Announcements

- WebWorK due tonight!
- A new webwork assigned for Friday
- I'm hoping to be able to post scores on Exam 2 by Friday, but probably will not be handing it back and discussing it until sometime next week
- I'll aim to get new updated grade reports up this weekend
- I have to slide out of office hours a bit early today at 4:30, but will be around from 3 until then
- Polling: `rembold-class.ddns.net`

Review Question

A parajumper leaps out of an airplane high in the air and accelerates towards the Earth. How does the absolute value of the work done by gravity compare to the work done by air resistance pushing back against the jumper?

A) $|W_{air}| > |W_g|$

B) $|W_{air}| = |W_g|$

C) $|W_{air}| < |W_g|$

D) Not enough information to tell

Stretchy Energy

Suppose you have a spring with a spring constant of 5 N/m (comparable to the springs you've used in lab). How much energy would it take to stretch such a spring from it's equilibrium point to 5 m stretched?

Putting it all Together

- Our goal with work was to quantify the energy entering or leaving a system
- We can now write the Energy Principle as

$$\Delta E_{sys} = W_{surr} + \text{ other inputs}$$

or, in a more iterative manner:

$$E_{sys,f} = E_{sys,i} + W_{surr} + \text{ other inputs}$$

- These are generally going to be the major methods you use to approach Energy Principle problems

Sledding Energy

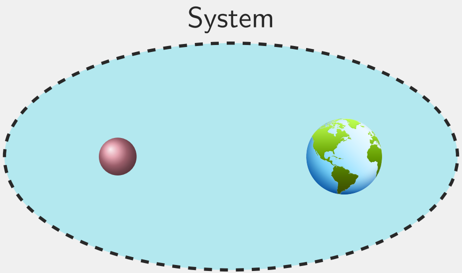
A sledder slides down a hill which exerts a frictional force of 500 N. If the sledder has a mass of 65 kg and the hill makes an angle of 35° with the horizontal, how far does the sledder slide down the hill before coming to a stop? Assume the sledder starts with a speed of 10 m/s down the hill.

Upcoming:

A Question of Potential

Multiparticle Systems

- Thus far looked at only single particle systems
- How to treat multi-particles systems?



- No change in mass, but

$$\Delta K_{sys} = W_{surr} = 0 \text{ ???}$$

That can't be correct!

Internal Interactions

- Unlike forces, we can't get away with neglecting internal interactions
- Need a method to quantify the addition or subtraction of energy due to interactions inside our system
- These energy terms are generally called the potential energy (U) of the various intersystem interactions
  - Gravitational potential energy if it is gravity acting between your particles
  - Spring potential energy if it is a spring force acting between your particles
  - Etc
- A system of two particles would thus have a total energy that looks like

$$E_{sys} = \underbrace{m_1c^2 + K_1}_{E_1} + \underbrace{m_2c^2 + K_2}_{E_2} + U_{12}$$

Internal Work

- If there are no outside forces, a change in energy thus looks like:

$$\Delta E_{sys} = \Delta K_1 + \Delta K_2 + \Delta U_{12} = 0$$

- Imagine that particle 1 is not moving, just to make the math a bit easier

$$\Delta E_1 = \Delta K_1 = 0$$

- Looking at the system of *just* particle 2:

$$\Delta E_2 = \Delta K_2 = W_{surr} = W_{1 \text{ on } 2}$$

- Plugging in:

$$\Delta E_{sys} = 0 + W_{1 \text{ on } 2} + \Delta U_{12} = 0$$

Potential Energy

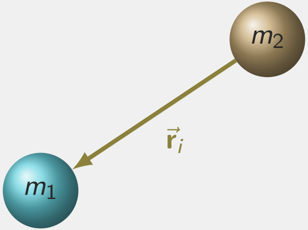
- We define the *change* in the potential energy to be

$$\Delta U = -W_{int}$$

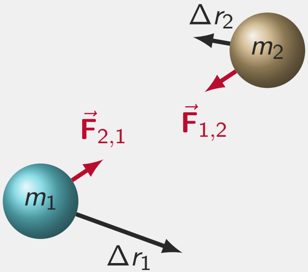
- We can lump all different interactions that might be happening between object pairs into a single potential
- Say you have 3 particles interacting via spring or gravitational forces, you could describe the total potential of the system as

$$U = U_{12} + U_{13} + U_{23}$$

  where each potential would describe the iterations between those two objects
- But how to find U?



