

A Bayesian Information Gain (BIG) Approach for HCI Applications

Abby Wanyu Liu

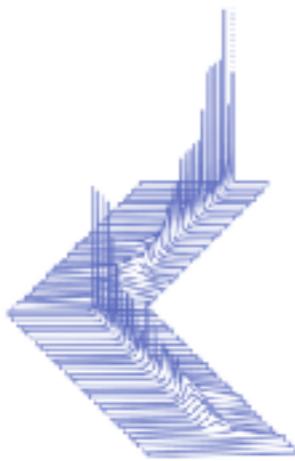
IRCAM Centre Pompidou



About Me



- * Postdoc at IRCAM Centre Pompidou, Paris
- * Working with A horizontal row of nine circular portraits of diverse individuals, likely colleagues or research partners, arranged side-by-side.
- * Keywords: HCI, information theory, computational interaction, bayesian, movement sonification



5th Summer School on Computational Interaction

Inference, optimization and modeling for the engineering of interactive systems

July 29th - August 2nd, 2019

Columbia University and Stony Brook University, USA



2015: University of Glasgow



2016: Aalto University, Helsinki



2017: ETH Zurich



2018: University of Cambridge

EDITED BY
ANTTI OULASVIRTA, PER OLA KRISTENSSON,
XIAOJUN BI, & ANDREW HOWES

[COMPUTATIONAL INTERACTION]



OXFORD

Agenda

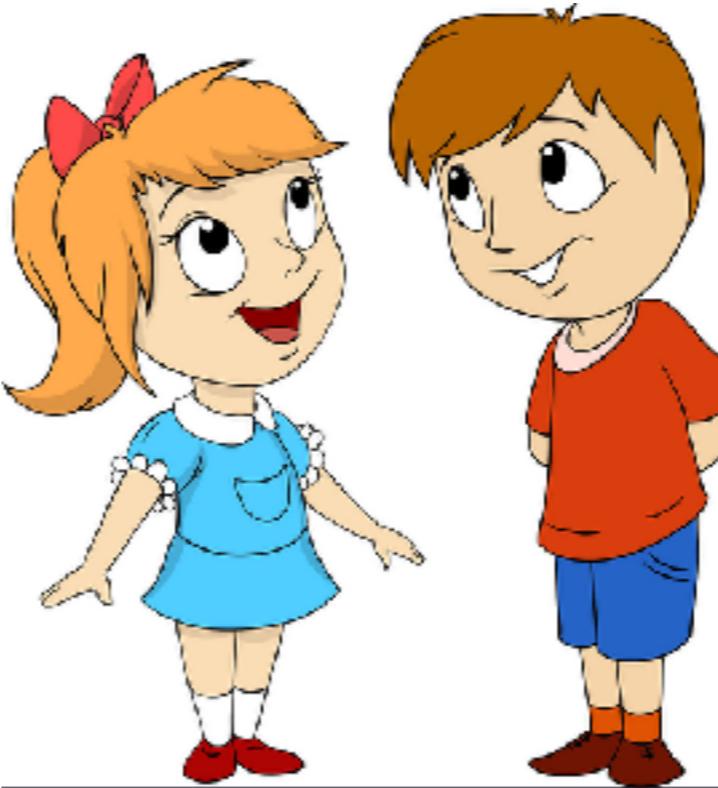
- * Information in human-computer interaction
- * Information Theory to quantify information
- * Bayesian Information Gain (BIG)
- * Information-theoretic measures to characterize interaction

Slides and Notebook exercises can be found on:

<https://github.com/wanyuliu/5thComputationallInteraction>



- Information is everywhere

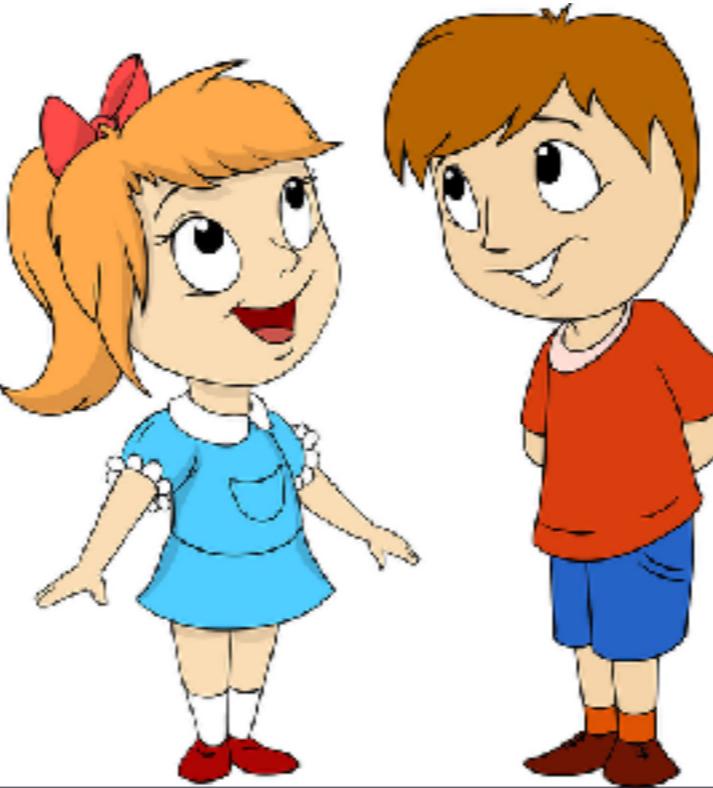


Jean-Pierre: Where do we go for dinner tonight? Pizza or burger?

Chantal: Let's have pizza!



- Information is everywhere



Jean-Pierre: Where do we go for dinner tonight? Pizza or burger?

Chantal: I don't know. We had pizza the other day so I would feel more like burger tonight. But I really like pizza. Btw Louise asked us if we want join her for dinner at Vincent's place. Vincent just came back from Sri Lanka and took some cooking courses there. But I'm not big fan of spicy food so I don't know. What do you reckon?

No Information !

- Information is everywhere

Information is any entity or form that provides the answer to a **question** of some kind or resolves **uncertainty**.

Source: <https://en.wikipedia.org/wiki/Information>

- Information in human-computer interaction

Characters



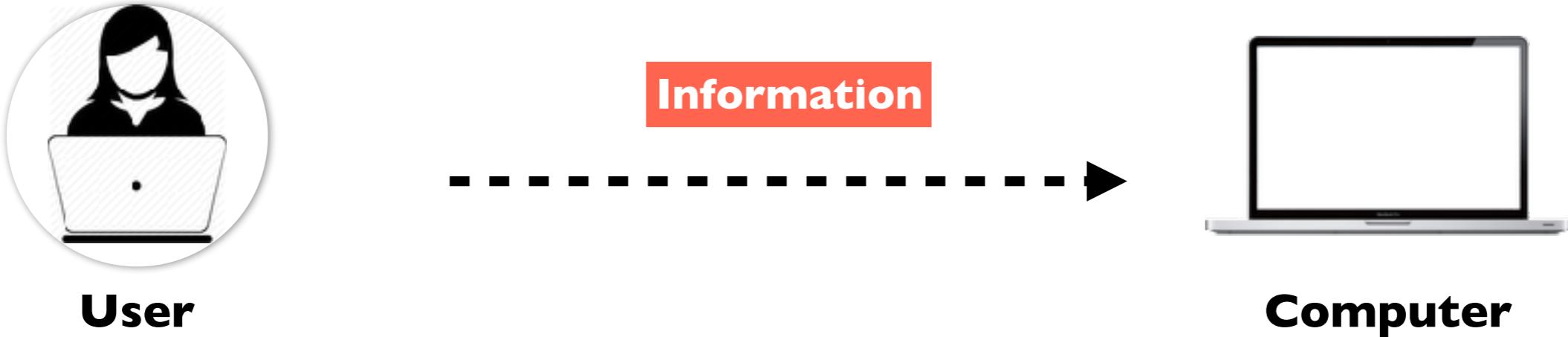
Words



Gestures



- Information in human-computer interaction

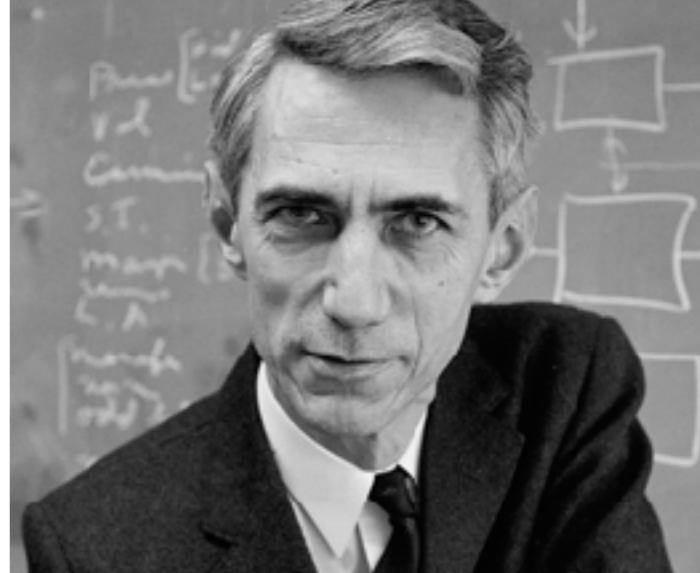


Can we quantify information?

Information in a more general sense?

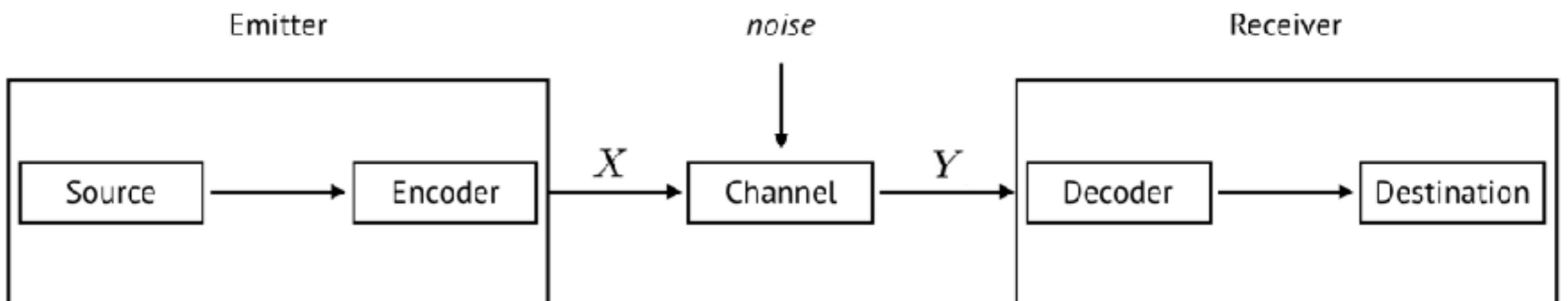
HCI as information transmission in *What Is Interaction?* (Kasper Hornbæk & Antti Oulasvirta, CHI'17)

- Information Theory to quantify information



Claude Shannon

A mathematical theory
of communication (1948)



Elements of information theory. Cover, T. M., & Thomas, J. A. (2012).

- Information Theory to quantify information

Mathematics, statistics, computer science, physics, neurobiology, electrical engineering, statistical inference, natural language processing, cryptography, neurobiology, human vision, the evolution and function of molecular codes (bioinformatics), model selection in statistics, thermal physics, quantum computing, linguistics, plagiarism detection, pattern recognition, anomaly detection, gambling, music composition....

Source: https://en.wikipedia.org/wiki/Information_theory

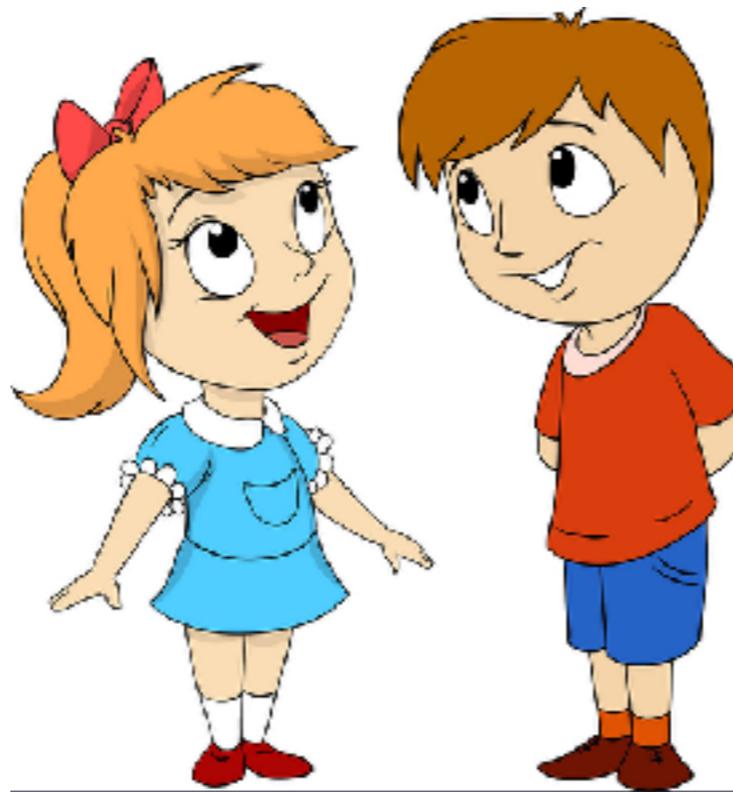
- Information Theory to quantify information

Random variable X can take values in $\{x_1, x_2, \dots, x_n\}$

$\uparrow \quad \uparrow \quad \uparrow$
 $P_1 \quad P_2 \quad \dots \quad P_n$

Entropy: $H(X) = -\sum_{i=1}^n P_i \log_2 P_i \quad 0 \leq H(X) \leq \log N$

- Information Theory to quantify information



Jean-Pierre: Where do we go for dinner tonight? Pizza or burger?

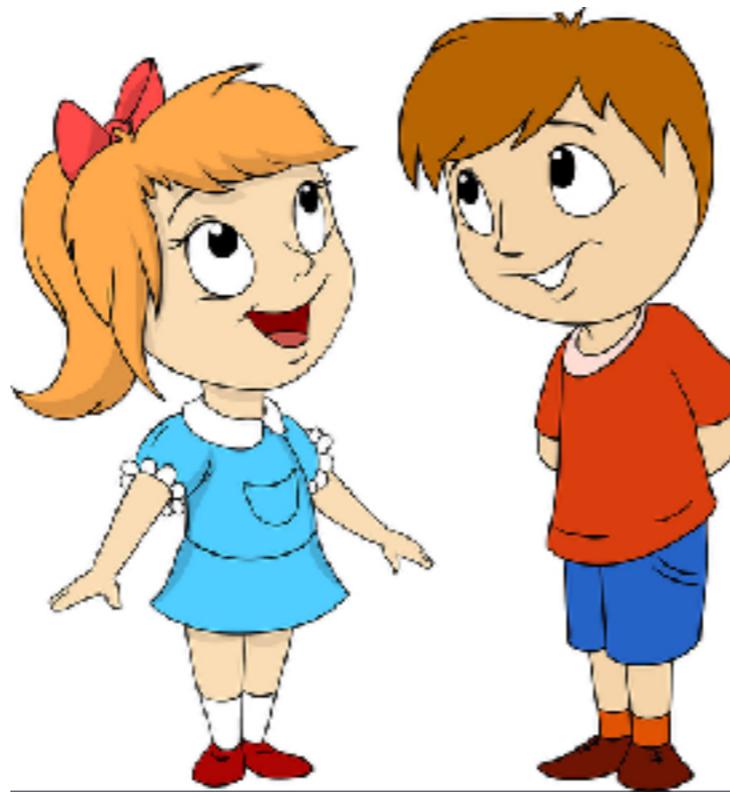
Chantal: Let's have burger!

$$\begin{matrix} \uparrow & \uparrow \\ x_1 = 0 & x_2 = 1 \end{matrix}$$

Before: $H(X) = \log 2 = 1$ bit

After: $H(X)' = 0$

- Information Theory to quantify information



Jean-Pierre: Where do we go for dinner tonight? Pizza or burger?

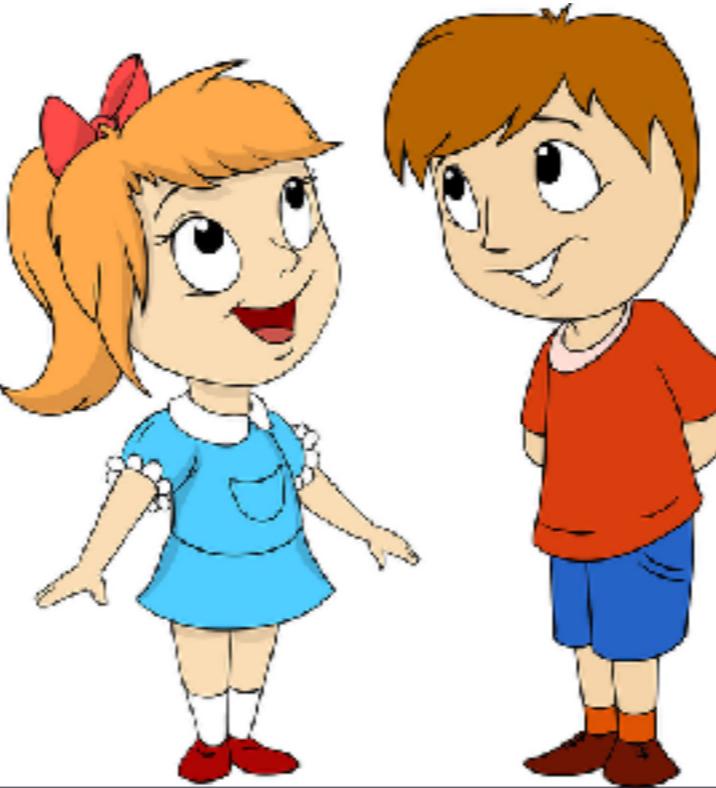
Chantal: Let's have pizza!

$$\begin{array}{c} \uparrow \quad \uparrow \\ x_1 = 1 \quad x_2 = 0 \end{array}$$

Before: $H(X) = - (0.2 \log 0.2 + 0.8 \log 0.8) = 0.72 \text{ bit}$

After: $H(X)' = 0$

- Information Theory to quantify information

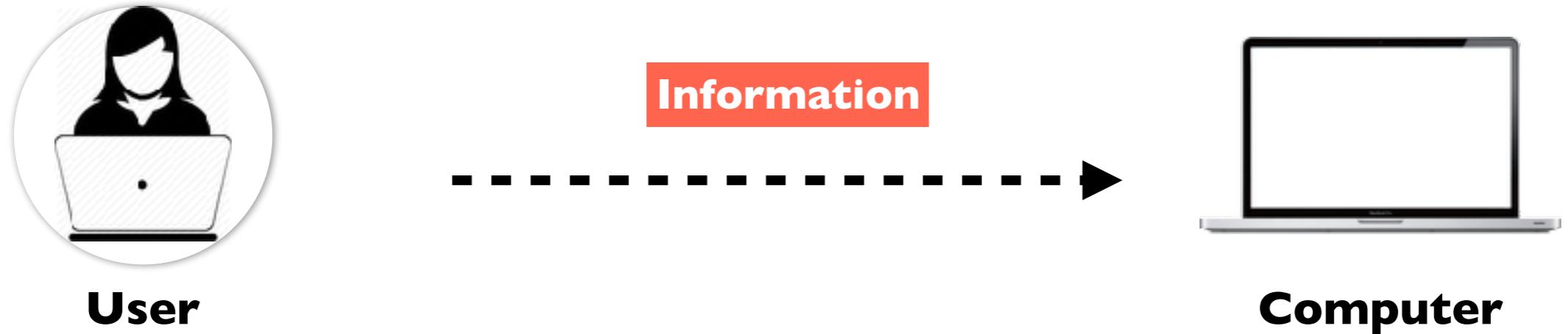


Jean-Pierre: Where do we go for dinner tonight? Pizza or burger?

Chantal: I don't know. We had pizza the other day so I would feel more like burger tonight. But I really like pizza. Btw Louise asked us if we want join her for dinner at Vincent's place. Vincent just came back from Sri Lanka and took some cooking courses there. But I'm not big fan of spicy food so I don't know. What do you reckon?

There is still uncertainty !

- Information Theory to quantify information



Characters

Words

Gestures

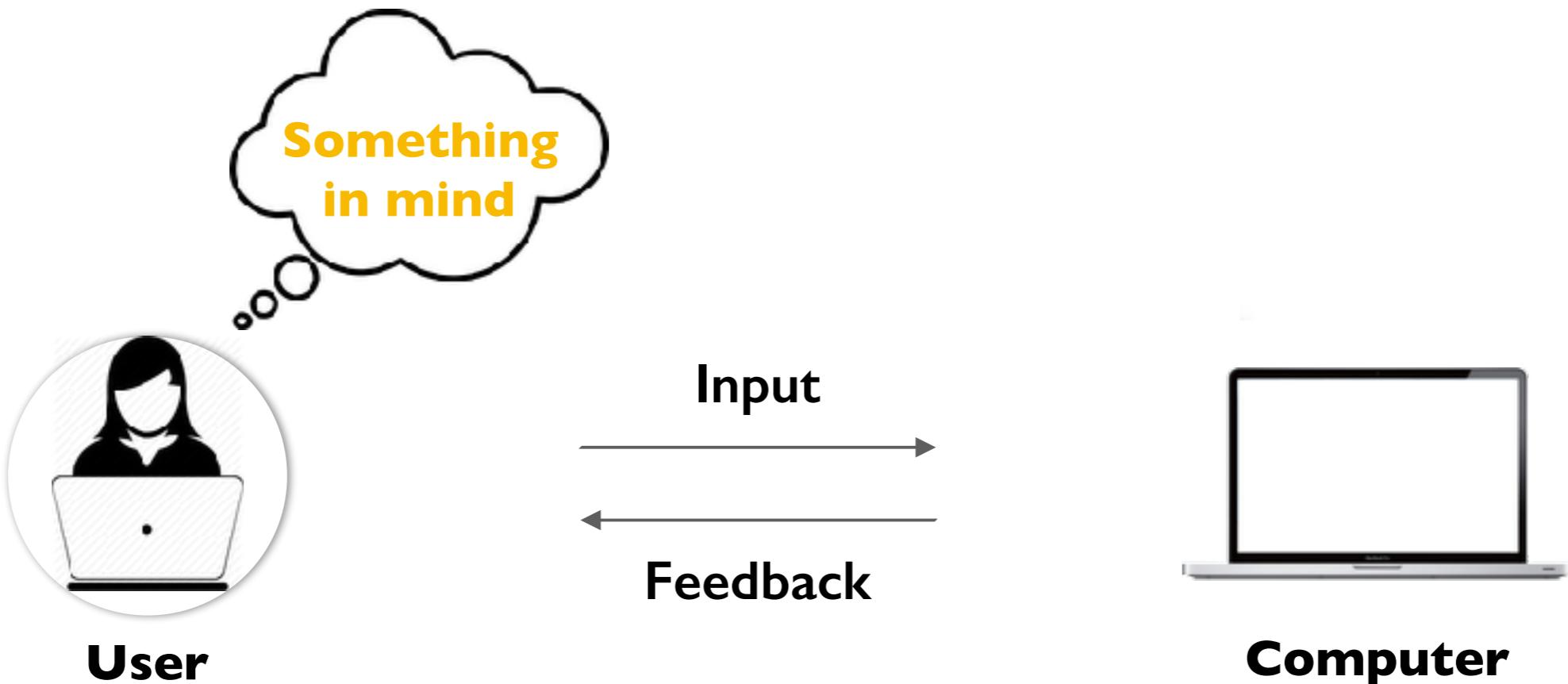
Commands

Any input?

- Bayesian Information Gain (BIG)

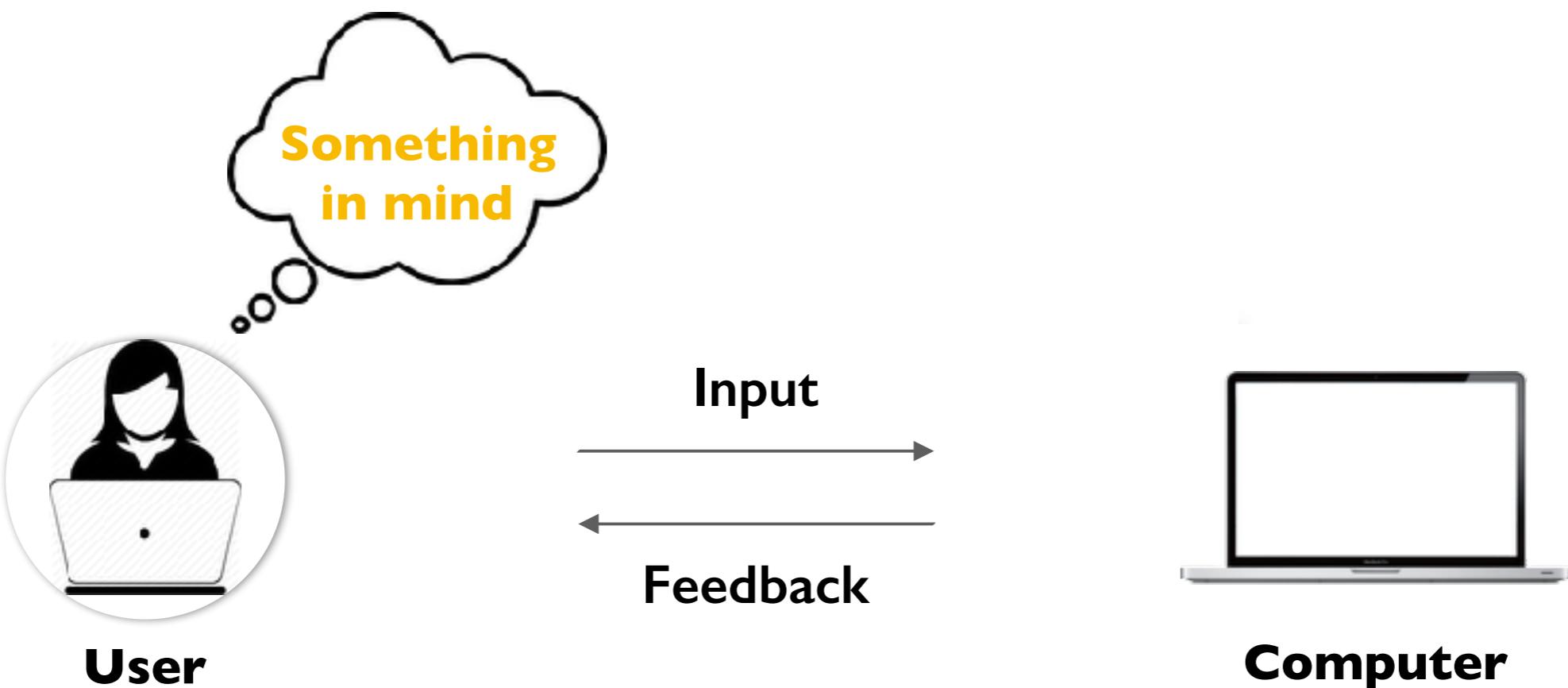
The computer gains **information** from the **user input** to reduce its **uncertainty** about the user's goal.

