“Pass the Pigs” Data Analysis

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the full project, including data and code, is available at https://github.com/jrehfus1

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Motivation

Does your family have a favorite game? The kind for which participation is semi-mandatory at holiday gatherings? The kind that gets more competitive than it should? If your family is anything like mine, then the answer to all of the above is “yes”. Unfortunately for me, I lose this game. A lot. Sometimes so badly that everyone else is cackling like hyenas at my ineptitude. So, in an effort to minimize familial humiliation, I decided to apply a numerical approach to determine a winning strategy for the pig-shaped dice game called *Pass the Pigs*[1].

How does the game work?

Pass the Pigs is a dice game, but instead of having 6-sided cubical dice, there are two dice shaped like pigs. The orientations in which the pigs land are worth different point values. Players alternate taking turns rolling the pigs and each player’s score is the sum of the scores they amass by the end of each turn. The winner is the player with the highest score at the end of the game. Given the unpredictable nature of pig-shaped dice, *Pass the Pigs* is a game of chance.

Outline of the game

During each round, except for the final round, every player takes their turn according to a prescribed order. At the end of each player’s turn, their score for that turn is added to the scores from their previous turns to generate a running total. As soon as one player’s running total exceeds 100 points (or a different previously agreed upon total), the game enters the final round. In the final round, the remaining players get one final turn to try to beat the leader’s total. At the end of the final round, the player with the highest running points total is the winner (**Figure 1**).

round	John	Mick	Stevie	
1	+ 0	+ 15	+ 25	
2	+ 20	+ 35	+ 56	
	= 20	= 50	= 81	
3	+ 12	+ 52	+ 0	Mick
	= 32	= 102	= 81	breaks 100
4	+ 0		+ 30	final
	= 32		= 111	round

Figure 1 Example scorecard from a game of *Pass the Pigs*. There are three players in this game: John, Mick, and Stevie. Each player takes one turn per round until one of the players scores more than 100 points. That player’s score is locked in, but the rest of the players get one more round to try to beat it. In this case, Stevie wins the game because she ended up with the highest total score.

[1] PASS THE PIGS ® is a registered trademark of David Moffat Enterprises © 1977, 1984, 1991, 1992, 1999, 2009, 2014 David Moffat Enterprises. All rights reserved.

Taking a turn

A player rolls both pigs at the beginning of their turn. Each of the possible pig landing orientations has a corresponding point value. For example, if both pigs land on their feet (a “double trotter”), the player scores 20 points and is allowed to roll again to try to add to their score. However, if both pigs land on their sides, but not on the same side (a “pig out”), the player’s turn is over, and they lose all of their points for that turn. Thus, there is risk and potential reward associated with each roll. A player must choose whether to stop rolling and take the points in hand or to risk all of their current turn’s points for the prospect of a higher return. A greedy player may begin accumulating many points during their turn, only to ultimately lose them all to a pig out (**Figure 2**).

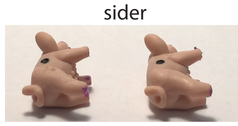
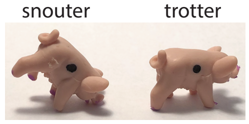
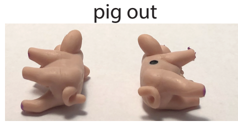
roll	pig orientations	score
1	trotter razorback 	+ 10
2	sider 	+ 1 = 11
3	snouter trotter 	+ 15 = 26
4	pig out 	= 0, end turn

Figure 2 Example of a player’s turn. Here, the player scores 10 points on their first roll, 1 on their second, 15 on their third, then loses all of their points and ends their turn after “pigging out” on their fourth roll.

