

**Files:** The accompanying files for this assignments are `assignment3_1.html`, `assignment3_2.html`, and `assignment3.3.html`.

**Delivery:** upload the modified HTML files and any other necessary files to the Racó. All explanations and/or answers to the problems should be included in the HTML files.

**Problem 1.** Write a program to draw a **cubic Lagrange polynomial** that interpolates any four points, using:

1. Uniform parameter values.
2. Non-uniform parameter values, using the distance between consecutive interpolated points.

You can draw both curves at the same time, using different colors.

**Problem 2.** In this problem we want to explore Runge's phenomenon. You will write a program to draw a **Lagrange polynomial** to interpolate  $n + 1$  points, and use it to interpolate samples from the function  $f(x) = \frac{1}{1+25x^2}$ , for different values of  $n$ . The points to be interpolated will be samples from the graph of the function  $f(x) = \frac{1}{1+25x^2}$  taken uniformly from the interval  $[-1, 1]$ , that is:  $x_i = 2i/n - 1$ .

1. Write a program to draw a Lagrange polynomial to interpolate  $n + 1$  points.
2. Use the Lagrange polynomial to interpolate the points for different values of  $n$  (make  $n$  a user-defined parameter that can be changed interactively).
3. Draw the graph of  $f(x)$  in the background, in order to compare better the different interpolating curves.

Note: you should translate and rescale your canvas or curve appropriately to be able to see it in your program.

**Problem 3.** Write a program to draw a **cubic Hermite polynomial** that interpolates two points  $P_0, P_1$ , allowing the user to see and control the two points and the two tangent vectors at  $P_0$  and  $P_1$ .

**Problem 4.** Using the program from Problem 3, consider the particular case  $P_0 = (200, 200)$ ,  $P_1 = (400, 300)$ , and tangent vectors  $\vec{v}_0 = (100, 100)$  and  $\vec{v}_1 = (100, 0)$ .

1. Compute the position of the curve at  $t = 1/2$ .
2. How should the tangent vectors be modified in order to: keep the same directions at  $P_0$  and  $P_1$ , and at the same time go through  $(300, 300)$  at  $t = 1/2$ ?

Solve the problem first, and then illustrate your result with your program. Write down your answers in `assignment3.3.html`.