Stochastic Modelling and Optimisation - Final Project

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1 Introduction

2 DP formulation

2.1 Variable definitions

Links (approaches):	$z \in Z$
Junctions:	$j \in J$
Set of inflowing links:	I_j
Set of outflowing links:	O_j
Cycle time (We assume $C_j = C$ for all junctions) :	C_j
Total lost time:	L_j
Set of stages:	F_j
Set of stages where link z has r.o.w.:	v_z
Saturation flow for link z:	S_z
Turning rates for inflowing link z and outflowing link w:	$t_{z,w}$
Green time of stage i at junction j:	$g_{j,i}$
Control interval:	T
Period intervals:	$[kT, (k+1)T]$
Demand flow:	d_z
Exit flow:	s_z

2.2 Constraints and indentities

Green light constraints:	$g_{j,i} \in [g_{j,i,\min}, g_{j,i,\max}]$
Cycle time constraint:	$\sum_{i \in F_j} g_{j,i} + L_j = C$
Exit flow constraint:	$s_z(k) = t_{z,0} q_z(k)$
Inflow to link z:	$q_z(k) = \sum_{w \in I_M} t_{w,z} u_w(k)$
Outflow from link z:	$u_z = \begin{cases} S_z & \text{if has r.o.w} \\ 0 & \text{otherwise} \end{cases}$
(Assuming space available in downstream link and $x_z > S_z$	
Average value for outflow from z:	$u_z(k) = S_z G_z(k)/C$
Effective green time:	$G_z(k) = \sum_{i \in v} g_{j,i}(k)$
Steady state demand:	$(1 - t_{z,0})\, q_z^N + d_z^N - u_z^N = 0$
(Assuming nominal green times that lead to steady-state link queues under non-saturating constant nominal demand)	

2.3 DP equations

Dynamics:

$$x_z(k+1) = x_z(k) + T\left[q_z(k) - s_z(k) + d_z(k) - u_z(k)\right] \tag{1}$$

Substituting gives:

$$x_z(k+1) = x_z(k) + T\left[(1-t_{z,0})\sum_{w\in I_M}\frac{t_{w,z}S_w\left(\sum_{i\in v_w}\Delta g_{M,i}(k)\right)}{C} + \Delta d_z(k) - \frac{S_z\left(\sum_{i\in v_z}\Delta g_{N,i}(k)\right)}{C}\right] \tag{2}$$

In vector notation:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\Delta\mathbf{g}(k) + \mathbf{T}\Delta\mathbf{d}(k) \tag{3}$$

where $\mathbf{x}$ is the state vector of the numbers of vehicles $\mathbf{x}_z$ within links $z \in Z$ and $\Delta g_{j,i} = g_{j,i} - g_{j,i}^{\mathrm{N}}$ is the vector of deviations from the steady-state green times and $\Delta d_z = d_z - d_z^{\mathrm{N}}$ is the deviation from the steady-state demand flows. $\mathbf{A} = \mathbf{I}$, $\mathbf{B}$ and $\mathbf{T}$ are the state, input, and disturbance matrices, respectively. The input matrix B reflects the specific network topology, fixed staging, cycle, saturation flows, and turning rates. For this problem we assume demand is in its steady-state: $\Delta \mathrm{d}(k) = 0$.

Cost:

$$\mathcal{J} = \frac{1}{2}\sum_{k=0}^{\infty}\left(\|\mathbf{x}(k)\|_{\mathbf{Q}}^2 + \|\Delta\mathbf{g}(k)\|_{\mathbf{R}}^2\right) \tag{4}$$

Here $\mathbf{Q}$ and $\mathbf{R}$ are non-negative definite, diagonal weighting matrices.