Finding a winning strategy

The outcome of a game of *Pass the Pigs* comes down to the scores produced by each roll during each of the turns a player gets. Since the number of turns in the game is limited, and the final round can occur at any time, it is important to maximize each turn’s score. Maximizing points per turn serves two functions. First, it ends the game before other players have a chance to build up their scores. Second, it increases the deficit faced by other players, pressuring them to make riskier decisions. The goal of this analysis was to determine how points can be maximized for each turn. I set out to come up with a winning strategy based on the rules of pig rolling probability.

Data requirement

- probabilities of pig landing orientations
- point values of pig landing orientations

Data source(s)

- the outcomes of hundreds of test rolls

Pertinent script(s)

- passing_pigs_v1.1.1

Collecting probability data

The point values for each pig landing orientation are outlined in the rules of *Pass the Pigs*. However, the probability of pigs landing in each orientation are not provided. To find out what the odds of pigs landing in different orientations are, I simulated 300 rolls with real game pigs and recorded the landing position of each pig. The probabilities that I determined are listed below (**Table 1**).

landing orientation	per roll probability
spot side up	0.240
spot side down	0.365
trotter	0.058
razorback	0.320
snouter	0.017
leaning jowler	0.0
oinker	0.0
piggy back	0.0

Table 1 After rolling both pig-shaped dice 300 times, I determined the probability of a pig landing in each possible orientation.

The expected score (ES) for a random roll is given by **Equation 1**:

$$[\mathbf{E1}]\ ES = \sum p(orientation_i) * s(orientation_i)$$

Where *p(orientation$_i$)* is the probability of the pigs landing in orientation *i* and *s(orientation$_i$)* is the corresponding score for orientation *i*. The summation is over all orientations *i*.

The probability of pigging out (p_{pig_out}) can be determined by the joint probability of one pig landing spot side up while the other lands spot side down (0.240*0.365 = 0.0876). However, since the order of the pigs does not matter, we have to multiply the joint probability by two (p_{pig_out} = 0.0876*2 = 0.175). There is a 17.5% chance of pigging out on any given roll.

Omitting the probability of pigging out, we are left with the expected score for a roll that scores points (ES_{npo}, or expected score with no pig out), as expressed by **Equation 2**:

$$[\mathbf{E2}]\ ES_{npo} = {}^{ES}/_{(1 - p_{pig\_out})}$$

The probability of stringing together *n* consecutive rolls without pigging out is given by **Equation 3**:

$$[\mathbf{E3}]\ p(n) = (1 - p_{pig\_out})^n$$

Scores are accumulated only by rolling the pigs successively without pigging out. The expected score from strings of consecutive rolls of different lengths, along with the corresponding probability of obtaining such strings of consecutive rolls, is plotted below (**Figure 3**). The expected score for *n* consecutive rolls scales with *n*, while the probability of rolling *n* times in a row without pigging out asymptotically approaches 0.

