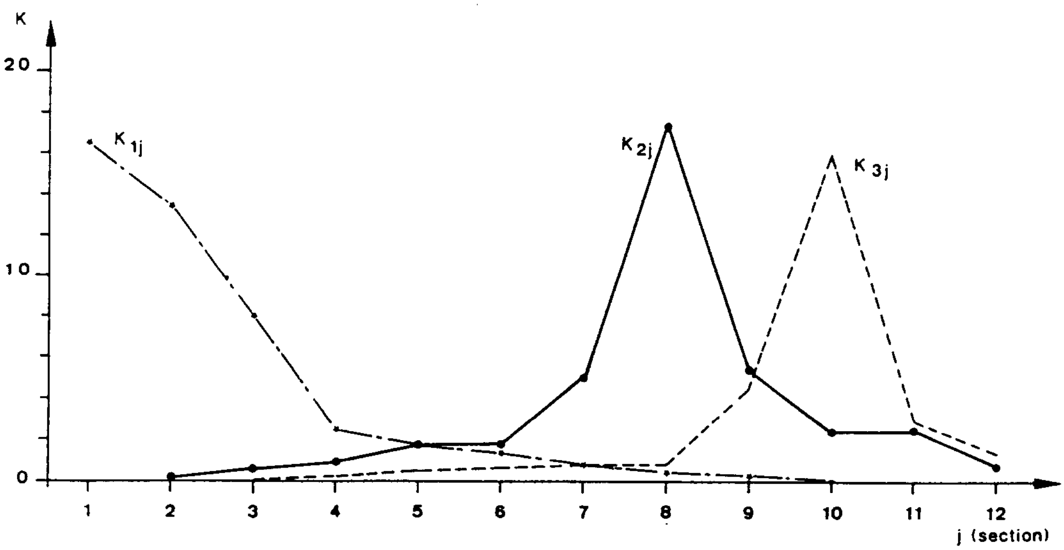

Fig. 2. Spatial distribution of gain matrice K_{LQ}.

may be performed with heuristically specified gain matrices, provided the spatial distribution of Fig. 2 and Appendix 1 is roughly preserved. Further experimentation might be necessary in this case. As a general rule, increasing the elements of the gain matrices leads to a more rapid reaction of the control system but, after a certain limit, behaviour of the closed-loop system may become oscillatory. Experimentation by simulation indicates that there is a broad region of values for which the control system is rapid enough without being oscillatory (high robustness).

Due to its structure, LQ control (2) *balances* the deviations of on-ramp volumes versus densities from their respective desired values in case of a constant disturbance (e.g. higher upstream demand). On the other hand, LQI-control *eliminates* the deviation of three preselected densities (the bottleneck densities) from their desired values in case of a constant disturbance by modifying sufficiently the on-ramp volumes. In other words, LQI control tolerates on-ramp volume deviations in order to avoid deviation of the three preselected bottleneck densities from their desired values. In practice, however, the two control laws (LQ and LQI) lead to similar results in the simulation tests of the next section.

Nevertheless, there exists an important difference in the *utilisation* of the two control laws. LQ-control (2) appears simpler but it requires prespecification of desired values for *all* controllable on-ramps ($\underline{r}^d$) and *all* densities ($\underline{\rho}^d$). On the contrary, LQI-control requires desired density values of the bottleneck densities *only* (see (3)). Thus, a modification of the desired state during operation is easier for LQI-control as compared to LQ-control and in this sense, utilisation of LQI is similar to utilisation of local feedback control which also requires specification of desired densities for the on-ramp sections only. For above reasons, LQI rather than LQ control is recommended for practical use on the Boulevard Périphérique.

Due to the multivariable (coordinated) control law, congestion in a given freeway section is more or less "visible" from each controllable on-ramp. Hence, LQ and LQI control may be more efficient in eliminating congestion as compared to local feedback. It should be recalled, however, that due to the linearisation performed in the design procedure, and due to the ignorance of constraints in the LQ problem formulation, LQ and LQI control may, under circumstances, also be insufficient to eliminate severe congestion.

