Traffic signal control – discrete-time linear quadratic control problem with an infinite-horizon

Monika, Jordi and Sebastian

Barcelona Graduate School of Economics

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Overview

Motivation

- Saturated road conditions call for a constant improvement in the traffic control
- Limitiations include:
  - Constantly increasing traffic
  - Limited urban space
  - Cost of building new transportation junctions
- Traffic light control of existing infrustructe pays therefore a key role in the urban optimisation of traffic
- How can it be modelled? BY DYNAMIC PROGRAMMING techniques

DP model - Dynamics

$$x_z(k+1) = x_z(k) + T\left[q_z(k) - s_z(k) + d_z(k) - u_z(k)\right] \tag{1}$$

Using traffic control variables:

$$x_z(k+1) = x_z(k) + \tag{2}$$

$$+T\left[(1-t_{z,0})\sum_{w\in I_M}\frac{t_{w,z}S_w\left(\sum_{i\in v_w}\Delta g_{M,i}(k)\right)}{C} + \Delta d_z(k) - \frac{S_z\left(\sum_{i\in v_z}\Delta g_{N,i}(k)\right)}{C}\right]$$

Finally in matrix notation:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\Delta\mathbf{g}(k) + \mathbf{T}\Delta\mathbf{d}(k) \tag{3}$$

DP model - Cost

$$\mathcal{J} = \frac{1}{2} \sum_{k=0}^{\infty} \left(\|\mathbf{x}(k)\|_{\mathbf{Q}}^2 + \|\Delta \mathbf{g}(k)\|_{\mathbf{R}}^2 \right) \tag{4}$$

Here $\mathbf{Q}$ and $\mathbf{R}$ are non-negative definite, diagonal weighting matrices

DP model - Solution

The discrete-time dynamic Riccati equation of this problem:

$$X = Q + A^T XA - \left(A^T XB\right)\left(R + B^T XB\right)^{-1}\left(B^T XA\right) \tag{5}$$

Solution to this problem is therefore given by a matrix (called the control) **L**:

$$\mathbf{L} = \left(B^T XB + R\right)^{-1} B^T XA \tag{6}$$

Putting it into DP framework:

$$\Delta g^* = -\left(B^T XB + R\right)^{-1}\left(B^T XA\right) x_{k-1} \tag{7}$$

And can equivalently by written as:

$$\mathbf{g}(k) = \mathbf{g}^{\mathrm{N}} - \mathbf{L}\mathbf{x}(k) \tag{8}$$

where $\Delta\mathbf{g} = \mathbf{g}(k) - \mathbf{g}^{\mathrm{N}}$.

Simulation - Network

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Paragraphs of Text

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Bullet Points

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- Aliquam blandit faucibus nisi, sit amet dapibus enim tempus eu
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- Nam cursus est eget velit posuere pellentesque
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Blocks of Highlighted Text

Block 1

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Multiple Columns

Heading

1. Statement
2. Explanation
3. Example

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Table

Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table: Table caption

Theorem

Theorem (Mass–energy equivalence)

$E = mc^2$

Verbatim

Example (Theorem Slide Code)

```
\begin{frame}
\frametitle{Theorem}
\begin{theorem}[Mass--energy equivalence]
$E = mc^2$
\end{theorem}
\end{frame}
```

Figure

Uncomment the code on this slide to include your own image from the same directory as the template .TeX file.

Citation

An example of the `\cite` command to cite within the presentation:

This statement requires citation [**?**].

References

John Smith (2012)

Title of the publication

Journal Name 12(3), 45 – 678.

The End