The Masy Joule Balance:
A new look at NIST's LEGO Watt Balance

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Abstract

Many units of measure have previously been defined in terms of physical artifacts. These physical artifacts are the only references defined with zero uncertainty. In mass metrology, the International Prototype Kilogram (IPK) has been used to define the kilogram since 1879. However, with the passing of time and the effects of the environment in which it is stored, it has been discovered that the IPK has been losing and gaining mass. Since this discovery, a global effort has been made to redefine the International System of Units (SI) kilogram. In the redefined system, the kilogram will be defined as a fixed value of the Planck constant h.

I. INTRODUCTION

Many units of measure have previously been defined using physical artifacts, which are the only references defined to have zero uncertainty. However, due to the passing of time and the effects of the environment in which the artifact is stored, it is impossible to guarantee that the artifact remains in its original condition. In mass metrology, the International Prototype Kilogram (IPK) has been used to define the kilogram since 1879. However, it has been discovered that the IPK's mass has been changing. Since this discovery, a global effort has been made to redefine the International System of Units (SI) kilogram. In the redefined system, the kilogram will be defined as a fixed value of the Planck constant h.

To accomplish this relationship between mass and the Planck constant h, the gravitational force of an object will be balanced with the electromagnetic force generated by a current-carrying solenoid using the Watt Balance, thus providing the relationship between mass and electrical constants.

Our Masy Joule Balance, on the other hand, equates the energy dissipated by the current-carrying solenoid and the change in the gravitational potential energy of the mass, and was developed at Masy BioServices' Mass Calibration Laboratory, hence the name, "Masy Joule Balance". Like the NIST Watt Balance, the Masy Joule Balance finds the relationship between mass and electrical values by balancing the different forces acting on each side of the balance arm, but differs in its calculations and implementations. On the surface, both arms seem very similar, but when we examine the calculations used to explain our implementation, we will quickly see the differences.

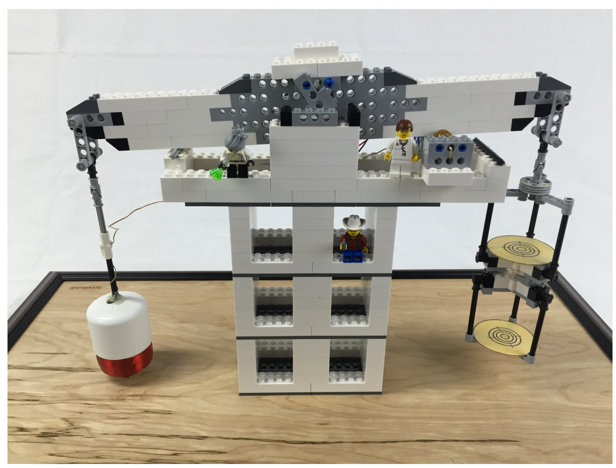

FIG. 1. Picture of completed Masy Joule Balance. The electronics are hidden on the underside of the base.

Like NIST, we were able to build our balance using LEGO for under $650. The parts required for our balance are very similar to those of NIST's LEGO Watt Balance and the differences are noted throughout this journal. We encourage others to improve upon our design and increase its sensitivity and functionality, or even design a balance entirely different, like what we have done with NIST's LEGO Watt Balance.

II. BALANCE THEORY

As the name implies, the LEGO Masy Joule Balance balances the different energies. The change in gravitational potential energy of the mass being measured is balanced against the energy consumed by the current-carrying solenoid.

$$E_{coil} = \Delta U_{mass} \tag{1}$$

Energy is the integral of two different forces acting on the opposite ends of the balance arm from 0, the "down" position of the Masy Joule balance, to *D*, the "balanced" position of the Masy Joule balance, with respect to distance. The energy dissipated by the current-carrying solenoid is the integral of the Lorentz force it produces and the change in gravitational energy experienced by the mass is the integral of the gravitational force it experiences.

$$\int_0^D F_B(x)dx = \int_0^D F_g\,(x)dx \tag{2}$$

The magnetic force of the current-carrying solenoid and the gravitational force experienced by the mass are substituted in, and the constants are brought out of the integral.