- Bayesian Information Gain (BIG)



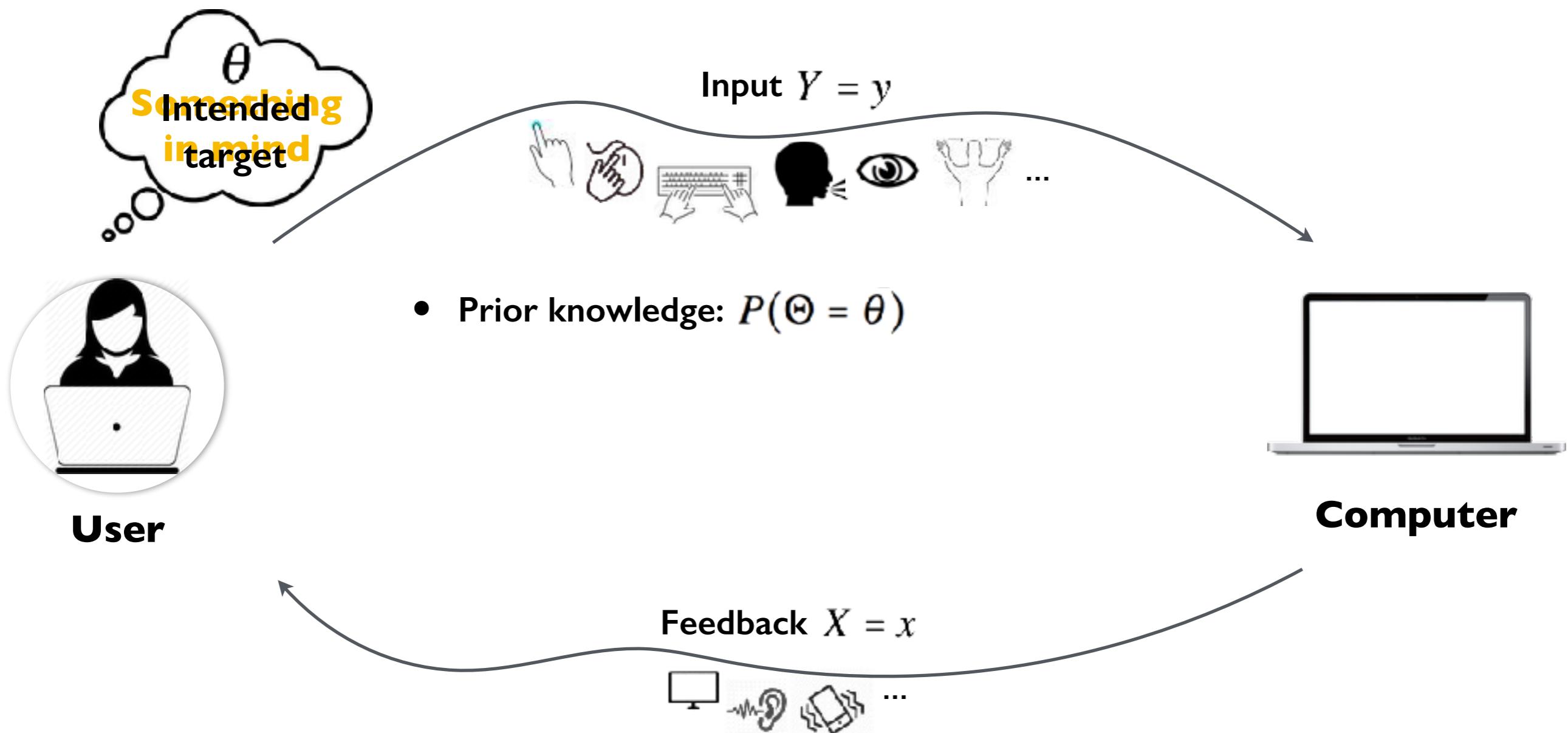
- To look for a restaurant
- To type a word
- To draw a gesture
- To select an icon
- To do something
-

- Bayesian Information Gain (BIG)

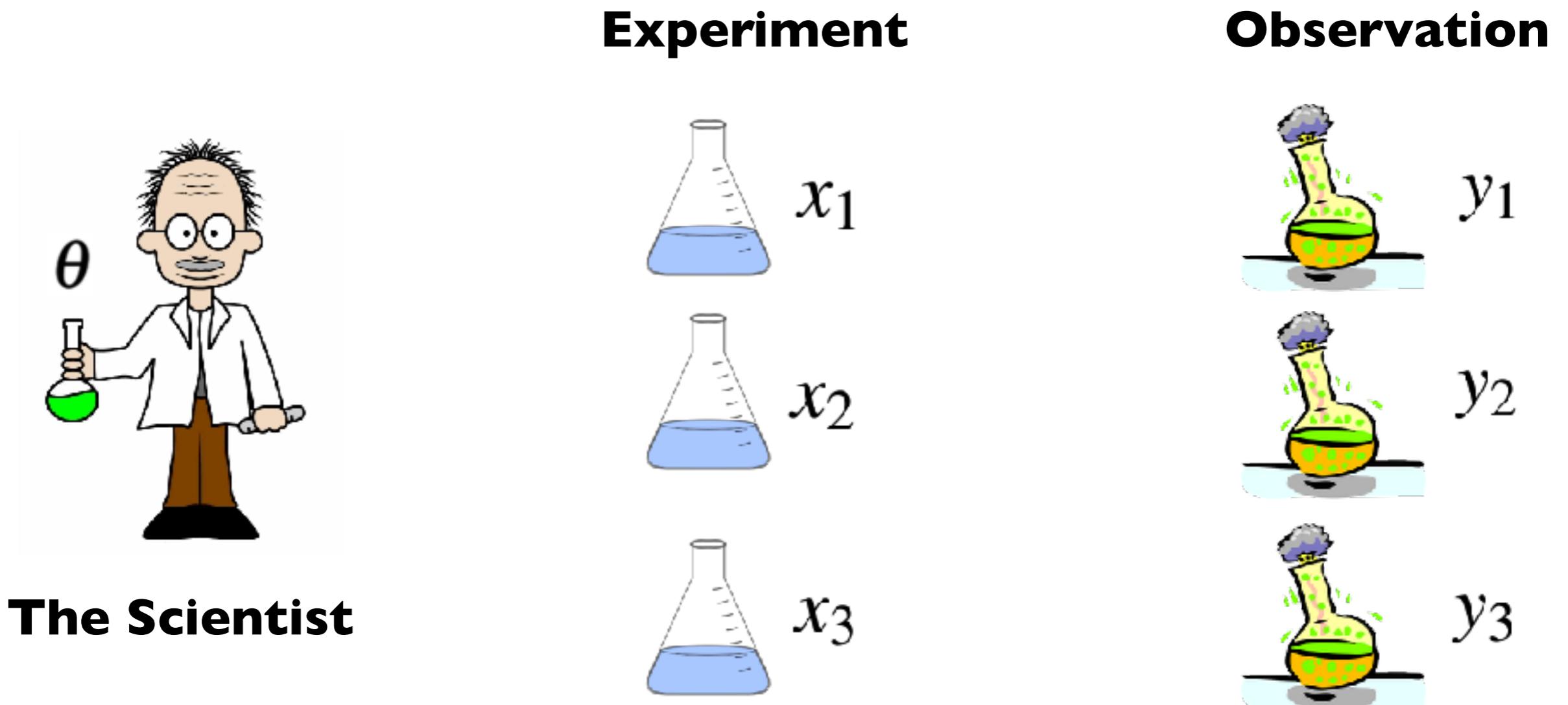


- Uncertainty about this something
- Uncertainty reduces gradually when receiving user input

- Bayesian Information Gain (BIG)

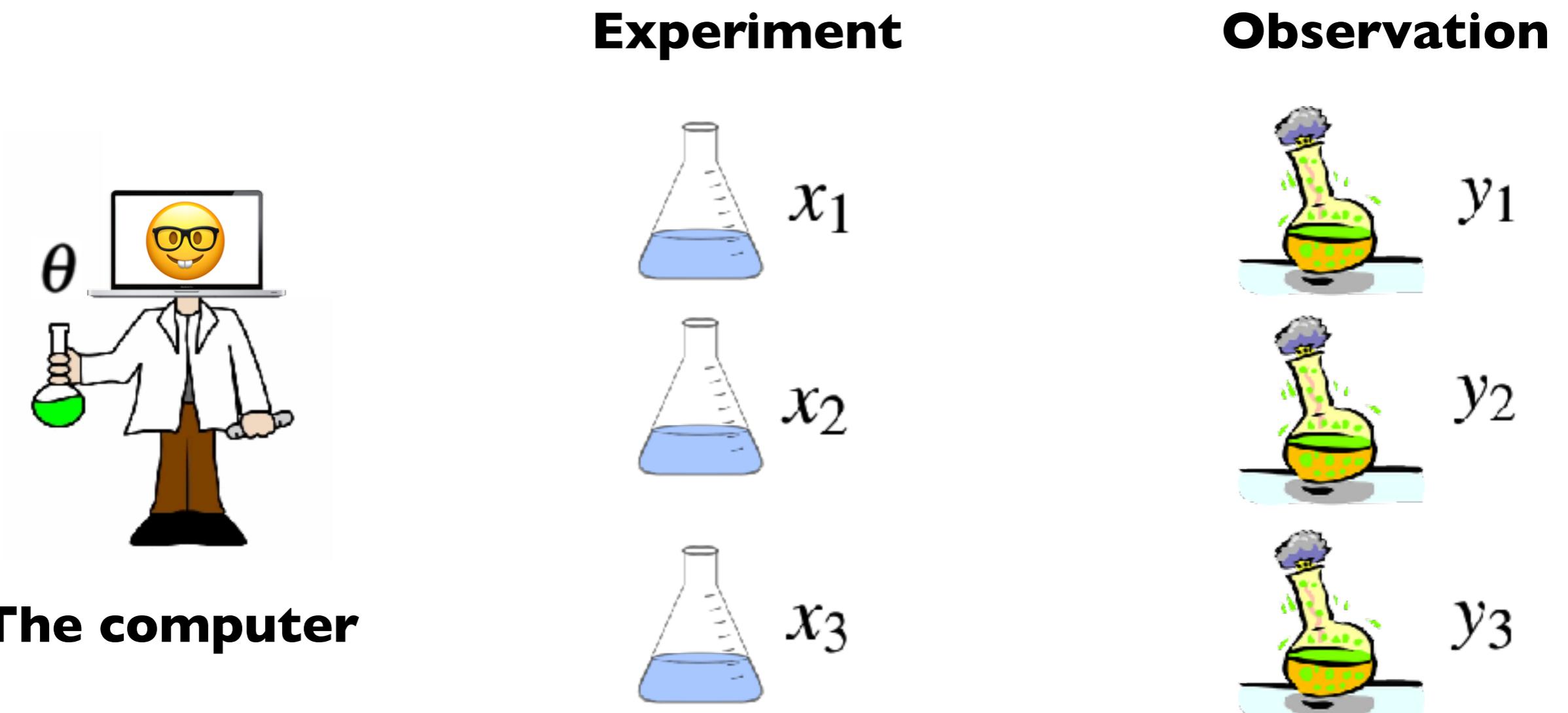


- Bayesian Information Gain (BIG)



On a Measure of the Information Provided by an Experiment (Lindley 1956)

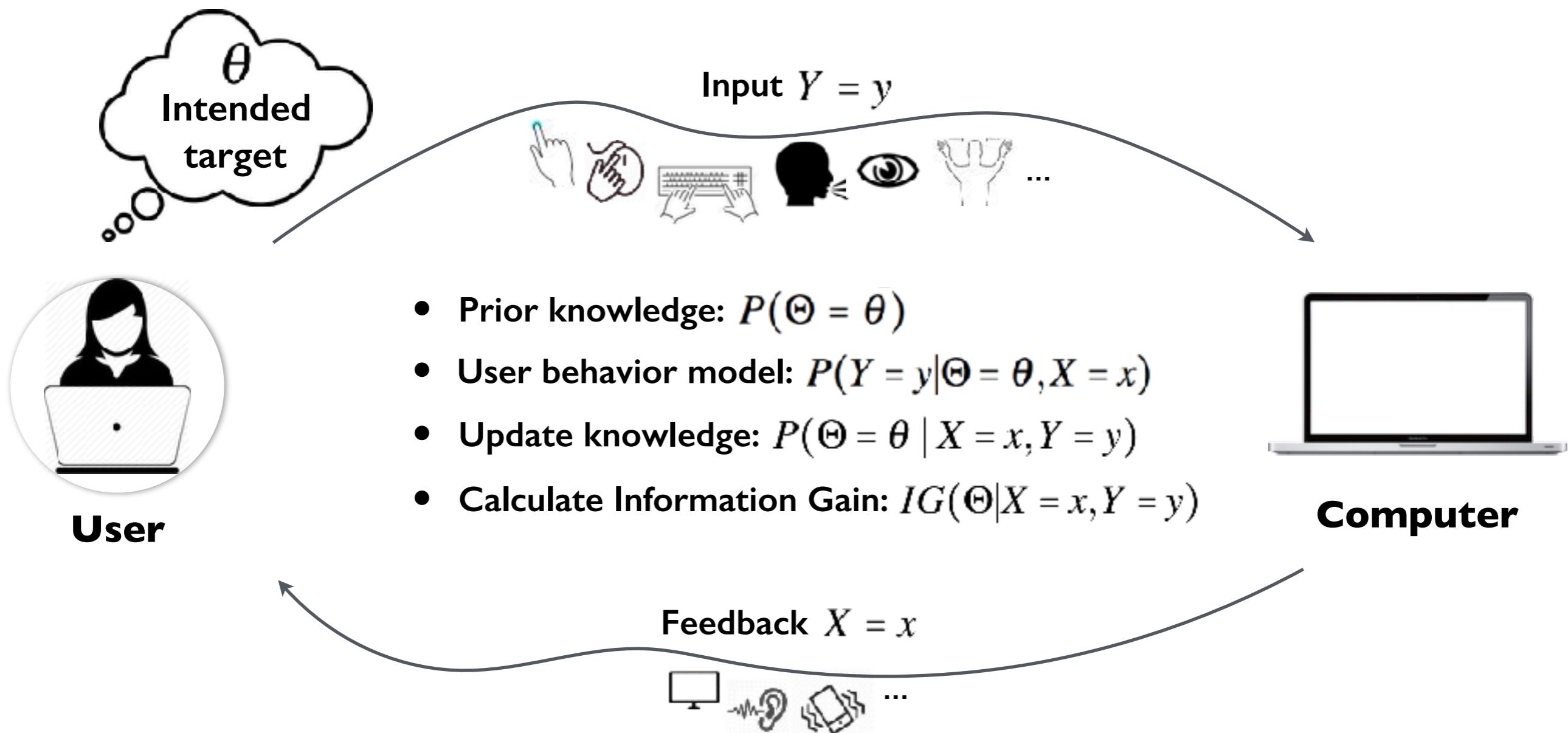
- Bayesian Information Gain (BIG)



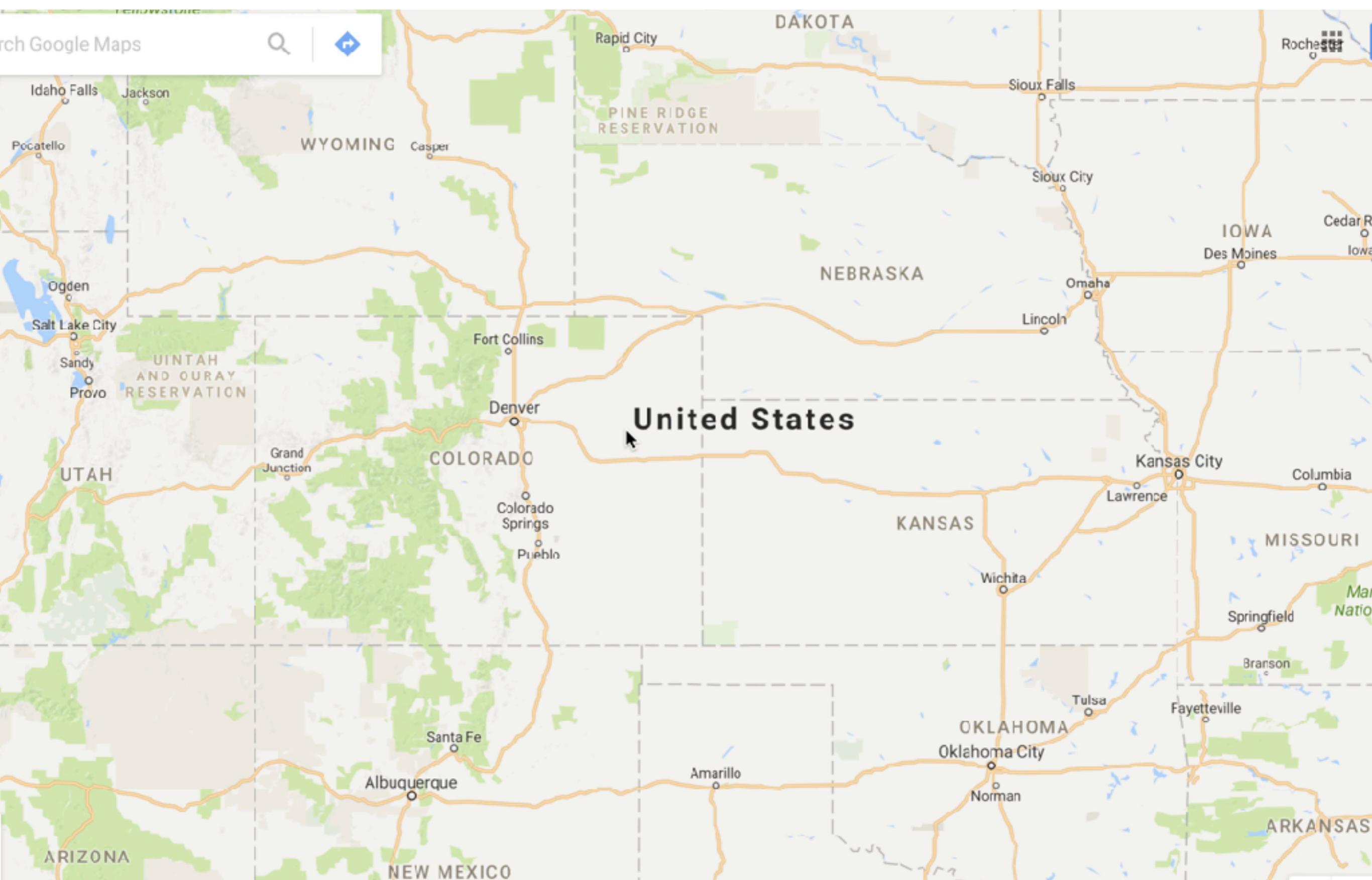
On a Measure of the Information Provided by an Experiment (Lindley 1956)

- Bayesian Information Gain (BIG)

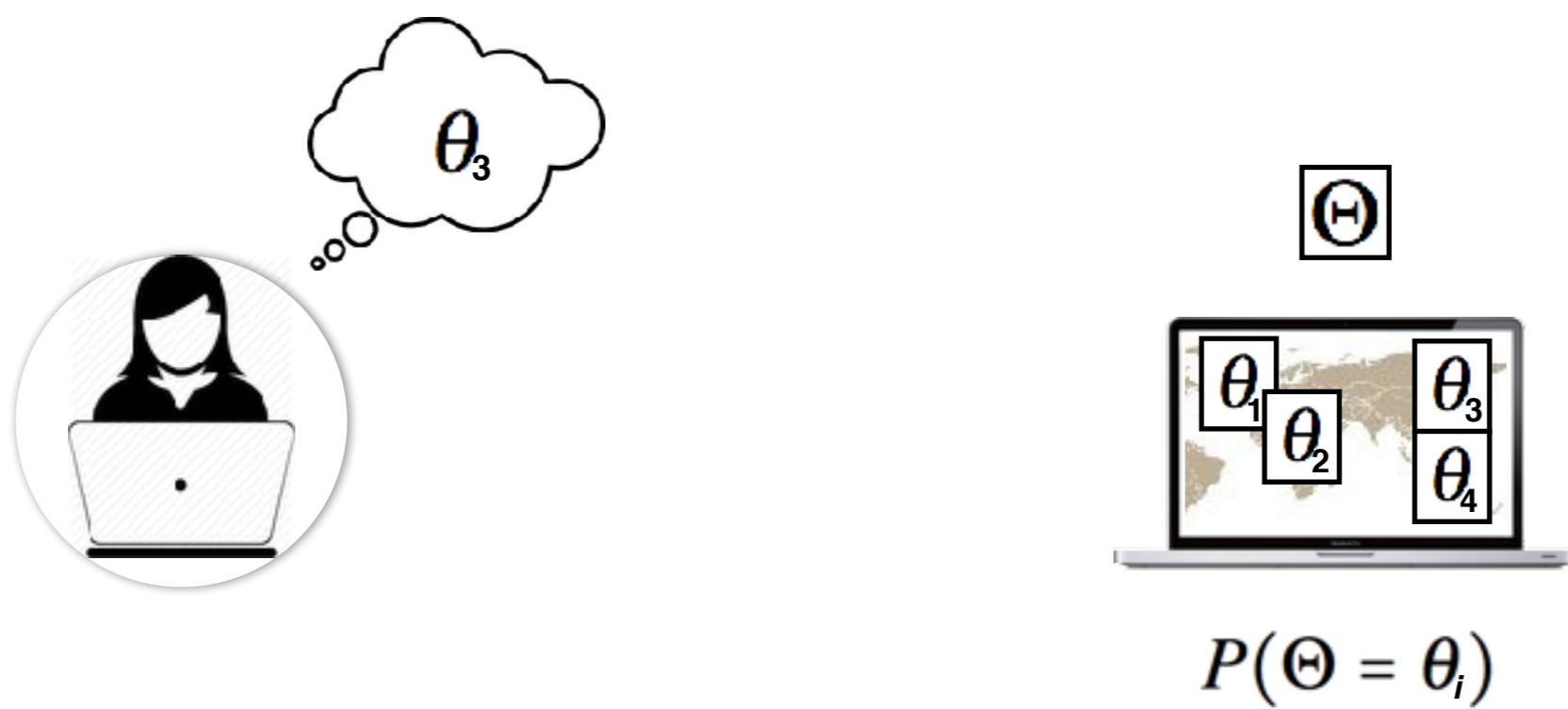
- Executes the user input only **Multiscale navigation**
- Maximizes the expected information gain $IG(\Theta|X = x, Y)$ **BIGnav**
- Leverages the expected information gain **BIGFile, Entrain**



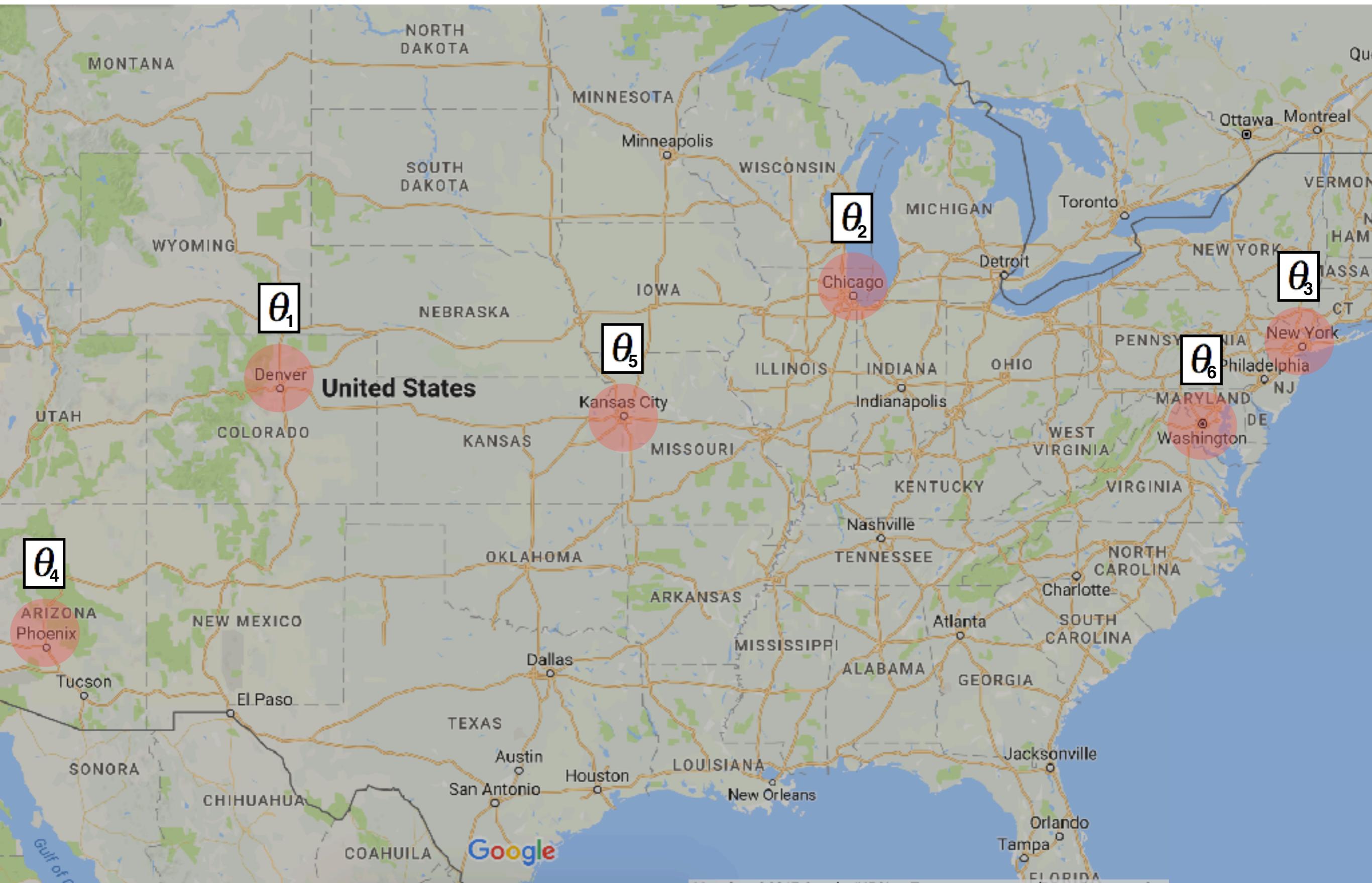
- Bayesian Information Gain (BIG) - Standard navigation



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- Bayesian Information Gain (BIG) - Standard navigation

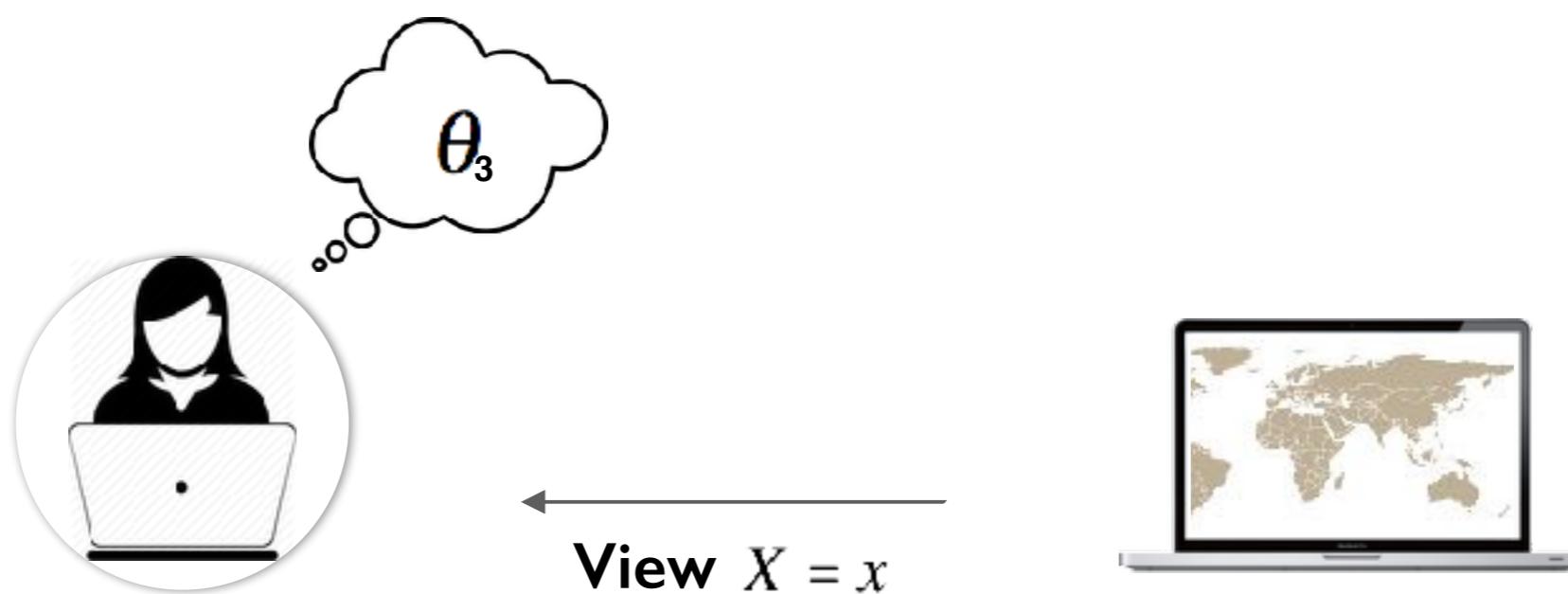


- Bayesian Information Gain (BIG) - Standard navigation

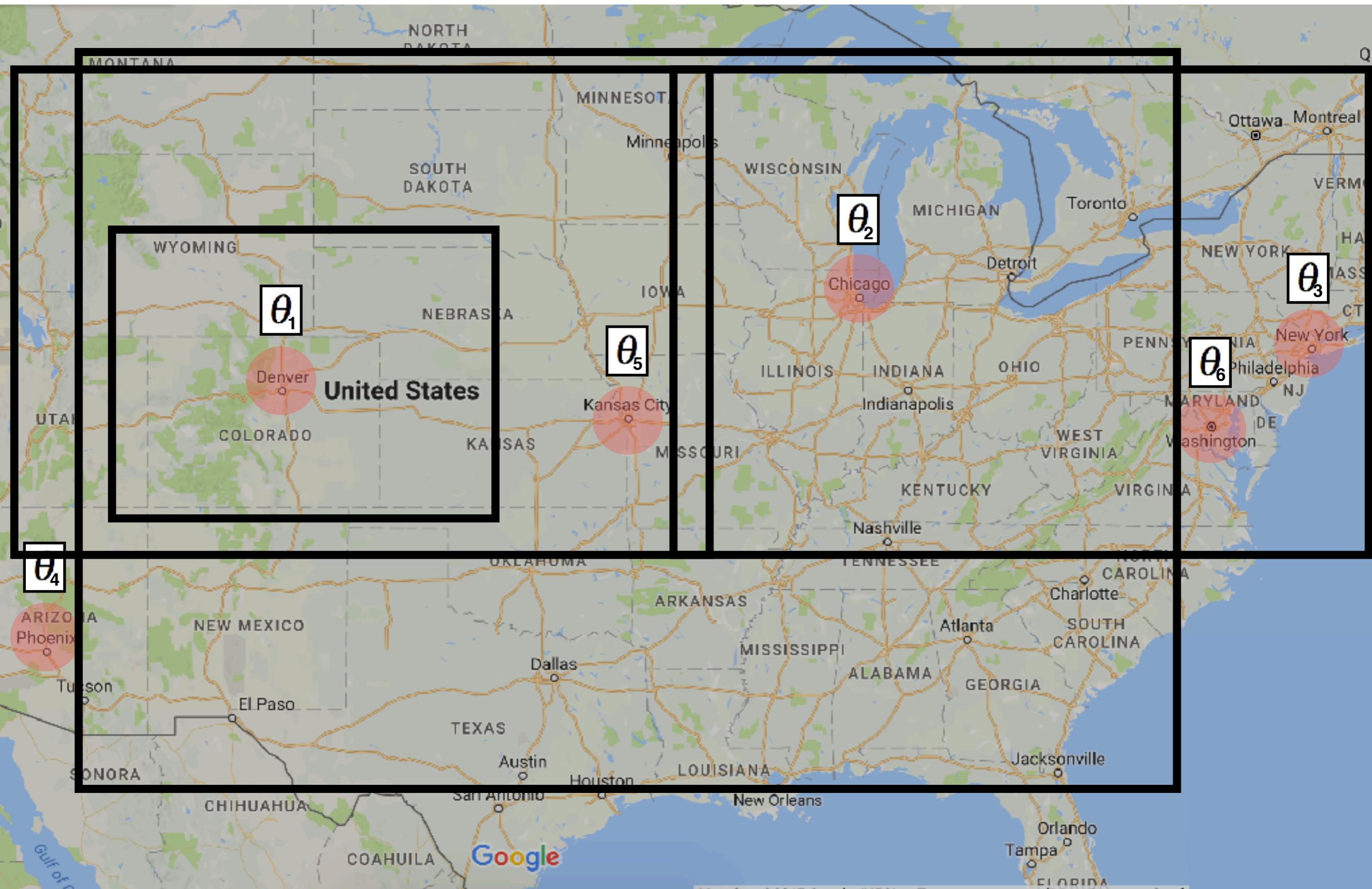
- * The computer's **Uncertainty** about the user's goal

$$H(\Theta) = -\sum_{i=1}^n P(\Theta = \theta_i) \log_2 P(\Theta = \theta_i)$$

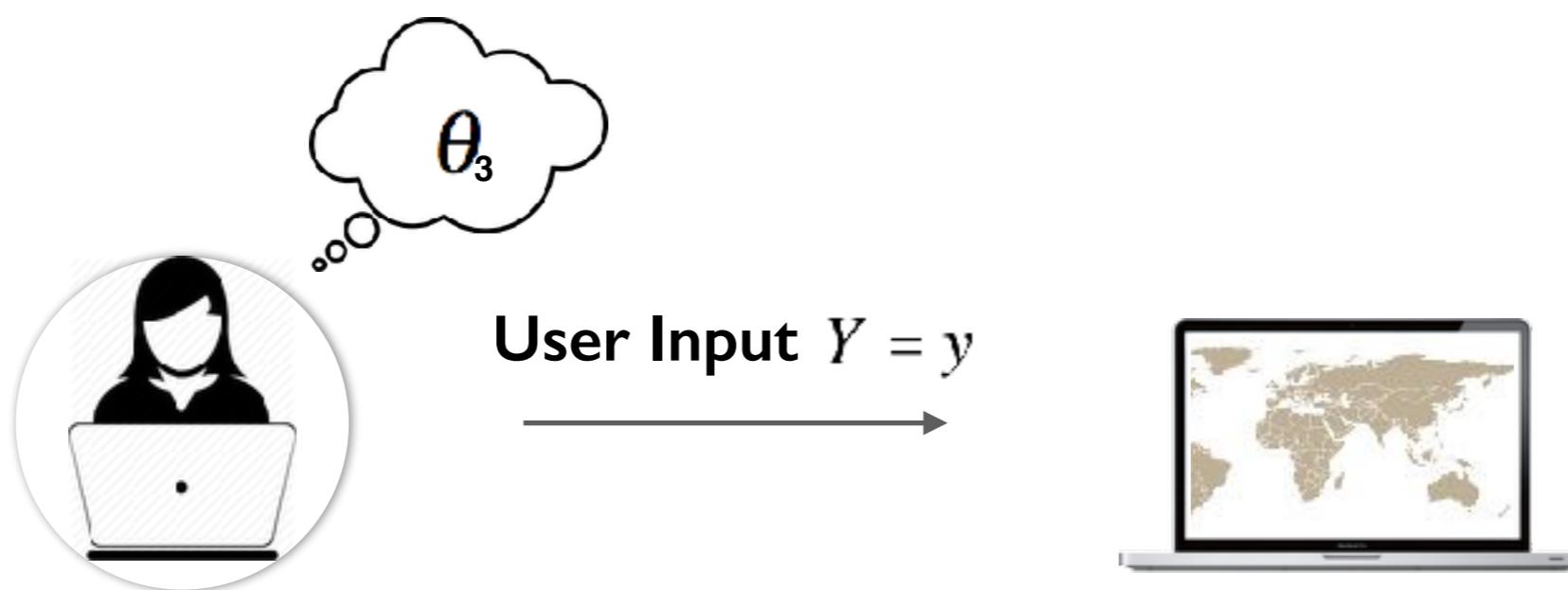
- Bayesian Information Gain (BIG) - Standard navigation