$$
\begin{aligned}
\Delta U &= -W_{int} \\
&= -(\vec{\mathbf{F}}_{2,1} \cdot \Delta\vec{\mathbf{r}}_1 + \vec{\mathbf{F}}_{1,2} \cdot \Delta\vec{\mathbf{r}}_2) \\
&= -\vec{\mathbf{F}}_{2,1} \cdot (\Delta\vec{\mathbf{r}}_1 - \Delta\vec{\mathbf{r}}_2) \\
&= -\vec{\mathbf{F}}_{2,1} \cdot \Delta\vec{\mathbf{r}}
\end{aligned}
$$



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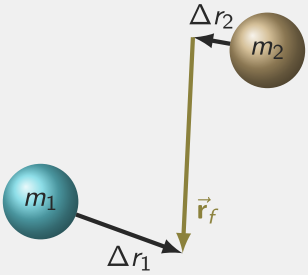
\end{aligned}
$$

Searching for U



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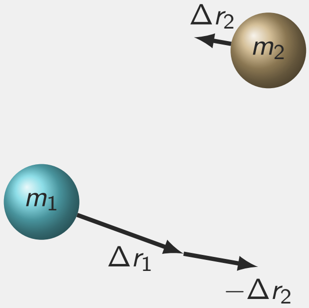
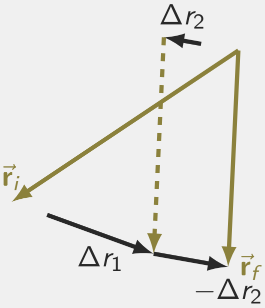
&= -\vec{\mathbf{F}}_{2,1} \cdot \Delta\vec{\mathbf{r}}
\end{aligned}
$$



$$\begin{aligned}\Delta U &= -W_{int}\\ &= -(\vec{\mathbf{F}}_{2,1}\cdot\Delta\vec{\mathbf{r}}_1 + \vec{\mathbf{F}}_{1,2}\cdot\Delta\vec{\mathbf{r}}_2)\\ &= -\vec{\mathbf{F}}_{2,1}\cdot(\Delta\vec{\mathbf{r}}_1 - \Delta\vec{\mathbf{r}}_2)\\ &= -\vec{\mathbf{F}}_{2,1}\cdot\Delta\vec{\mathbf{r}}\end{aligned}$$



$$
\begin{aligned}
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&= -\vec{\mathbf{F}}_{2,1} \cdot (\Delta\vec{\mathbf{r}}_1 - \Delta\vec{\mathbf{r}}_2) \\
&= -\vec{\mathbf{F}}_{2,1} \cdot \Delta\vec{\mathbf{r}}
\end{aligned}
$$

Potentials from Forces

- Written another way:
$$F_r = -\frac{\Delta U}{\Delta r}$$
where F_r is the component of force in the direction of $\vec{\mathbf{r}}$.
- Looks a lot like a derivative!!
$$F_r = -\frac{\mathrm{d}U}{\mathrm{d}r}$$
- Gives us a method to work backwards (via an integral) to find the potential that corresponds to a force!!
$$U(r) = -\int_0^r F_r\,dr$$

Spring Potential

We know that the force of a spring is given by:

$$F = -k_s s$$

where s is the distance from equilibrium. Suppose a stretch a spring from equilibrium to a stretched distance x. What would the potential energy look like?

Useful Potential Energies

- Gravitation Potential Energy (near Earth)

$$U = mgy$$

- Gravitational Potential Energy (everywhere)

$$U = -G\frac{m_1 m_2}{r}$$

- Electric Potential Energy

$$U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$$

- Spring Potential Energy

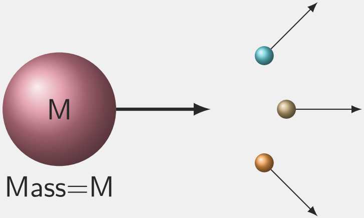
$$U = \frac{1}{2}k_s s^2$$

Moon Problems

Suppose the Moon were to suddenly lose all its tangential speed, causing it to plummet towards the Earth. How fast would it be traveling when it struck the Earth's surface? The mass of the Earth is 5.97×10^{24} kg, the mass of the Moon is 7.35×10^{22} kg, the radius of the Earth is 6371 km, the radius of the Moon is 1737 km, and the distance between them is 385 000 km.

Changing up the Rest Energy

- So far we have seen instances where the mass of the object did not change
- This is not always the case for atomic particles!
- If there is a changing mass, be careful to consider the *entire* energy when looking at your ΔE, don't get in the bad habit of assuming ΔE just equals ΔK!



$$m_1 + m_2 + m_3 < M$$

Atomic Example

A neutron ($1.674\,929 \times 10^{-27}$ kg) at rest can spontaneously decay into a proton ($1.672\,623 \times 10^{-27}$ kg), an electron ($9.109\,390 \times 10^{-31}$ kg) and an antineutrino (massless). What is the total kinetic energy of the system after the decay? Can a proton decay into a neutron, electron and antineutrino?