Announcements

- CompDay 6 and 7 due tonight
- Homework 7 due Monday
- Read thru Ch 8.4 for Friday
- Responses: `rembold-class.ddns.net`

Today's Objectives

- To understand how to interpret equilibrium points and stability from Lagrangian differential equations
- To appreciate when using Lagrange multipliers might be useful/appropriate
- To be able to solve simple systems with Lagrange multipliers

Q1

Last class we looked at a bead on a rotating parabola. Solving for the equation of motion would have given:

$$\ddot{\rho} = \frac{(-4g + \omega^2 - 16\dot{\rho}^2)\rho}{16\rho^2 + 1}$$

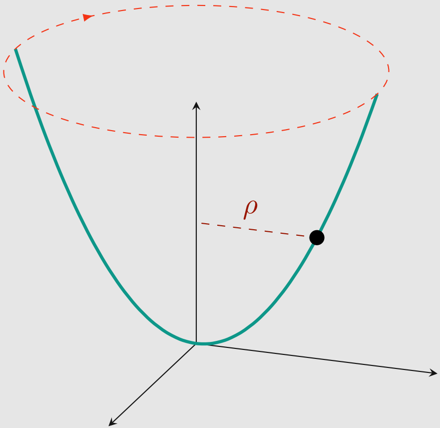
Where will any equilibrium positions exist for our particle on the wire?

A) At $\rho = 0$ and at $\rho = \pm 1/4$

B) At $\rho = 0$ and when $\omega = 2\sqrt{g}$

C) At $\rho = 0$

D) There are no equilibrium positions



Q2

As a reminder, our equation of motion is still:

$$\ddot{\rho} = \frac{(-4g + \omega^2 - 16\dot{\rho}^2)\rho}{16\rho^2 + 1}$$

Would $\rho = 0$ be a stable or unstable equilibrium point?

A) Stable

B) Unstable

C) It depends

D) This question makes no sense

Q3

What about the other possibility, when $\omega = 2\sqrt{g}$. Any ρ at rest would be at equilibrium here, but would be be stable or unstable?

A) Stable

B) Unstable

C) It depends

D) This question makes even less sense!

Q4

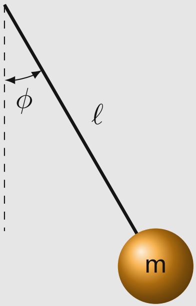
When should you use Lagrangian multipliers?

A) When the constraint force is not perpendicular to the motion
B) When you want to know how a constraint force effects the object's motion
C) When the constraint force is non-conservative
D) When you want to know the magnitude of the constraint force

Q5

Take the situation of a normal pendulum. Without constraints, the Lagrangian in polar coordinates could be described as:

$$\mathcal{L} = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\phi}^2 + 2gr\cos(\phi)\right)$$



What would the differential equation for r look like with the method of Lagrangian multipliers?

A) $\ddot{m}r = -mg\cos\phi + mr\dot{\phi}^2 - \ell\lambda$

B) $\ddot{m}r = mg\cos\phi + mr\dot{\phi}^2 - \lambda$

C) $\ddot{m}r = mg\cos\phi + mr\dot{\phi}^2 + \lambda$

D) $\ddot{m}r = mg\cos\phi + mr\dot{\phi}^2 + \ell\lambda$

Q6

Taking the same situation then, where the Lagrangian is:

$$\mathcal{L} = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\phi}^2 + 2gr\cos(\phi)\right)$$

What would the magnitude of the tension force on the string be?

A) $m\left(g\cos\phi + r\dot{\phi}^2\right)$

B) $m\ell\left(g\cos\phi + r\dot{\phi}^2\right)$

C) $m\ell\left(g\cos\phi - r\dot{\phi}^2\right)$

D) $m\left(-g\cos\phi + r\dot{\phi}^2\right)$