Description: The exam has two types of problems. Problems 1–3 should be answered on paper and delivered to the professor when you are done. Problems 4–5 are practical and must be delivered at `https://examens.fib.upc.edu` before the exam ends. Note that problems 4 and 5 include writing some explanation in the HTML file.

Duration: 3:30 hours.

Publication of final grades: Wednesday, January 24, through the Racó.

Revision: Anyone wishing to revise the grades obtained should: 1) send the professor an e-mail by Friday, January 26; 2) show up in the following room on Tuesday, January 30 at 16:00: Omega Building, 4th floor, office 422.

Problem 1. [1 point] Deliver on paper. Suppose that you are asked to add a new functionality to a drawing software based on Bézier curves. The new functionality is to allow the user to split curves into two. That is, the user will select a curve, then click a point on the curve, and then the program should break the existing curve into two at the selected point.

What method for Bézier curves would you use to implement this? Why? Explain how the method works.

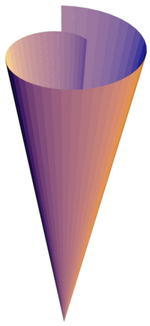
Problem 2. [1 point] Deliver on paper. Suppose that you need to construct a software to design 3D objects using piece-wise curves, to be used to produce high-quality images. The software will also take care of producing the necessary projections from 3D to 2D.

Which of the curves studied in class would you use? Why?

Problem 3. [1.5 points] Deliver on paper. Let $C(t)$ and $Q(t)$ be two B-spline curves defined using the same control points, but with two different knot vectors. The knot vector of $C(t)$ is given by $[0, 0, 0, 0.1, 0.2, \ldots, 0.8, 0.9, 1, 1, 1]$, while the knot vector of $Q(t)$ is given by $[0, 0, 0, 1, 2, \ldots, 8, 9, 10, 10, 10]$.

Prove that $C(t)$ and $Q(t)$ are the same curves.

Problem 4. Deliver through the Racó. For a 3D scene depicting a chestnut stand, we want to model a paper cone like the one shown below (left).



1. **[2 points]** Give a parametrization of the surface of the cone, formed by the segments that connect the apex (i.e., bottom vertex) of the cone to the (partial) spiral curve defining the top edge of the cone. Assume that the apex is at point $(0, 0, -h)$ (for some $h > 0$), and that the spiral lies on the plane $z = 0$.

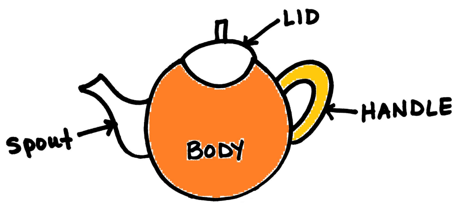
   Justify in detail how you obtained the parametrization in the HTML file.

2. **[1 point]** Write a program to draw such a cone on the screen.

Problem 5. [3.5-4.5 points] Deliver through the Racó. Write a program to draw part of the outer surface of a *Utah teapot*, similar to the one below (left). **You must draw at least the body and the handle** [3.5 points] (see figure, right). You can get up to one extra point by drawing also the spout and the lid.

You don't need to worry about the connection of the different parts to the body.

3D view of Utah teapot



Main parts of the teapot

Important: In the HTML file that you will deliver, include a brief description of the strategy that you have followed and the design decisions made.