$$I_{input}L\int_0^D B(x)dx = mg\int_0^D dx \tag{3}$$

The issue with Eqs. (3) is that it is very difficult to accurately determine the integral of the magnetic field as a function of distance with respect to distance. To solve this, we will introduce the equation that describes the voltage induced in a wire of length *L* when it is moved through a magnetic field *B* at a constant velocity in a separate mode of operation that we will call "charge mode".

$$V_{induced}(t) = B(x)L\frac{dx}{dt} \tag{4}$$

By rearranging Eqs. (4) and taking the integral of both sides, we get Eqs. (5).

$$\int_0^T V_{induced}(t)dt = L\int_0^D B(x)dx \tag{5}$$

By substituting Eqs. (4) into Eqs. (3), we can eliminate the magnetic field from our equation.

$$I_{input}\int_0^T V_{induced}(t)dt = mg\int_0^D dx \tag{6}$$

Now, although our equation no longer contains the integral of the magnetic field, we now must find the integral of the induced voltage. We can do this by substituting current in using Ohm's Law. Voltage as a function of time is related to current as a function of time with constant resistance through Ohm's Law.

$$V(t) = I(t)R \tag{7}$$

Then by substituting Eqs. (7) into Eqs. (6), we get Eqs. (8).

$$I_{input}R\int_0^T I_{induced}(t) = mg\int_0^D dx \tag{8}$$

Then, by completing the integrals on both sides of Eqs. (8), we get Eqs. (9).

$$I_{input}RQ_{induced} = mgD \tag{9}$$

Where $Q_{induced}$ is the total charge that accumulates from the current passing through the current-carrying solenoid during "charge mode".

Finally, by using Ohm's law from Eqs. (7) again, we can substitute for the input current and reduce the equation even further

$$V_{input}Q_{induced} = mgD \tag{10}$$

Now we have completed the Masy Joule Balance equation. However, there are still issues in measuring these variables found in the equation accurately.

V_{input}, D, and g can all be measured direcly, and m is what we are looking for. What remains is an accurate method to measure the total charge Q that passes through the current-carrying solenoid during "charge mode". We can do this with Riemann Sum approximations. By taking the ratio of the difference between the $Q_{calculated}$ and the Q_{approx} and the $Q_{calculated}$, we can determine the number of samples we need to take for a given time to obtain the total accumulated charge $Q_{induced}$ with acceptable accuracy.

$$A \geq \left| \frac{\int_0^T I(t)dt - \sum_{i=0}^{n} \frac{T}{n} I(t_i)}{\int_0^T I(t)dt} \right| \times 100\% \tag{11}$$

Where n is the number of samples required for this desired % accuracy A, t_i is the time at which the current is being measured, and T is the total time of the experiment. For our purposes, we will be setting our desired % accuracy A to 1%.

The issue with Eqs. (11) is that we are not able to take the integral or Riemann Sum of the unknown function $I(t)$. We must find a way of representing it as a function of time t, so we can numerically determine n.

Again, Ohm's Law from Eqs. (7) is used to to replace the current $I(t)$ in the equation to voltage $V(t)$ to enable further substitution and eventual calculations.

$$A \geq \left| \frac{\frac{1}{R}\int_0^T V(t)dt - \frac{1}{R}\sum_{i=0}^{n} \frac{T}{n} V(t_i)}{\frac{1}{R}\int_0^T V(t)dt} \right| \times 100\% \tag{12}$$

Now we must observe the relationship between the voltage induced $V(t)$ and time t in order to complete our integration and Riemann Sums.