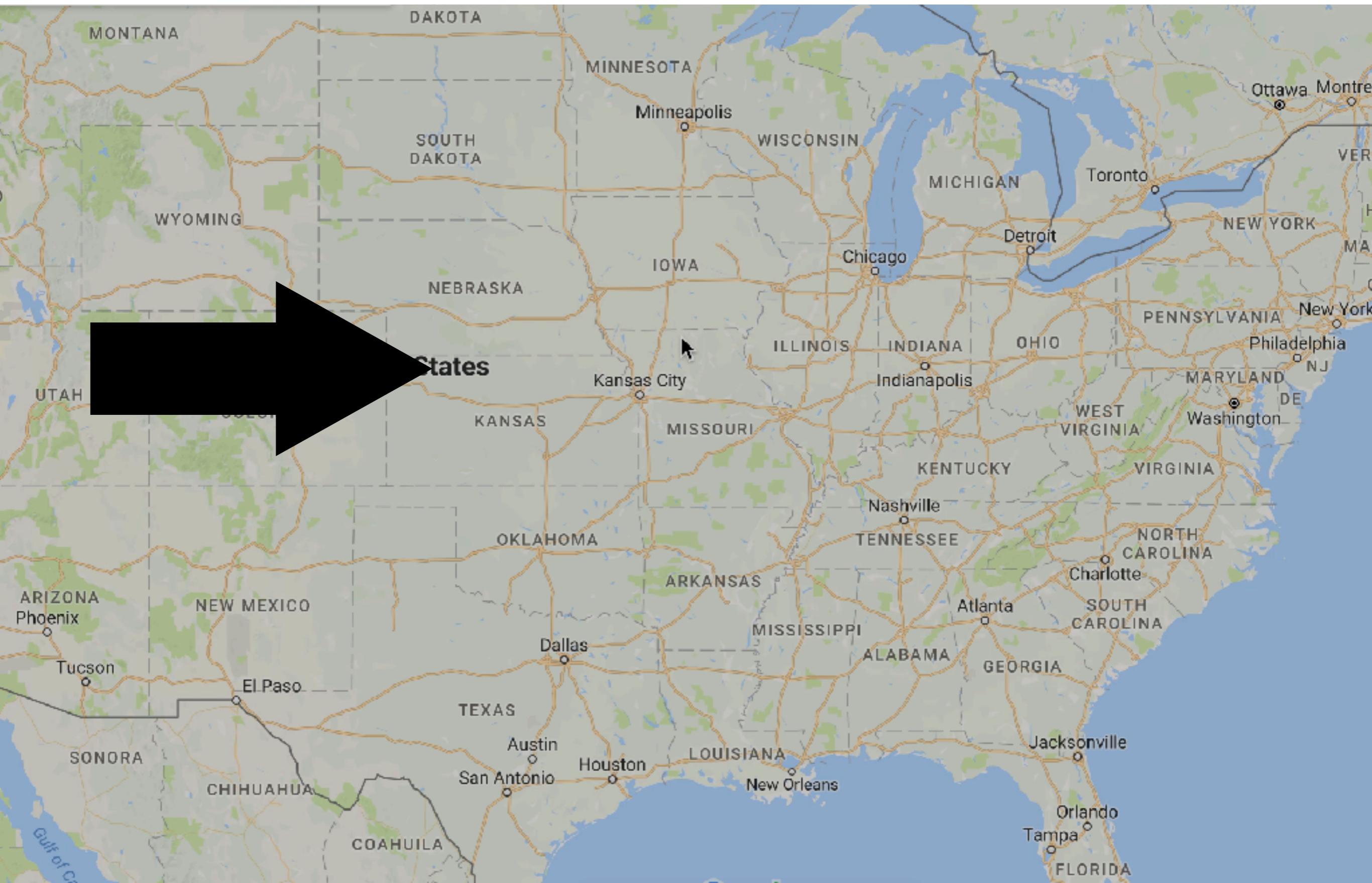
- Bayesian Information Gain (BIG) - Standard navigation



- Bayesian Information Gain (BIG) - Standard navigation



- Bayesian Information Gain (BIG) - Standard navigation



- Bayesian Information Gain (BIG) - Standard navigation



View $X = x$

$$P(Y = y | \Theta = \theta, X = x)$$

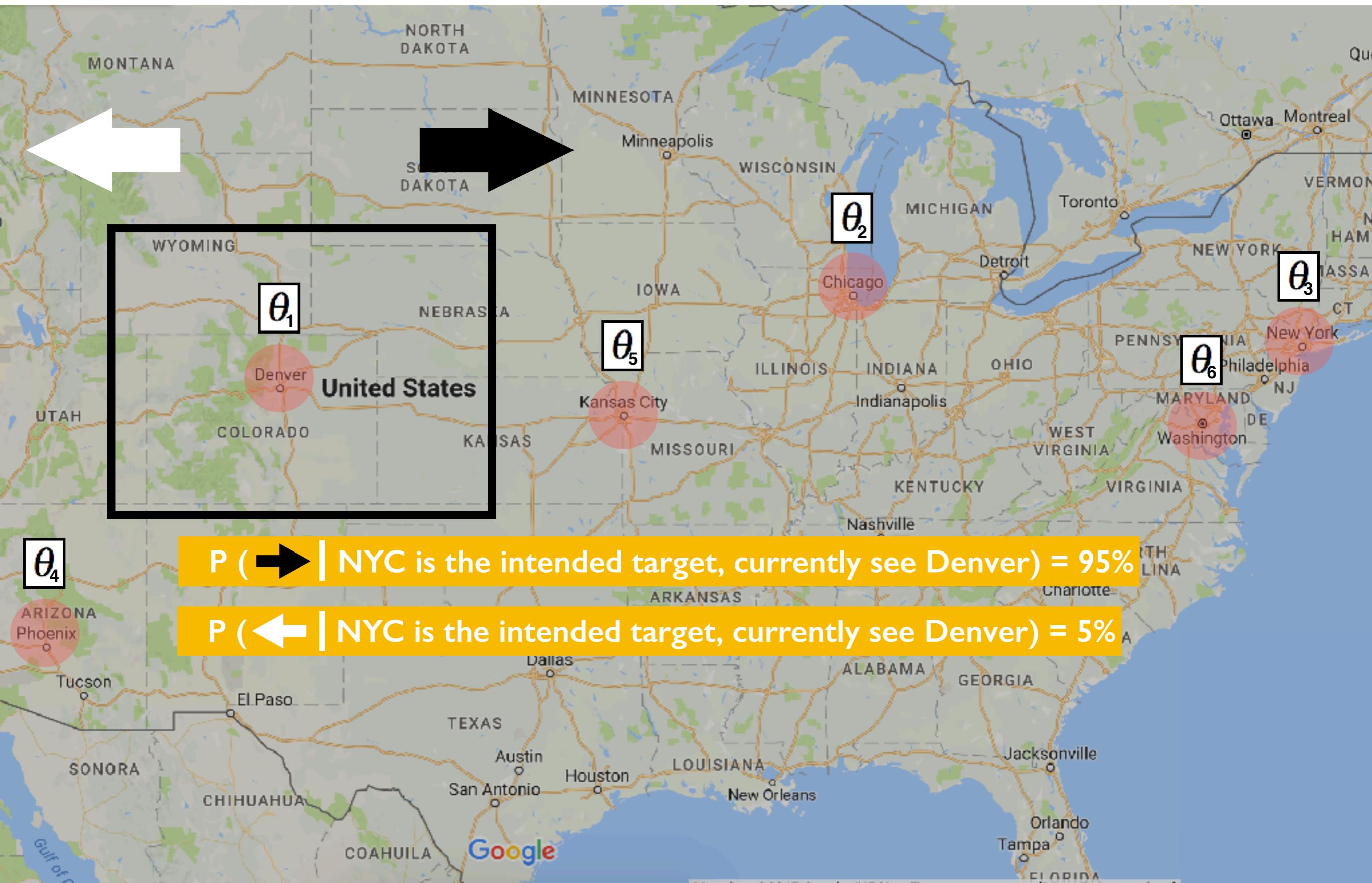
Interpret User Input



$$P(\Theta = \theta_i)$$

User Input $Y = y$

- Bayesian Information Gain (BIG) - Standard navigation



- Bayesian Information Gain (BIG) - Standard navigation



View $X = x$

$$P(\Theta = \theta | X = x, Y = y)$$

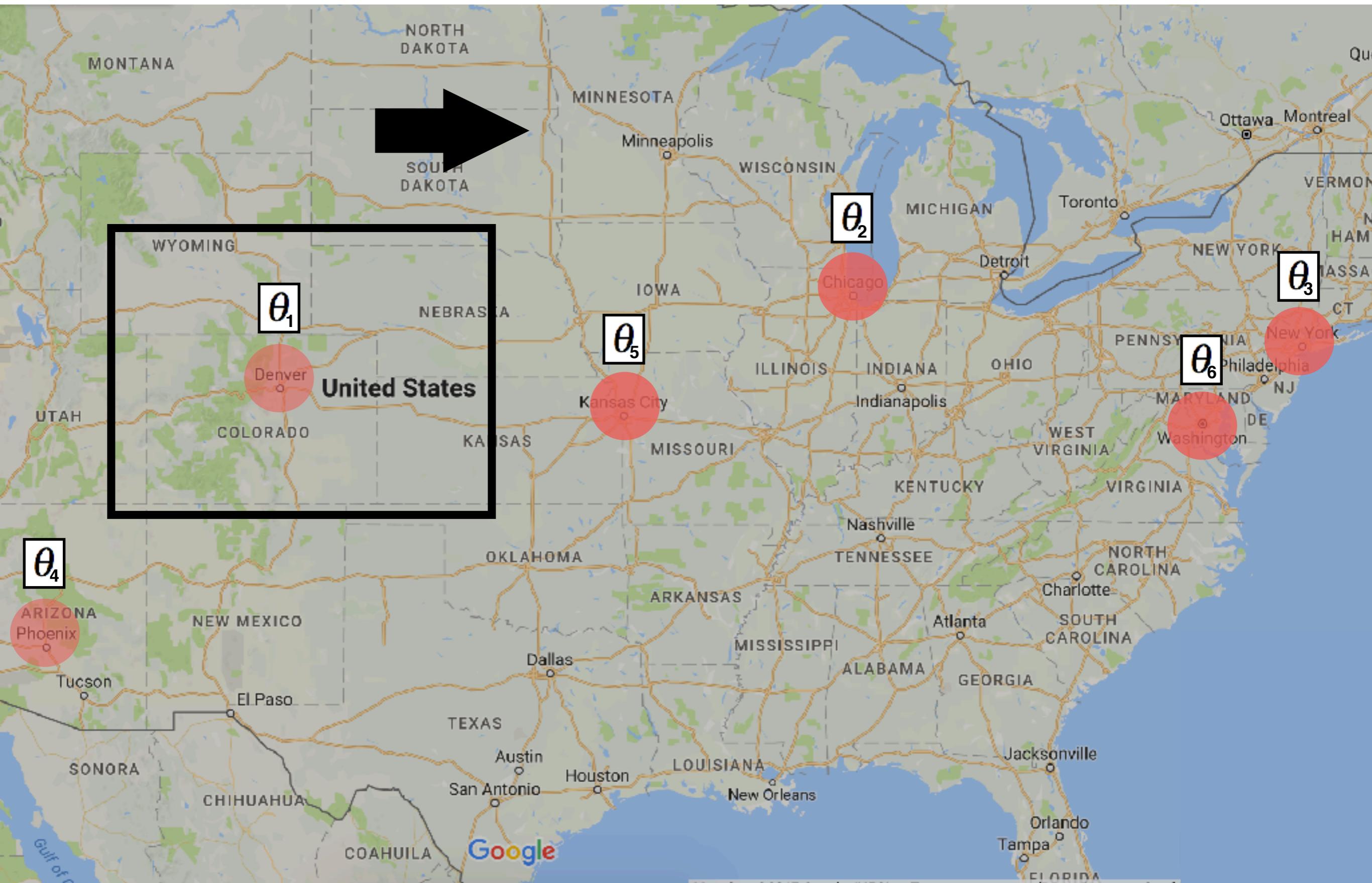
Update its knowledge



$$P(\Theta = \theta_i)$$

User Input $Y = y$

- Bayesian Information Gain (BIG) - Standard navigation



- Bayesian Information Gain (BIG) - Standard navigation

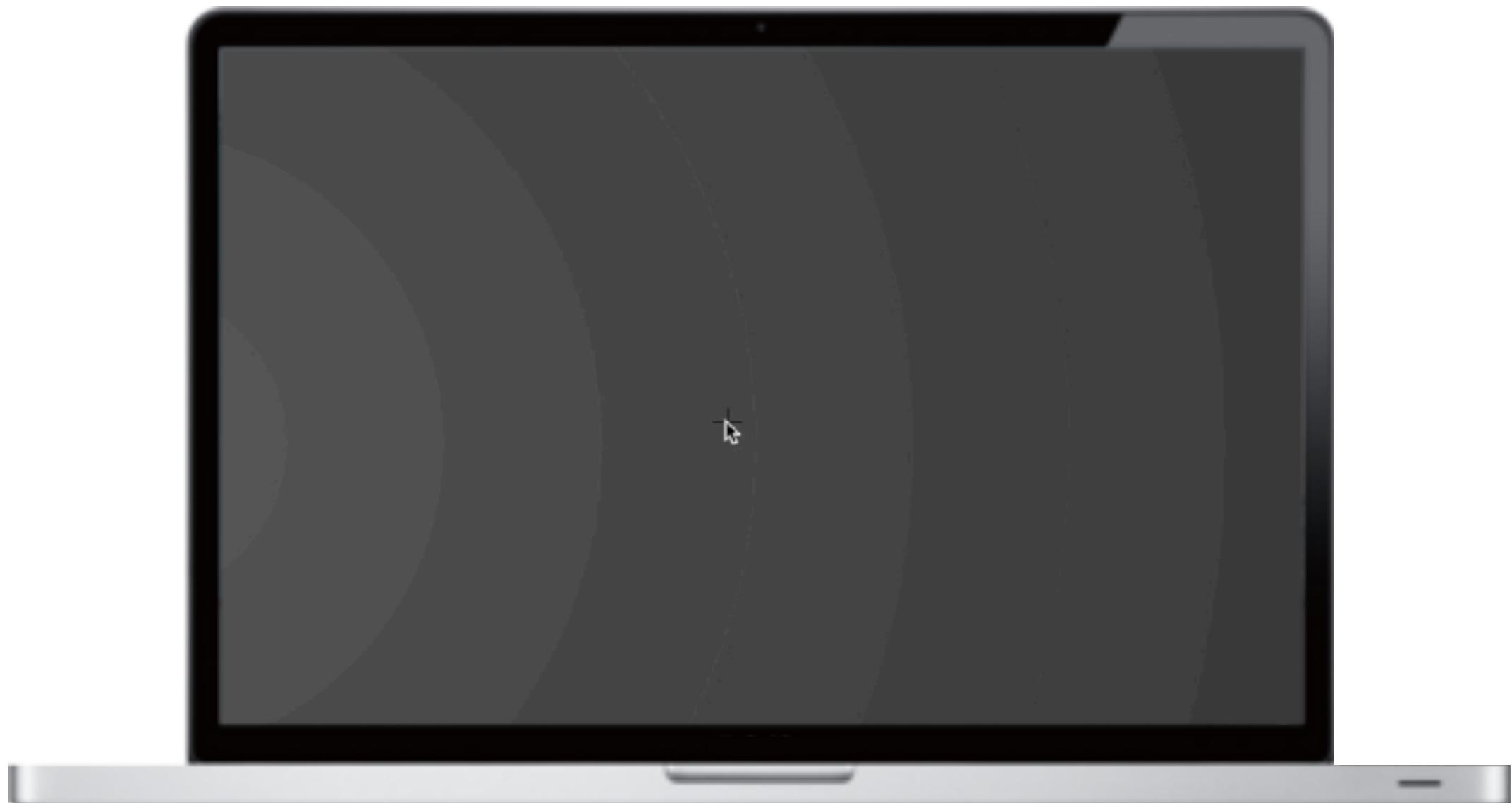
- * The computer's **Uncertainty** about the user's goal

$$H(\Theta) = -\sum_{i=1}^n P(\Theta = \theta_i) \log_2 P(\Theta = \theta_i)$$

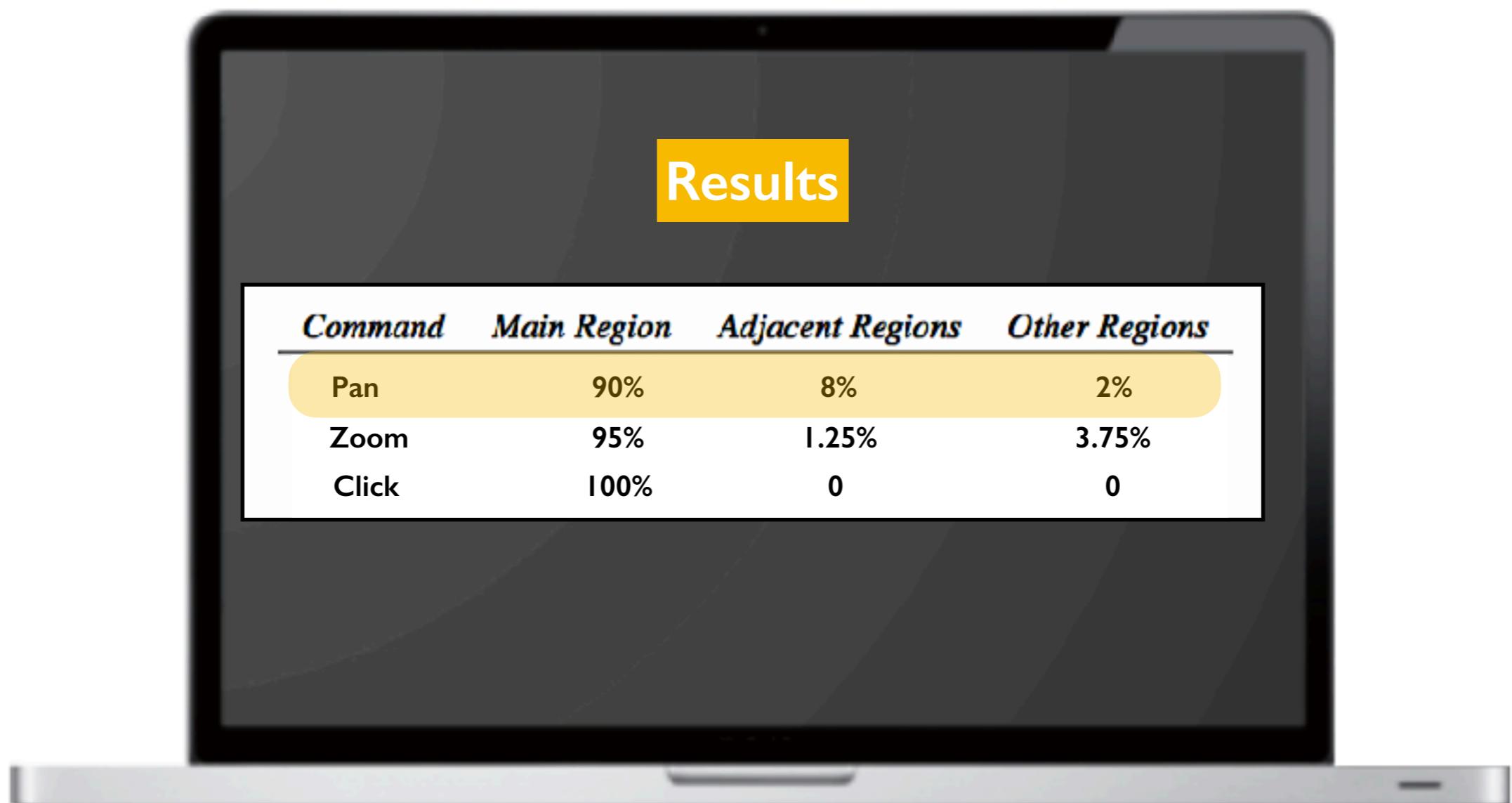
- * The computer's **updated knowledge** about the user's goal

$$P(\Theta = \theta | X = x, Y = y) = \frac{P(Y = y | \Theta = \theta, X = x)P(\Theta = \theta)}{P(Y = y | X = x)}$$

- Bayesian Information Gain (BIG) - Standard navigation
- * A calibration session to understand user behavior $P(Y = y | \Theta = \theta, X = x)$



- Bayesian Information Gain (BIG) - Standard navigation



- Bayesian Information Gain (BIG) - Standard navigation

- * The computer's **Uncertainty** about the user's goal

$$H(\Theta) = -\sum_{i=1}^n P(\Theta = \theta_i) \log_2 P(\Theta = \theta_i)$$

- * The computer's **updated knowledge** about the user's goal

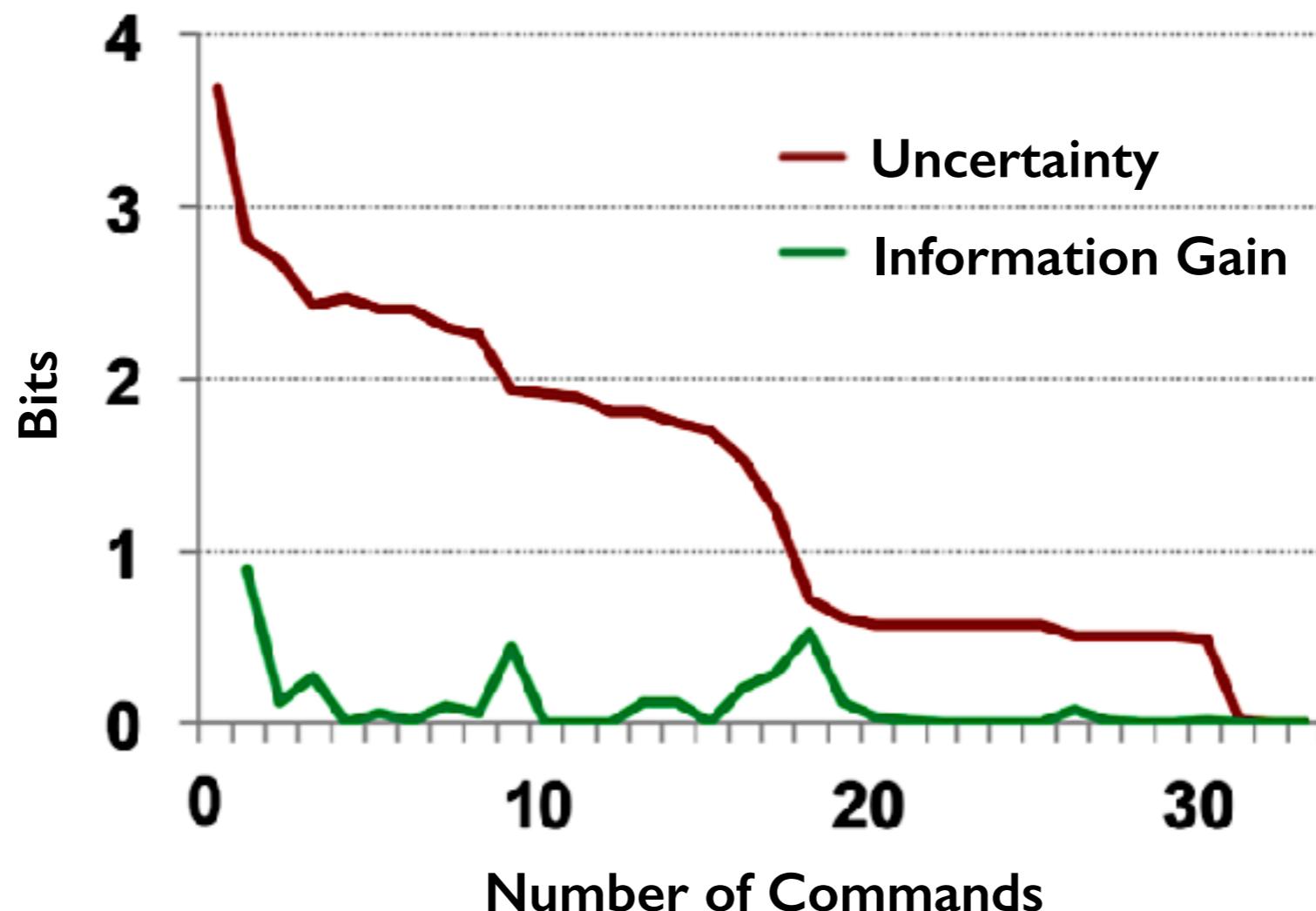
$$P(\Theta = \theta | X = x, Y = y) = \frac{P(Y = y | \Theta = \theta, X = x)P(\Theta = \theta)}{P(Y = y | X = x)}$$

- * The **information** in the user's input for reducing the computer's uncertainty

$$IG(\Theta | X = x, Y = y) = H(\Theta) - H(\Theta | X = x, Y = y)$$

- Bayesian Information Gain (BIG) - Standard navigation

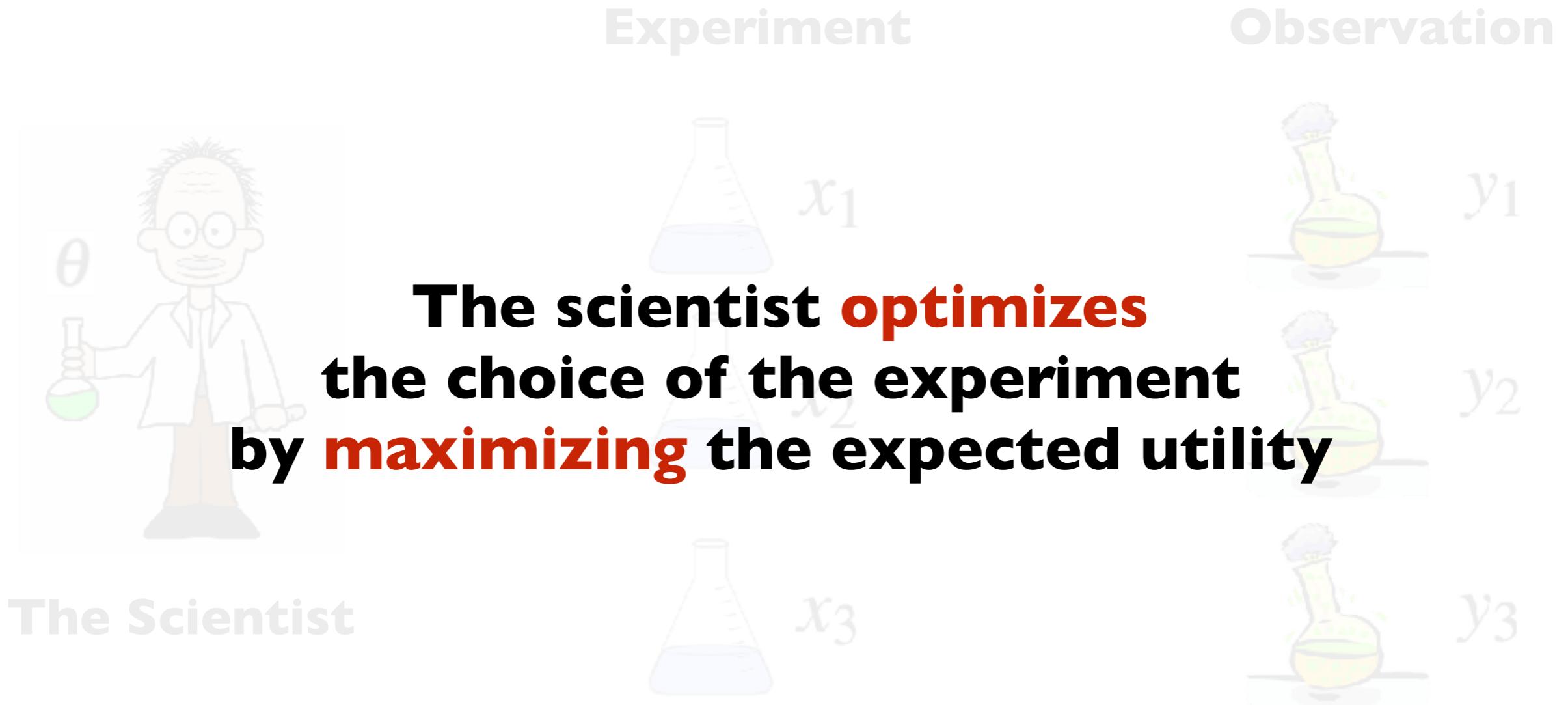
- * Executes the user input only Multiscale navigation
- * Each user input does not provide much information for the computer to know her goal



- Bayesian Information Gain (BIG) - BIGnav

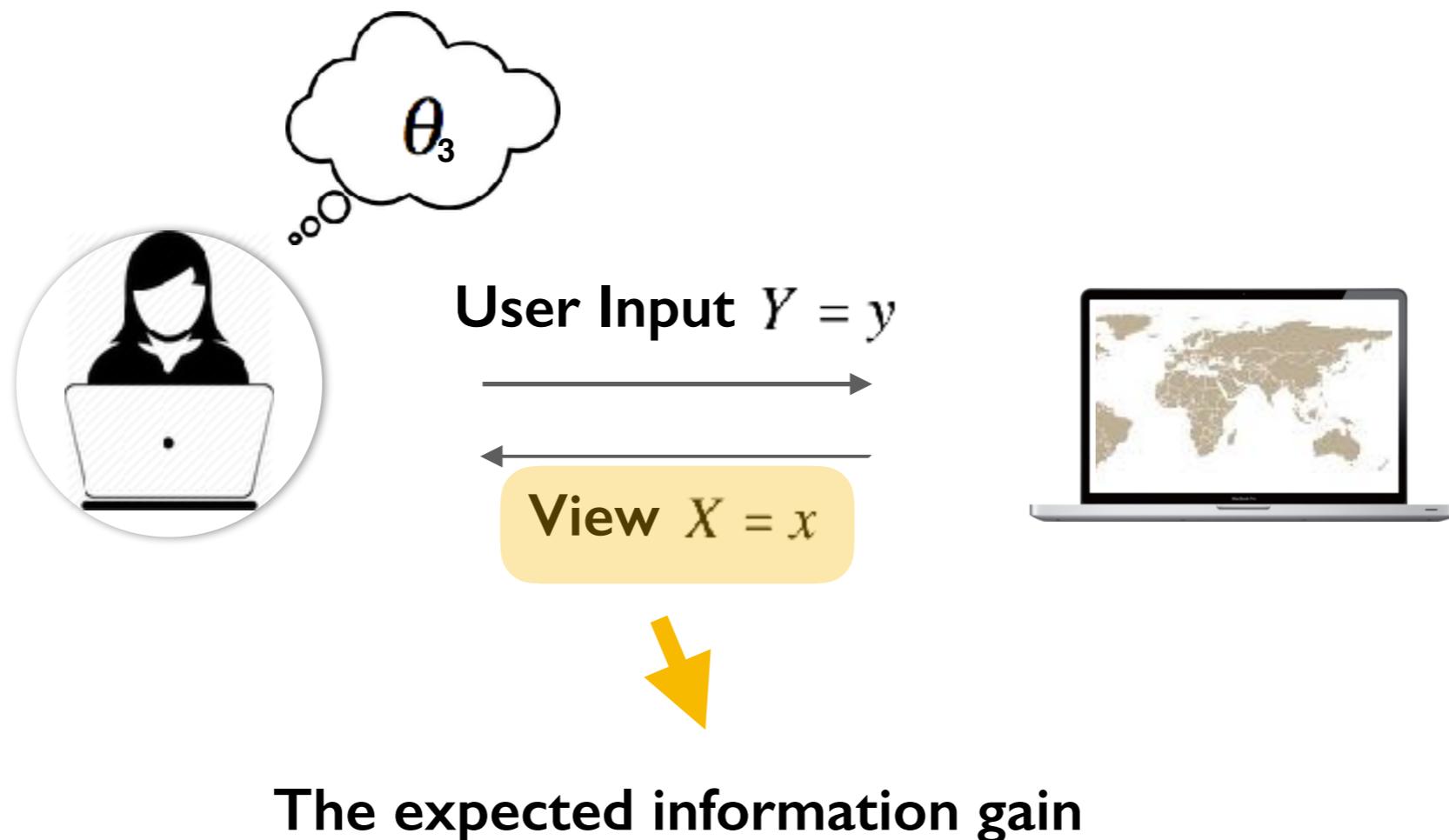
**Can we challenge users
to give more information?**