Both control laws (2) and (3) are applicable using traffic occupancy instead of density measurements. In this case, desired occupancy (instead of density) values should be provided by the user. This is particularly attractive in a real life (instead of simulation) environment since loop detectors provide direct occupancy measurements. A slight modification of the gain matrices is necessary in this case (see Papageorgiou, 1988, for details). Furthermore, it should be noted that control laws (2) and (3) are applicable even if electrical sensitivities of individual detector stations are not compatible with each other.

Final adjustment of the gain matrices to real life conditions can be readily performed following the guidelines given in Papageorgiou (1988).

4. SIMULATION TESTS

Validation and comparison of control strategies is performed on the basis of several demand scenaria using the simulation model of Part I. The simulation tests indicate the way of functioning, the principal consequences, and the qualitative comparative performance of the involved control strategies. However, in view of the unmodelled complicated diversion phenomena, simulation results should not be understood as an accurate description of real phenomena notedly in what concerns lengths of on-ramp queues, and interaction with the surface network. Detailed investigation of these phenomena is postponed to the real-life experimentation period.

Rush hour congestion

Figures 3 and 4 present simulation results obtained by application of no control and LQI control, respectively under realistic demand conditions including a rather primitive diversion model.

Under no control conditions, a solid block of congestion appears in sections 1-8 after t = 40 min. The congestion lasts until the end of the simulation horizon similarly to what has been observed in real measurements. Diversion occurs mainly at the mainstream entry and A6 but lower diverting volumes are also observed at on-ramps Italie and Chatillon.

LQI-control reduces total time spent by 38% and causes diversion of a slightly lower number of cars as compared to the uncontrolled case. The congestion area diminishes and does not exceed section 6. At the same time, mean speeds in the congestion area are roughly doubled as compared to the uncontrolled case. On-ramp volume Chatillon is reduced to some 950 veh/h. In view of the high demand at this on-ramp, the corresponding diversion rate reaches 20% of the original demand. A complete elimination of the rush hour congestion does not succeed even in the controlled case due to override applying to on-ramp Italie in Fig. 4. A complete elimination of congestion and a further increase of the serving rate may be achieved if A6 is assumed controllable (see Papageorgiou, 1988).

Differences between local control and LQI-control are marginal for this simulation test. This is due to the fact that the gradual building up of congestion proceeds slowly enough for local control to adapt to changing conditions in a similar way as coordinated control.

Elimination of nonrecurrent congestion

The second test scenario aims at pointing out the advantage of coordinated LQI-control versus local control. Throughout the simulation horizon of four hours, demand is slightly lower than the desired volume conditions but an initial congestion (e.g. due to an accident) is assumed present in the eleventh section at time zero: $\rho_{11}(0)$ = 400 veh/km. Test results for no control, local control, and LQI-control are given in Figs. 5, 6, and 7 respectively. Results are summarized as follows:

(i) Under no control, the initial congestion mounts in upstream direction and reaches section 1 at t = 15 min. Width of congestion increases rapidly during the first 15 minutes and diminishes very slowly later on. At the end of the simulation horizon of 4 hours, congestion is still present in the first two BP sections. As a result, a mainstream entry queue of at maximum 380 veh. is built up.

(ii) Under local control, each on-ramp reacts only if the congestion is present in its own section. Since this is not enough, the congestion mounts and holds in the second section until t = 115 min. Maximum queues of 180 veh. are observed at on-ramp Chatillon.

(iii) Under coordinated LQI-control, ramp metering is generally activated earlier because the congestion is "visible" before having reached a certain on-ramp section. In fact, in an initial phase the congestion is mounting but it is soon dissolved (t = 30 min) by a coordinated action of on-ramps at Italie and Chatillon. Maximum queues at on-ramp Chatillon do not exceed 120 veh. Hence, coordinated control succeeds in dissolving the congestion earlier although it turns out to be less restrictive as compared to local control. Efficacy of this elegant reaction to nonrecurrent congestion is confirmed by Table 2 which gives the total waiting time and total time spent achieved by each strategy for this test scenario.

Further demand scenarios are considered in the report by Papageorgiou (1988).