By observing the relationship between the induced voltage and the magnetic field from Eqs. (4), we can conclude that voltage $V(t)$ is proportional to the magnetic field $B(x)$.

$$V(t)\ \alpha\ B(x) \tag{13}$$

And the magnetic field B decays according to the Inverse Cube Law, since the magnet is a dipole source.

$$B(x)\ \alpha\ \frac{1}{x^3} \tag{14}$$

And because the current-carrying solenoid is moved at a constant velocity during "charge mode", we can use the relationship between velocity, time, and distance.

$$x = vt \tag{15}$$

We finally get our relationship between desired % accuracy A, the total time of the experiment T, and the number of samples required n in Eqs. (16).

$$A \geq \frac{1}{R} \left| \frac{\int_0^T \frac{1}{t^3} dt - \sum_{i=0}^{n} \frac{T}{n} \frac{1}{{x_i}^3}}{\int_0^T \frac{1}{t^3} dt} \right| \times 100\% \tag{16}$$

Thus, we are able to take the integral and Riemann Sums and determine the number of samples n that we need over an experiment time T for our desired % accuracy A. From Eqs. (16) and the Masy Joule Balance equation from Eqs. (10), we can effectively find the relationship between our measured mass and a voltage supplied to the current-carrying solenoid.

III. BALANCE MECHANICS

Because the Masy Joule Balance is made of LEGO bricks, it is much more easily influenced by imperfections. Thus, we decided to adjust our Masy Joule Balance equation and absorb all constants into a single constant C. By doing this we can find the linear relationship between the mass m and the voltage v, and the slope of the trendline of our dataset would be that constant C.

The Masy Joule Balance was initially modeled after NIST's LEGO Watt Balance. Thus, the Masy

Joule Balance shares many features of NIST LEGO Watt Balance. However, several changes have been made to improve the accuracy and stability of the device.

Firstly, we decreased the mass of the arm itself to decrease the rotational inertia of the balance. This allows us to measure much smaller masses with increased resolution. Also, rather than using a symmetrical design, the Masy Joule Balance utilizes a single coil immersed in a radial magnetic field while the opposite arm of the balance utilizes two weighing pans, one on top of the other. This decreases the possibility of magnetizing our test weights since the distance from the neodymium magnets to the weight pans has increased. The top weighing pan is used to hold tare weights to balance out the arm when measuring lower masses, while the bottom pan holds the actual item or mass being weighed. The use of tare weights improves the range of masses that the balance can accurately measure from 0-13g to 0-105g.

The coil was made using a standard 1-inch PVC coupling with a 1.25-inch PVC cap secured on one end. The wire was wound onto the PVC pipe using a low-speed electric hand drill and secured with epoxy. A counter was created using an Arduino Uno to count the number of winding around the coil. The coil had approximately 3000 windings of AWG-36 magnet wire, and the total resistance of the coil was approximately 450Ω. A pair of neodymium (N48) ring magnets was secured to the base with a brass. The magnets are oriented on the bolt so that they are touching each other and are attracted to each other. The specific orientation (North up or south up) does not matter. Nuts were placed on either side of the bolt securing the magnets to ensure they remain stationary throughout our experiments.

A hole was drilled into the PVC cap and a LEGO cross axle with two LEGO Wedge Belt Wheels were passed through the hole. The assembly was then bonded with hot glue. This was then connected to another LEGO cross axle, which was then connected to the beam using the LEGO universal joint system.

The central pivot was a 4 by 1 technic brick secured to the beam with a connector peg. The technic brick was secured so that it balanced on its edge as it sits on a smooth surface. This knife edge-like balance point has much lateral movement due to rotation and thus much higher precision than the T brick used on the NIST LEGO Watt Balance. A series of LEGO bricks were stacked on either side of the tower to prevent the balance from moving out of alignment.

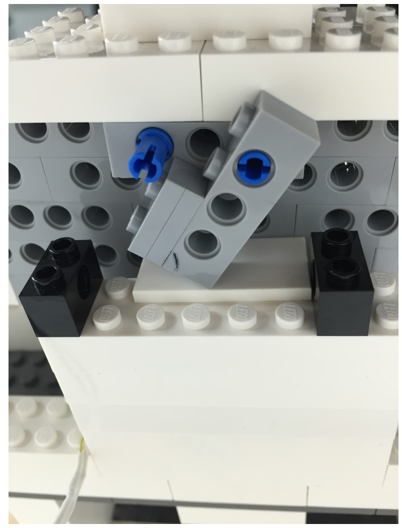

FIG. 2. The balance rests on a knife edge created by turning a 4 by 1 technic brick on its side.