- Bayesian Information Gain (BIG) - BIGnav

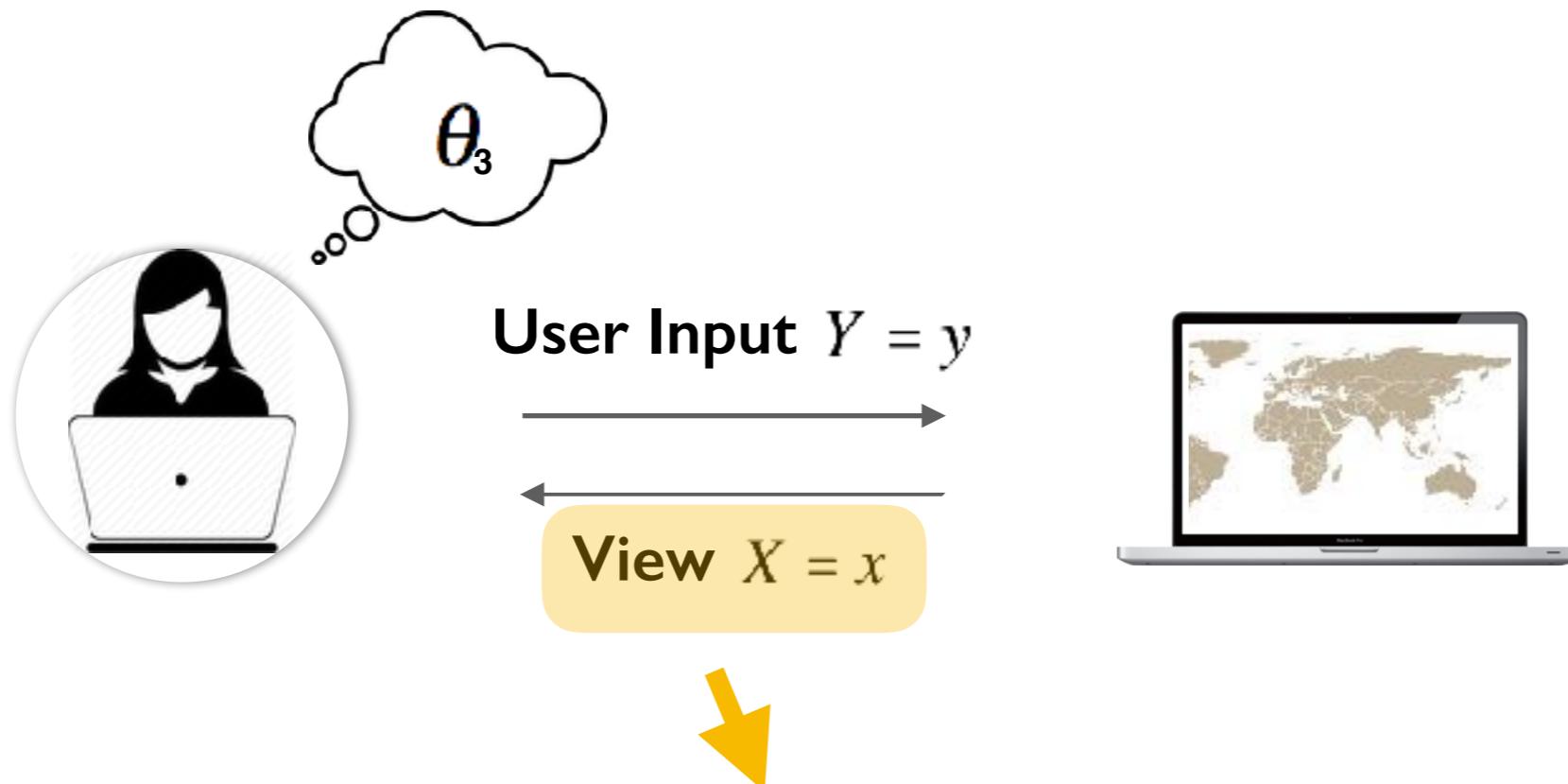


On a Measure of the Information Provided by an Experiment (Lindley 1956)

- Bayesian Information Gain (BIG) - BIGnav

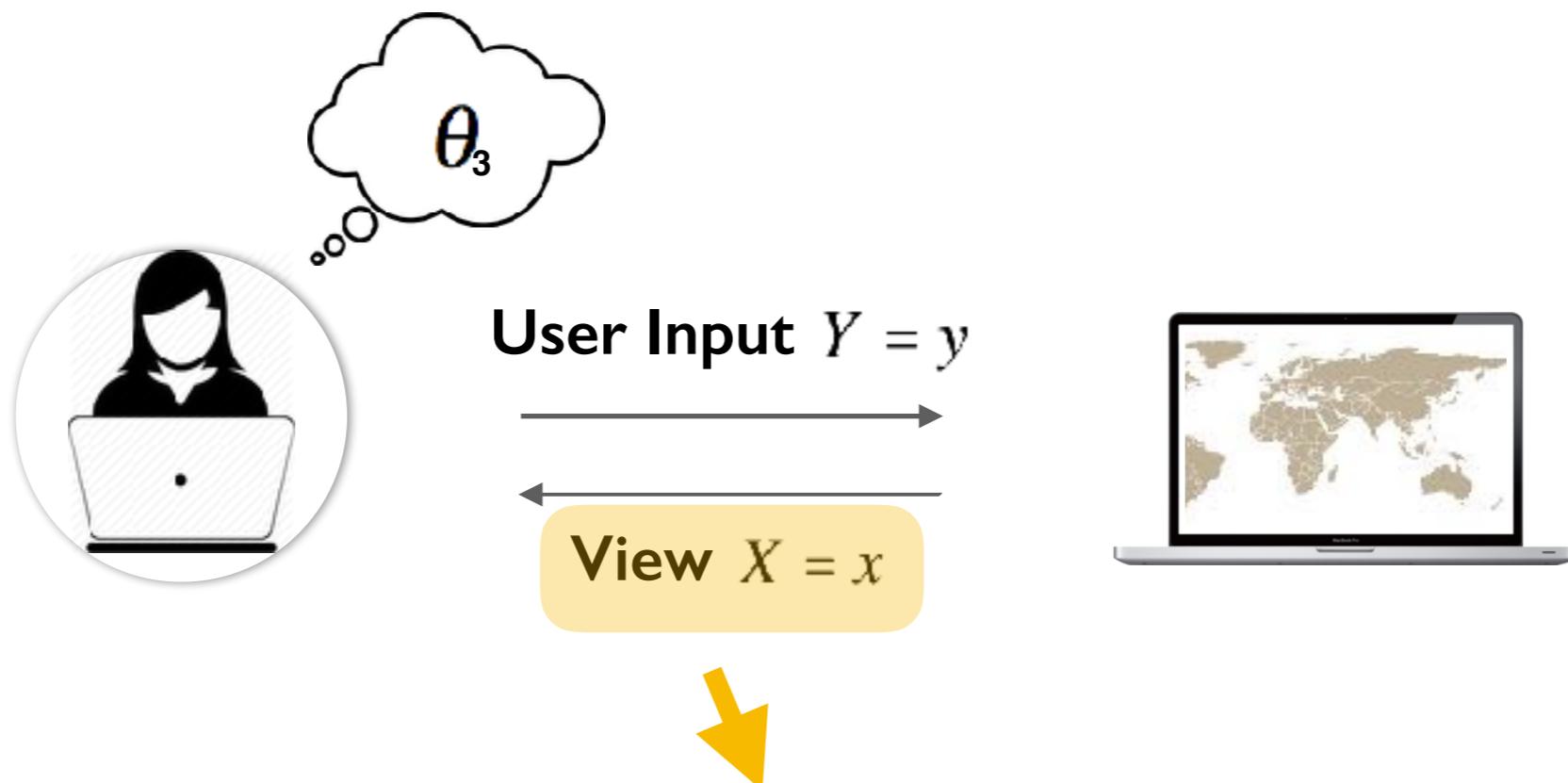


- Bayesian Information Gain (BIG) - BIGnav



**Choose the feedback (a view) that
maximizes the expected
information gain from the user's
subsequent input**

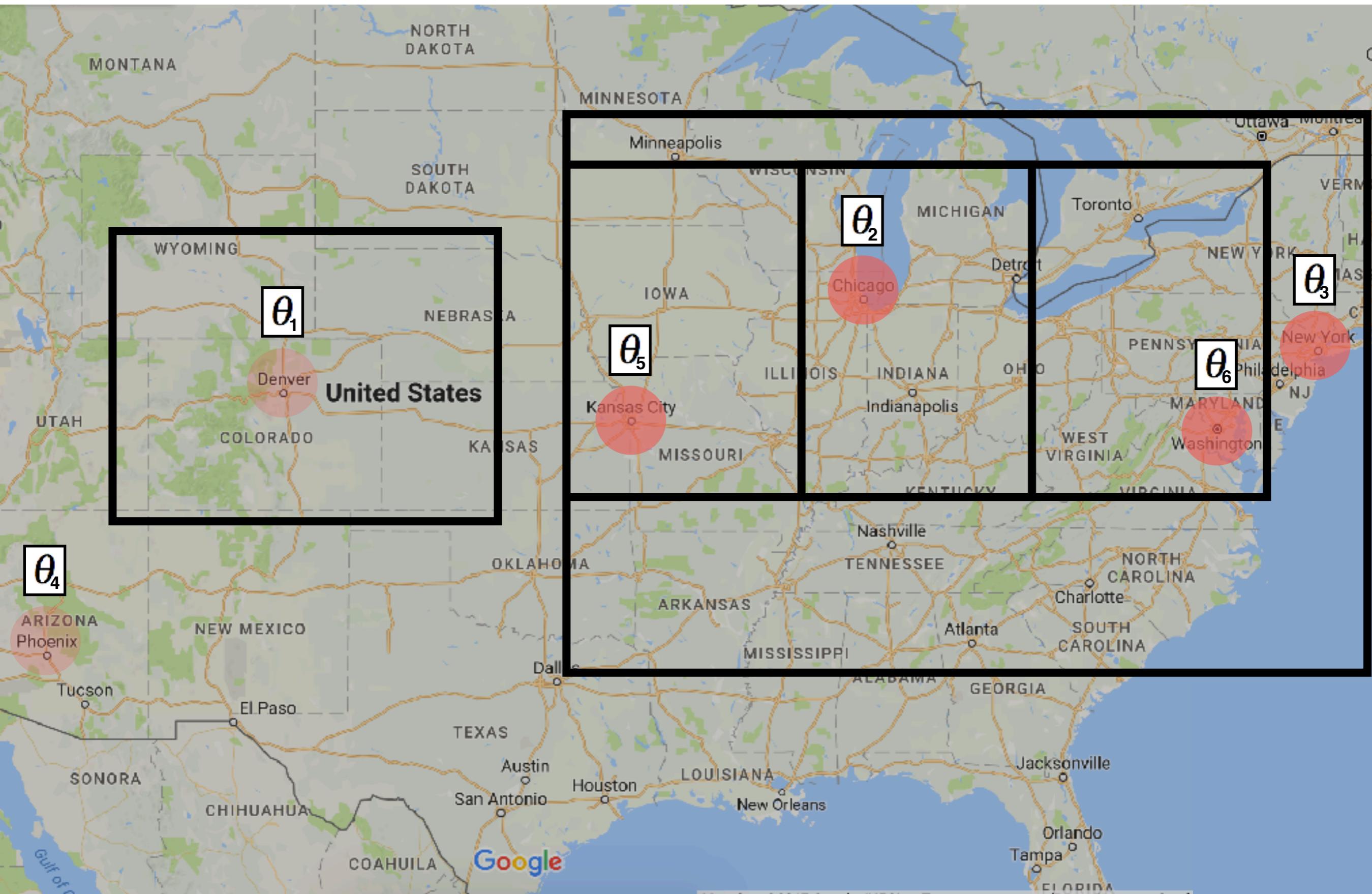
- Bayesian Information Gain (BIG) - BIGnav



**Go over all possible feedback,
and find the one that maximizes
the expected information gain**

$$IG(\Theta|X = x, Y) = H(\Theta) - H(\Theta|X = x, Y)$$

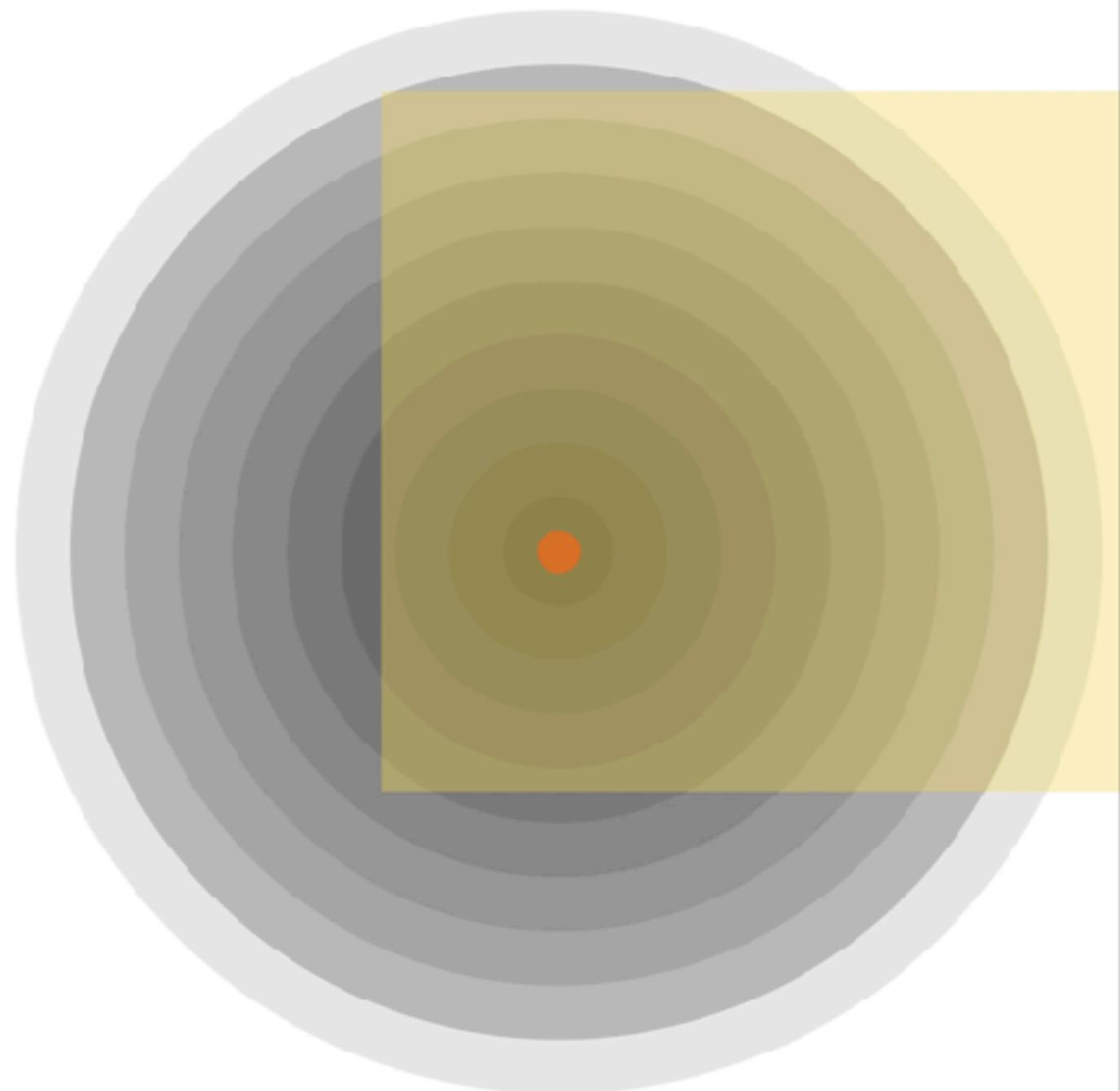
- Bayesian Information Gain (BIG) - BIGnav



- Bayesian Information Gain (BIG) - BIGnav

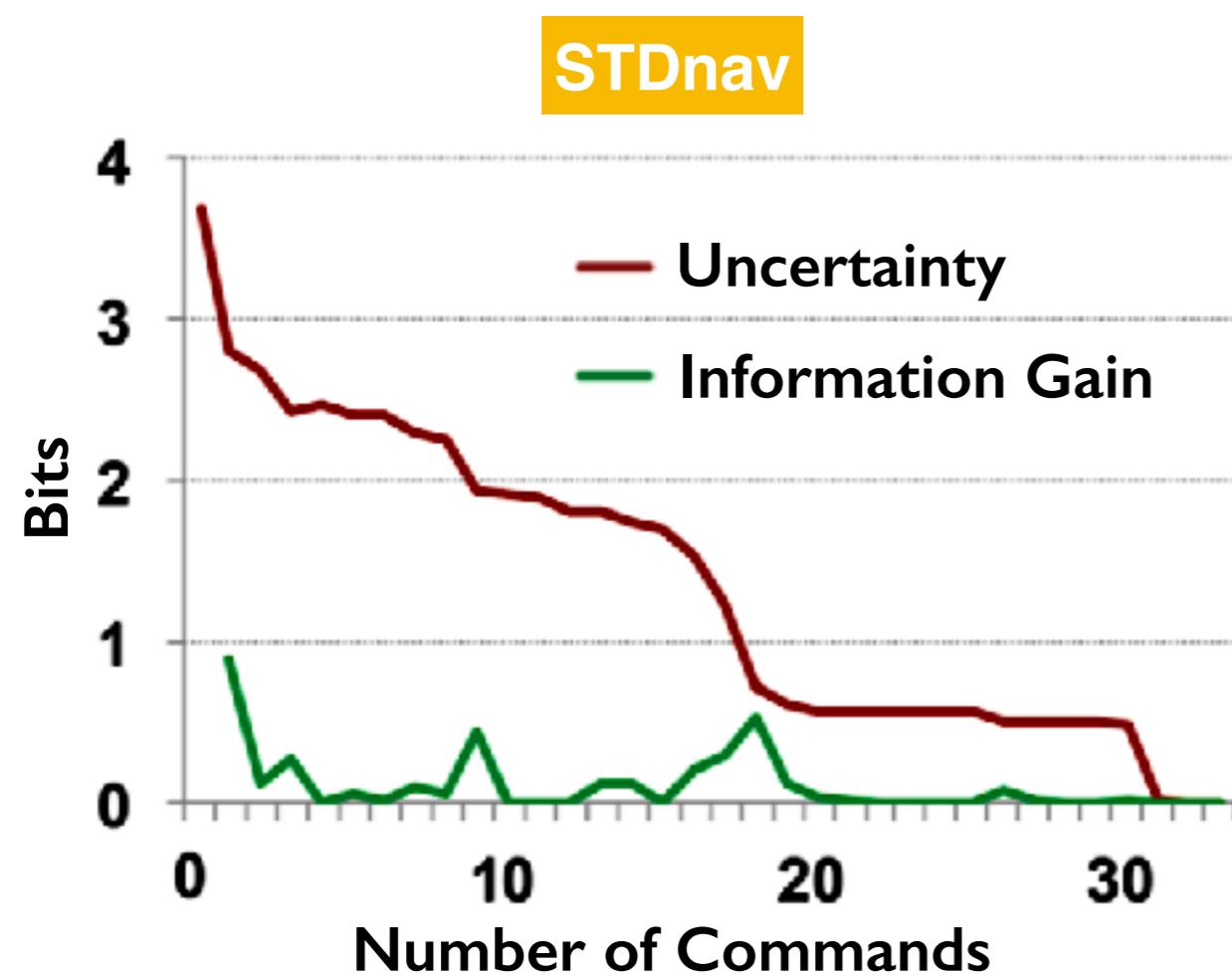


View



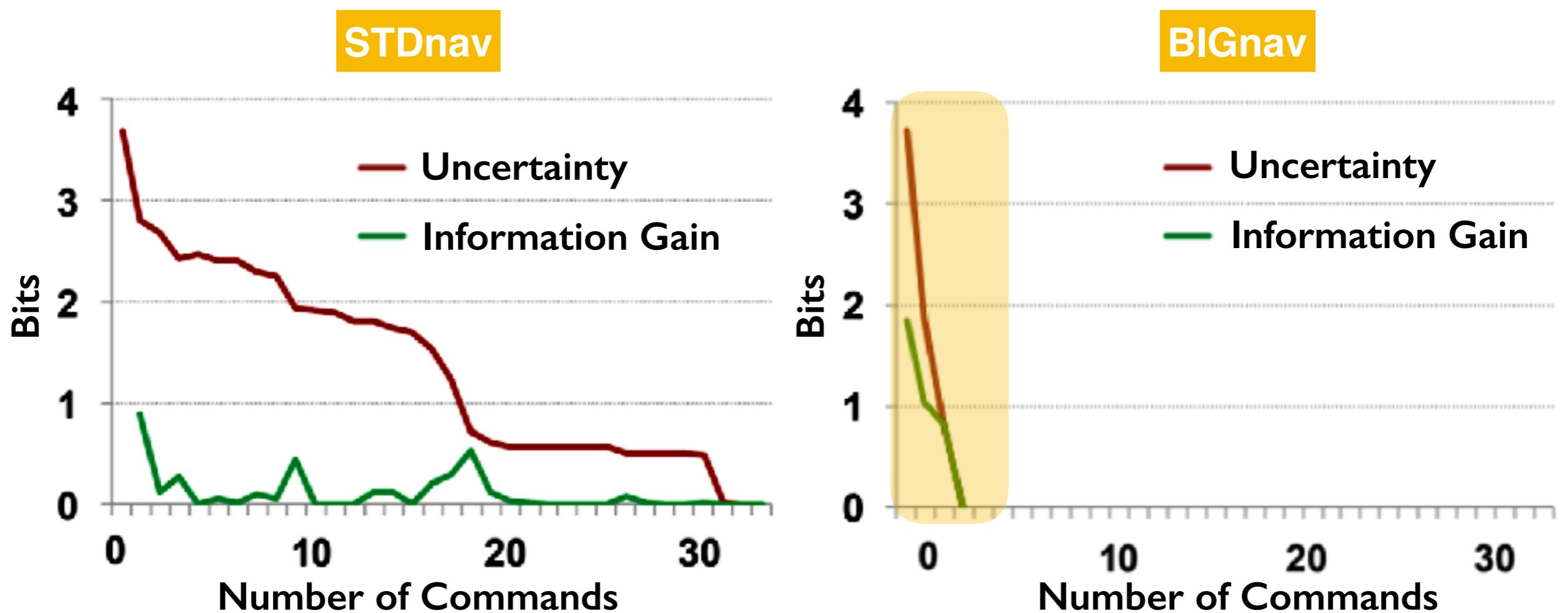
- Bayesian Information Gain (BIG) - BIGnav

- * **BIGnav** gains maximum information from each user input



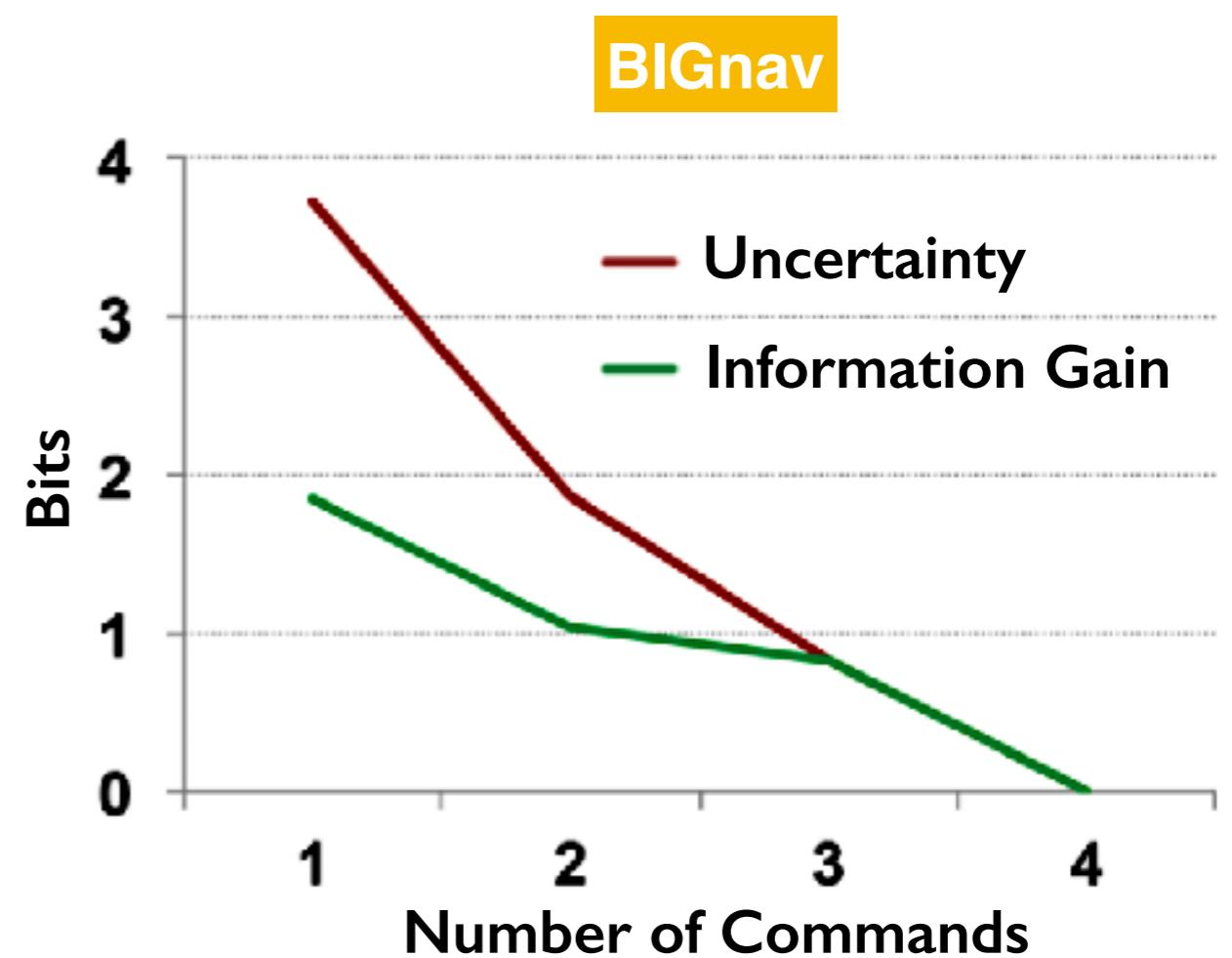
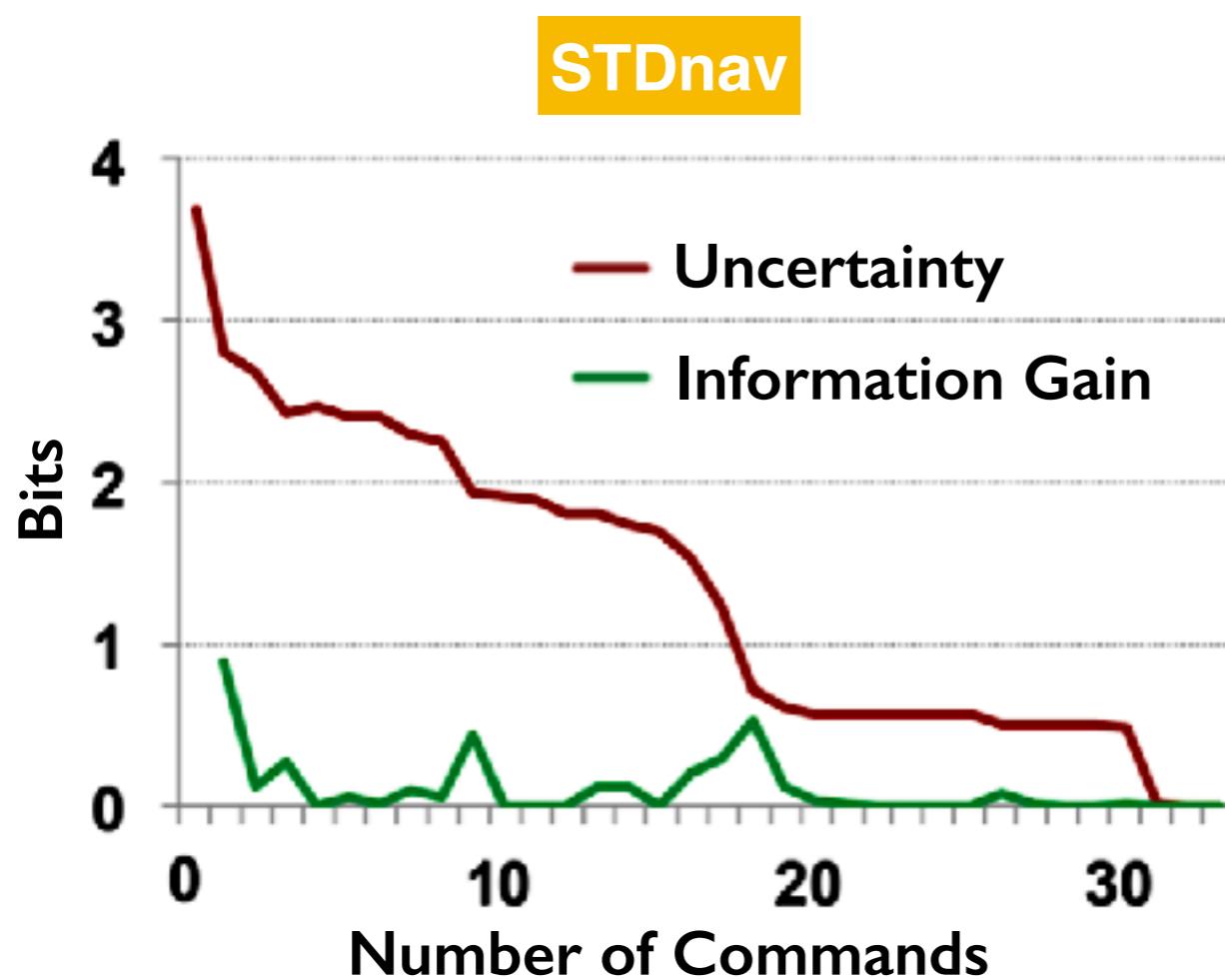
- Bayesian Information Gain (BIG) - BIGnav

- * **BIGnav** gains maximum information from each user input



- Bayesian Information Gain (BIG) - BIGnav

- * **BIGnav** gains maximum information from each user input





BIGmap

A map application - “3 steps to go to Paris”.

