Announcements

- Homework
  - Video Homework due tonight
  - Webwork 15 due on Wednesday
  - Nothing due Friday. Study up!
  - Also nothing due the Monday after Spring break. Relax!
- Test 2 on Friday!
  - You get a new notecard
  - Study materials are posted
  - Same general style as Test 1
  - Will probably be on an honor system, where I will send it out and you have like 1.25 hours to complete, photograph, and email it back to me.
- Tutoring is still happening. See Campuswire for info!
- Need to ask questions or talk with me personally? Email me and we can set up a time to meet in person or via Zoom.
- Polling: `rembold-class.ddns.net`

Capacitor Addition

In Parallel

- Each plate of the capacitor connected to other capacitor plates
- As if all the plates were touching, forming one large plate
- Since capacitance in proportional to A, we can get the total capacitance by adding

$$C_{eq} = C_1 + C_2 + C_3 + \cdots$$

In Series

- Negative plates connected to positive plates
- Neutral $\Rightarrow$ same charge on each
- Conservation of energy thus says

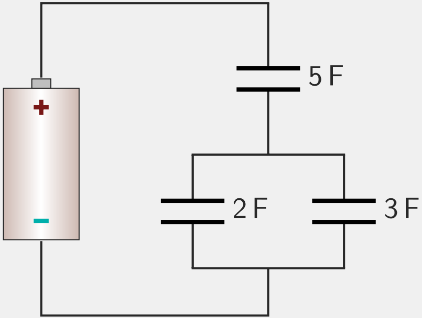
$$\mathcal{E} - \frac{Q}{C_1} - \frac{Q}{C_2} - \frac{Q}{C_3} = 0$$

- Same as

$$\mathcal{E} - Q\underbrace{\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)}_{1/C_{eq}} = 0$$

Capacitor Circuits

What is the equivalent capacitance of the circuit shown to the right?

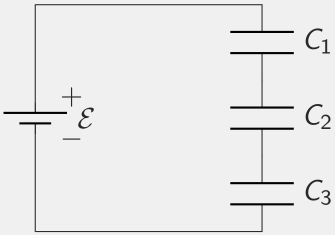


Solution: 2.5 F

Understanding Check

Given the circuit to the right, what amount of charge is on the middle capacitor when the circuit reaches steady state? You are told that

$$\mathcal{E} = 5\,\mathrm{V} \quad C_1 = 2\,\mathrm{F}$$
$$C_2 = 4\,\mathrm{F} \quad C_3 = 2\,\mathrm{F}$$



A) 8 C
B) 4 C
C) 2 C
D) 0.8 C

Solution: 4 C

Energy Considerations

- Would like to be able to talk about the energy stored in a capacitor in term of its capacitance
- Recall that

$$\Delta U = \frac{1}{2}\epsilon_0 E^2(\text{volume})$$

- We can thus write:

$$\begin{aligned}\Delta U &= \frac{1}{2}\epsilon_0\left(\frac{Q}{A\epsilon_0}\right)^2(As) = \frac{1}{2}\epsilon_0\frac{Q^2}{A^2\epsilon_0^2}(As)\\ &= \frac{1}{2}\frac{Q^2 s}{\epsilon_0 A} = \frac{1}{2}\frac{(C\Delta V)^2}{C}\\ &= \frac{1}{2}C(\Delta V)^2\end{aligned}$$

So, about those dielectrics now...

- Recall that, with the dielectric inserted, the electric field inside the capacitor is

$$E_{insulator} = \frac{E_{w/o}}{K}$$

- Then the potential difference is

$$\Delta V = E_{insulator} s = \frac{E_{w/o} s}{K} = \frac{\Delta V_{w/o}}{K}$$

- So the capacitance is

$$C = \frac{Q}{\Delta V} = \frac{Q}{\frac{\Delta V_{w/o}}{K}} = K C_{w/o}$$

- So inserting a dielectric *increases* the capacitance of our capacitor!

Glass Example

Suppose we have a capacitor with a capacitance of $100\,\mu\text{F}$ with a vacuum between the charged plates. We charge it to $10\,\text{V}$ and then insert a piece of glass with a dielectric constant of 8 between the plates. How much additional charge can be placed on the capacitor plates? How much more energy can the capacitor store now?

Solution: $7\,\text{mC}$, $35\,\text{mJ}$

Resistance

- We've already been talking about resistance
  - In terms of:
    - electron density
    - electron mobility
    - cross-sectional area
- This is great when taking a microscopic view, but not as useful for an everyday, macroscopic perspective
- Want to start eliminating or separating out the various dependencies

Current Density

We define the current density J to be

$$J = \frac{\mathrm{I}}{A}$$

with units of A/m^2.

Conductance

- In our current equation:

$$I = |q|nAuE$$

  q, n and u are all basically properties of the metal
- Would be useful to wrap them all up together so we just have one lump "property" to consider
- Just realize this technically hides information from us!

Conductance

We define the conductivity (σ) of a metal to be

$$\sigma = |q|nu$$

with units of A/V, also called Siemens (S)

A Macroscopic Take

- All told, we now can write:

$$J = \frac{\mathrm{I}}{A} = \sigma E$$

- Over a certain length of wire, we also know that:

$$|\Delta V| = EL$$

- Combining terms:

$$\frac{\mathrm{I}}{A} = \sigma\left(\frac{\Delta V}{L}\right)$$
$$\mathrm{I}\left(\frac{L}{\sigma A}\right) = \Delta V$$

Join the Resistance

Resistance

We define the constants in parentheses to be the resistance:

$$R = \frac{L}{\sigma A}$$

where R has units of ohms (Ω) or V/A

- Contains all the electronic and geometric properties of the wire/resistor all wrapped up in a nice single parameter
- Easy to relate the current to the potential difference:

$$\Delta V = IR$$

Wire Resistance

We looked at several problems where we were using short copper wires. What is the resistance of a 30 cm long, 1 mm diameter copper wire? We know that, for copper, $n = 8.5 \times 10^{28}\,\mathrm{e^-/m^3}$ and $u = 4.4 \times 10^{-3}\,(\mathrm{m/s})/(\mathrm{N/C})$.

Solution: $6.38\,\mathrm{m\Omega}$

Going Zen: Ohhhmmmmm...

- Resistance is only useful to us as a mostly fixed parameter
  - Means n and u shouldn't change much depending on current
- Materials where n and u are mostly constant with current are called ohmic.
- By comparison, non-ohmic materials have their electric properties change significantly depending on current (and temperature)
- Most metals are only roughly ohmic, so long as the currents are low
- Semiconductors: seriously non-ohmic
  - n varies greatly on voltage
  - Low voltage means n is incredibly small, $\Rightarrow$ basically zero current
  - Double the voltage and n and get huge, $\Rightarrow$ lots of current!
  - Let's you turn on/off segments of wire depending on voltage