



# Announcements

- Homework
  - Video Homework 3 due by midnight tonight
  - WebWork 7 due on Wednesday
- Hoping to start Ch 5 on Wednesday.
- Polling: `rembold-class.ddns.net`



## Review Question

Mystery material A has a Young's modulus of 132 GPa whereas mystery material B has a Young's modulus of 110 GPa. Without knowing anything else about the materials, what could you deduce about the intermolecular spring strengths for each of the materials?

- A) A's spring constant is 1.2 times greater than material B's.
- B) A's spring constant is greater than material B's, but less than 1.2 times as much.
- C) B's spring constant is greater than material A's.
- D) There is not enough information to determine which material's intermolecular spring constant will be greater.

$$E_3 = 3E_1$$

$$E_2 = 4E_1$$

$$E_1$$

$$d$$

$$x$$

$$h^2 = u^2 E_1$$

$$|d\vec{E}| = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2}$$

$$= \frac{\lambda}{4\pi\epsilon_0} \frac{dx}{r^2}$$

$$dE_y = \frac{\lambda}{4\pi\epsilon_0} \frac{dx}{r^2} \frac{y}{r}$$

$$dE_x = \frac{\lambda}{4\pi\epsilon_0} \frac{dx}{r^2} \frac{x}{r}$$

$$\cos\theta = \frac{y}{r}$$

$$d\vec{E} = |d\vec{E}| \cos\theta \hat{y}$$

$$dE_y = \frac{\lambda}{4\pi\epsilon_0} \frac{dx}{r^2} \frac{y}{r}$$

$$dE_x = \frac{\lambda}{4\pi\epsilon_0} \frac{dx}{r^2} \frac{x}{r}$$

$$\lambda_1 = \frac{u_1}{f}; \lambda_2 = \frac{u_2}{f}$$

$$\sin\theta_2 = \frac{\lambda_1}{\lambda_2} \sin\theta_1$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\lambda_2}{\lambda_1} = \frac{u_1}{u_2} = \frac{n_2}{n_1}$$

$$U = F_e r = F_r \sin\theta = F_L$$

$$v = v_0 \sin\theta$$

$$F_n x + F_g x = ma$$

$$F_n x = 0; F_g x = F_n \sin\theta$$

$$= mg \sin\theta$$

$$a_x = g \sin\theta$$

$$z = \sqrt{z^2 + x^2}$$

$$v^2 = 2g \sin\theta \Delta x$$

$$v^2 = 2gh$$

$$v_s = \sqrt{2gh} \sin\theta$$

# Relations to Macroscopic Forces

$$|U|^2 = A^2 \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

$$B(x) = \frac{\sqrt{x}}{\sqrt{\pi}} e^{-\sigma^2(x-x_0)^2}$$

$$E(\psi) = A \cos(k_0 x - \omega t)$$

$$|F| = \frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$$

$$= \frac{mv^2}{r}$$

$$E_{pot} = -\frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$$

$$E_{kin} = \frac{1}{2} mv^2$$

$$= \frac{1}{2} \frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$$

$$E_{pot} = -2E_{kin}$$

$$E_{pot} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{ze(e)}{r}$$

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

$$\frac{A'B'}{AB} = \frac{s'}{s}$$

$$F_2 = \frac{F_L}{2n}$$

$$E = F_2 \cdot s$$

$$= \frac{F_L}{n} \cdot u \cdot h$$

$$F_L = F_L \cdot h$$

$$= m \cdot g \cdot h$$

$$s = u \cdot h$$

$$b = b_2 + \mu_0 u = b_2 (1 + \mu_0 u)$$

$$= \mu_0 \epsilon_0 h J = \mu_0 J$$

$$m_1 v_{1x} + m_2 v_{2x} = m_1 v_{1x}' + m_2 v_{2x}'$$

$$\frac{1}{2} m_1 v_{1x}^2 + \frac{1}{2} m_2 v_{2x}^2 = \frac{1}{2} m_1 v_{1x}'^2 + \frac{1}{2} m_2 v_{2x}'^2$$

$$= \frac{1}{2} m_1 v_{1x}^2 + \frac{1}{2} m_2 v_{2x}^2$$

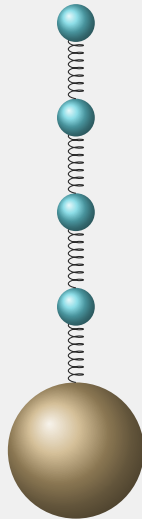
$$\tan\theta = \frac{ax}{g}; a = g \tan\theta$$

$$F_s = \frac{mg}{\cos\theta}; |F_s| = \frac{mg}{\sin\theta}$$



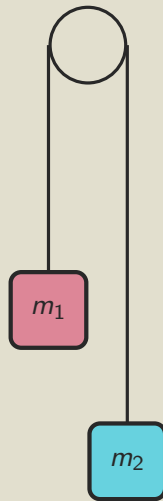
# Tension

- Force exerted by wires or strings
- Propagates up the sequence of atomic springs
- If atomic masses small compared to the end mass, then force equal along entire length
  - In this situation, tension just redirects forces