Europe map featuring large cities with their population as distribution.



A map application - “Navigate to Helsinki”.

Europe map featuring large cities with their population as distribution.

Exercise



Coffee Break



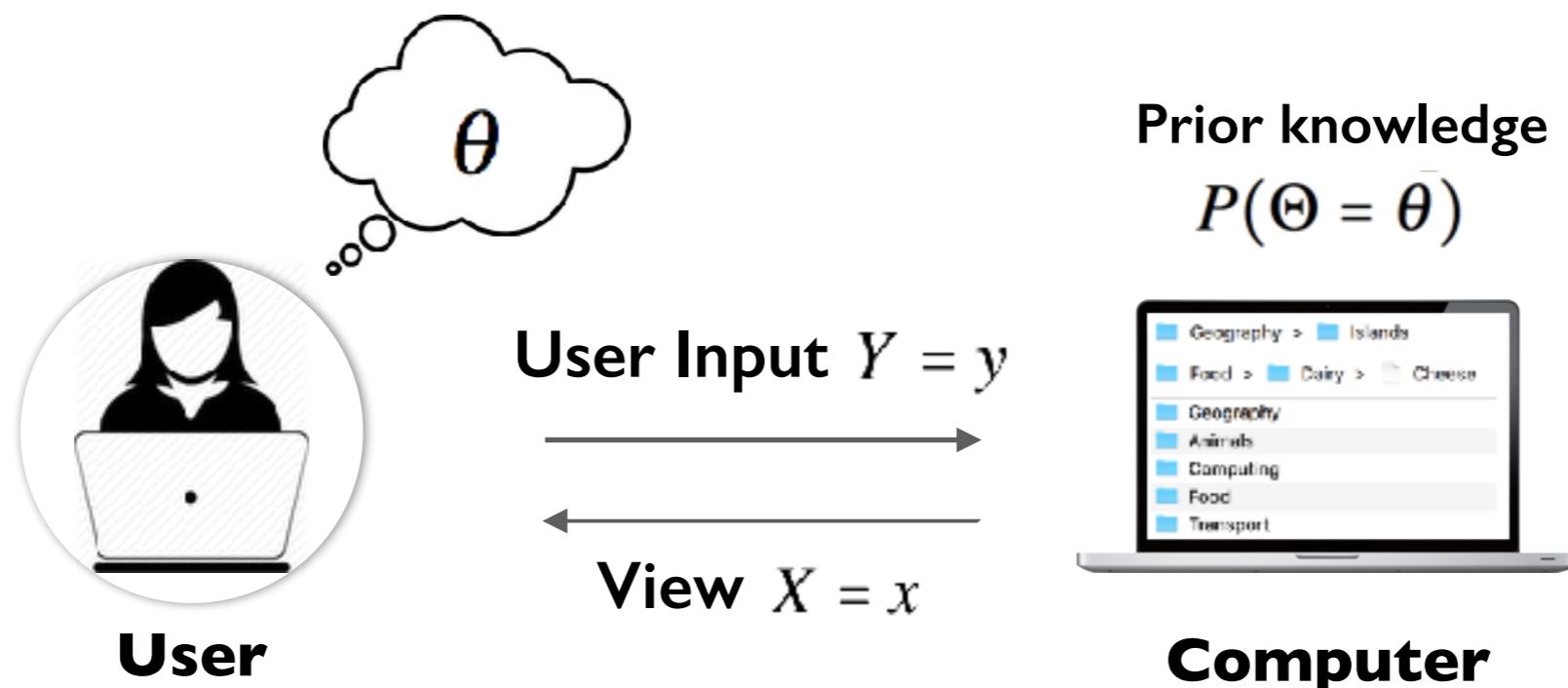
- Bayesian Information Gain (BIG) - BIGnav

More efficient but...
Higher cognitive load

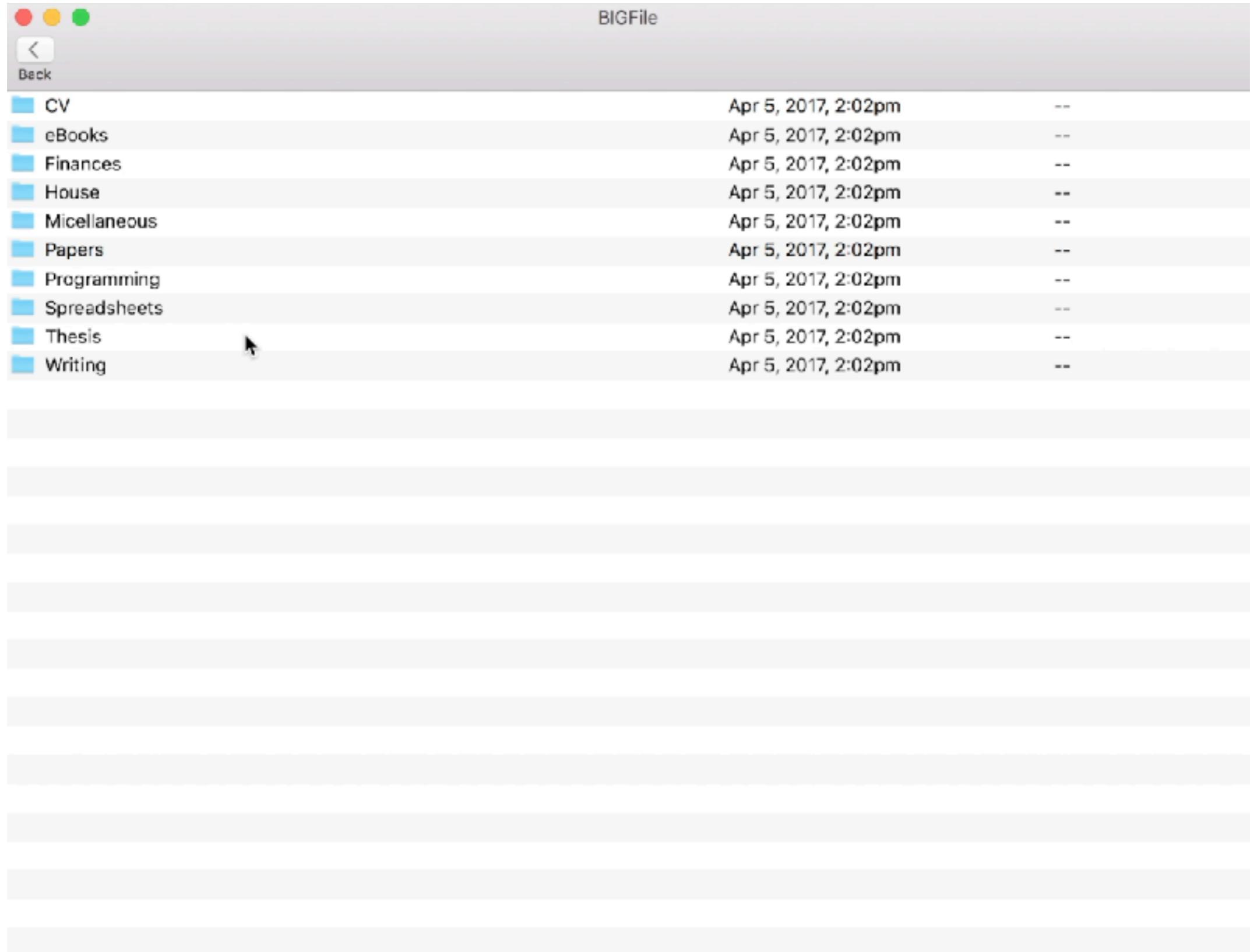


- Bayesian Information Gain (BIG) - BIGFile

- * Leverages the expected information gain $IG(\Theta|X = x, Y)$



- Bayesian Information Gain (BIG) - BIGFile



- Bayesian Information Gain (BIG) - BIGFile

BIGFile

Back

Geography > Islands > Tropical > Touristic > Large > Hawaii

Food > Dairy > Cheese

History > Inventions

Education > Curriculum > Masters > German

Estimated shortcuts

Geography	Apr 5, 2017, 2:02pm	--
Animals	Apr 5, 2017, 2:02pm	--
Computing	Apr 5, 2017, 2:02pm	--
Food	Apr 5, 2017, 2:02pm	--
Transport	Apr 5, 2017, 2:02pm	--
Health	Apr 5, 2017, 2:02pm	
Entertainment	Apr 5, 2017, 2:02pm	
History	Apr 5, 2017, 2:02pm	--
Plants	Apr 5, 2017, 2:02pm	--
People	Apr 5, 2017, 2:02pm	--
House & Home	Apr 5, 2017, 2:02pm	--
Education	Apr 5, 2017, 2:02pm	--
Budget	Apr 5, 2017, 2:02pm	60k
Essay	Apr 5, 2017, 2:02pm	60k
Paper	Apr 5, 2017, 2:02pm	60k
Article	Apr 5, 2017, 2:02pm	60k
Fireman	Apr 5, 2017, 2:02pm	60k
Building	Apr 5, 2017, 2:02pm	60k
Watch	Apr 5, 2017, 2:02pm	60k
Plan	Apr 5, 2017, 2:02pm	60k
Footstep	Apr 5, 2017, 2:02pm	60k
Camera	Apr 5, 2017, 2:02pm	60k
Cardboard	Apr 5, 2017, 2:02pm	60k
Photo	Apr 5, 2017, 2:02pm	60k
Brick	Apr 5, 2017, 2:02pm	60k

The usual hierarchy



Back

Geography >	Islands >	Tropical >	Touristic >	Large >	Hawaii
Food >	Dairy >	Cheese			
History >	Inventions				
Education >	Curriculum >	Masters >	German		

Geography	Apr 5, 2017, 2:02pm	--
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Photo	Apr 5, 2017, 2:02pm	60k

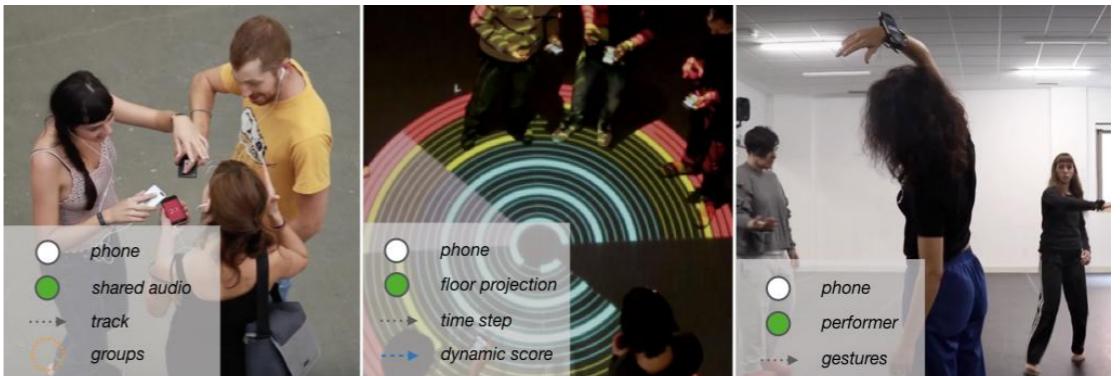
Having direct access to the target

- Bayesian Information Gain (BIG) - BIGnav, BIGFile

**From goal-directed interaction
To computational creativity**



• Collective Music Making

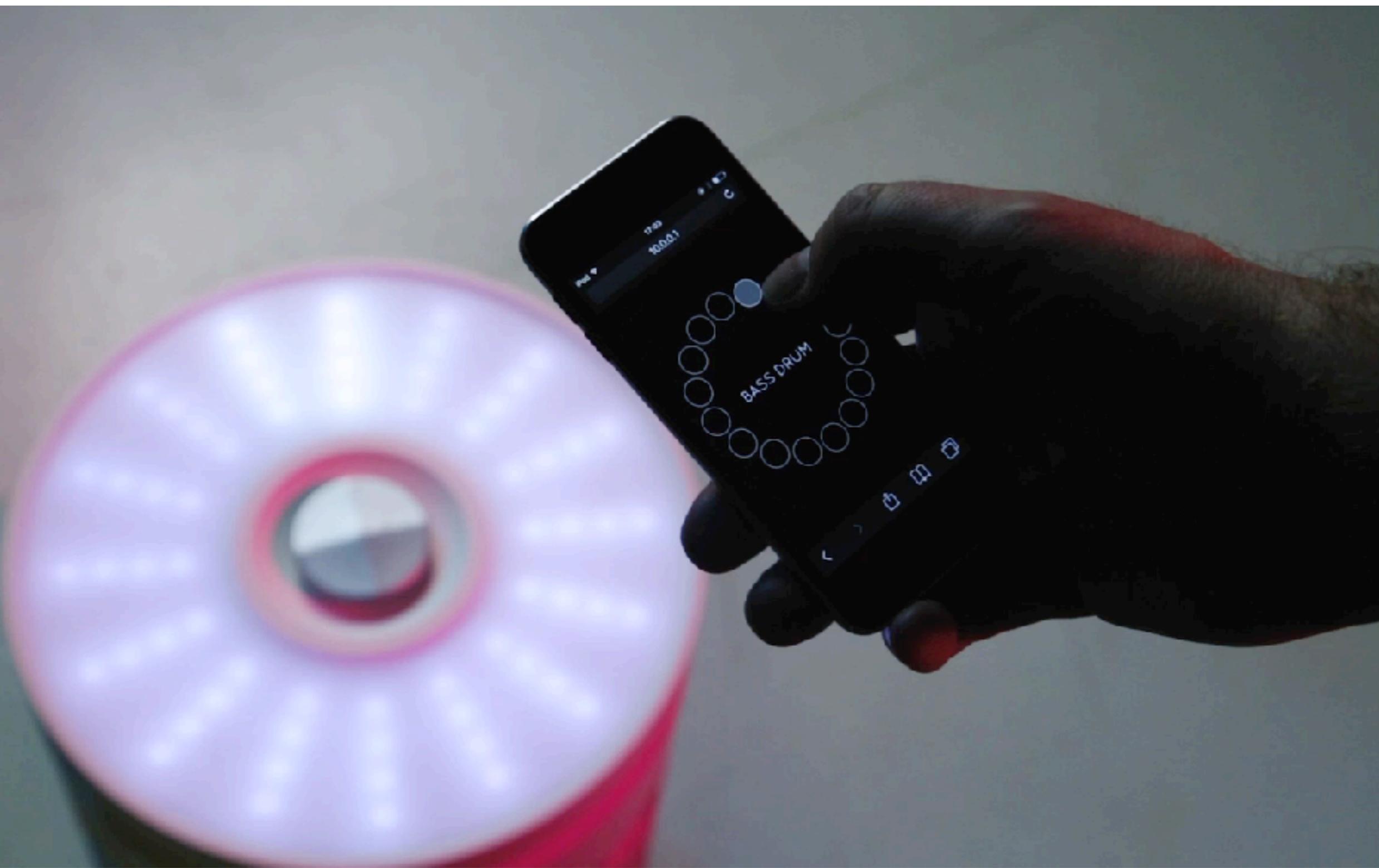


<http://cosima.ircam.fr>

coord. Norbert Schnell



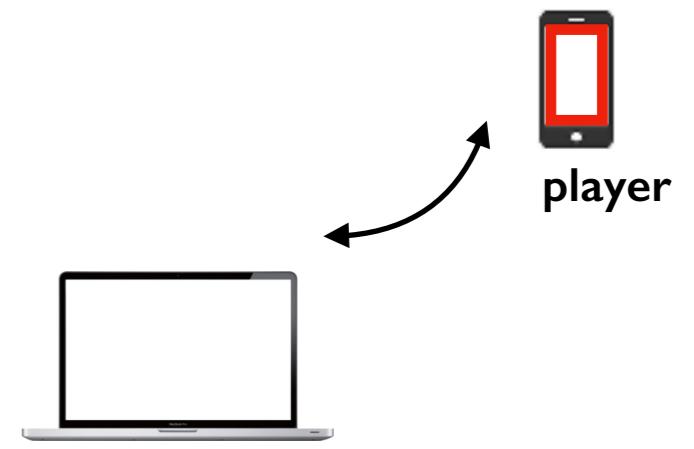
- Coloop (2017) - collective sequencer



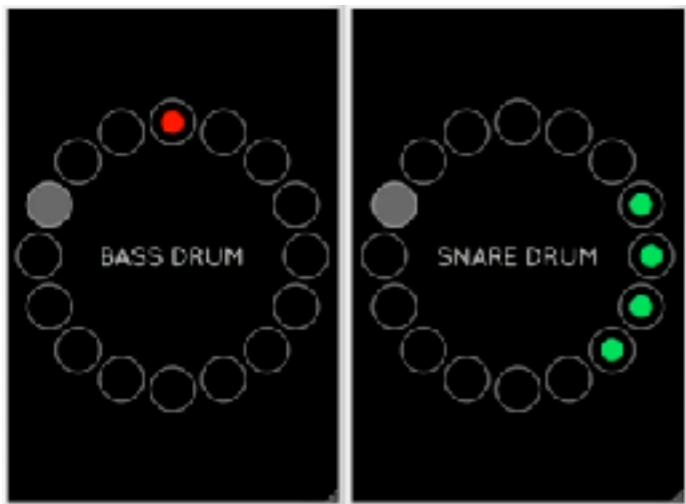
- Coloop (2017) - collective sequencer



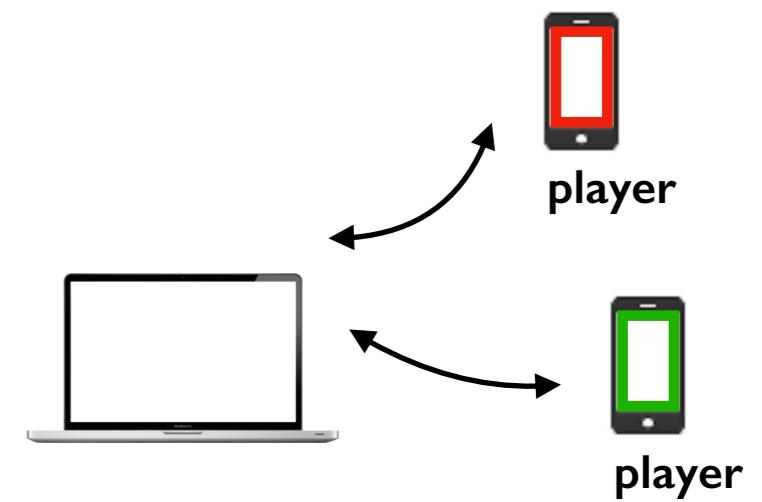
individual player interface



- Coloop (2017) - collective sequencer



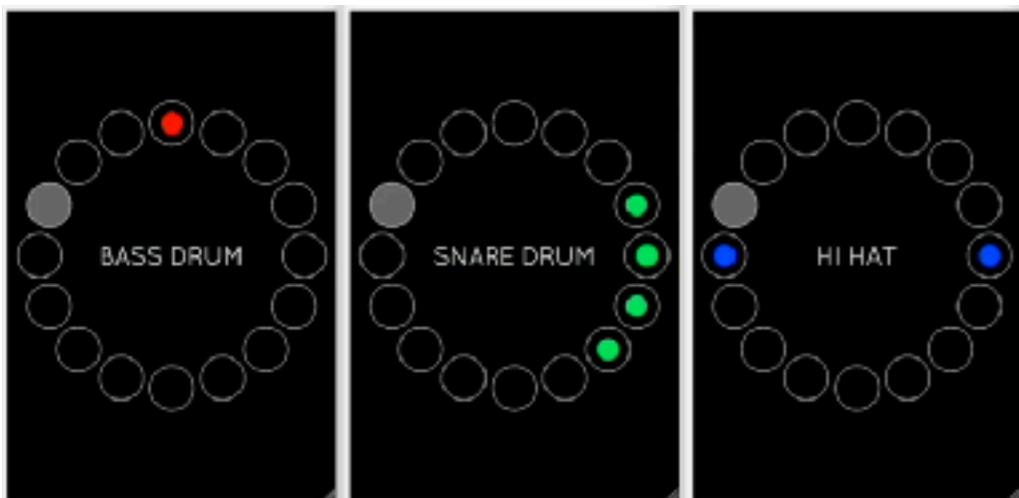
individual player interface



Synchronized clock

<https://github.com/collective-soundworks>

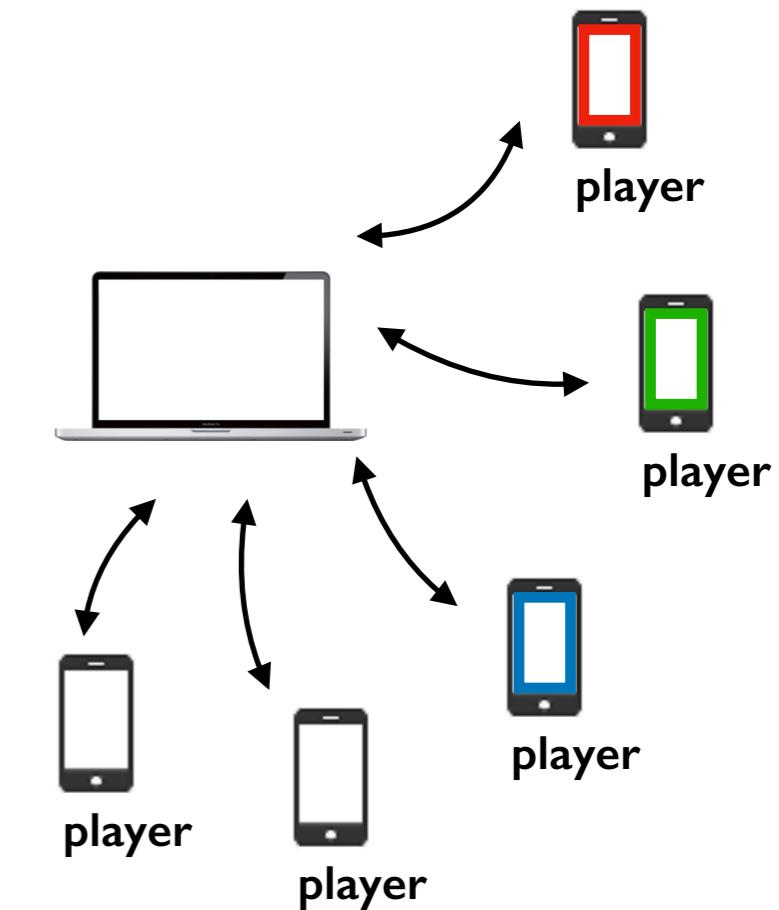
- Coloop (2017) - collective sequencer



individual player interface



Shared music object



Synchronized clock

<https://github.com/collective-soundworks>

- Coloop (2017) - collective sequencer

soirées sonores
centre pompidou
2018

- Coloop (2017) - collective sequencer

musical outcome is **collective**...

how to encourage social interaction?

...but most participants remained **individual**

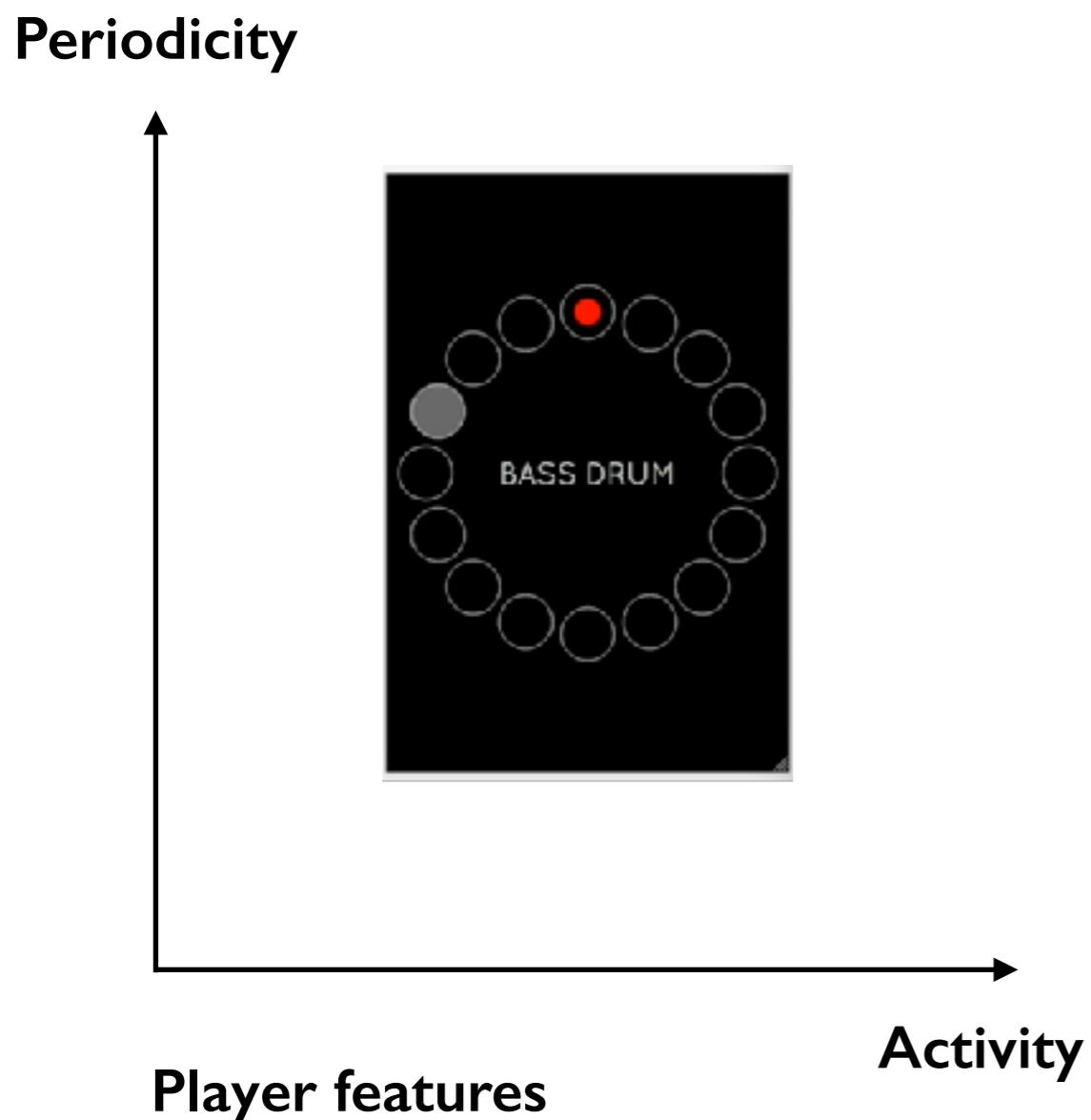
- Coloop (2017) - collective sequencer



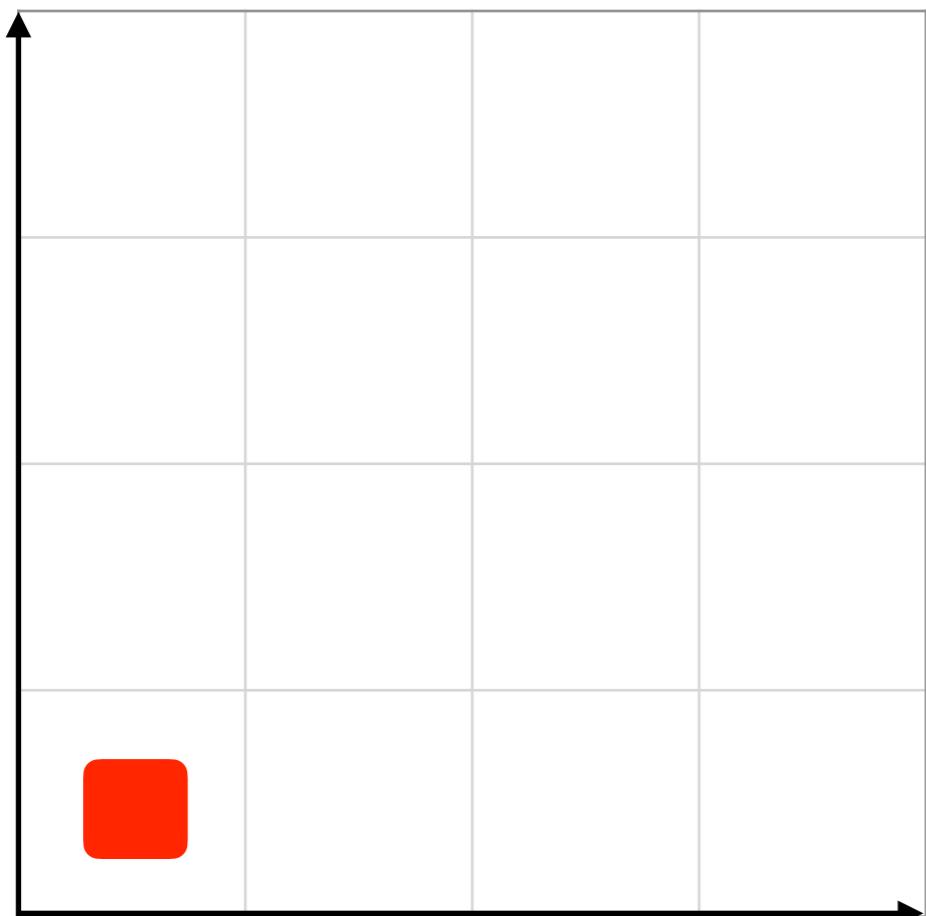
- Entrain (2019) - adaptive musical agent