5. CONCLUSIONS

Two alternative linear, traffic responsive , coordinated feedback strategies for on-ramp control have been derived using linear quadratic optimisation theory. The first control strategy results from a typical LQ approach whilst the second one includes integration for some bottleneck densities. Results obtained with both control strategies in the simulation tests are similar but LQI-control is finally recommended for application due to easier utilisation. Both control strategies are known to have excellent robustness properties due to their feedback structure. Sensitivity with respect to stochastic zero mean measurement errors is low. The control laws resulting from application of LQ optimisation theory to the linearized equations of META have been given the name METALINE.

Applications of the METALINE in a real-time (on-line) environment is particularly simple. In the case of BP Sud, twelve occupancy measurements (from twelve detector stations located in distances of 500 m) are connected to the three controllable on-ramp volumes by use of a 3×12 matrix. The connection is to be effectuated every 40 s, the duration of a traffic cycle. In other words, 36 multiplications are needed every 40 s for realisation of the kernel of the control strategy in real-time. Real life experiments are planned for the next future.

METALINE tolerates different electrical sensitivities of the detector stations which provide occupancy measurements. Furthermore, adaptation of control parameters (desired conditions, gain matrix) to real-life conditions is easily performed. METALINE is compared to (i) no control, (ii) fixed time control (no use of measurements), and (iii) local feedback con-

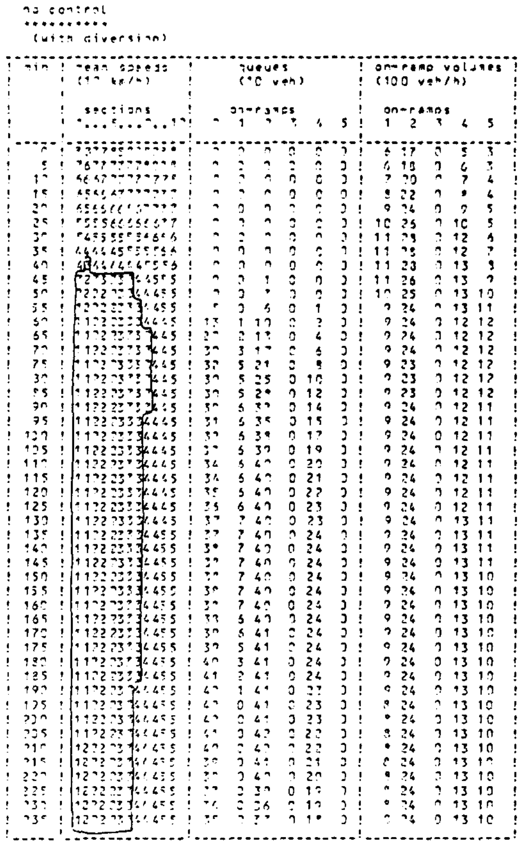

Fig. 3. No control.

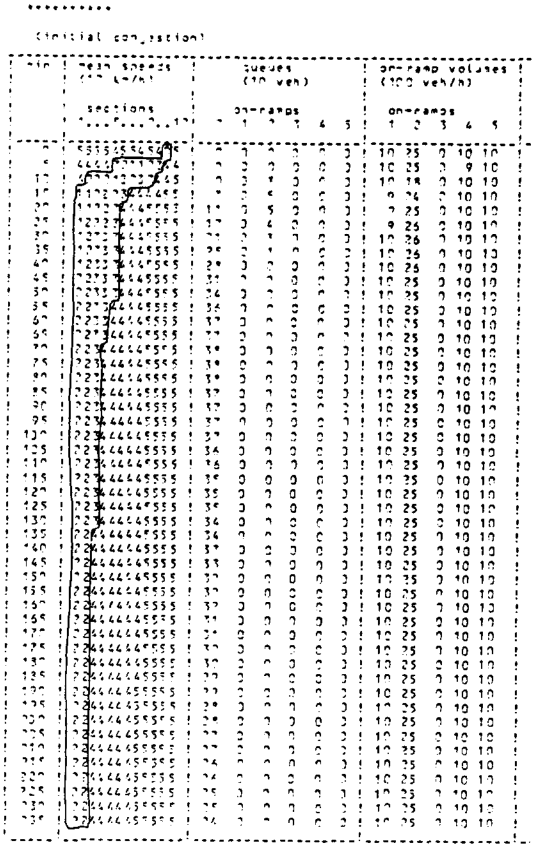

Fig. 5. No control.