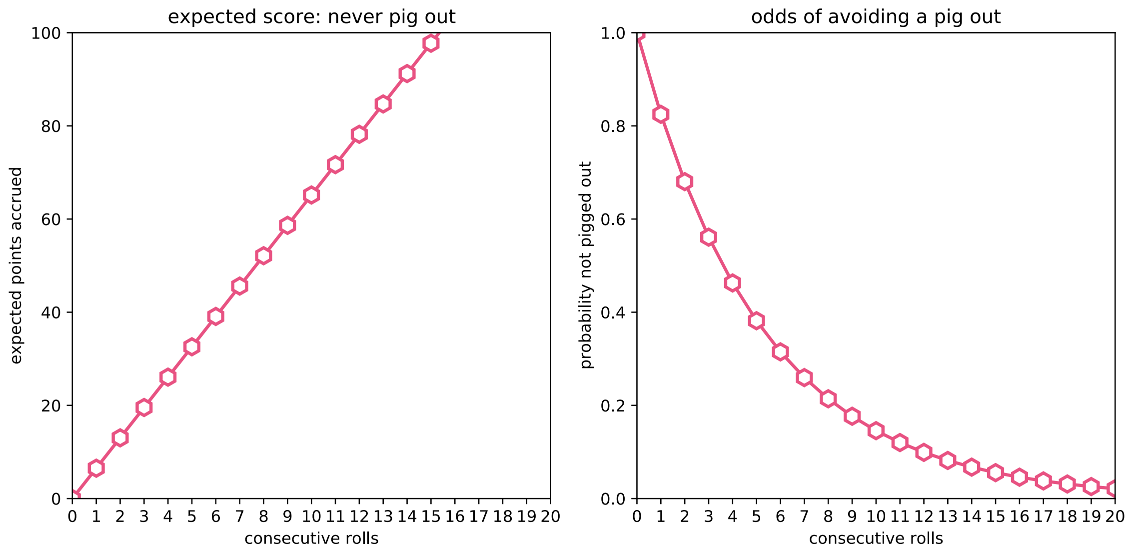


Figure 3 *Left* The expected total number of points accrued during a player's turn if they never pig out (ES_{npo}). Note that ES_{npo} scales linearly. *Right* The probability of generating consecutive rolls without pigging out (p(n)). Note that p(n) asymptotically approaches 0.

Winning Strategy #1: roll a set number of times for every turn

The first strategy for maximizing the number of points scored in a given turn involves the player always rolling the same number of times, regardless of the score. The logic behind Winning Strategy #1 (WS #1) is that the expected score from a string of successive rolls increases linearly, while the probability of stringing together successive rolls approaches zero as the number of rolls increases. As a result, the expected score for turns consisting of different numbers of rolls first increases and then decreases (**Figure 4**).

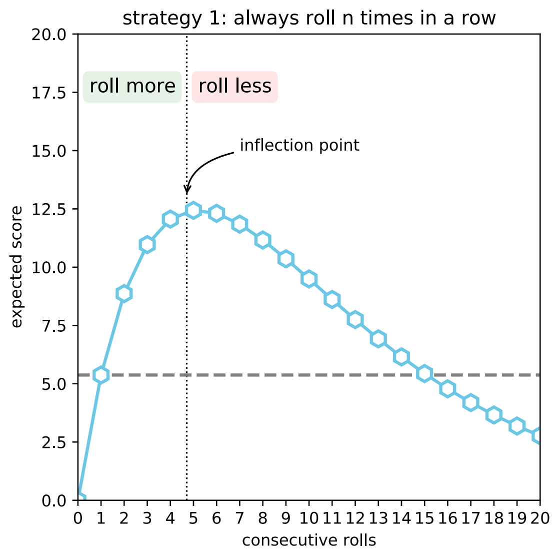


Figure 4 The expected score after taking into account the probability of not pigging out for the given number of consecutive rolls (blue). The inflection point of the indicates the number of consecutive rolls at which the expected score is maximal. Rolling the pigs more or fewer times, on average, will not produce the maximal score for a turn. For reference, the gray dashed line indicates the expected score from a single roll of the pigs.

As an example, consider rolling the pigs 3 times in a row versus 10 times in a row. The ES_{npo} for 3 rolls is 19.6 points, while that for 10 rolls is 65.2 points. However, the probability of successfully rolling 3 times in a row without pigging out is 56.2%, while the corresponding probability for 10 rolls is just 14.6%. As a result, the expected score for a turn in which the player rolls only 3 times is 11 points, as opposed to 9.5 points for rolling 10 times in a row. Perhaps counter-intuitively, rolling more times eventually *decreases* a player's score for a given turn. In fact, rolling the pigs 16 times is expected to produce a lower score than rolling them just once!