- Entrain (2019) - adaptive musical agent



- Entrain (2019) - adaptive musical agent

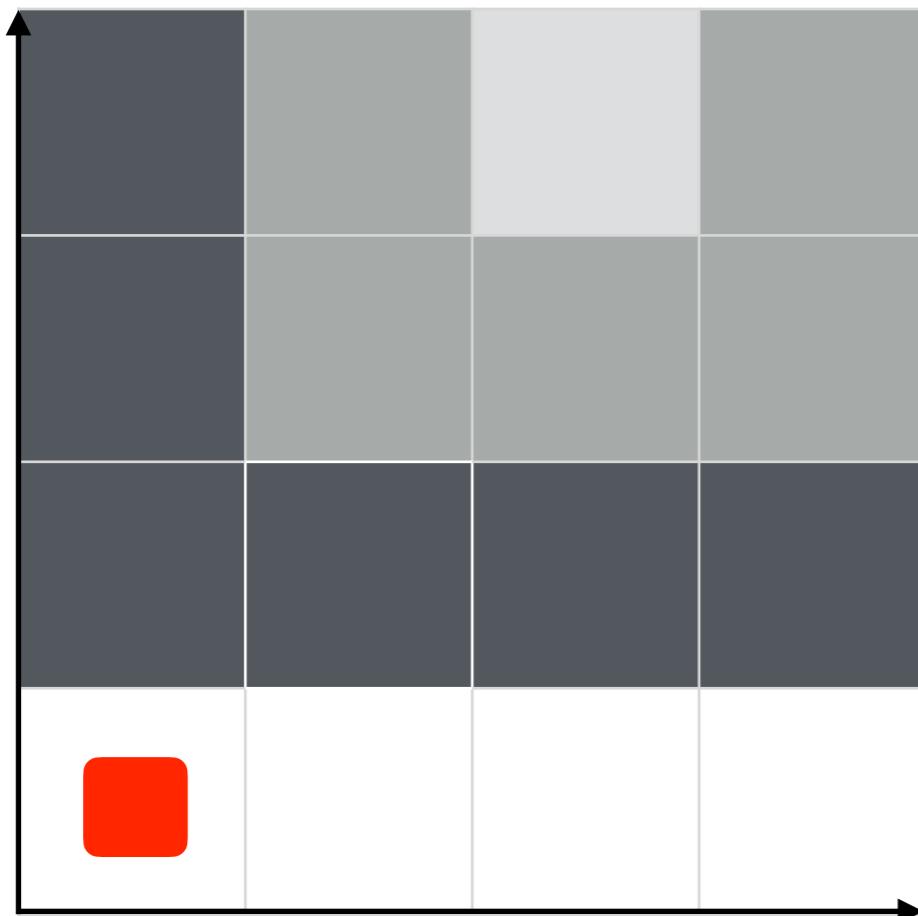


Player features

$$Y = y$$

Player state

- Entrain (2019) - adaptive musical agent

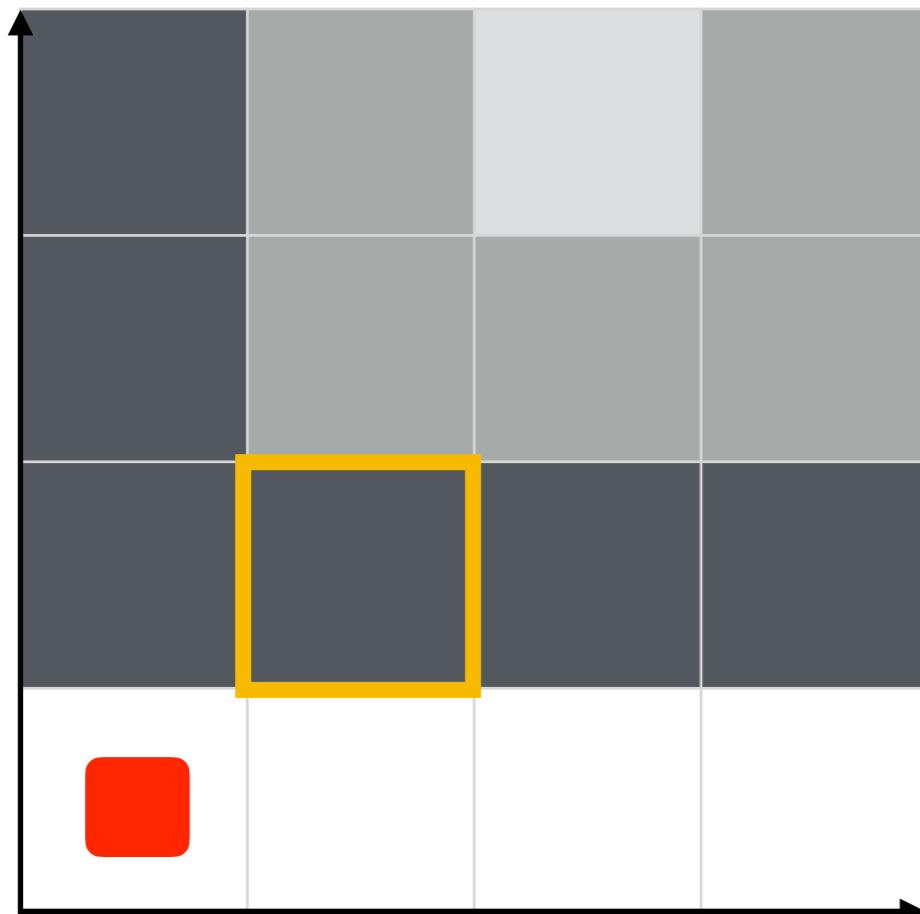


Player features

$P(\Theta = \theta)$ Probabilistic model

$Y = y$ Player state

- Entrain (2019) - adaptive musical agent



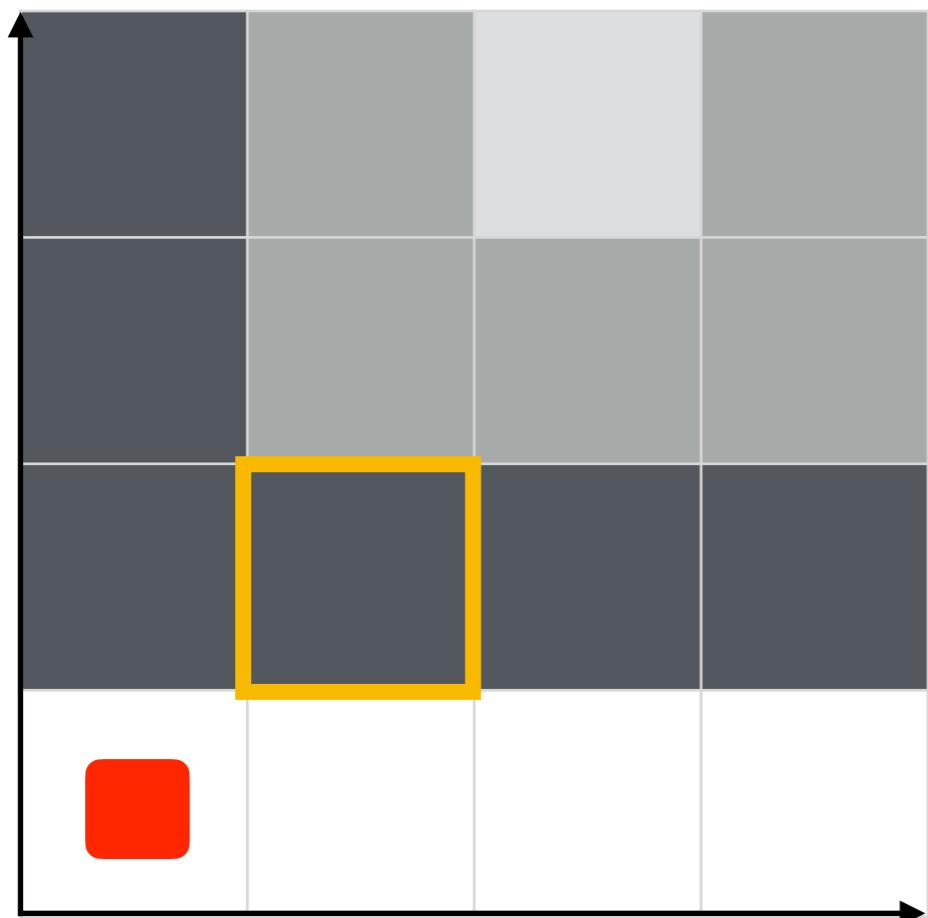
Player features

$P(\Theta = \theta)$ Probabilistic model

$X = x$ Special state

$Y = y$ Player state

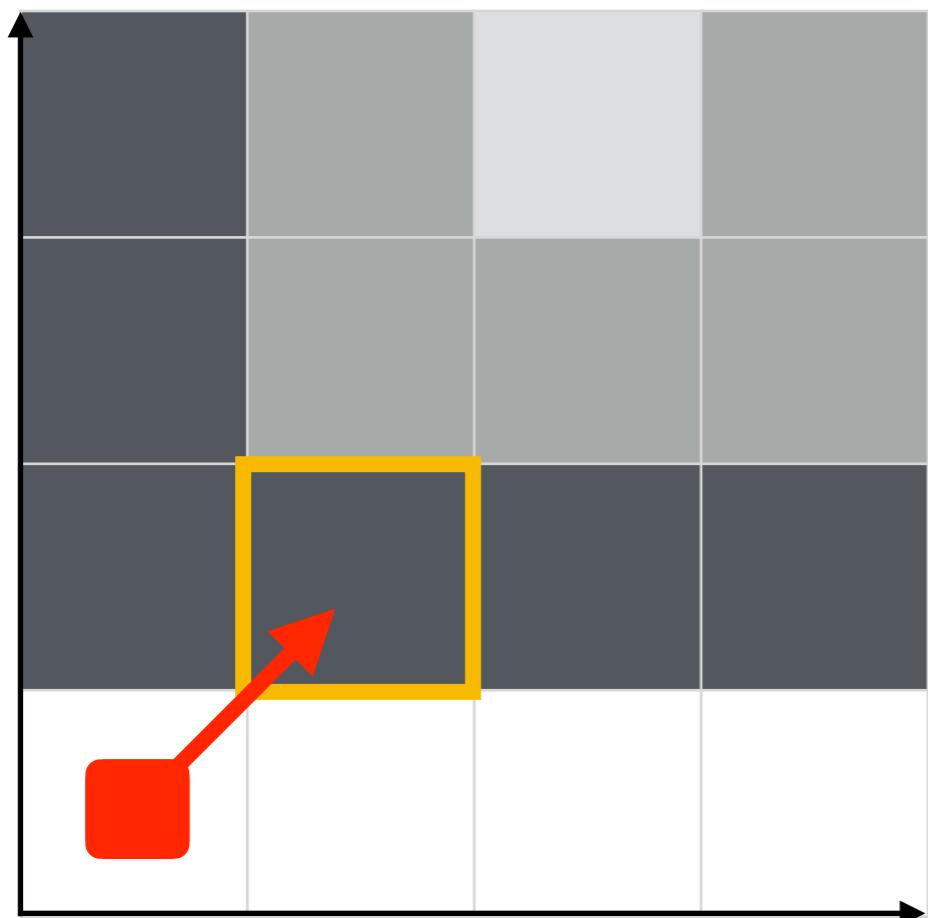
- Entrain (2019) - adaptive musical agent



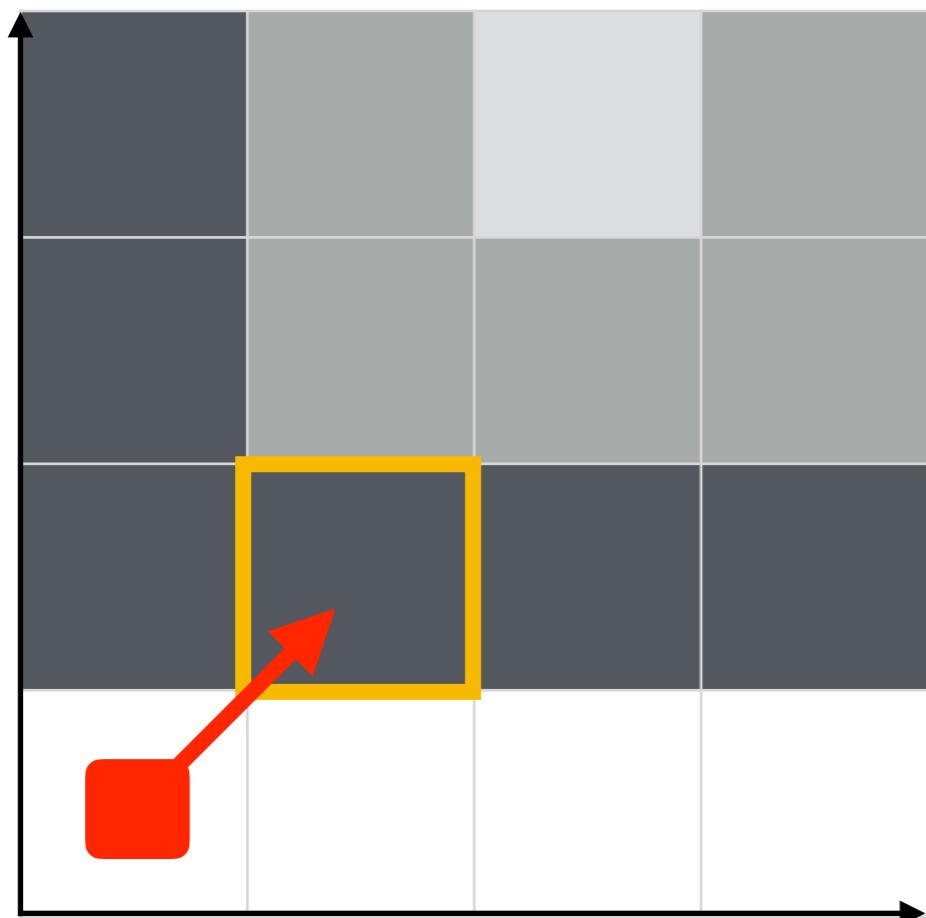
Player features



- Entrain (2019) - adaptive musical agent



- Entrain (2019) - adaptive musical agent

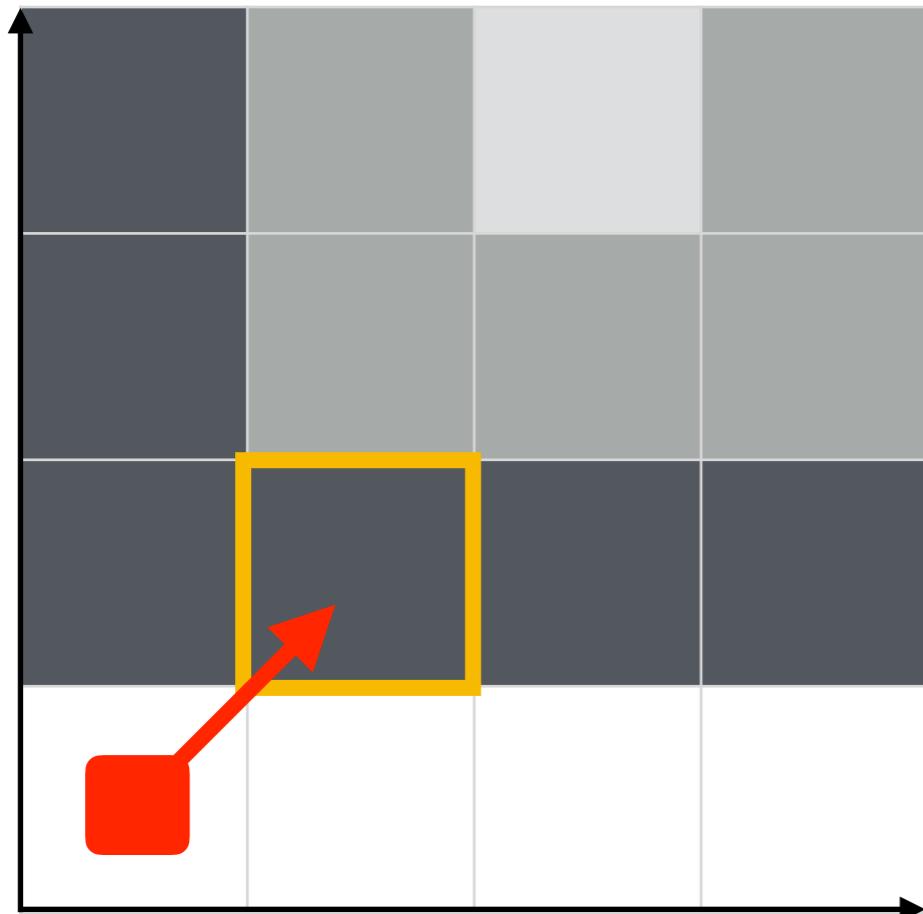


Player features



Shared visual effect

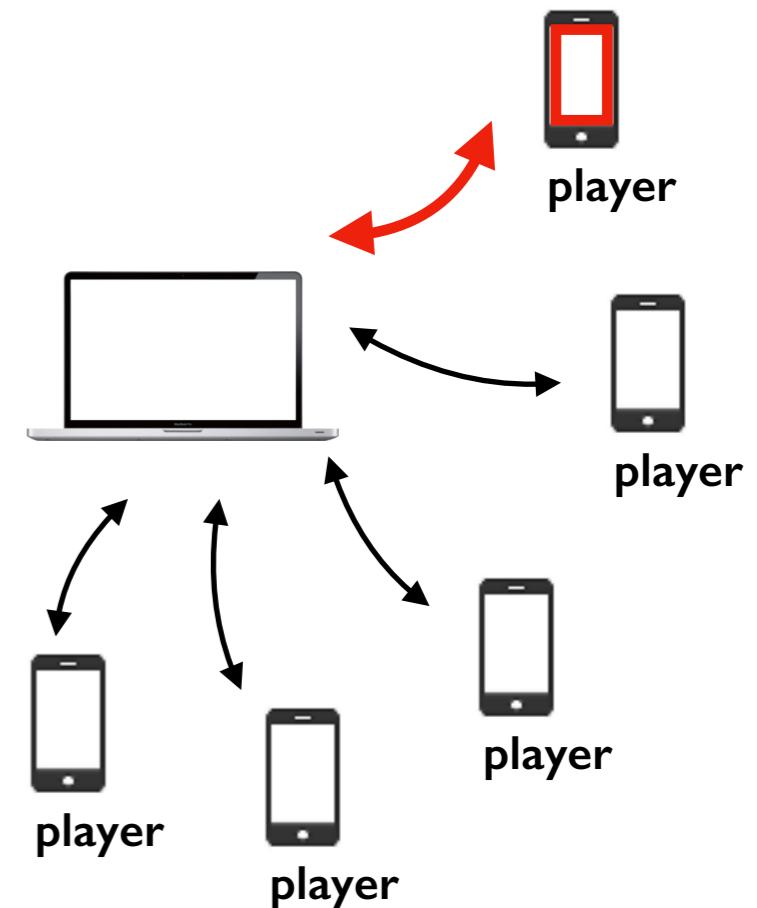
- Entrain (2019) - adaptive musical agent



Player features



Shared visual effect



Shared audio effect

- Entrain @ SIGGRAPH'19



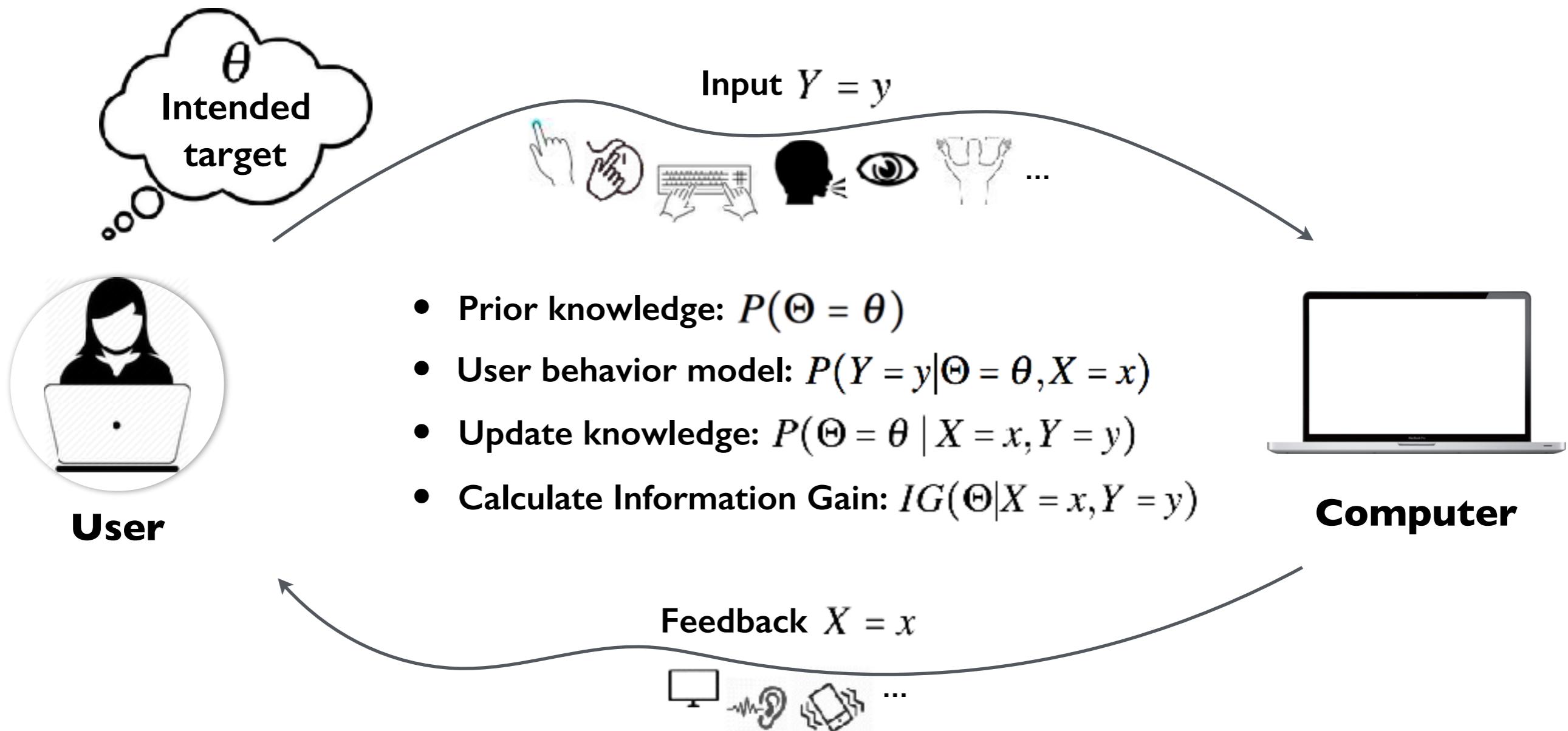
- Entrain @ Computational Interaction Summer School

Connect yourself to

192.168.1.100:8000

- Bayesian Information Gain (BIG)

- Executes the user input only **Multiscale navigation**
- Maximizes the expected information gain $IG(\Theta|X = x, Y)$ **BIGnav**
- Leverages the expected information gain **BIGFile, Entrain**



Recap

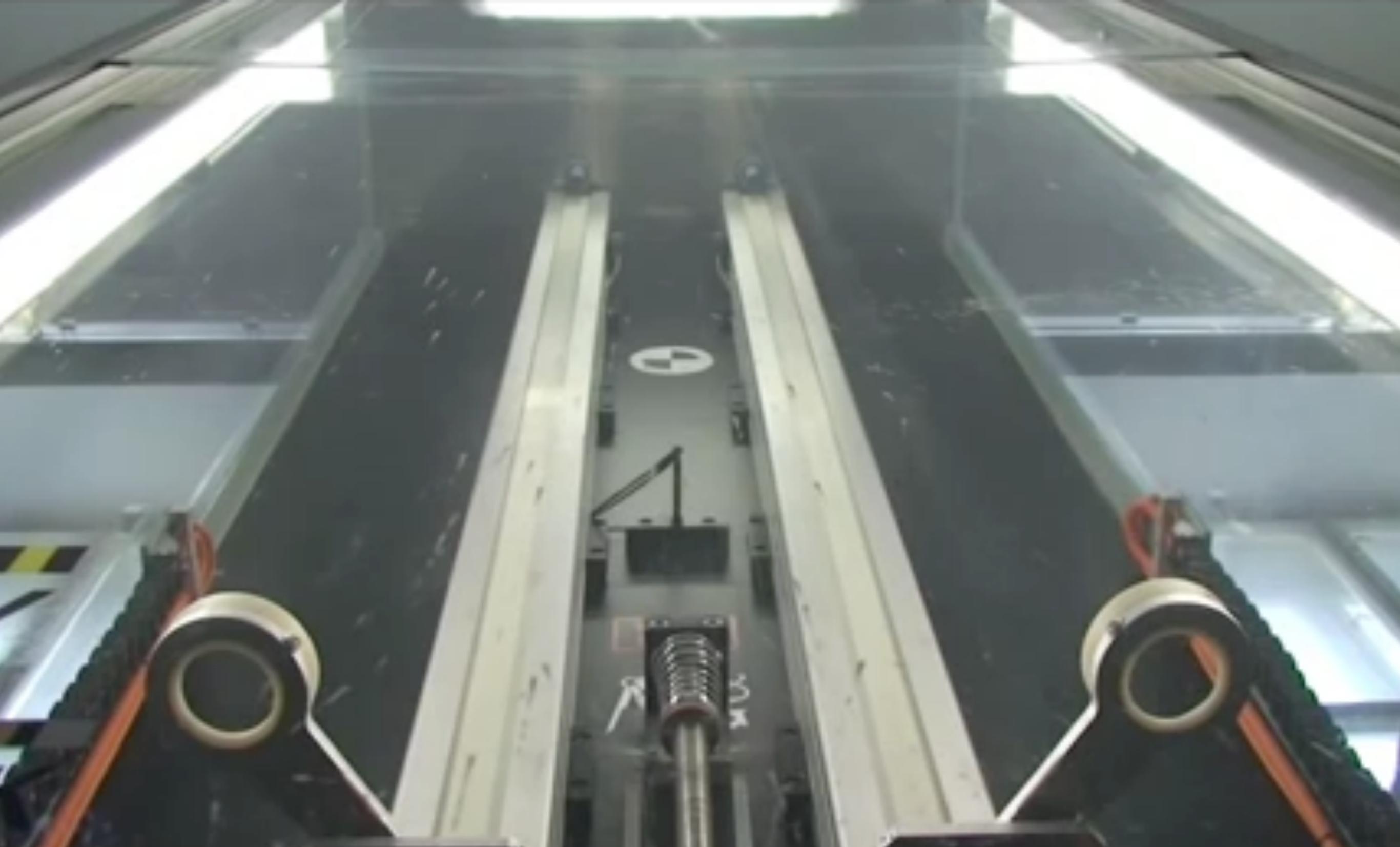
- * Information can be quantified in bits
- * Information reduces uncertainty
- * Improved information gain (BIG)
- * Information-theoretic measures to characterize interaction

Email: AbbyWanyu.Liu@ircam.fr
Web: abbywanyuliu.com

THE ULTIMATE MACHINE

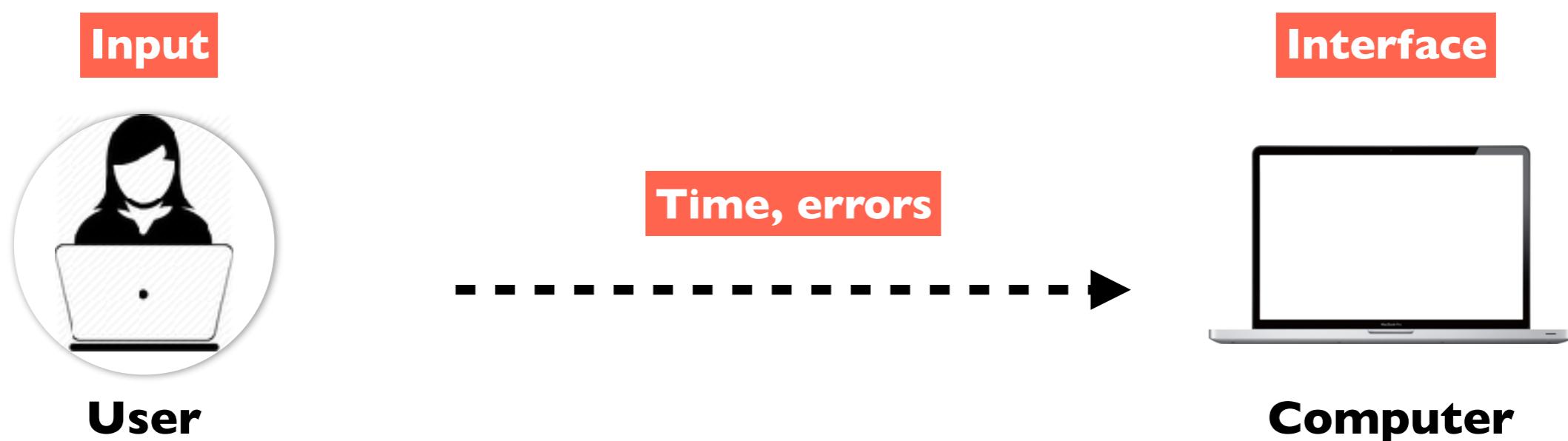
by Claude Elwood Shannon

Email: AbbyWanyu.Liu@ircam.fr
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Email: AbbyWanyu.Liu@ircam.fr
Web: abbywanyuliu.com

- Information-theoretic measures to characterize interaction



- Information-theoretic measures to characterize interaction

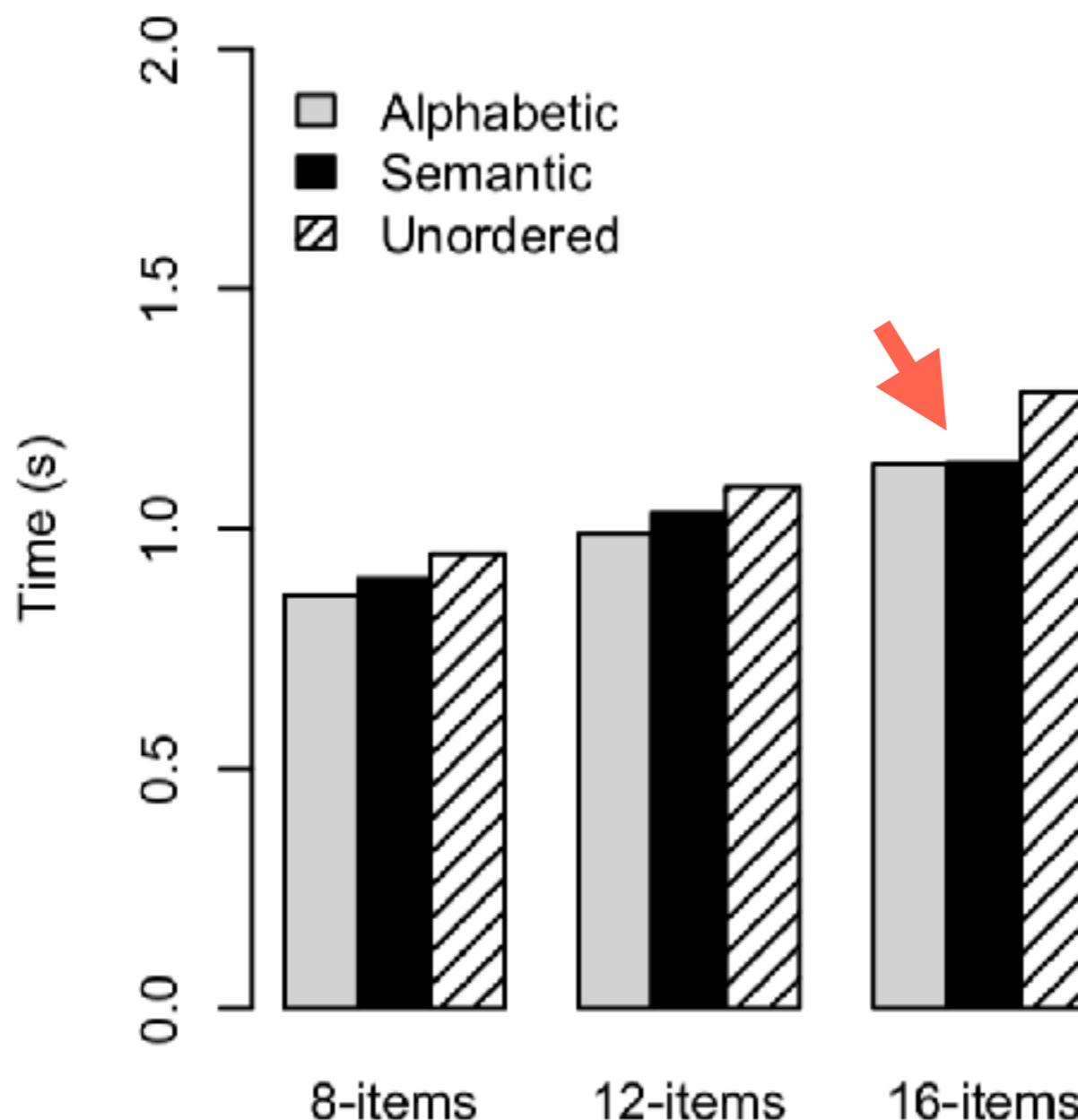
Drawback I: Speed-accuracy tradeoff

- Information-theoretic measures to characterize interaction

Solution : Control errors.

- * Control error rate under 4 %.
- * Remove errors from data analysis.