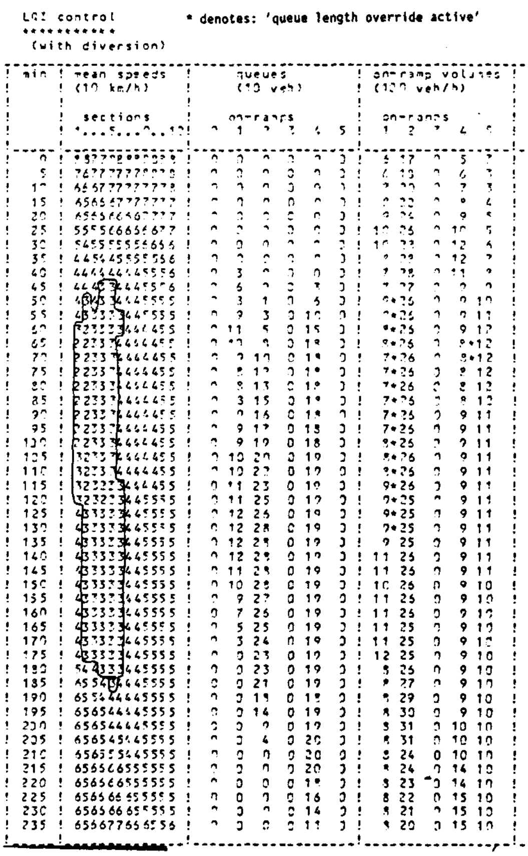

Fig. 4. LQI control strategy.

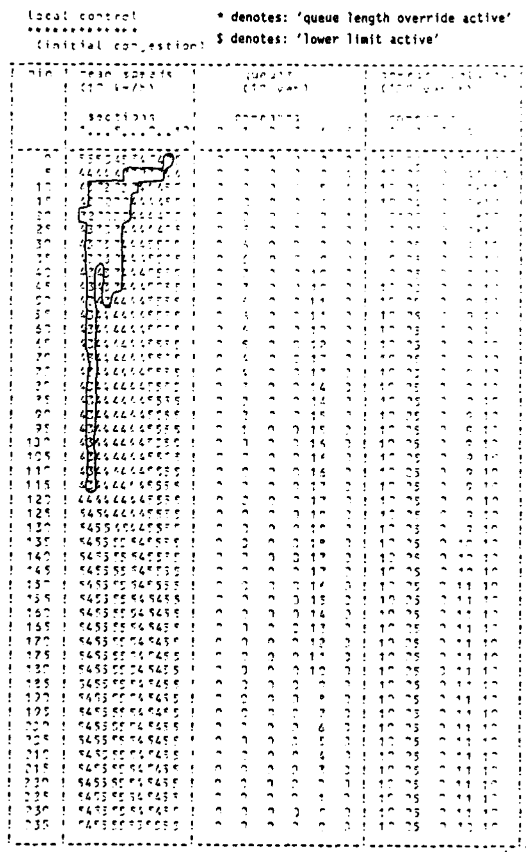

Fig. 6. Local control.

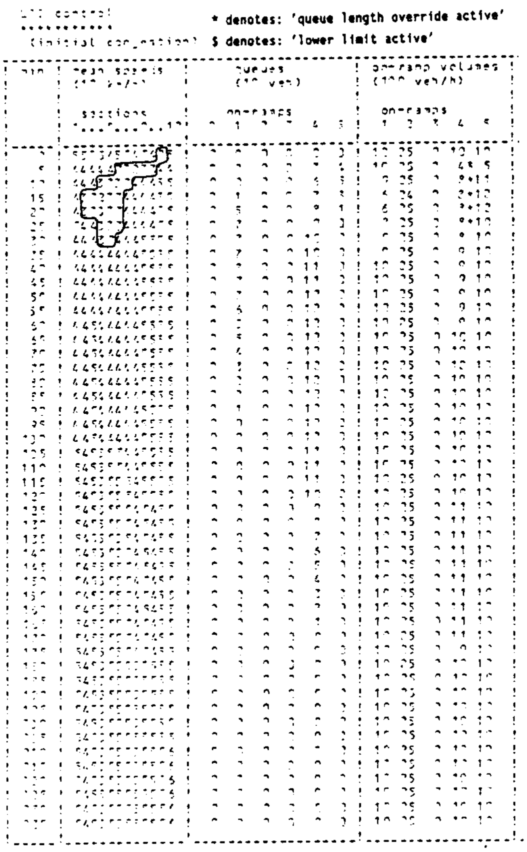

Fig. 7. LQI control.

trol applied to each controllable on-ramp individually.