To find out when a player should stop rolling the pigs, we need to calculate when the expected score for $n+1$ rolls is less than that of n rolls. This value can be determined from **Equation 4**:

$$\textbf{[E4]}\ (ES_{npo})(n+1)\left(1-p_{pig_{out}}\right)^{n+1} \leq (ES_{npo})(n)(1-p_{pig\_out})^{n}$$

After simplifying **Equation 4**, we can solve for the optimal number of times to roll the pigs ($n_{optimal}$) by using **Equation 5**:

$$\textbf{[E5]}\ n_{optimal} = {(1-p_{pig\_out})}\Big/{p_{pig\_out}}$$

According to p_{pig_out} calculated above, the $n_{optimal}$ for a turn is 4.71 rolls, corresponding to the inflection point in **Figure 4**. The corresponding expected score for a turn in which none of the 4.71 rolls pig out is 30.67 points. An obvious limitation associated with implementing this strategy is that a player cannot roll the pigs for anything other than an integer number of times.

Winning Strategy #2: keep rolling until reaching a threshold score for every turn

The second strategy for maximizing the number of points scored in a given turn is to stop rolling only after a threshold score is achieved. In this strategy, the number of rolls varies for each turn.

For example, if the first few rolls each yield small point totals, the player may end up rolling several more times (barring a pig out) before reaching the threshold score. On the other hand, the player may amass the threshold score in only a few rolls, at which point they stop rolling and take the score in hand. The logic of Winning Strategy #2 (WS #2) is illustrated below (**Figure 5**). At a certain point, the current score exceeds the expected score for rolling the pigs one more time.

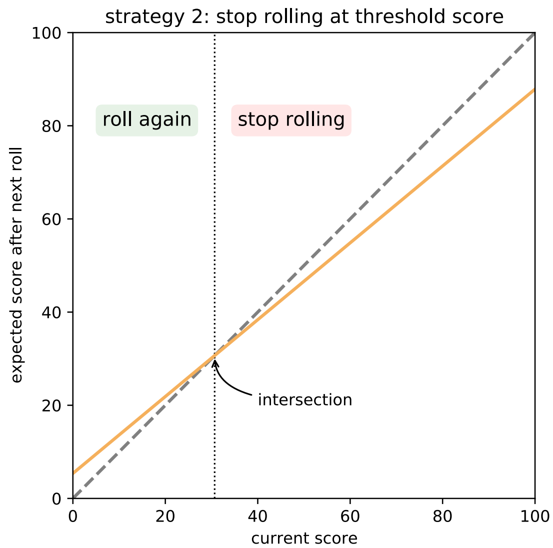


Figure 5 The expected score following one more roll of the pigs given a current score (orange). For reference, the gray dashed line indicates equal values for the current score and the expected score. The intersection of the lines indicates the current score at which rolling one more time is expected to produce no score change. At lower current scores, rolling is favorable. At higher scores, rolling is unfavorable.

As an illustrative example, consider cases in which a player has either 10 points or 60 points. The expected score given by rolling the pigs one more time is given by **Equation 6**:

$$\textbf{[E6]}\ ES_{one\_more\_roll} = (s_{curent} + ES_{npo})(1 - p_{pig\_out})$$

In the case that the player only has 10 points, the expected score from one more roll is 13.63 points. Thus, rolling again is expected to increase the players score, despite the potential for pigging out. However, if the player has already amassed 60 points, rolling the pigs again is expected to yield a score of only 54.9 points. In this scenario, the odds of scoring 0 points due to a pig out dictate that it would be wiser to take the points in hand than to risk what the player already has in an effort to make a relatively meager gain.

In order to determine the threshold score at which a player should stop rolling, we must know when the current score plus the expected score for the next roll is lower than the current score. The threshold score can be determined by solving for $s_{current}$ in **Equation 7**:

$$\textbf{[E7]}\ (s_{curent} + ES_{npo})(1 - p_{pig\_out}) \le s_{curent}$$

Plugging in the p_{pig_out} and ES_{npo} calculated above, we find that the threshold score is 30.67 points. Based on the ES_{npo}, the threshold score will be achieved after, on average, 4.71 rolls. Note that the number of rolls necessary for implementing WS #2 is identical to the number required for WS #1.