- Information-theoretic measures to characterize interaction



Model of visual search and selection time in linear menus. (Bailly et al. CHI'14)

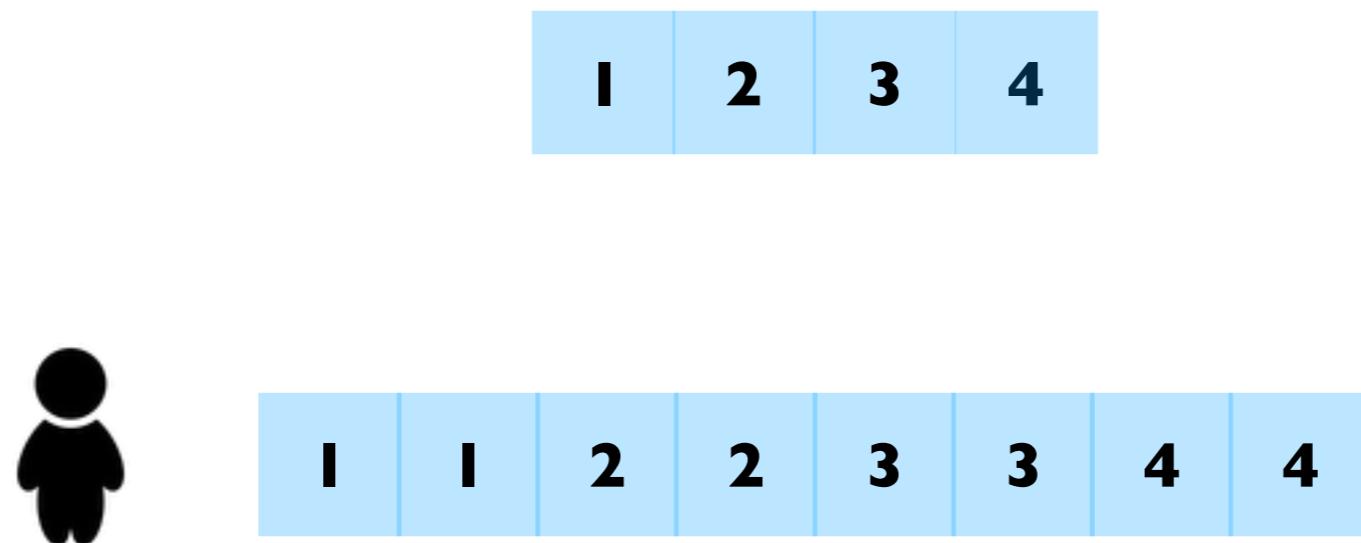
- Information-theoretic measures to characterize interaction

Drawback 2: The treatment of errors

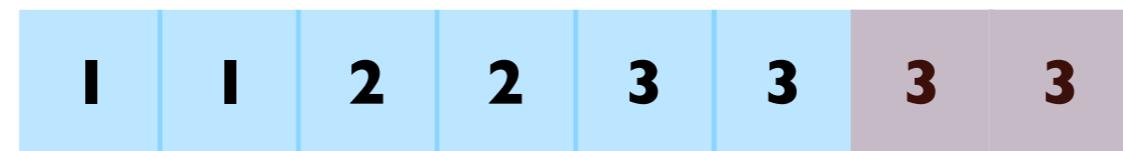
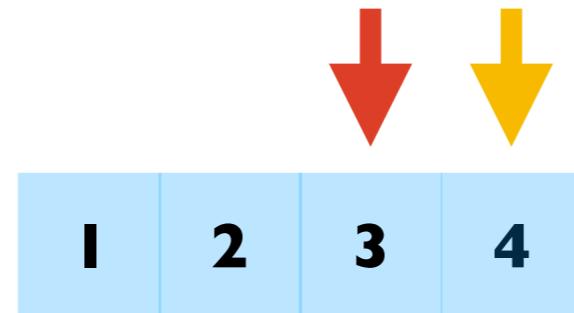
- Information-theoretic measures to characterize interaction



- Information-theoretic measures to characterize interaction

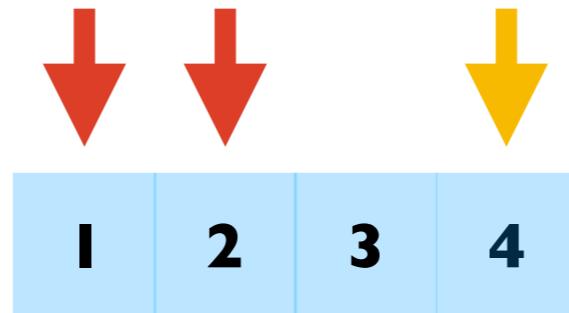


- Information-theoretic measures to characterize interaction



Error rate: $2 / 8 = 25\%$

- Information-theoretic measures to characterize interaction

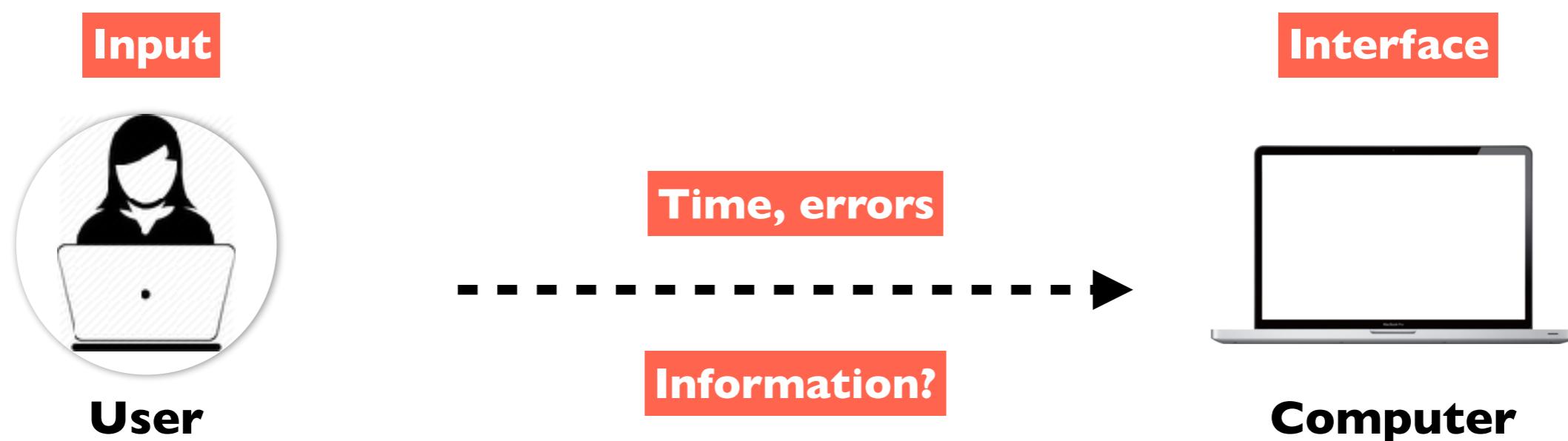


Error rate: $2 / 8 = 25 \%$



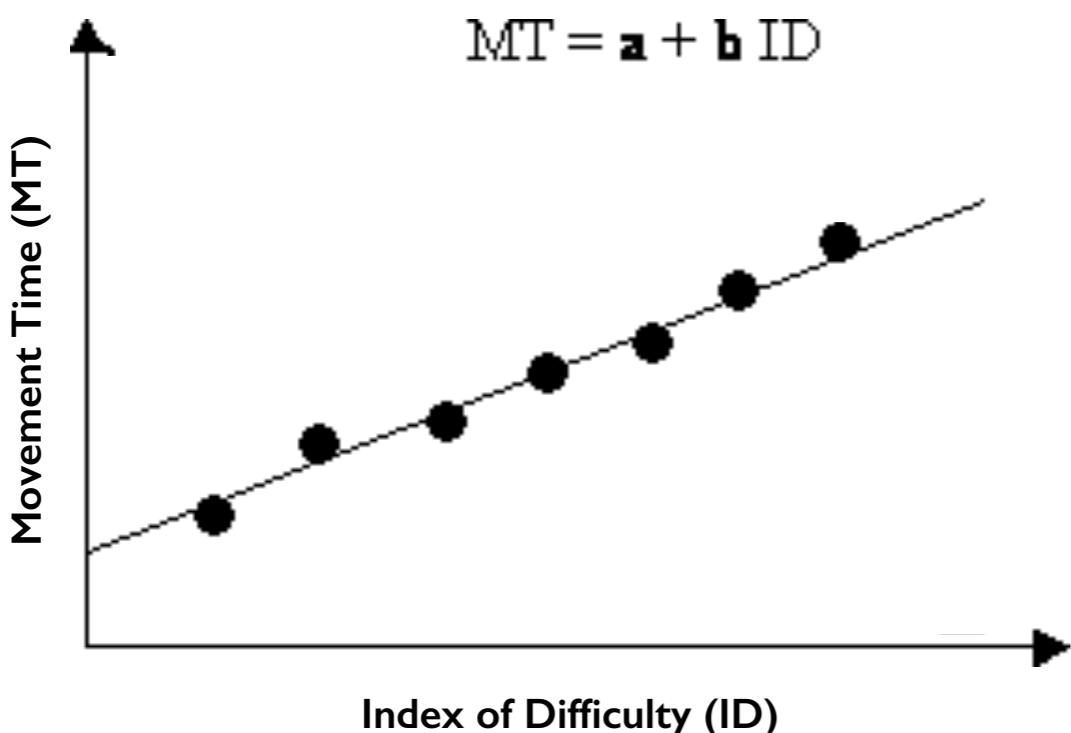
Error rate: $2 / 8 = 25 \%$

- Information-theoretic measures to characterize interaction



- Information-theoretic measures to characterize interaction

Fitts' law



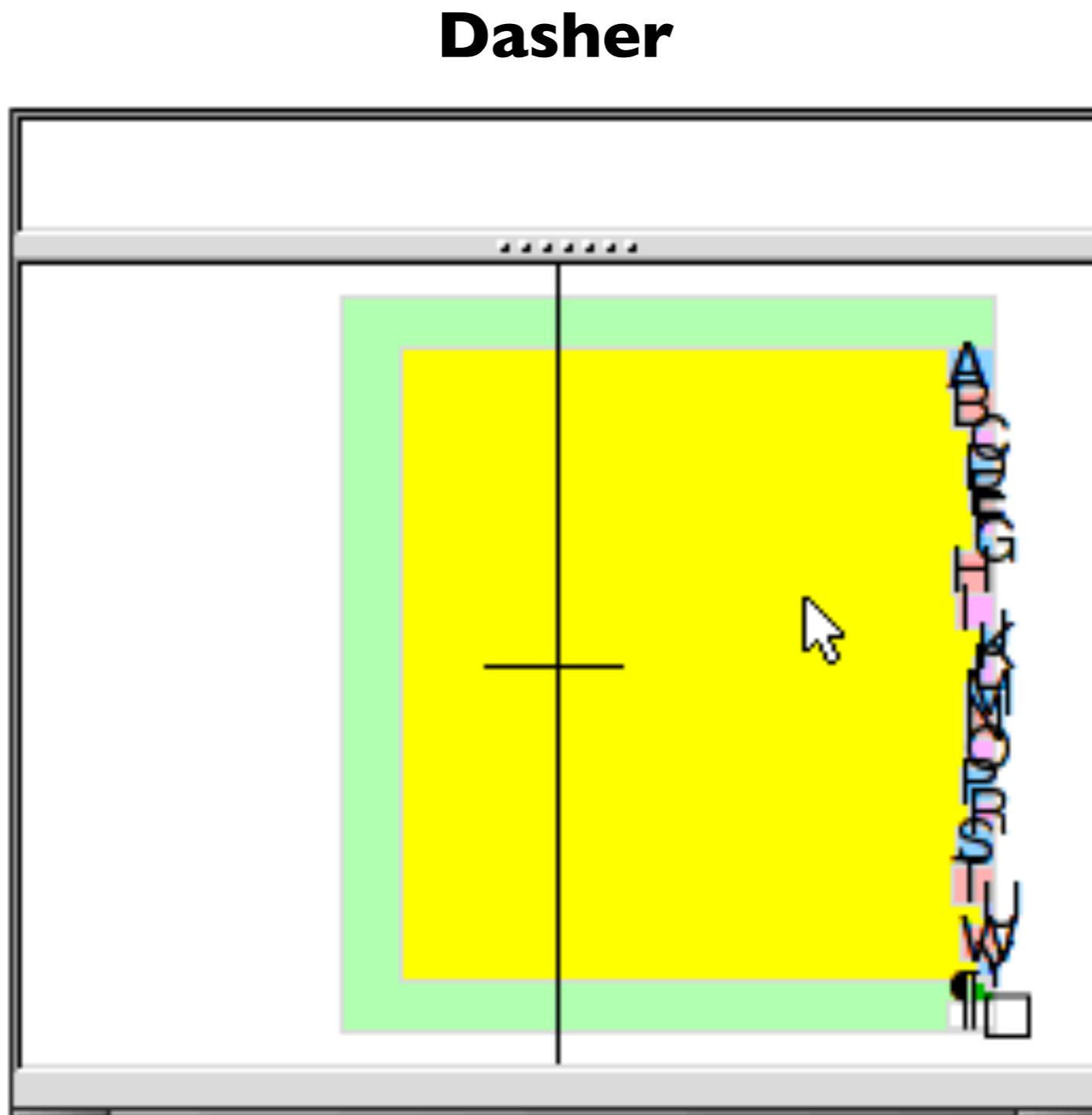
Movement Time:

$$T = a + b \log_2 \left(1 + \frac{D}{W} \right)$$

Channel Capacity:

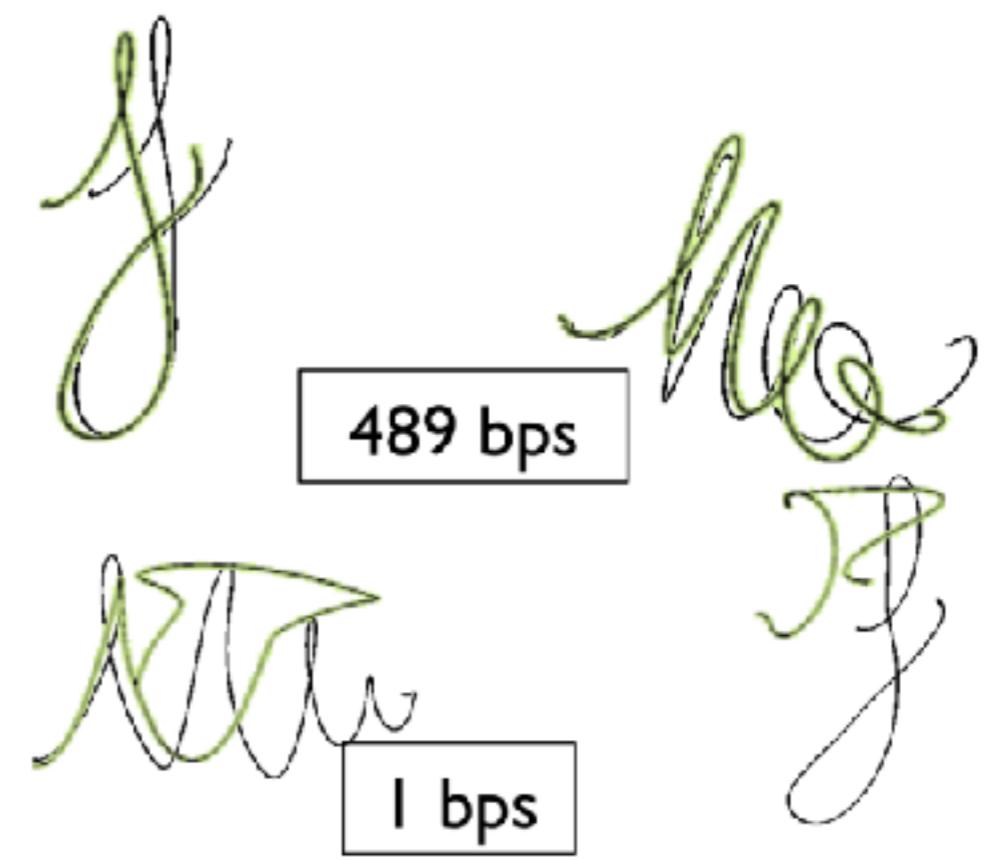
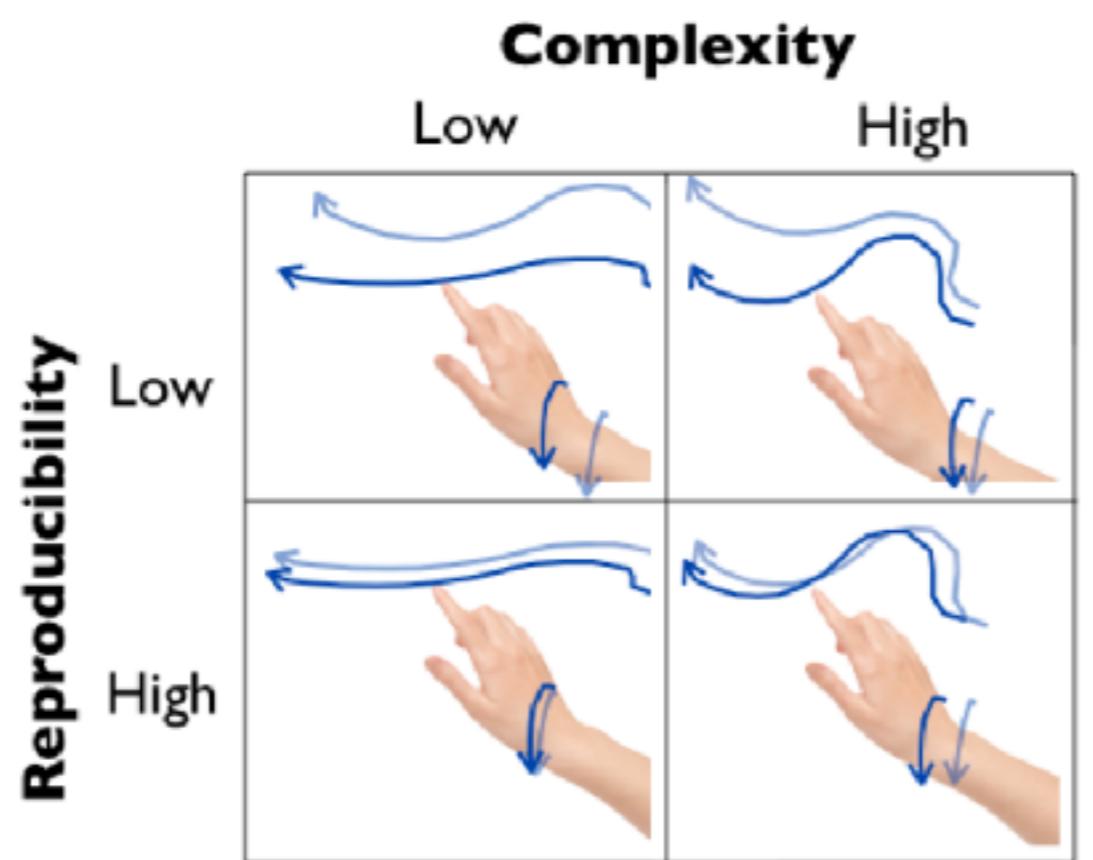
$$C = B \log_2 \left(\frac{S + N}{N} \right)$$

- Information-theoretic measures to characterize interaction



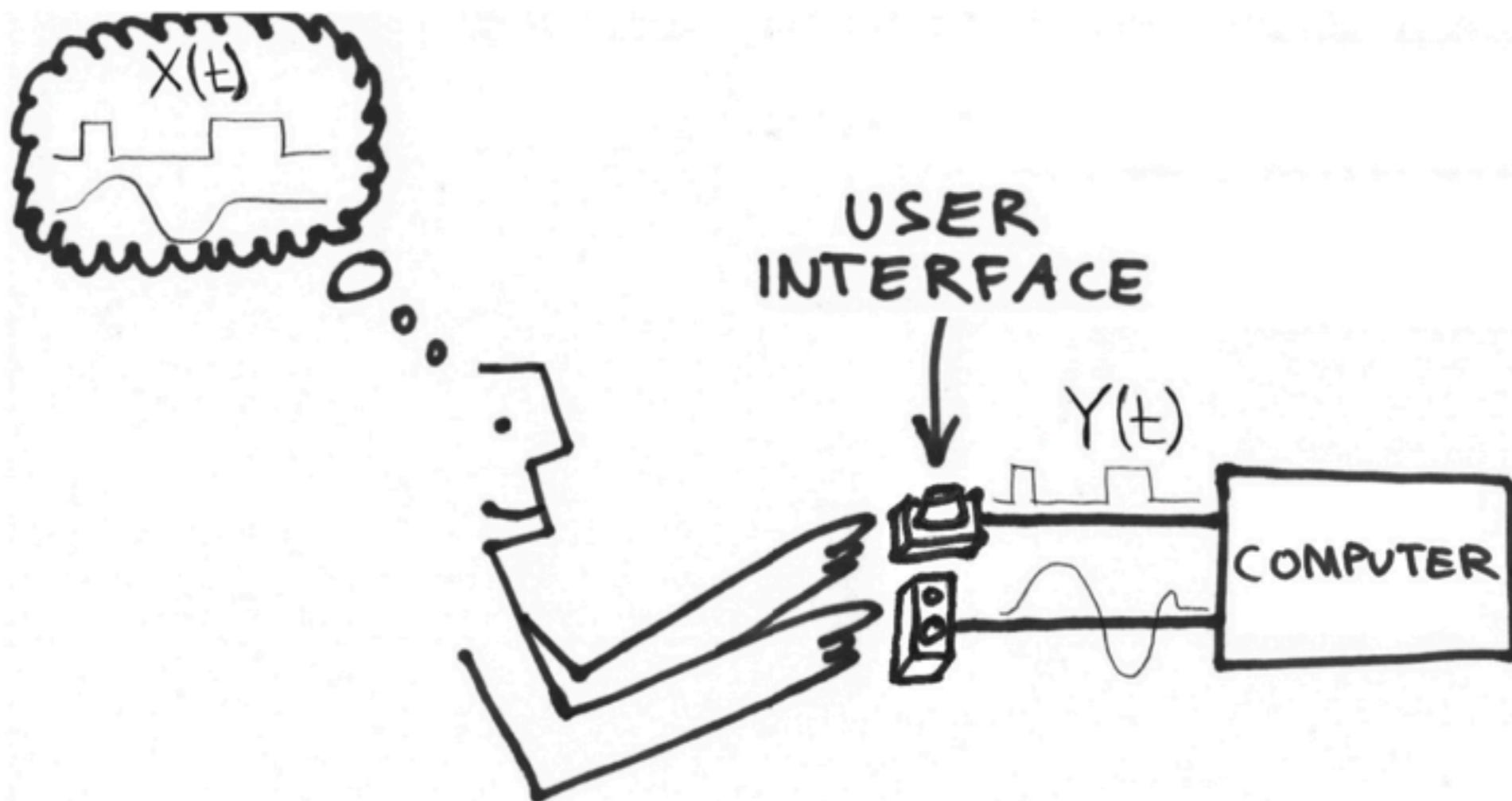
Source: <http://www.inference.org.uk/dasher/>

- Information-theoretic measures to characterize interaction



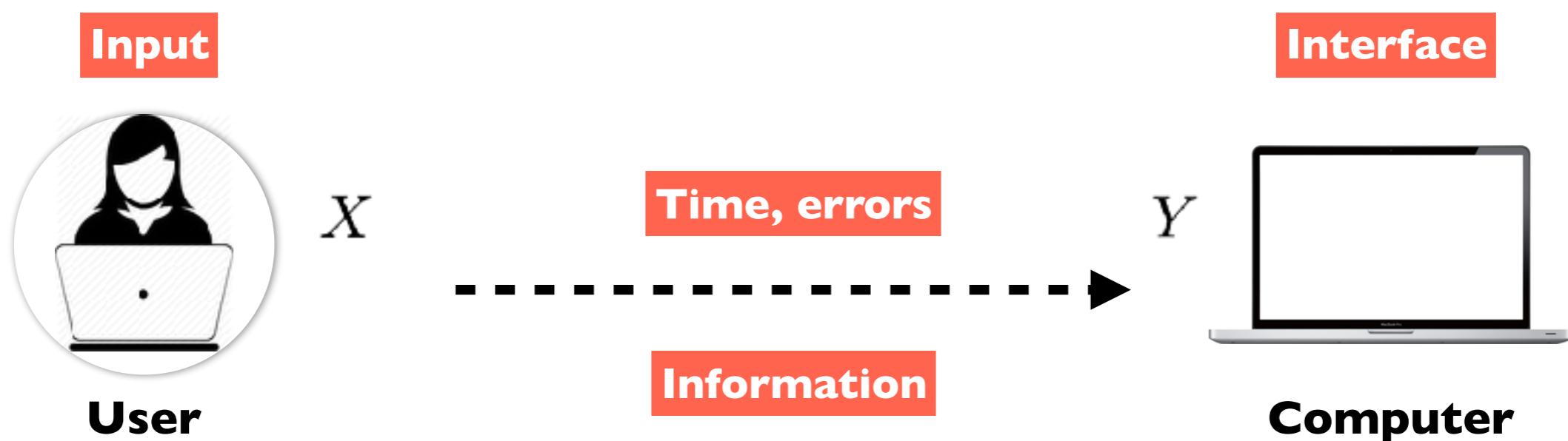
Information Capacity of Full-body Movements. (Oulasvirta et al. CHI'13)

- Information-theoretic measures to characterize interaction



An Approach for Using Information Theory to Investigate Continuous Control of Analog Sensors by Humans.
(Berdahl et al. AM'16)

- Information-theoretic measures to characterize interaction



- Information-theoretic measures to characterize interaction



X : A set of all possible messages that a user can transmit,
representing the intended inputs.

X takes values in

1	2	3	4
---	---	---	---

 $x_1 \quad x_2 \quad x_3 \quad x_4$

- Information-theoretic measures to characterize interaction



$P(X)$: The probability distribution of the intended inputs.

X takes values in

1	2	3	4
---	---	---	---

 $p(x_1) \ p(x_2) \ p(x_3) \ p(x_4)$

- Information-theoretic measures to characterize interaction



Input entropy: How much information could be transmitted.

Corresponding to **input size** and the **probability distribution**.



$$H(X) = - \sum_{i=1}^n P_i \log_2 P_i$$

- Information-theoretic measures to characterize interaction



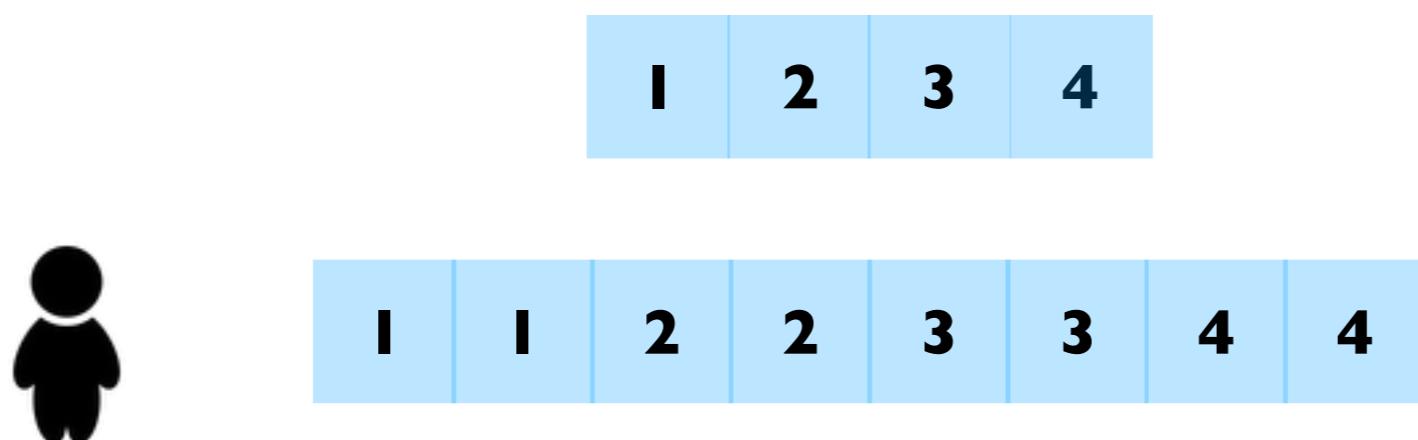
Y : The actual input received by the computer.



- Information-theoretic measures to characterize interaction



Y : The actual input received by the computer.



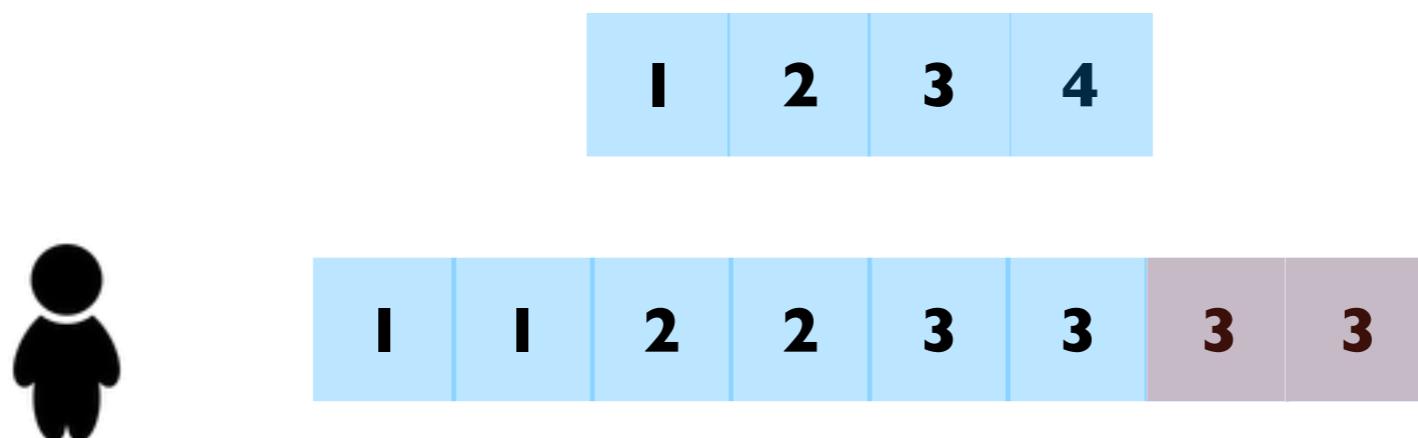
$$H(X) = 2 \text{ bits}$$

- Information-theoretic measures to characterize interaction



$I(X; Y)$: Mutual information between the intended input and the actual input.

It describes how much information actually gets transmitted.