The comparison is effectuated on the basis of simulation tests using META (see Part I) as a simulation model. The main conclusions derived from the simulation tests are:

(i) Suitable utilisation of on-ramp metering ameliorates traffic conditions as compared to the no control case. In particular, congestion area is restricted in space and time and mean speed is generally increased. As a consequence, total time spent in the system (including total waiting time at the on-ramp queues) and total fuel consumption are clearly decreased, the precise rate of decrease depending upon the particular application conditions. Override of control strategy decisions in case of excessive queue lengths avoids interference with surface street traffic.

(ii) Traffic responsive, feedback control strategies are clearly superior to fixed-time control. Fixed-time control may partially overload and partially underutilise the existing infrastructure.

(iii) Coordinated on-ramp control (METALINE) is superior to local feedback control (ALINEA) in case of unexpected incidents. Both feedback control strategies lead to roughly the same results under normal conditions.

(iv) Ramp metering may redistribute traffic diversion in space and time but the final load of surface streets may be reduced as compared to the uncontrolled case due to the higher served volume.

(v) The two main entering volumes of the present application, namely Autoroute A6 and mainstream entrance, are not controllable. Consequently, any amelioration of traffic conditions on BP will be primarily in favour of these entering streams.

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Table 2. Results of nonrecurrent congestion

Strategy	Total waiting time (veh · h)	Total time spent (veh · h)	Total time spent: Increase w.r.t. no control (%)
No control	1,192	4,180	–
local	519	3,102	−25.8
LQI	348	2,823	−32.5

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APPENDIX 1

The values of the gain matrices derived for on-ramp control of the Southern part of Boulevard Périphérique of Paris are the following

Matrix K_{LQ} (LQ-control): 3 × 12

$$\begin{bmatrix} 16.5 & 13.5 & 8.0 & 2.6 & 1.8 & 1.4 & 0.3 & 0.4 & 0.3 & 0.1 & 0 & 0 \\ 0.2 & 0.1 & 0.6 & 0.9 & 1.7 & 1.8 & 5.0 & 17.4 & 5.4 & 2.4 & 2.5 & 0.7 \\ 0 & 0 & 0.1 & 0.2 & 0.5 & 0.6 & 0.3 & 0.9 & 4.4 & 16.0 & 3.0 & 1.3 \end{bmatrix}$$

Matrix K^1_{LQI} (LQI-control): 3 × 12

$$\begin{bmatrix} 39.0 & 49.8 & 15.5 & 2.8 & 0.9 & 0.5 & 0.4 & 0.4 & 0.1 & -0.1 & -0.1 & 0 \\ -0.1 & -0.5 & 1.0 & 2.3 & 5.3 & 8.3 & 18.0 & 47.2 & 14.3 & 7.0 & 2.6 & 0.2 \\ 0 & 0.1 & -0.6 & -0.8 & -1.4 & -2.1 & -2.1 & -1.2 & 14.6 & 42.8 & 10.2 & 2.7 \end{bmatrix}$$

Matrix K^2_{LQI} (LQI-control): 3 × 3

$$\begin{bmatrix} 9.0 & 0.1 & 0 \\ -0.1 & 8.9 & 1.5 \\ 0.1 & -1.7 & 8.9 \end{bmatrix}$$

If autoroute A6 is also controllable, we obtain (for additional bottleneck section 6):