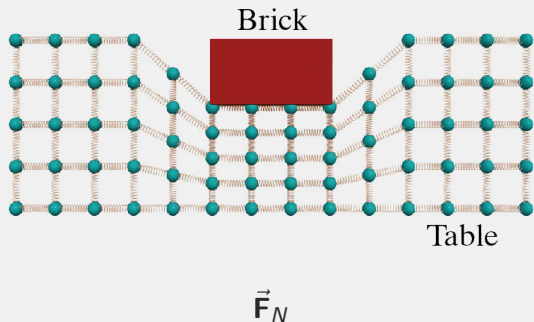


An Attwood machine consists of a pulley over which a rope hangs with two masses attached to each end. Suppose the mass of the rope and pulley are tiny compared to the masses. If  $m_1 = 10 \text{ kg}$  and  $m_2 = 14 \text{ kg}$  and both start from rest, how quickly are they moving 0.5 s later?





# Normal Forces

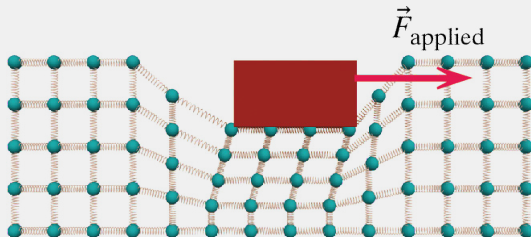


- When a solid object is placed atop another, they influence the atomic springs
- “Table” compresses until spring forces counteract gravity
- Force due to compression of the table atoms
- Points perpendicular to the surface
  - Hence called the normal force



# Friction

- Movement of brick moving across table forces down new atoms
- Can also visualize the sideways atomic springs pushing back
- “Depth” that it has sunk plays a role
  - Related to the normal force in some fashion
- Springs left behind will bounce back and jiggle, raising temperature
- Force is parallel to the surface!



$$|\vec{F}_{friction}| \approx \mu |\vec{F}_N|$$



# A Tale of Two Frictions

- In general, friction will not depend on the velocity
- One large exception: if the velocity is zero
  - Static Friction: friction on a non-moving object

$$F_{friction} \leq \mu_s F_N$$

- Kinetic Friction: friction on objects moving at any other speed

$$F_{friction} \approx \mu_k F_N$$





# Sliding Block

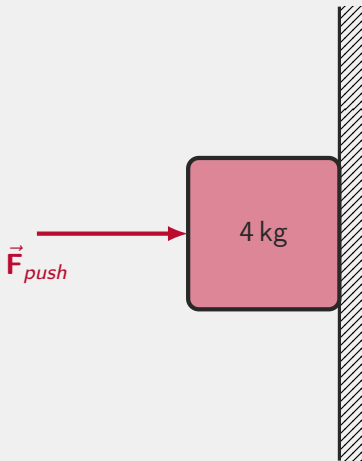
Say I pull a 10 kg block across a level surface at a constant speed. If I pull with a constant force of 80 N, what is the coefficient of kinetic friction?



# Understanding Check

Suppose you are pressing a 4 kg block against a wall. If the coefficient of static friction between the block and wall is 0.7, how hard must you press to ensure the block doesn't begin to slide?

- A) 5.71 N
- B) 27.4 N
- C) 39.2 N
- D) 56 N



# Understanding Analytic Springs



- Can write the momentum principle in differential form:

$$\Delta \vec{p} = \vec{F}_{net} \Delta t \quad \Rightarrow \quad \frac{d\vec{p}}{dt} = \vec{F}_{net}$$

- Want to find an expression for the location of a mass on a spring at any time

- Assumptions:

- No friction or air resistance
- Only force of the spring present
- Mass of the spring is negligible compared to the attached masses
- Origin is located at the relaxed length

- Implies that:

$$\frac{d\vec{p}}{dt} = \vec{F}_{net} \quad \Rightarrow \quad \frac{d(mv_x)}{dt} = -k_s x$$
$$m \frac{d^2 x}{dt^2} = -k_s x$$



# Trig Functions to the Rescue

- Need a function which is it's own double derivative
  - sin or cos will fit the bill!
- Add in some constants to generalize things:

$$x(t) = A \cos(\omega t)$$

where

$A$  = amplitude

$$\omega = \sqrt{\frac{k_s}{m}} = \text{angular frequency}$$

- Can also talk about

$$\text{Period: } T = \frac{2\pi}{\omega}$$

$$\text{Frequency: } f = \frac{1}{T} = \frac{\omega}{2\pi}$$



# Atomic Periods

We previously found that lead had an atomic weight of  $207.2 \text{ g/mol}$  and an atomic spring constant of about  $4.98 \text{ N/m}$ . How long does it take a displaced atom to oscillate back to the same position?



