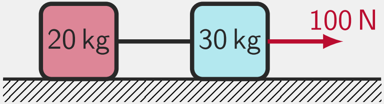
Announcements

- Homework
  - WebWorK 9 due tonight
  - WebWorK 10 posted later this afternoon and due Friday
  - May have a WebWorK over the weekend instead of VHW, we'll see
- Test 2
  - A week from today
  - Should finish with content on Friday
  - Chapters 3–5
  - You get a new notecard
- I have something at 4:30pm today, so will have to cut office hours short, though I'll be available from 3pm until then.
- Polling: `rembold-class.ddns.net`

Review Question

In the system to the lower right, two boxes are connected with a rope and then the rightmost box is pulled with a force of 100 N purely horizontally. The connecting rope can withstand a force of 50 N before in snaps. As the boxes travel along the frictionless surface, does the rope snap?

A) Yup, they pulled too hard!
B) Nope, the rope is fine.
C) I don't have enough info to decide
D) There is a rope?



The Parallel and the Perpendicular

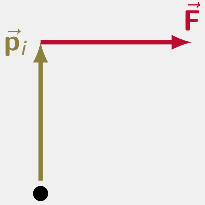
- We've been handling curving just as a natural consequence of the momentum principle
  - Eg: gravitational orbits
- This is fine if going from forces to motion, not as useful if going the other direction
- Need to understand what parts of force create curvature
- Need ways to quantify "curvature"

The Parallel

- What we already know:
  - Applying a force in the same direction of motion increases $|\vec{\mathbf{p}}|$
  - Applying a force in the opposite direction of motion decreases $|\vec{\mathbf{p}}|$
  - In both cases the force is parallel ($\parallel$) to $\vec{\mathbf{p}}$.
- Neither case causes any curving
- Only causes changes is the magnitude of the momentum

The Perpendicular

What does a force perpendicular ($\perp$) to the momentum do?



$$\Delta t = 1\,\mathrm{s}$$

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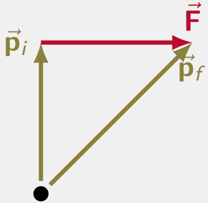

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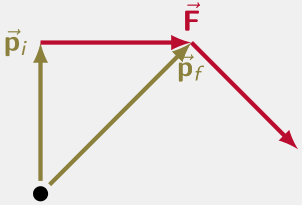

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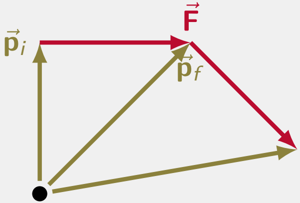

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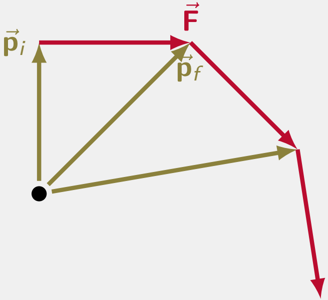

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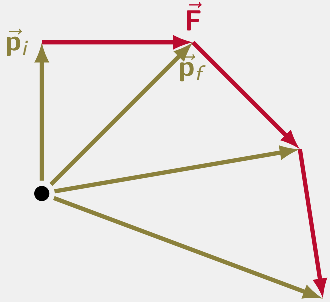

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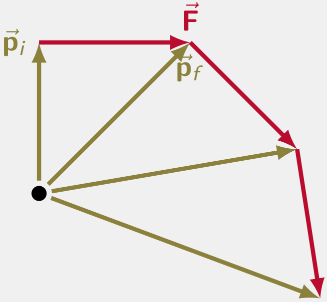

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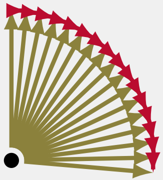


$\Delta t = 0.1\,\mathrm{s}$

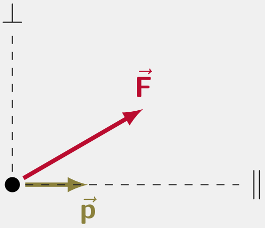
What Affects What

- So we have that:
  - Forces $\|$ to the $\vec{\mathbf{p}}$ change the magnitude, but not the direction
  - Forces $\perp$ to the $\vec{\mathbf{p}}$ change the direction, but not the magnitude
- Because these two are perpendicular to one another, we can write any force as

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_{\|} + \vec{\mathbf{F}}_{\perp}$$

- This makes it easier for us to understand how different forces will effect the motion
- The main key then is being able to divide forces into the $\|$ and $\perp$ parts

The · Product



- Can use same cosine method as we've been using

$$\vec{\mathbf{F}}_{\parallel} = \left|\vec{\mathbf{F}}\right| \cos(\theta)\hat{\mathbf{p}}$$

$$F_{\perp} = \left|\vec{\mathbf{F}}\right| \cos(\phi)$$

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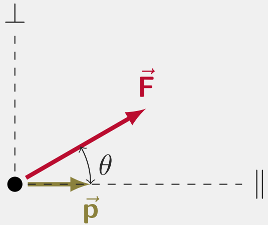

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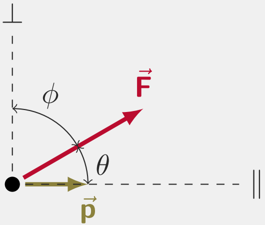

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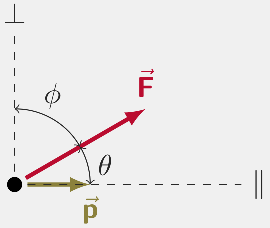
The $\cdot$ Product



