

1. Griffiths gives an experimental value for  $\alpha/4\pi\epsilon_0$  in Table 4.1 (Be careful with this table's units!). Here we'll take a closer look at the implications of the atomic polarizability of atomic hydrogen.
  - (a) Using the experimental value, estimate the atomic radius of hydrogen. How well does this radius correspond to other methods of measuring a hydrogen atom's radius, say for example the Bohr radius?
  - (b) What physical assumptions or simplifications does Griffiths make when talking about the physical distribution of negative charge inside the atom? Does that seem entirely realistic? Why or why not?
  - (c) Suppose you have a parallel plate capacitor with plates possessing a potential difference of 100 V and separated by 1 mm. What is the electric field between the plates? (Realize this is basically a 1D Laplace question.)
  - (d) Now place our hydrogen atom from above into this electric field, and determine the resulting "separation distance"  $d$  between the electron cloud and the proton nucleus. What fraction of the atomic radius (from part 2) is this? Would this seem likely to ionize the hydrogen atom?
  - (e) Estimate what voltage would be necessary to fully ionize the hydrogen atom in this capacitor. (You can say that the atom will be fully ionized if you pull the nucleus outside of the electron cloud.)
2. Consider a dielectric sphere of radius  $a$  that has a polarization that is directed radially outwards from the center of the sphere:

$$\vec{\mathbf{P}} = P_0 \vec{\mathbf{r}}$$

- (a) Determine the bound charges at the surface,  $\sigma_b$ , and in the volume of the sphere,  $\rho_b$ .
  - (b) Calculate the electric field everywhere. (*You can do this without finding the potential first!*)
  - (c) Sketch (by hand) the electric field lines inside and outside the sphere.
3. When a neutral dielectric is polarized, no new charges are created or destroyed, so the total charge must still be zero. The charge density on the surface is given by

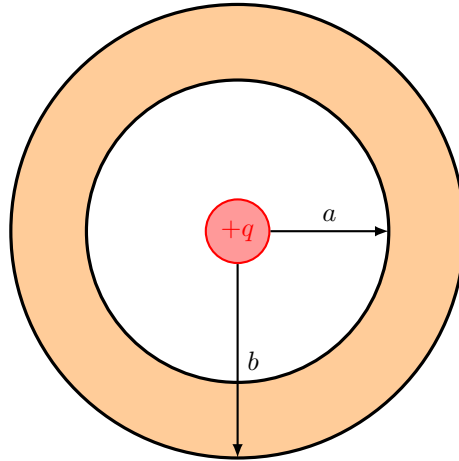
$$\sigma_b = \vec{\mathbf{P}} \cdot \hat{\mathbf{n}}$$

and the charge density in the bulk is given by:

$$\rho_b = -\nabla \cdot \vec{\mathbf{P}}$$

Using the definitions (and maybe some fundamental theorems...), show that the total charge for any neutral dielectric with polarization  $\vec{\mathbf{P}}$  is zero.

4. A point charge  $+q$  is at the center of a spherical silicon shell with inner radius  $a$  and outer radius  $b$ . The shell is a linear dielectric, with a dielectric constant of  $\epsilon_r$ . The shell is electrically neutral.



- (a) Compute  $\vec{E}$ ,  $\vec{D}$ , and  $\vec{P}$  everywhere.
  - (b) How is  $\vec{E}$  in the silicon different from what it would have been if the silicon were not present? (Explain why/how this difference arises physically.)
  - (c) Make a plot in Jupyter of the magnitude of the electric field as it varies from  $r = 0.5$  to  $r = 3$ . You can let  $q = 1 \mu\text{C}$ ,  $a = 1 \text{ m}$  and  $b = 2 \text{ m}$ .
  - (d) Using the same constants, make a plot of the magnitude of  $\vec{D}$  over the same interval.
5. A solid sphere (radius  $R$ ) of a linear dielectric material (dielectric constant  $\epsilon_r$ ) has been “injected” with a uniform free charge density  $\rho_f$  throughout its volume.
- (a) Find the potential at the center of the sphere (setting  $V(\infty) = 0$ ). (*You may want to find  $\vec{D}$  first*)
  - (b) Does your answer come out larger or smaller than for a simple sphere with uniform charge density  $\rho_f$ ? Does this result make physical sense to you? Explain.
  - (c) What do you get in the limit of an infinite dielectric constant? What physical situation would this correspond to?