Matrix K^1_{LQI} (LQI-control): 4 × 12

$$\begin{bmatrix} 39.0 & 49.8 & 15.9 & 3.3 & 1.5 & 1.7 & 0.4 & 0 & -0.1 & 0 & 0 & 0 \\ 0.2 & -0.8 & 6.4 & 13.3 & 34.6 & 52.3 & 15.2 & 6.9 & 2.5 & 0.5 & 0 & 0 \\ -0.1 & -0.2 & -0.7 & -0.3 & 1.1 & 1.1 & 17.2 & 47.0 & 14.5 & 7.2 & 2.7 & 0.3 \\ 0 & 0 & -0.1 & -0.2 & -0.3 & -0.9 & -1.1 & -1.3 & 14.3 & 42.7 & 10.2 & 2.7 \end{bmatrix}$$

Matrix K^2_{LQI} (LQI-control): 4 × 4

$$\begin{bmatrix} 9.0 & 0.2 & 0 & 0 \\ -0.3 & 9.0 & 1.0 & 0 \\ 0.1 & -1.2 & 8.8 & 1.5 \\ 0 & 0.1 & -1.8 & 8.9 \end{bmatrix}$$

APPENDIX 2

LQI-Control for coordinated on-ramp metering

Theory

Consider a nonlinear, dynamic process described by the state equation (A1) and output equation (A2)

$$x(k+1) = f[x(k), u(k)] \quad \text{(A1)}$$

$$y(k) = c[x(k)] \quad \text{(A2)}$$

with $x \in R^n$, $u \in R^m$, $y \in R^p$ the state, control, and output vectors respectively. Assume existence of a desired steady-state (x^d, u^d, y^d). Linearisation of (A1), (A2) around the desired steady-state yields

$$\Delta x(k+1) = A\,\Delta x(k) + B\,\Delta u(k) \quad \text{(A3)}$$

$$\Delta y(k) = C\,\Delta x(k) \quad \text{(A4)}$$

where $\Delta \underline{\cdot} = \underline{\cdot} - \underline{\cdot}^d$ and

$$A = \partial f/\partial x|_d,\ B = \partial f/\partial u|_d,\ C = \partial c/\partial x|_d.$$

We assume that $[A,B]$ is completely controllable. We augment the system equation (A1) by the integral equation

$$z(k+1) = z(k) - y_d + c[x(k)] \quad \text{(A1')}$$

where $z \in R^p$ are additional state variables. Linearisation of (A1') with $z_d = O$ gives

$$z(k+1) = z(k) + C\,x(k). \quad \text{(A3')}$$

Assumption: The matrix $\begin{vmatrix} AB \\ CO \end{vmatrix}$ has range $n+p$.

Let us now consider minimisation of the quadratic criterion

$$J = \frac{1}{2} \sum_{k=0}^{\infty} [\|\Delta x(k)\|_Q^2 + \|z(K)\|_S^2 + \|\Delta u(k)\|_R^2] \quad \text{(A5)}$$

where Q, S, R are weighting matrices. Minimisation of (A5) subject to the augmented system equations (A3), (A3') is accomplished by the control law

$$\Delta u(k) = -K_P \, \Delta x(k) - K_I \, z(k) \qquad \text{(A6)}$$

where K_p, K_I are gain matrices to be calculated by solution of the corresponding steady-state Riccati equation. Equation (A6) may be written

$$u(k) = u(k-1) - K_p[x(k) - x(k-1)] - K_I[y(k-1) - y_d] \qquad \text{(A7)}$$

which is an LQI-multivariable feedback regulator. It is easy to show that the LQI-regulator leads to zero offset with respect to the output vector in case of a constant additive disturbance in the system equation (A3) provided the above Assumption holds. It should be reminded that a LQ-regulator leads to nonzero offset in case of a constant additive disturbance.

Application to coordinated ramp metering

A necessary condition for the Assumption of last section to hold is that $p \leq m$, (i.e. the number of control variables should not be less than the number of output variables). This means that for the present application, one may select one density value (bottleneck density) for each controllable on-ramp.

Assuming availability of density measurements only, mean speeds in the feedback law are replaced by $\Delta v = -\Delta \rho/3$ which corresponds to approximation of the $V(\rho)$-relationship by a straight line. Although rather rude, this modification does not influence the control results substantially due to control robustness.

Further details on application of LQ and LQI control theory to coordinated ramp metering in the southern part of Boulevard Périphérique may be found in Papageorgiou (1988).