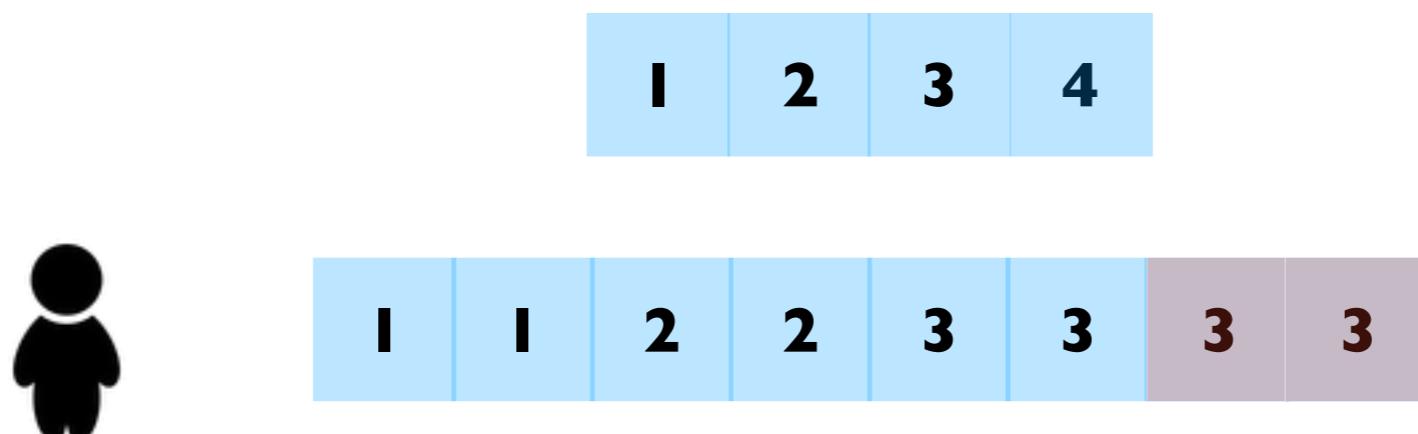
$$I(X; Y) = H(X) - H(X|Y)$$

- Information-theoretic measures to characterize interaction



$I(X; Y)$: Mutual information between the intended input and the actual input.

It describes how much information actually gets transmitted.



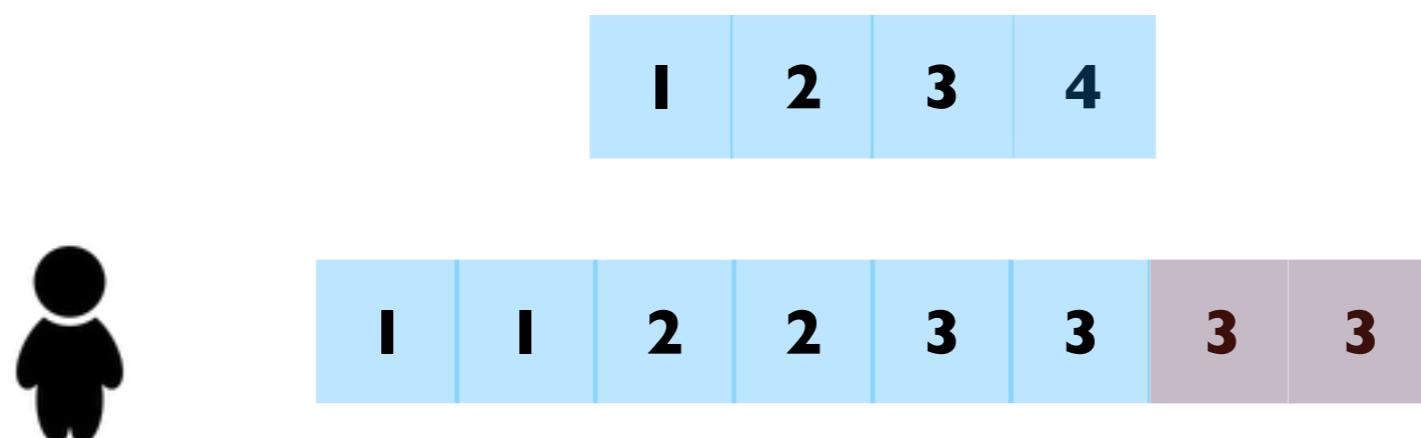
$$I(X; Y) = \sum_x \sum_y P(X = x, Y = y) \log_2 \frac{P(X = x, Y = y)}{P(X = x)P(Y = y)}$$

- Information-theoretic measures to characterize interaction



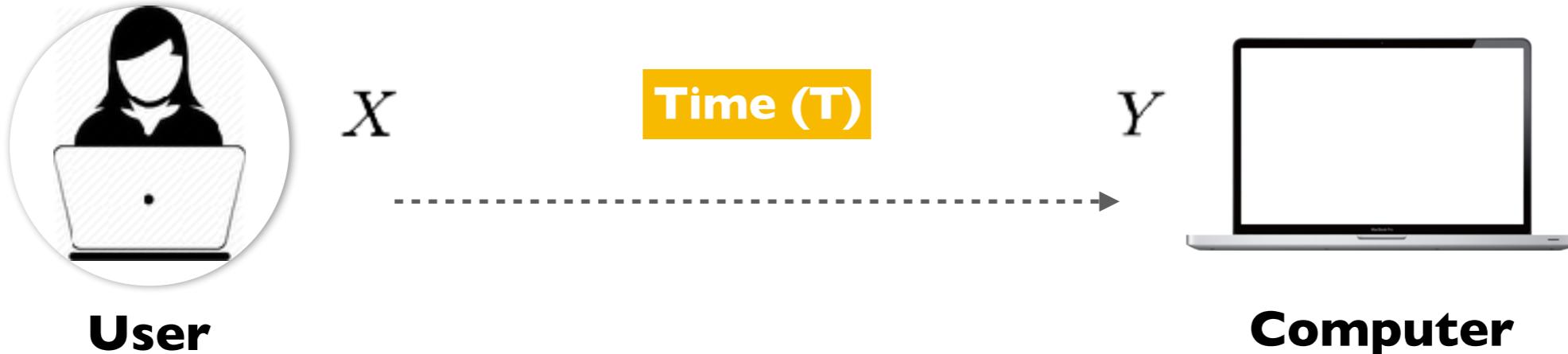
$I(X; Y)$: Mutual information between the intended input and the actual input.

It describes how much information actually gets transmitted.

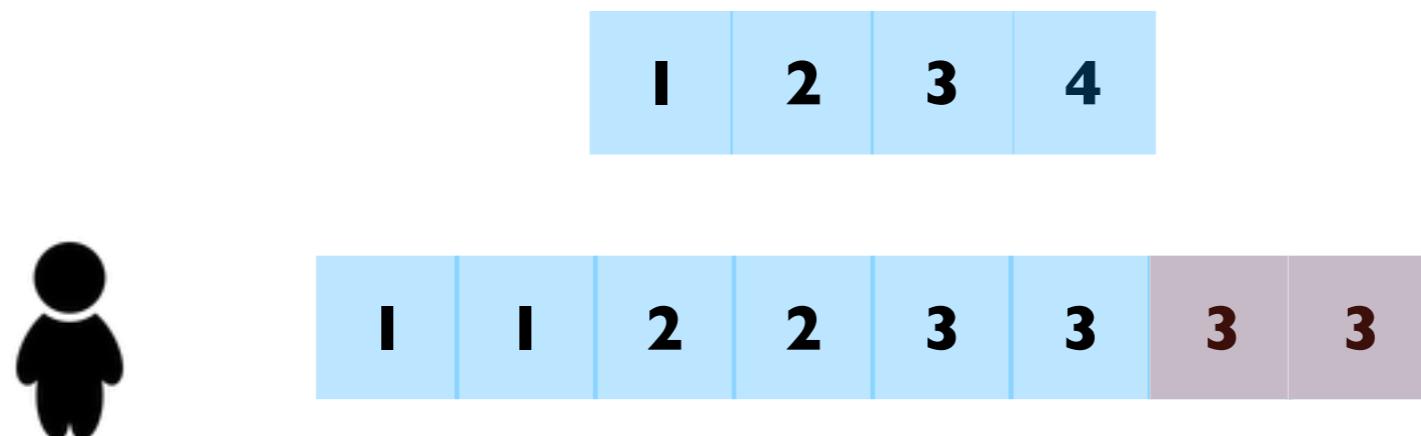


$$I(X; Y) = 1.5 \text{ bits}$$

- Information-theoretic measures to characterize interaction

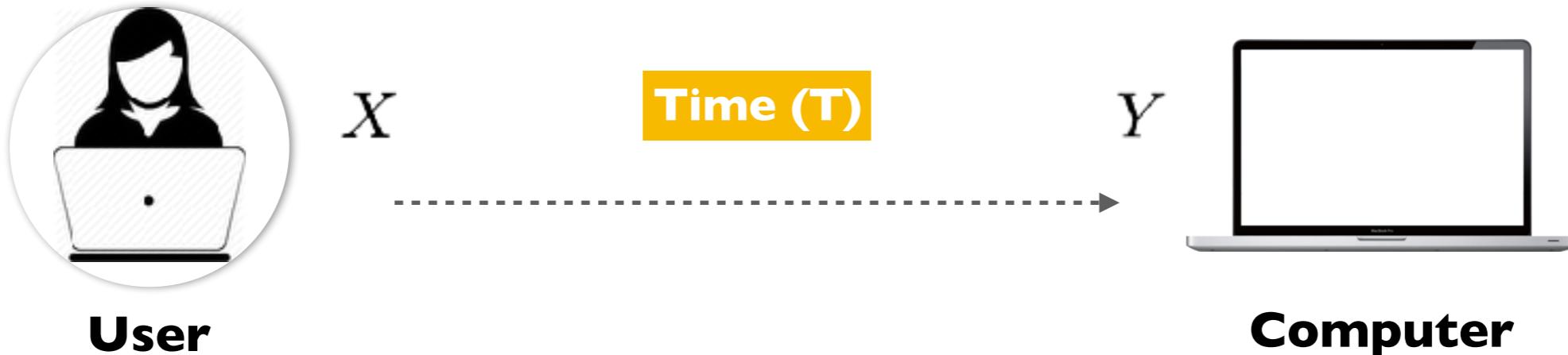


$TP = \frac{I(X;Y)}{T}$: Throughput describes information transmission efficiency.

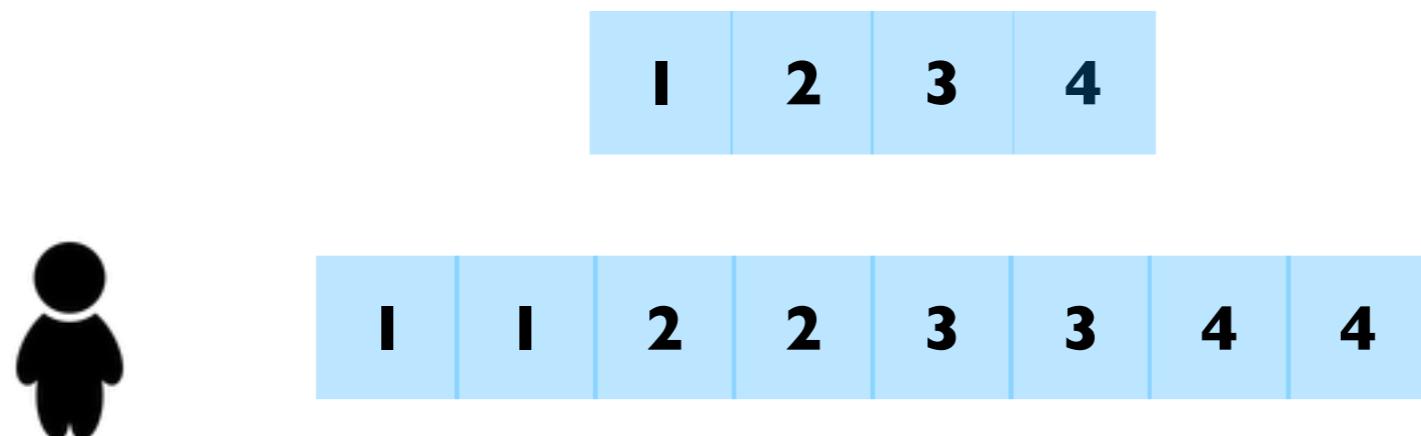


$$TP = I(X;Y)/T = 1.5/1.5 = 1 \text{ bit / s}$$

- Information-theoretic measures to characterize interaction



$TP = \frac{I(X;Y)}{T}$: Throughput describes information transmission efficiency.



$$TP = I(X;Y)/T = H(X)/T = 2/1.5 = 1.3 \text{ bit / s}$$

- Information-theoretic measures to characterize interaction

Advantage I:

Standard language to describe interaction

- Information-theoretic measures to characterize interaction

Advantage I:

Standard language to describe interaction

$H(X)$: how much information could be transmitted.

$I(X;Y)$: how much information actually gets transmitted.

$H(X|Y)$: how much information gets lost, related to how users make errors.

TP: information transmission efficiency.

- Information-theoretic measures to characterize interaction

Advantage 2:
More consistent measure

- Information-theoretic measures to characterize interaction

Advantage 2:

More consistent measure

	$H(X)$	$I(X; Y)$	$H(X Y)$	TP_i
$N_{of X} \uparrow$	\uparrow	\uparrow	-	\uparrow
$p(X) \uparrow$	\downarrow	\downarrow	-	\downarrow
$P_e \uparrow$	-	\downarrow	\uparrow	\downarrow
$T \uparrow$	-	-	-	\downarrow

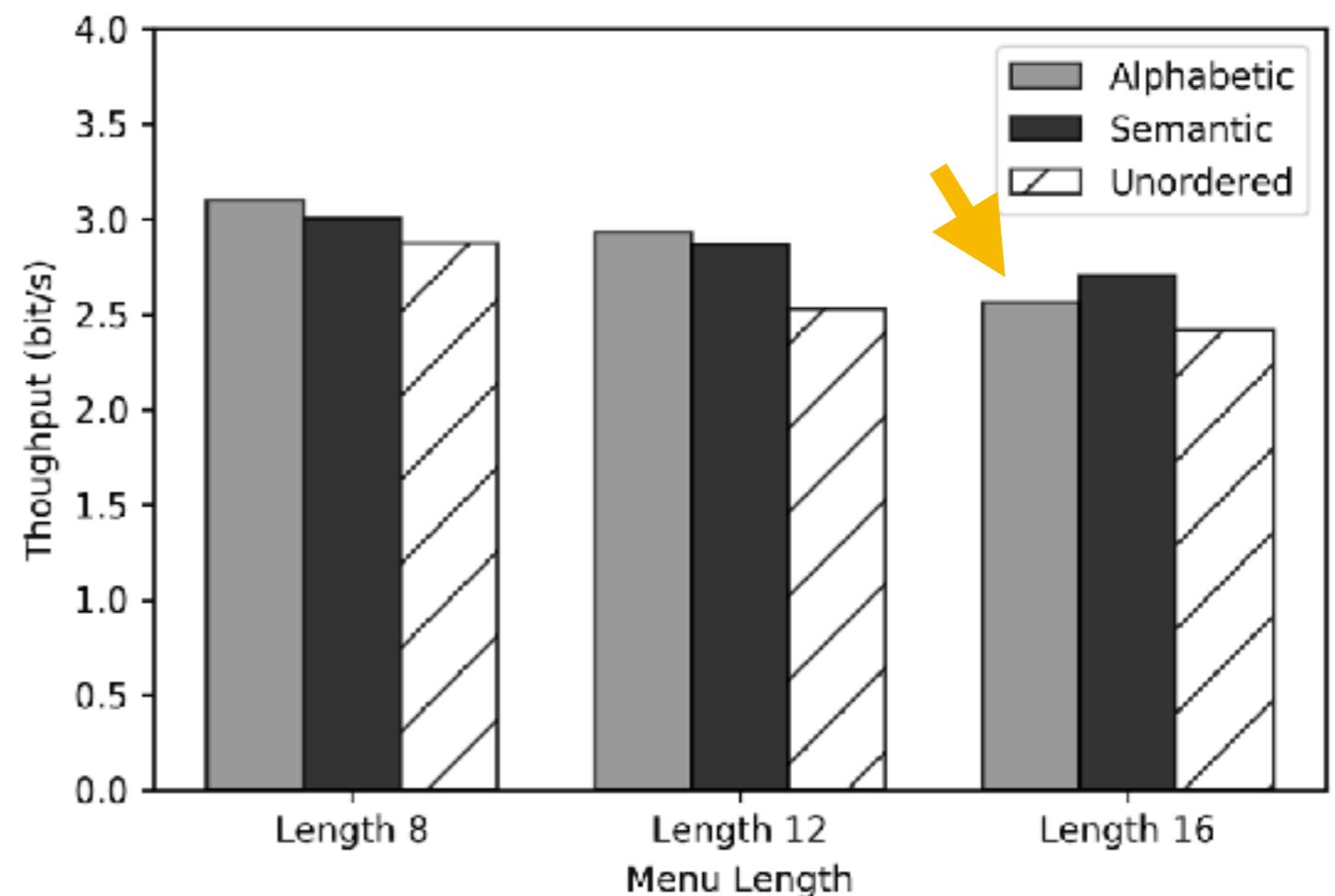
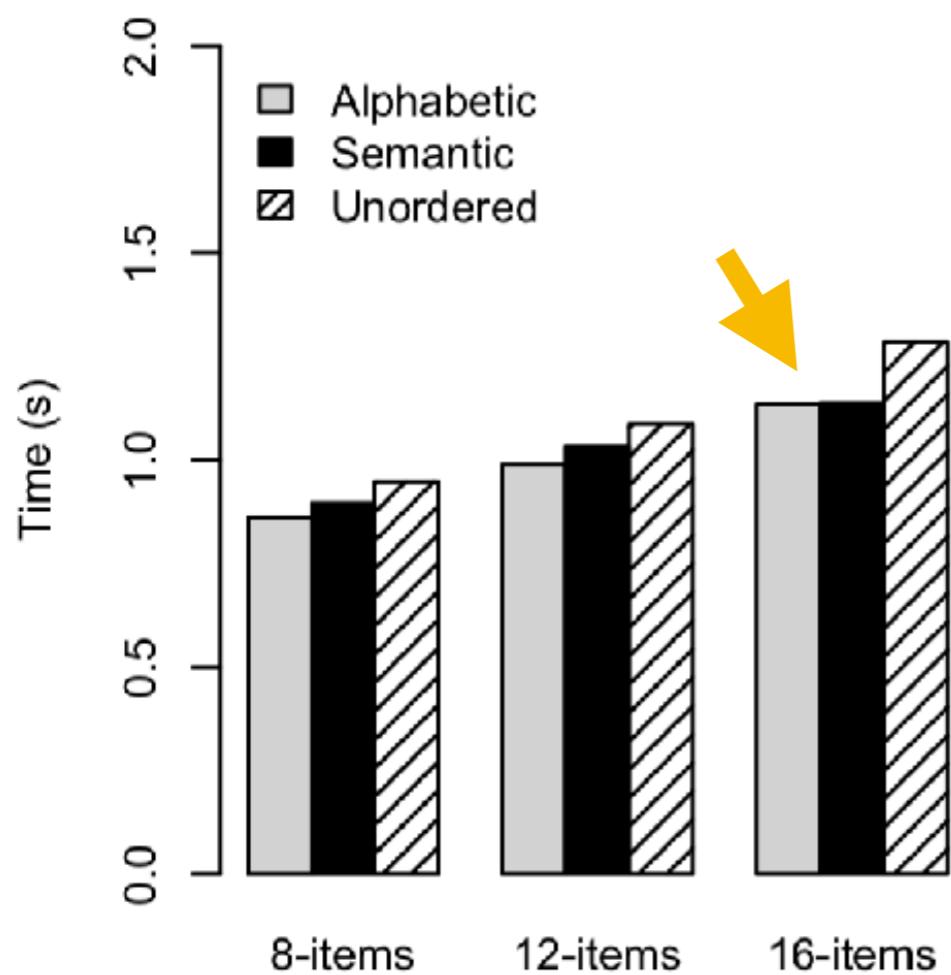
- Information-theoretic measures to characterize interaction

Advantage 3:
Speed-accuracy tradeoff

- Information-theoretic measures to characterize interaction

Advantage 3:

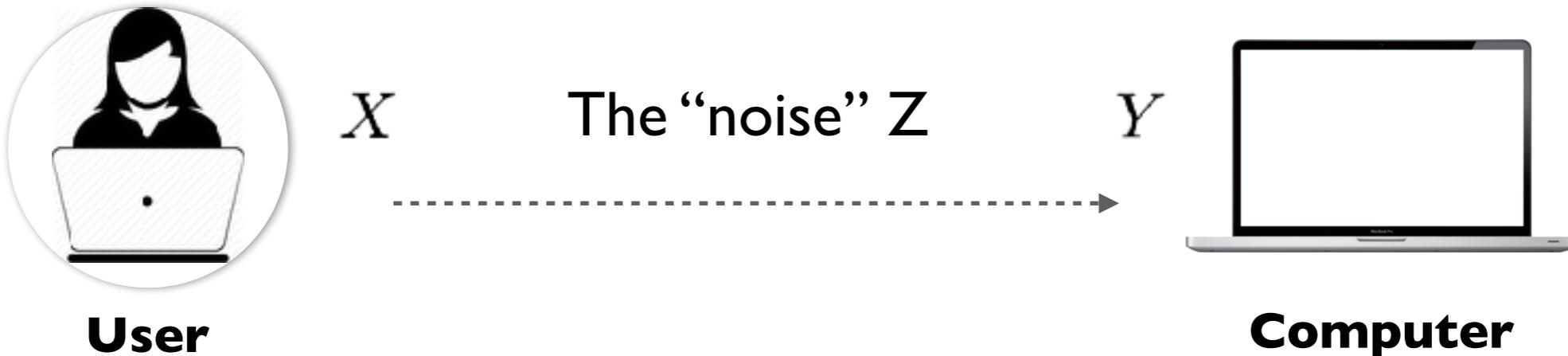
Speed-accuracy tradeoff



- Information-theoretic measures to characterize interaction

Advantage 4:
**Equivocation provides information about
how users make errors**

- Information-theoretic measures to characterize interaction



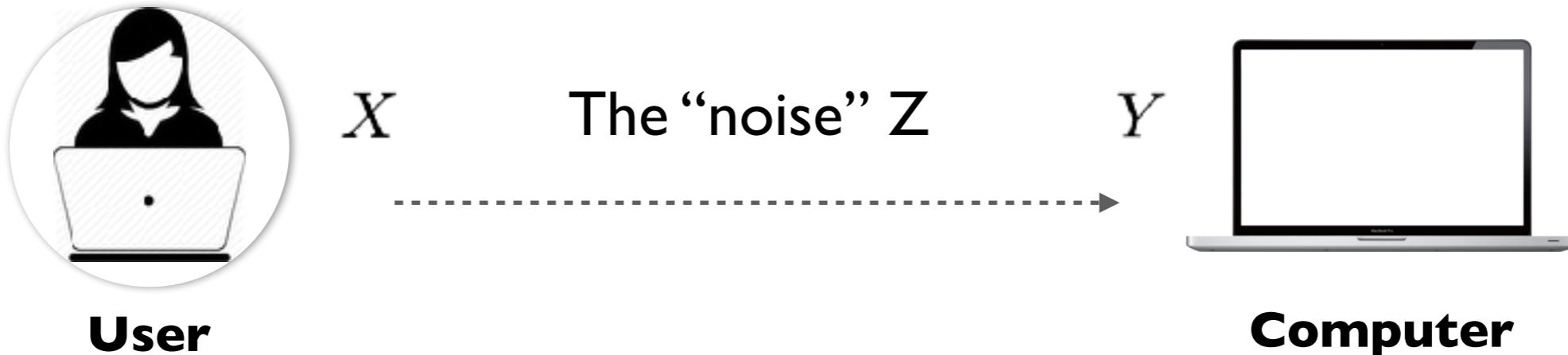
The error random variable:

$$E = \begin{cases} 0 & \text{if } X = Y; \\ 1 & \text{if } X \neq Y. \end{cases}$$

The error rate:

$$P_e = P(X \neq Y)$$

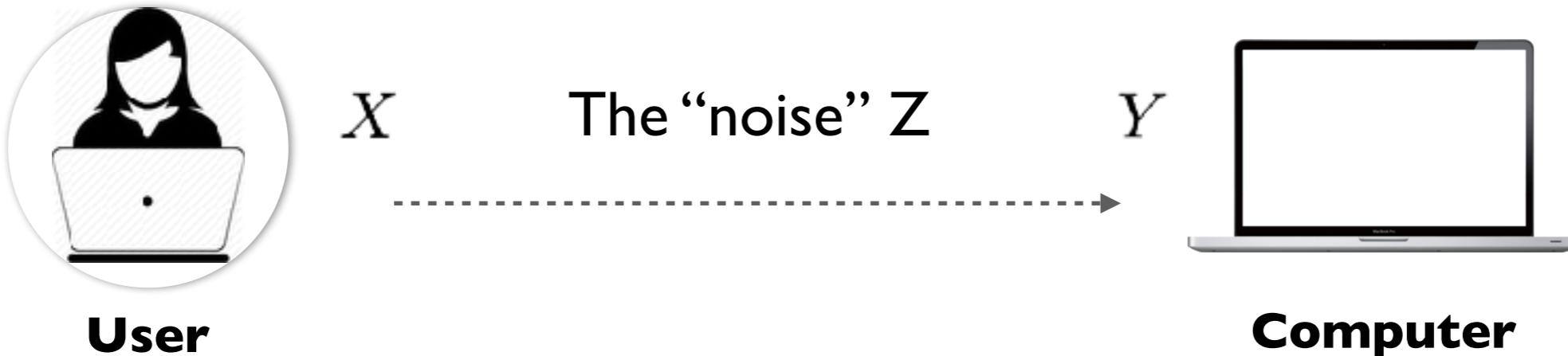
- Information-theoretic measures to characterize interaction



The error rate P_e has binary entropy:

$$H(E) = -P_e \log_2 P_e - (1 - P_e) \log_2(1 - P_e)$$

- Information-theoretic measures to characterize interaction



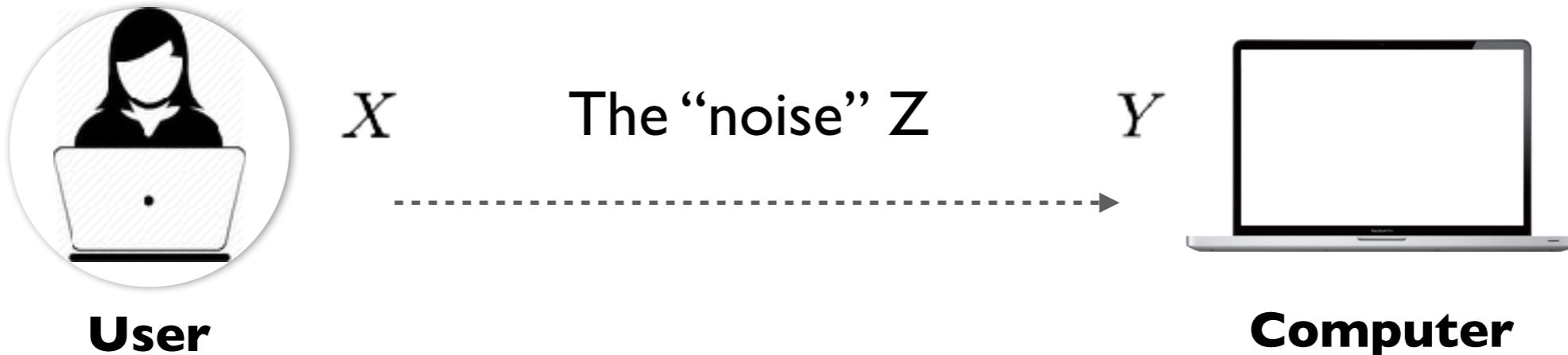
$$I(X; Y) = H(X) - H(X|Y)$$

$$H(X|Y) \leq H(E) + P_e \times H(Z|E = 1)$$

Fano's inequality.

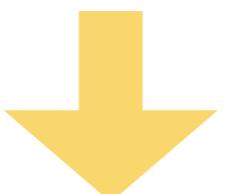
[Theorem 2.10.1] *Elements of information theory*. Cover, T. M., & Thomas, J. A. (2012).

- Information-theoretic measures to characterize interaction



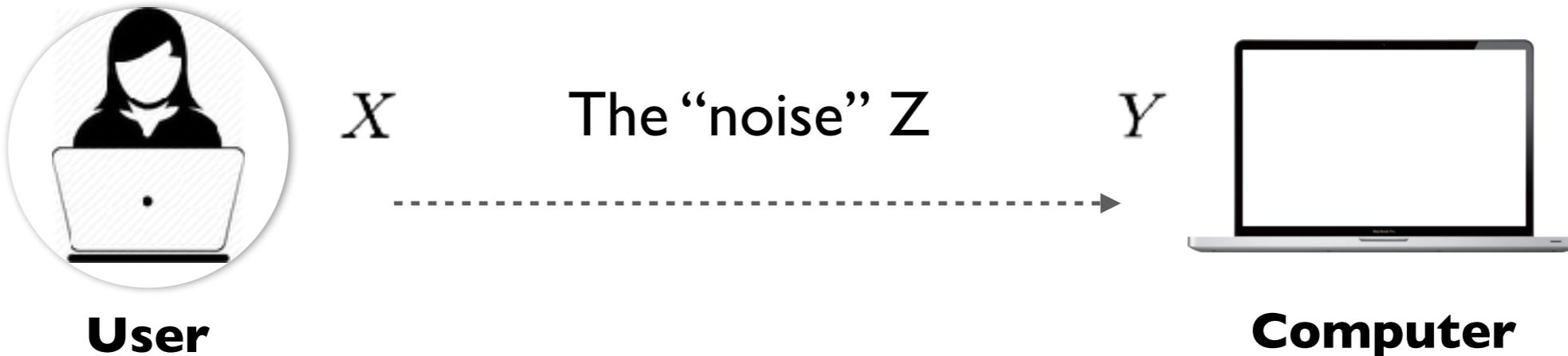
$$I(X; Y) = H(X) - H(X|Y)$$

$$H(X|Y) \leq H(E) + P_e \times H(Z|E = 1)$$



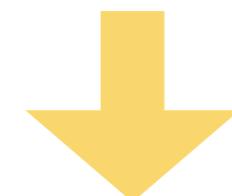
The fact that users make errors. At most 1 bit.

- Information-theoretic measures to characterize interaction



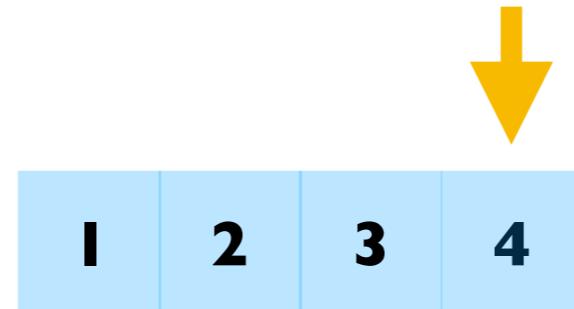
$$I(X; Y) = H(X) - H(X|Y)$$

$$H(X|Y) \leq H(E) + P_e \times H(Z|E = 1)$$



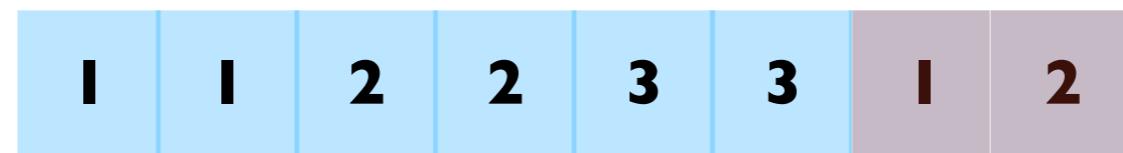
How they make errors.

- Information-theoretic measures to characterize interaction



Error rate: $2 / 8 = 25 \%$

$$\begin{aligned} H(X|Y) &= 0.5 \text{ bits} \\ TP &= I(X;Y)/T \\ &= 1.5/1.5 = 1 \text{ bit / s} \end{aligned}$$



Error rate: $2 / 8 = 25 \%$

$$\begin{aligned} H(X|Y) &= 0.7 \text{ bits} \\ TP &= I(X;Y)/T \\ &= 1.3/1.5 = 0.8 \text{ bit / s} \end{aligned}$$

Exercise

