Announcements

- Homework 5 due on Monday!
  - It should all be doable after today, so don't save it all till Monday!
  - Heavy use of plotting throughout the homework, but I'm trying to convey that you should be comfortable and familiar with visualizing results.
- Polls up for HW4 time and preferred Test 1 type on Campuswire! Don't dally as they expire today!
- Have read Ch 3.3.1 and at least part of Ch 3.3.2 by Monday

Just chill

Q1

Consider a function $f(x)$ that is both continuous and differentiable over some domain. Given a step size of a, which could be an approximate derivative of this function somewhere in that domain?

$$\frac{\mathrm{d}f}{\mathrm{d}x} \approx$$

A. $\dfrac{f(x_i + a) - f(x_i)}{x_i}$

B. $\dfrac{f(x_i + a) - f(x)}{a}$

C. $\dfrac{f(x_i) - f(x_i - a)}{a}$

D. More than one of these

Q1

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C. $\dfrac{f(x_i) - f(x_i - a)}{a}$

D. More than one of these (B and C)

Just chill

Q2

Say we want to use

$$\frac{\mathrm{d}f}{\mathrm{d}x} \approx \frac{f(x_i + a) - f(x_i)}{a}$$

What point best describes the location at which we are computing the approximate derivative?

A. a

B. x_i

C. $x_i + a$

D. Somewhere else

Q2

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What point best describes the location at which we are computing the approximate derivative?

A. a

B. x_i

C. $x_i + a$

D. Somewhere else $(x_i + a/2)$

Q3

Taking a second derivative is as simple as applying the same discrete derivative equation again, at the location of the first derivative.

$$f''(x_i) \approx \frac{f'(x_i + a/2) - f'(x_i - a/2)}{a}$$

What is the value of the second derivative then in terms of f?

A. $\dfrac{f(x - a/2) - 2f(x + a/2) + f(x + 3a/2)}{a^2}$

B. $\dfrac{f(x - a/2) + f(x + 3a/2)}{a^2}$

C. $\dfrac{-2f(x + a/2)}{a^2}$

D. $\dfrac{f(x - a/2) + 2f(x + a/2) + f(x + 3a/2)}{a^2}$

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Implementing Relaxation

A. Break up region of interest into discrete chunks
B. Set boundary conditions
C. Set initial guess at all other starting values
D. Choose max iterations and target accuracy
E. Start relaxing!
    - Update all non-boundary terms with average of neighbors
    - Calculate difference from last iteration
    - Compare to target accuracy to see if keep iterating or target reached!
F. Plot up those sweet sweet results

Just chill

Q4

To investigate if we have converged to a solution, we must compare our estimate of V before and after the averaging calculation. For our 1D relaxation code, V will be a 1D array. For the kth estimate, we can compare V_k against the previous value by taking the difference. If this difference is stored as `err`, what is the type of `err`?

A. A scalar

B. A 1D array

C. A 2D array

D. A string

Q4

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Demo!

For the rest of class I'll walk you through how I'd approach putting together a method of relaxation solver for the 1D case. A video of this will be available, but the notebook itself will not! Feel free to follow along on your laptops and ask questions as we go!

Our problem: Solve the 1D Laplace equation where $V(x = 0) = -5$ and $V(x = 50) = 10$. We'd like to be accurate at least to within mV.