- Can use same cosine method as we've been using

$$\vec{\mathbf{F}}_{\parallel} = \left|\vec{\mathbf{F}}\right| \cos(\theta)\hat{\mathbf{p}}$$

$$F_{\perp} = \left|\vec{\mathbf{F}}\right| \cos(\phi)$$

- Can use Dot product
- Dot product gives component of first vector in the direction of second vector

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = A_x B_x + A_y B_y + A_z B_z$$

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \left|\vec{\mathbf{A}}\right|\left|\vec{\mathbf{B}}\right| \cos(\theta)$$

- So

$$\vec{\mathbf{F}}_{\parallel} = (F_x p_x + F_y p_y + F_z p_z)\,\hat{\mathbf{p}}$$

$$\vec{\mathbf{F}}_{\perp} = \vec{\mathbf{F}} - \vec{\mathbf{F}}_{\parallel}$$

Dot Product Example

An object is traveling with momentum $\langle 1, 1, 0 \rangle$ kg m/s. A $\langle 3, 0, 0 \rangle$ N acts on the object. Divide the force up into the parallel and perpendicular components to describe the motion.

Bringing it All Back to Momentum Principle

- How will this help us?
- Recall we can write

$$\vec{\mathbf{p}} = |\vec{\mathbf{p}}|\hat{\mathbf{p}}$$

- Thus the change in momentum is a product rule

$$\frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t} = \frac{\mathrm{d}|\vec{\mathbf{p}}|}{\mathrm{d}t}\hat{\mathbf{p}} + |\vec{\mathbf{p}}|\frac{\mathrm{d}\hat{\mathbf{p}}}{\mathrm{d}t}$$

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$$\frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t} = \underbrace{\boxed{\frac{\mathrm{d}|\vec{\mathbf{p}}|}{\mathrm{d}t}\hat{\mathbf{p}}}}_{\text{changing speed}} + \underbrace{\boxed{|\vec{\mathbf{p}}|\frac{\mathrm{d}\hat{\mathbf{p}}}{\mathrm{d}t}}}_{\text{changing direction}}$$

Relating Like to Like

- Knowing what the momentum principle looks like, we can break things up so that

$$\frac{\mathrm{d}|\vec{\mathbf{p}}|}{\mathrm{d}t}\hat{\mathbf{p}} = \vec{\mathbf{F}}_{\parallel}$$
$$|\vec{\mathbf{p}}|\frac{\mathrm{d}\hat{\mathbf{p}}}{\mathrm{d}t} = \vec{\mathbf{F}}_{\perp}$$

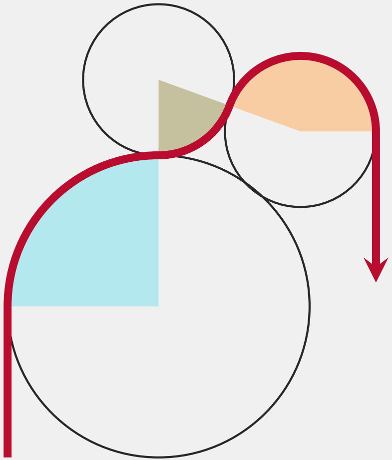
- We can deal with the magnitude of momentum terms, but what about this $\frac{\mathrm{d}\hat{\mathbf{p}}}{\mathrm{d}t}$ term?

Make the Circles Kiss

- In general, the form of $\frac{\mathrm{d}\hat{\mathbf{p}}}{\mathrm{d}t}$ would vary
- Can always express a bit of curve as moving along a piece of a circle though
- For uniform circular motion:
  - For a given speed v and radius R:

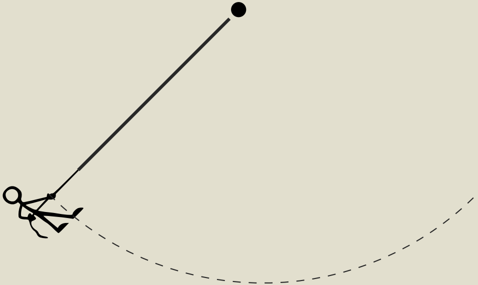
$$\left|\frac{\mathrm{d}\hat{\mathbf{p}}}{\mathrm{d}t}\right| = \frac{v}{R}$$

  - Direction points towards center of circle



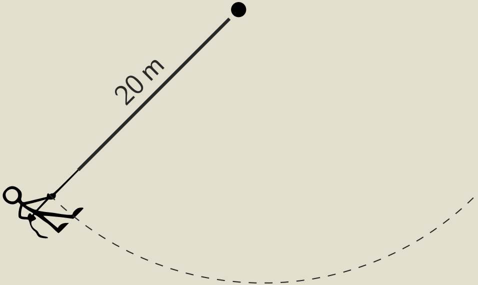
Swing Example Conceptual

Consider the situation where a person is swing back and forth on a rope swing. They start out at some angle and then swing back and forth. Investigate the $\perp$ and $\parallel$ net forces at various instances throughout the motion.



Swing Example Numbers

The rope can take a maximum tension force of 800 N. If the swinger is 65 kg and the rope is 20 m long, what is the maximum speed they could be traveling at the bottom of the swing?



Do the Loop!

Real rollercoasters don't mess around with safety equipment and instead rely purely on physics to provide their thrills. How fast does a 500 kg cart need to be traveling to complete a 10 m loop without leaving the track?