Game simulations reveal that WS #2 is preferable

It turns out that both strategies are identical, in principal. Examining **Equation 4** reveals why. WS #1 solves this equation for n, while WS #2 solves it for $ES_{npo}*n$. Despite this identity, the actual implementation of each strategy may lead to differences in performance.

Consider that the ideal implementation of WS #1 involves rolling the pig dice 4.71 times for each turn. Of course, only integer values can be used for the number of rolls in a turn. Thus, practical implementation of WS #1 requires rolling five times per turn. Similarly, the number of points that are amassed in a given turn is always an integer value. While the threshold total for WS #2 is 30.67 points, the practical implementation of WS #2 would require a player to stop rolling after accruing 37 or more points.

The question now becomes whether or not these subtle differences in practical implementation result in one strategy being more reliable than the other. To find out, I simulated 10,000 games of *Pass the Pigs* using each strategy. The number of turns required to achieve a winning score of 100 points was recorded for each game, and the results are plotted below (**Figure 6**).

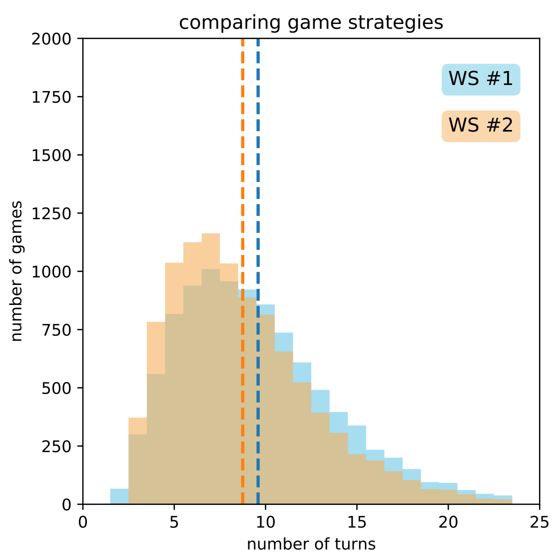


Figure 6 Histogram of the number of turns required to reach a total of 100 points in 10,000 simulated games. Bin widths are one turn. The results for Winning Strategy #1 are shown in blue, those for Winning Strategy #2 are shown in orange. The means of each distribution are indicated with a dashed line.

On average, WS #1 requires 9.53 turns to reach 100 total points. In contrast, WS #2 requires an average of 8.71 turns. Applying a t-test reveals that the mean number of turns required to reach 100 points is significantly lower ($p \sim 4*10^{-43}$) when WS #2 is used[2]. This discrepancy likely stems from a slight decrease in efficiency for WS #1 as a result of implementing a rule based on an integer number of rolls rather than an integer number of points as in WS #2. Both strategies,

[2] For a detailed explanation of the appropriateness of a t-test in this situation, please see Luley *et al* (Reference 1).

however, produce 100 points over a wide number of turns. In order to maximize the chances of victory over familial rivals, it is imperative that you play multiple games, perhaps in a “best of seven” series, for example.

Conclusions

Winning strategy #2 is the best *Pass the Pigs* strategy I have developed to date. When implemented in real games, I felt much more competitive compared to when I was implementing an ad hoc strategy before (I do not have data from previous competitions). However, given that *Pass the Pigs* is a game of chance, anything can happen, and victory is never assured. The pigs still have to bounce in your favor.

Open questions

One can imagine more complicated strategies, such as one that factors in the scores of your opponents. While such strategies are beyond the scope of the present work, they are certainly intriguing, and I may explore them at a later date. There are also other simple family games that would be fun to dominate. If I develop a bad enough record, I may be forced to quantitatively evaluate potential winning strategies of some of those games in the future.

References

1. Lumley T, Diehr P, Emerson S, Chen L (2002) The importance of the normality assumption in large public health data sets. *Annu Rev Public Health* 23: 151-169.