Announcements

- Homework
  - Webwork due tonight
  - Another Webwork due on Friday
- No class a week from today (SSRD)
- Test 3 a week from Friday! (Ch20-23)
  - Probably going to look similar to Test 2
  - I'm scrambling to try to have feedback on that test to you by Friday or Saturday
- Polling: `rembold-class.ddns.net`

Review Question

A magnetic field is pointing directly to the right and has a magnitude given by:

$$\left|\vec{\mathbf{B}}\right| = 5t^2 - 2t$$

A square loop measures 1 m per side and is oriented so that its normal points towards the right. If the loop has an internal resistance of $5\,\Omega$, what is the current in the loop at $t = 5$s?

A. 2.3 A

B. 4.6 A

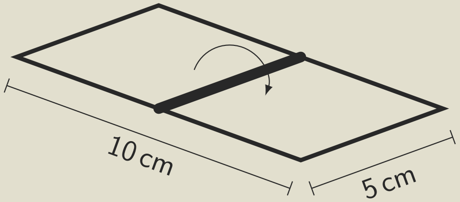
C. 9.6 A

D. 23 A

Solution: 9.6 A

Just keep Spinning

Suppose a wind turbine is rotating the loop shown to below at 10 rad/s. A magnetic field of 2 T points upwards. What is the emf in the loop? What is the peak voltage in the loop?



Solution: $\mathcal{E} = BA\omega \sin(\omega t)$, $\mathcal{E}_{peak} = 0.1\,\mathrm{V}$

The laws we've been discussing recently are the work of James Maxwell, and comprise the Maxwell Equations:

$$\oint \vec{\mathbf{E}} \cdot \hat{\mathbf{n}} dA = \frac{q_{enc}}{\epsilon_0}$$

$$\oint \vec{\mathbf{B}} \cdot \hat{\mathbf{n}} dA = 0$$

$$\oint \vec{\mathbf{E}} \cdot d\vec{\ell} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} = -\frac{\mathrm{d}}{\mathrm{d}t} \int \vec{\mathbf{B}} \cdot \hat{\mathbf{n}} dA$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 \mathrm{I}_{enc}$$

Summary so far: Maxwell's Equations

The laws we've been discussing recently are the work of James Maxwell, and comprise the Maxwell Equations:

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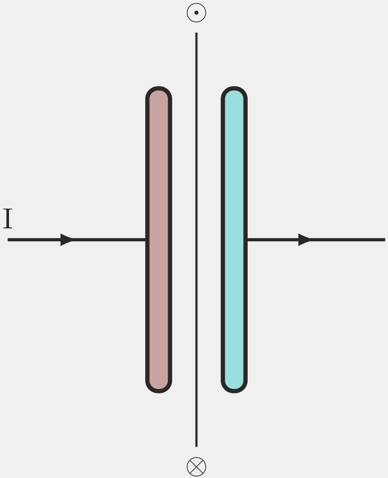
$$\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 \mathrm{I}_{enc} + \dots$$

What is Wrong?

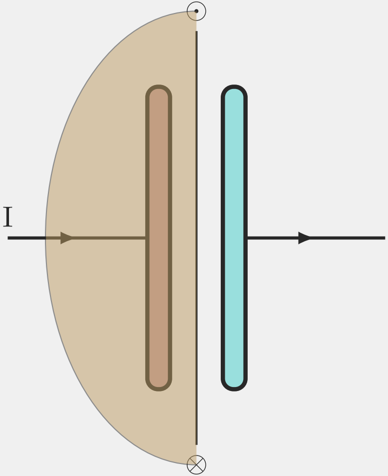
Why would we suspect a missing term?



No current pierces the soap bubble, so no current enclosed. $\Rightarrow$ No magnetic field around loop

What is Wrong?

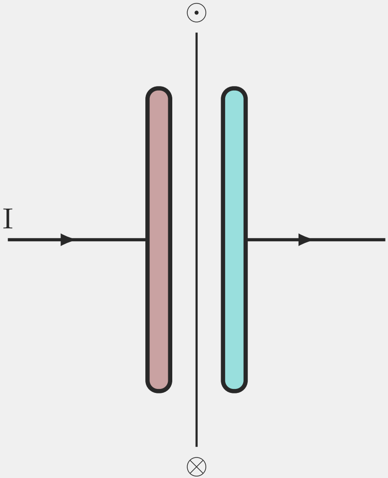
Why would we suspect a missing term?



Now current pierces the soap bubble, so current enclosed.
$\Rightarrow$ Magnetic field around loop?

What is Wrong?

Why would we suspect a missing term?



Crisis! Contradiction Reached!

The Missing Piece

- Presumably, the magnetic field around the loop *should* be non-zero
  - Only a tiny chunk of the wire missing
- Maxwell reasoned that:
  - Current causes the plates to charge up
  - Increased charge increased electric field between plates
  - Maybe the magnetic field is related to that changing electric field?
  - Heavily inspired by Faraday's recent discoveries
- So flux through that surface would be:

$$\Phi_E = \int \vec{\mathbf{E}} \cdot \hat{\mathbf{n}} dA$$

- $\vec{\mathbf{E}}$ and $\hat{\mathbf{n}}$ are parallel, so

$$\Phi_E = EA = \left(\frac{Q}{A\epsilon_0}\right) A = \frac{Q}{\epsilon_0}$$

Finding the new Current

- We are interested in the change in the flux over time, so

$$\frac{\mathrm{d}\Phi_E}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{Q}{\epsilon_0}\right)$$
$$= \frac{1}{\epsilon_0}\frac{\mathrm{d}Q}{\mathrm{d}t}$$
$$= \frac{\mathrm{I}}{\epsilon_0}$$

- So we can say that

$$\mathrm{I} = \epsilon_0 \frac{\mathrm{d}\Phi_E}{\mathrm{d}t}$$

so that, all together:

$$\oint \vec{\mathbf{B}} \cdot d\hat{\ell} = \mu_0 \left(\mathrm{I}_{enc} + \epsilon_0 \frac{\mathrm{d}\Phi_E}{\mathrm{d}t}\right)$$

All of E&M in 5 Equations

- Maxwell's Equations

$$\oint \vec{\mathbf{E}} \cdot \hat{\mathbf{n}} dA = \frac{q_{enc}}{\epsilon_0}$$

$$\oint \vec{\mathbf{B}} \cdot \hat{\mathbf{n}} dA = 0$$

$$\oint \vec{\mathbf{E}} \cdot d\vec{\ell} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 \mathrm{I}_{enc} + \mu_0 \epsilon_0 \frac{\mathrm{d}\Phi_E}{\mathrm{d}t}$$

- Lorentz Force Equation

$$\vec{\mathbf{F}} = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}\right)$$

Getting Wavy

- Changing magnetic flux creates an electric field
- Changing electric flux creates a magnetic field
- Seems circular? Can we set up a self-sustaining system (with no charges or currents)?
    - Answering this involves solving Maxwell's Equations
    - This can be difficult and involves a lot of vector calculus (Take Phys 345)
    - We'll assume a general configuration and then show that it works.
- A huge variety of changing electric and magnetic field possibilities we could guess
    - Most would not satisfy all of Maxwell's equations
    - Turns out we need the region of magnetic and electric field to be moving for them all to be satisfied

The Proposal

- The cyclical nature of Maxwell's equations suggests something like a wave might fit the description
- Take a region of space with the following field configuration and let it move: