



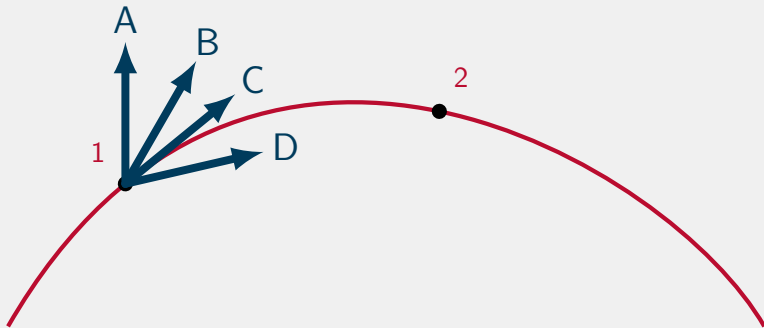
# Announcements

- Homework 2 is due by midnight tonight!
- Video Homework 1 will be posted today after class
  - Look over the objective
  - Create or find a problem that you feel solving would satisfy the objective
  - Solve your problem
  - Video ( $< 4$  min) yourself explaining your solution (at a level that someone else in the class would understand)
  - Samples posted on course webpage
- Starting Ch2 today and continuing on through next week
- No class on Monday!
- Polling: `rembold-class.ddns.net`



## Review Question!

A ball travels through the air, and part of its trajectory is shown in red below. Which arrow best represents the direction of the average velocity of the ball as it travels from location 1 to location 2?





# A Summary

- What we know thus far:
  - How to describe an object's position in a universally understood fashion
  - How to describe a moving object's velocity and acceleration
  - How to find an object's momentum (at low speeds)
  - That there is a relationship between changes in momentum and in outside interactions that must be acting on the object



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- What we know thus far:
  - How to describe an object's position in a universally understood fashion
  - How to describe a moving object's velocity and acceleration
  - How to find an object's momentum (at low speeds)
  - That there is a relationship between changes in momentum and in outside interactions that must be acting on the object
- Today:
  - Vector components and angles
  - Predicting the future
  - Force and the Momentum Principle

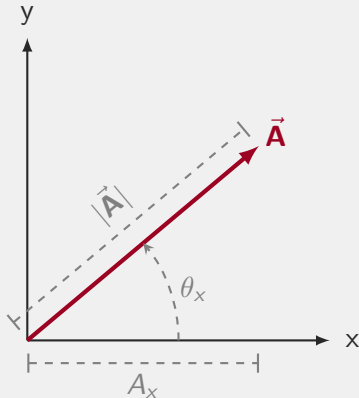


# Components and Angles

- Frequently may have information in the form of angles and magnitudes, rather than vector components
- Can relate angles to components using cosines

$$\cos \theta_x = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{A_x}{|\vec{\mathbf{A}}|}$$

$$\Rightarrow A_x = |\vec{\mathbf{A}}| \cos \theta_x$$



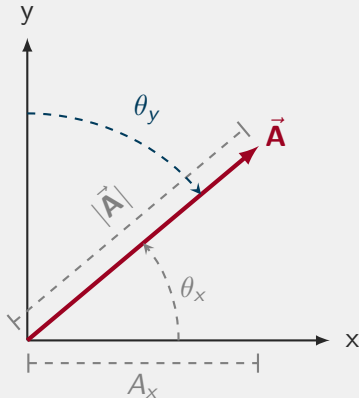


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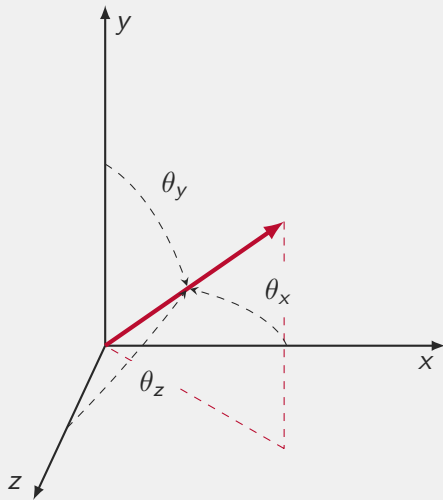
$$\cos \theta_x = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{A_x}{|\vec{\mathbf{A}}|}$$

$$\Rightarrow A_x = |\vec{\mathbf{A}}| \cos \theta_x$$





# Direction Cosines



- All other axes work the same way!
- Can write any vector then as:

$$\vec{r} = \langle |\vec{r}| \cos \theta_x, |\vec{r}| \cos \theta_y, |\vec{r}| \cos \theta_z \rangle$$

