Announcements

- WebWorK 4 due tonight!
- Test 1 on Friday!
  - Old test and solutions posted on the main webpage
  - Polling questions are also a good source of review
  - Bring a calculator, or email me if you want to borrow one on test day
  - Show up on time, as I can't let you stay around late because of the next class
- There will be no homework over the weekend, as I think you always need a break after a test
- Polling: `rembold-class.ddns.net`

Gravity's Final Form

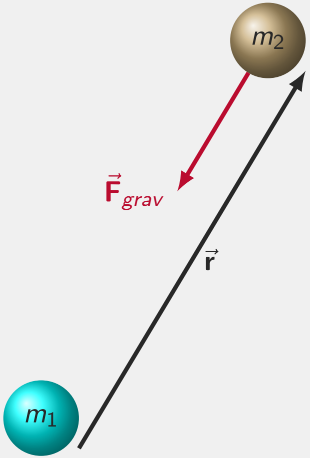
- We've seen that near the surface of the Earth the force of gravity can be approximated as

$$\vec{\mathbf{F}}_{gravity} \approx m\vec{\mathbf{g}}$$

where $\vec{\mathbf{g}} = \langle 0, -9.8, 0 \rangle$ N/kg

- More accurately, the force of gravity is defined as

$$\vec{\mathbf{F}}_{\text{grav on 2 by 1}} = -G\frac{m_1 m_2}{|\vec{\mathbf{r}}|^2}\hat{\mathbf{r}}$$



Break it Down

$$\vec{\mathbf{F}}_{\text{grav on 2 by 1}} = -G\frac{m_1 m_2}{|\vec{\mathbf{r}}|^2}\hat{\mathbf{r}}$$

- m_1 and m_2 are the masses of the two objects
- $|\vec{\mathbf{r}}|$ is the distance between the objects
- $\hat{\mathbf{r}}$ is a direction pointing from the mass to the object it is interacting with
  - Points *to* the object of interest
- G is a measure of the strength of gravity

$$G = 6.67 \times 10^{-11}\,\mathrm{Nm^2/kg^2}$$

Dragging ya down

Let's calculate the force of the Earth on me, a 75 kg individual. For reference, the Earth has a mass of 5.972×10^{24} kg and an average radius of 6371 km. Compare this force to our earlier approximation.

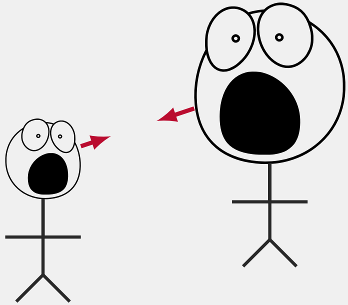
Bobby has a big head with a mass of 10 kg located at $\langle 2, 10, 3\rangle$ m. Beth has an even bigger head with a mass of 20 kg located at $\langle 12, 5, -2\rangle$ m. What is the gravitational force exerted on Beth's head by Bobby's head? For the sake of simplicity here, just let the gravitational constant $G = 1\text{Nm}^2/\text{kg}^2$.

A) $\langle 1.08, -.544, -.544\rangle$ N

B) $\langle 13.3, -6.66, -6.66\rangle$ N

C) $\langle -1.08, .544, .544\rangle$ N

D) $\langle -10.8, 2.72, 2.72\rangle$ N



Example

A rogue asteroid is observed with a velocity of $\langle 4000, 0, 0\rangle$ m/s when it is located at $\langle 0, 5\times10^{11}, 0\rangle$m (assuming the Sun is at the origin). If the asteroid has a mass of 2000 kg and the Sun has a mass of 2×10^{30} kg, determine the shape of its orbit. Use a time step of 1 day.

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$$\vec{\mathbf{F}}_{gravity} = \langle 0, -1.067, 0 \rangle \text{ N}$$

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$$\vec{\mathbf{p}}_{future} = \left\langle 8 \times 10^6, -92\,206.1, 0 \right\rangle \text{N s}$$

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$$\vec{\mathbf{p}}_{future} = \left\langle 8\times 10^{6}, -92\,206.1, 0\right\rangle \text{ N s}$$

$$\vec{\mathbf{r}}_{future} = \left\langle 3.46\times 10^{8}, 4.999\,96\times 10^{11}, 0\right\rangle \text{ m}$$

Iterate Smarter, Not Harder

- We could keep on doing this by hand
  - Oh goodness the pain though...
- Part of the power of iterations is in using computers
- The setup is simple:
  - Set up a loop in the computer (for or while)
  - That continually evaluates and updates our 3 iterative steps

The Gameplan

- Define constants and initial conditions

```
mSun = 2e30        # kg
mAst = 2000        # kg
G = 6.67e-11       # Nm^2/kg^2

r = vec(0,5e11,0)  # m
v = vec(4000,0,0)  # m/s
p = mAst*v         # kg m/s

t=0                # s
dt=86400           # s (1 day)
```