IV. BALANCE ELECTRONICS AND DATA ACQUISITION

Similar to the NIST LEGO Watt Balance, the Masy Joule Balance utilized a photodiode and a line laser to determine the balance position and a Phidgets 1002 Voltage Output to produce the voltage supplied to the current-carrying solenoid. However, we chose to use an Arduino Uno as our voltage reference when reading from the photodiode and line laser system.

The photodiode and the line laser were mounted on opposite sides of the tower so that the arm could swing freely between them.

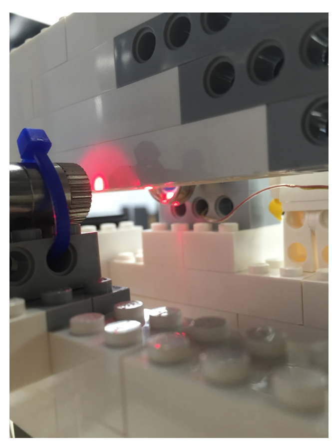

FIG. 3. The line laser is projected onto the arm and the photodiode behind.

During operation, the line laser is projected onto the arm and a portion of the laser would shine through to the photodiode. When the laser is half covered by the arm and half projected onto the photodiode, the arm is considered balanced. During operation, to ensure the beam is in the balanced position consistently during each run, the voltage across the photodiode must remain constant at the balanced voltage for a total of 20 consecutive samples gathered at a sampling rate of 80ms.

The program provided by NIST for the LEGO Watt Balance was not well suited for the Masy Joule Balance due to the differences in wiring and setup. Instead, a custom operating program was written in Java. The program now lives on Github as an open-source project.

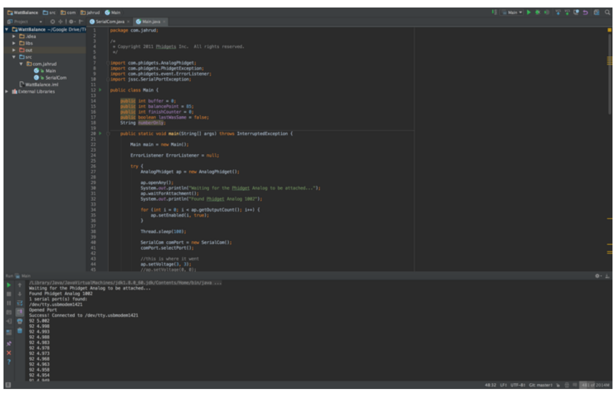

FIG. 4. The program was developed in Java and is run without a GUI.

This program utilizes the feedback loop from the laser-photodiode system to determine whether the arm is too far left or too far right, and thus determine whether the voltage supplied to the current-carrying solenoid from the Phidgets 1002 Voltage Output needs to increase or decrease.

V. MEASUREMENT

To ensure the Masy Joule Balance remains consistent for each experiment, alignment lines are drawn on the fulcrum and the top of the tower. Before each experiment, the mass is placed onto the weighing pan. When the test begins, the coil will produce a magnetic field that pulls the coil toward the magnets. The beam is then realigned using the alignment dots. The entire beam is then pulled toward the front face of the balance. This ensured that the beam is aligned and is in the same position for each measurement.

VI. ACKNOWLEDGMENTS

NIST's experience in building the LEGO Watt Balance was extremely useful in the initial design and planning of our Masy Joule Balance. Many of the parts used in our design were chosen due to their inclusion in NIST's LEGO Watt Balance physics journal. Our balance was heavily influenced by design choices made by the NIST in designing their LEGO Watt Balance.

Assistance from Masy BioServices is gratefully acknowledged. This project was completed as a summer internship at Masy BioServices' Mass Calibration Laboratory, and the hospitality and help received from Masy BioServices was a major encouragement to this project.

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