- Often, if things are given in terms of angles, some of them will be very simple
- If in 2d,  $\cos \theta_y = \sin \theta_x$

$U=3$   
 $U=2$   
 $U=1$

$E_3 = 3E_1$   
 $E_2 = 4E_1$   
 $E_1$

$d$

$h^2 = u^2 E_1$

$ma_g \downarrow$   
 $ma_g \rightarrow v_1$

$d\vec{r} = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2}$   
 $d\vec{r} = \frac{2}{4\pi\epsilon_0} \frac{dx}{r^2}$

$dE_y = \frac{2}{4\pi\epsilon_0} \frac{dx}{r^2} \frac{y}{r}$   
 $dE_x = \frac{2}{4\pi\epsilon_0} \frac{dx}{r^2} \frac{x}{r}$

$\Delta P = \epsilon_0 A (T_1 - T_0)$

$U = F_e r = F_r \sin \theta = F_L$   
 $v = v_0 \sin \theta$

$F_n \cdot x + F_g \cdot x = ma$   
 $F_n \cdot x = 0; F_g \cdot x = F_n \sin \theta$   
 $= mg \sin \theta$   
 $a_x = g \sin \theta$

$z = \sqrt{z^2 + x^2}$   
 $v^2 = 2gh$   
 $v_s = \sqrt{2gh} \cdot \sin$

# Looking Forward

$U^2 = A^2 \exp(-\frac{x^2}{2\sigma^2})$   
 $B(x) = \frac{\sqrt{x}}{\sqrt{\pi}} e^{-\sigma^2(x-x_0)^2}$

$\psi(\psi) = A \cos(k_0 x - \omega t)$

$|F| = \frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$   
 $= \frac{mv^2}{r}$

$U_H = -\int \vec{B} \cdot (d\vec{r})$   
 $U_H = E_H b = v d B b$   
 $J = \frac{n}{V} q v d A$   
 $b \frac{U}{V} = \frac{1}{A q v b} - \frac{1}{b d e v d}$   
 $= -\int \vec{B} \cdot d\vec{r}$

$E_{pot} = -\frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$   
 $E_{kin} = \frac{1}{2} m v^2 = \frac{1}{2} \frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$   
 $E_{pot} = -2 E_{kin}$

$E_{pot} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$   
 $= \frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$

$\frac{1}{s} = \frac{1}{s} + \frac{1}{s'}$   
 $\frac{A'B'}{AB} = \frac{s'}{s}$

$F_2 = \frac{F_L}{2u}$   
 $E = F_2 \cdot s$   
 $= \frac{F_L}{u} \cdot u \cdot h$   
 $F_L = F_L \cdot h$   
 $= m \cdot g \cdot h$   
 $s = u \cdot h$

$\vec{B} = \vec{B}_2 + \mu_0 \vec{J} = \vec{B}_2 (1 + \chi_{mag})$   
 $E = c \vec{B}$   
 $= \mu_0 c \vec{J} = \mu_0 \vec{J}$

$m_1 v_{1x} + m_2 v_{2x} = m_1 v_{1x}' + m_2 v_{2x}'$   
 $\frac{1}{2} m_1 v_{1x}^2 + \frac{1}{2} m_2 v_{2x}^2 = \frac{1}{2} m_1 v_{1x}'^2 + \frac{1}{2} m_2 v_{2x}'^2$   
 $\tan \theta = \frac{ax}{g}; a = g \tan \theta$   
 $F_s = \frac{mg}{\cos \theta}; F_L = \frac{mg}{\sin \theta}$