- Define constants and initial conditions
- Create a loop

```python
# Iterate 3000 times
for i in range(3000):
    rhat = r/mag(r)
    fnet = -G*mSun*mAst*rhat/mag(r)**2
    p = p + fnet*dt
    r = r + p/mAst*dt
    print('The position is ' + r)
```

The Gameplan

- Define constants and initial conditions
- Create a loop
- Calculate the net force

```python
# Iterate 3000 times
for i in range(3000):
    rhat = r/mag(r)
    fnet = -G*mSun*mAst*rhat/mag(r)**2
    p = p + fnet*dt
    r = r + p/mAst*dt
    print('The position is ' + r)
```

The Gameplan

- Define constants and initial conditions
- Create a loop
- Calculate the net force
- Update momentum

```
# Iterate 3000 times
for i in range(3000):
    rhat = r/mag(r)
    fnet = -G*mSun*mAst*rhat/mag(r)**2
    p = p + fnet*dt
    r = r + p/mAst*dt
    print('The position is ' + r)
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The Gameplan

- Define constants and initial conditions
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```

The Gameplan

- Define constants and initial conditions
- Create a loop
- Calculate the net force
- Update momentum
- Update position
- Print out anything of interest

```python
# Iterate 3000 times
for i in range(3000):
    rhat = r/mag(r)
    fnet = -G*mSun*mAst*rhat/mag(r)**2
    p = p + fnet*dt
    r = r + p/mAst*dt
    print('The position is ' + r)
```

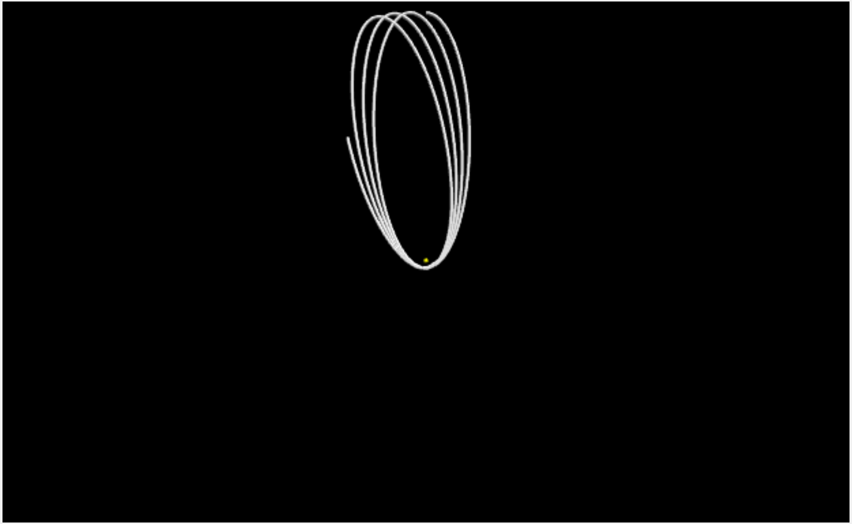
Make it Glorious!

- You can, of course, draw these objects in Glowscript
- Simply add a sphere, and then update it's position each iteration:

```
ast = sphere(pos=r/5e8, make_trail=True)

for i in range(3000):
    rate(100)
    rhat = r/mag(r)
    fnet = -G*mSun*mAst/mag(r)**2*rhat
    p = p + fnet*dt
    r = r + p/mAst*dt
    ast.pos = r/5e8
    print('The position is ' + r)
```

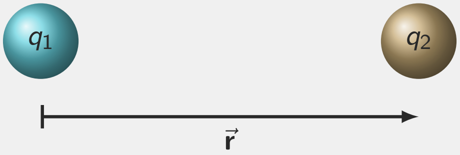
Putting it All Together



The Electric Force

- Technically, the electromagnetic force is comprised of both electric and magnetic bits
- We'll look at the electric bit here and save the magnetic bit till next semester
- Electric Force takes on a form *very* similar to gravity's!
- For two charged particles:

$$\vec{\mathbf{F}}_{\text{elec 1 on 2}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{\mathbf{r}}|^2}\hat{\mathbf{r}}$$



Comparing Forces

$$\vec{\mathbf{F}}_g = -G\frac{m_1 m_2}{|\vec{\mathbf{r}}|^2}\hat{\mathbf{r}}, \quad \vec{\mathbf{F}}_e = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|r|^2}\hat{\mathbf{r}}$$

- G and $\frac{1}{4\pi\epsilon_0}$ are both constants giving the strength of the force

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9\,\mathrm{Nm}^2/\mathrm{C}^2$$

- q and m are the properties of the objects (charge or mass)
    - Mass m is always positive and measured in kilograms (kg)
    - Charge q can either be positive or negative and is measured in coulombs (C)
- In both cases, $\vec{\mathbf{r}}$ points from the surrounding object to the system object

How much is a Coulomb?

Given the image below, compare the magnitude and direction of the electric force and the gravitational force.