# Leap Frogging Forwards

- We know that

$$\vec{p} = m\vec{v} \quad \text{and} \quad \vec{v} = \frac{\Delta\vec{r}}{\Delta t}$$

- We can combine these to be able to predict future positions based on current momentum!

$$\vec{p} = m\vec{v}$$

$$= m \frac{\Delta\vec{r}}{\Delta t}$$

$$\vec{p} \cdot \Delta t = m \cdot \Delta\vec{r}$$

$$\frac{\vec{p}}{m}(t_2 - t_1) = \vec{r}_2 - \vec{r}_1$$

$$\Rightarrow \vec{r}_2 = \vec{r}_1 + \frac{\vec{p}}{m}(t_2 - t_1)$$



# Being Future Oriented

Suppose I slip and fall while ice-skating. I have a mass of about 70 kg and I was traveling with a velocity of  $\langle 5, 0, 8 \rangle$  m/s when I fell. Assuming the ice is super slippery so I don't have any interactions with it, the momentum should stay the same ( $\Delta \vec{p} = 0$ ). If we take the location of my fall to be the origin, what will my position be in 1 s? In 10 s?



## Your Turn!

Suppose you have a constant velocity of  $\langle 2, 2, 0 \rangle$  m/s. At  $t = 15$  s you are located at the point  $\langle 10, 50, 20 \rangle$  m. Where were you located 10 s ago?

- A)  $\langle -10, 30, 20 \rangle$  m
- B)  $\langle 30, 70, 40 \rangle$  m
- C)  $\langle -10, 30, 0 \rangle$  m
- D) It is impossible to tell without a mass

$$E_3 = 3E_1$$

$$E_2 = 4E_1$$

$$E_1$$

$$d$$

$$x$$

$$h^2 = u^2 E_1$$

$$dE = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2}$$

$$dE_x = \frac{\lambda}{4\pi\epsilon_0} \frac{dx}{r^2}$$

$$dE_y = \frac{\lambda}{4\pi\epsilon_0} \frac{dx}{r^2} \frac{y}{r}$$

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$$U = F_e r = F_r \sin \theta = F_L$$

$$v = v_0 \sin \theta$$

$$U_1, U_2 = X \sin \theta$$

$$F_n, x + F_a, x = m a$$

$$F_n, x = 0; F_a, x = F_a \sin \theta$$

$$= m g \sin \theta$$

$$a_x = g \sin \theta$$

$$z = \sqrt{z^2 + x^2}$$

$$v^2 = 2 g \sin \theta \Delta x$$

$$v^2 = 2 g h$$

$$v_s = \sqrt{2 g h} \sin \theta$$

$$\Delta P = c \Delta T (T_1 - T_0)$$

$$\lambda_1 = \frac{u_1}{f}; \lambda_2 = \frac{u_2}{f}$$

$$\sin \theta_2 = \frac{\lambda_1}{\lambda_2} \sin \theta_1$$

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{\lambda_2}{\lambda_1} = \frac{u_1}{u_2} = \frac{n_2}{n_1}$$

$$v^2 = 2 g \sin \theta \Delta x$$

$$v^2 = 2 g h$$

$$v_s = \sqrt{2 g h} \sin \theta$$

$$U = F_e r = F_r \sin \theta = F_L$$

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# The Momentum Principle

$$U^2 = A^2 \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

$$B(x) = \frac{\sigma}{\sqrt{\pi}} e^{-\sigma^2(x-x_0)^2}$$

$$E(\psi) = A \cos(k_0 x - \omega t)$$

$$|F| = \frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$$

$$= \frac{mv^2}{r}$$

$$U_H = -\int \mathbf{B} \cdot \left(\frac{d\mathbf{r}}{dt}\right) d\mathbf{r}$$

$$U_H = E_H b = v d B b$$

$$J = \frac{n}{V} q v d A$$

$$b \frac{U}{V} = \frac{1}{A q v b} \int b d v d$$

$$= -\int \mathbf{B} \cdot d\mathbf{r} dU_H$$

$$E_{pot} = -\frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$$

$$E_{kin} = \frac{1}{2} m v^2$$

$$= \frac{1}{2} \frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$$

$$E_{pot} = -2 E_{kin}$$

$$E_{pot} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$$

$$\frac{A'B'}{AB} = \frac{s'}{s}$$

$$F_2 = \frac{F_L}{2}$$

$$E = F_2 \cdot s$$

$$= \frac{F_L}{2} \cdot u \cdot h$$

$$F_L = F_L \cdot h$$

$$= m \cdot g \cdot h$$

$$s = u \cdot h$$

$$b = b_2 + \mu_0 U = b_2 (1 + \lambda_{mag})$$

$$= \mu_{rel} \mu_0 H = \mu H$$

$$m_1 v_{1x} + m_2 v_{2x} = m_1 v_{1x}' + m_2 v_{2x}'$$

$$\frac{1}{2} m_1 v_{1x}^2 + \frac{1}{2} m_2 v_{2x}^2 = \frac{1}{2} m_1 v_{1x}'^2 + \frac{1}{2} m_2 v_{2x}'^2$$

$$= \frac{1}{2} m_1 v_{1x}^2 + \frac{1}{2} m_2 v_{2x}^2$$

$$F_s = \frac{m a}{\cos \theta}$$

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$$U_H = -\int \mathbf{B} \cdot \left(\frac{d\mathbf{r}}{dt}\right) d\mathbf{r}$$

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$$= -\int \mathbf{B} \cdot d\mathbf{r} dU_H$$



# Drawing the Line

- Need to distinguish between what we consider the object, and what we consider the stuff around the object that is interacting with it
  - System: The object we are interested in
    - Could be simple: tennis ball
    - Could be complex: Star
  - Surroundings: Everything outside the System
    - The racket, the air, the earth
    - Nearby stars in the galaxy, the ISM
- Can sound trivial but it is important you distinguish between the two
- Helps compartmentalize in your mind what types of information you may need from different objects



# The Momentum Principle

- We know the  $\Delta\vec{p}$  is related to this strength of interaction
- Want to quantify “strength of interaction”. Give it some numbers
- We do so with the concept of force

## The Momentum Principle

$$\Delta\vec{p} = \vec{F}_{net}\Delta t$$

- Things to note:
  - Force is a **vector** in the same direction as the change in momentum!
  - For a given interaction, the needed force depends on how fast you need the interaction to happen
    - $\Delta t$  small  $\rightarrow \vec{F}_{net}$  large
    - $\Delta t$  large  $\rightarrow \vec{F}_{net}$  smaller
  - The Force is constant for this  $\Delta t$  time interval
  - **net** is an important term in physics



# Not for Fishing

- Net means the total contribution of all surroundings
- Since force is a vector, this is a vector sum:

$$\vec{\mathbf{F}}_{net} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_3 + \cdots$$

- Note that the change in momentum is related only to the **net** force



## Example: Blackballing

Both balls at the front of the classroom have the same size and mass. We'll observe them both bounce when dropped from the same height, then answer the following: which ball experienced a larger force?

- A) Ball A
- B) Ball B
- C) They were the same
- D) It is impossible to tell





## Practice Time!

Return to the practice problem situation from last class: Suppose you bring a go-cart ( $m = 100 \text{ kg}$ ) and a semi-truck ( $m = 2000 \text{ kg}$ ) to a stop by pressing against them. Both initially have a velocity of  $\langle 10, 0, 15 \rangle \text{ m/s}$  and you manage to stop them in 3 s.

- What is the net force needed to stop the go-cart?
- What is the net force needed to stop the semi-truck?
- Say you can only apply a force with a magnitude of 300 N before you either start slipping or your bones break. How long a time frame would you need to spread this force over to stop the semi-truck safely (eg. How much longer than the current 3 s)?