# Analytic vs Iterative

Now that we've returned for a bit of analytic equations, let's return to some pros and cons between the analytic and iterative methods of doing things

## Analytic

- Let's us easily see general features about solutions
  - Period does not depend on amplitude
  - Increasing mass increases period
  - Increasing  $k_s$  decreases period
  - etc
- Only works for particular assumptions

## Iterative

- Easy to express in three dimensions
- Can easily add new variations
  - Adding in friction or air resistance
  - Springs with non-negligible mass
  - Springs with non-ideal spring behavior
  - etc

$U=3$   
 $U=2$   
 $U=1$

$E_3 = 3E_1$   
 $E_2 = 4E_1$   
 $E_1$

$d$

$h^2 = U^2 E_1$

$ma_g \downarrow$

$d\vec{r} = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2}$

$d\vec{r} = \frac{\lambda}{4\pi\epsilon_0} \frac{dx}{r^2}$

$d\vec{r} = \frac{\lambda}{4\pi\epsilon_0} \frac{dx}{r^2}$

$\Delta P = e\sigma A(T_1 - T_0)$

$U = F_e r = F_r \sin \theta = F_L$

$v = v_0 \sin \theta$

$U_{A,CH} = X_{CH}$

$\cos \theta$

$\lambda_1 = \frac{U_1}{f}$ ;  $\lambda_2 = \frac{U_2}{f}$

$\sin \theta_2 = \frac{\lambda_1}{AB'}$

$\sin \theta_1 = \frac{\lambda_2}{AB'}$

$\frac{\sin \theta_1}{\sin \theta_2} = \frac{\lambda_2}{\lambda_1} = \frac{U_1}{U_2} = \frac{v_1}{v_2}$

$h$

$F_{ax}$

$F_{ay}$

$F_{ax} + F_{ay} = ma$

$F_{ax} = 0$ ;  $F_{ay} = F_{ax} \sin \theta$

$= mg \sin \theta$

$a_x = g \sin \theta$

$v^2 = 2g \sin \theta \Delta x$ ;  $v^2 = 2gh$ ;  $v_s = \sqrt{2gh} \cdot \sin$

$U = F_e r = F_r \sin \theta = F_L$

$v = v_0 \sin \theta$

$U_{A,CH} = X_{CH}$

$z = \sqrt{z^2 + x^2}$

# The Speed of Information

$U^2 = A^2 \exp(-\frac{x^2}{2\sigma^2})$

$B(x) = \frac{\sqrt{x}}{\sqrt{\pi}} e^{-\sigma^2(x-x_0)^2}$

$\psi(\psi) = A \cos(k_0 x - \omega t)$

$|F| = \frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$

$= \frac{mv^2}{r}$

$U_H = -\int \vec{B} \cdot (d\vec{r})$

$U_H = E_H b = v d B b$

$J = \frac{n}{V} q v d A$

$b \frac{U}{V} = \frac{1}{A q v b} \frac{1}{b d v d}$

$= -\int \vec{B} \cdot d\vec{r}$

$\frac{1}{\gamma} = \sqrt{1 - (v/c)^2}$

$P_z^{(A)} = -P_z^{(A)}$

$F_2 = \frac{F_L}{2u}$

$E = F_2 \cdot s$

$= \frac{F_L}{u} \cdot u \cdot h$

$= F_L \cdot h$

$= m \cdot g \cdot h$

$s = u \cdot h$

$\frac{1}{\gamma} = \frac{1}{s} + \frac{1}{s'}$

$\frac{A'B'}{AB} = \frac{s'}{s}$

$\oint \vec{B} \cdot d\vec{r} = \mu_0 I + \mu_0 \epsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A}$

$\oint \vec{E} \cdot d\vec{A} = \frac{1}{\epsilon_0} q$

$\oint \vec{B} \cdot d\vec{A} = 0$

$\vec{B} = B_z + \mu_0 \vec{u} = B_z(1 + \lambda_{mag})$

$E = c \vec{B}$

$\vec{B} = B_z + \mu_0 \vec{u} = B_z(1 + \lambda_{mag})$

$E = c \vec{B}$

$\vec{B} = B_z + \mu_0 \vec{u} = B_z(1 + \lambda_{mag})$

$E = c \vec{B}$

$E_{pot} = -\frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$

$E_{kin} = \frac{1}{2} m v^2$

$E_{pot} = -\frac{1}{2} \frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$

$E_{pot} = -2 E_{kin}$

$E_{pot} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$

$= \frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$

$\frac{A'B'}{AB} = \frac{s'}{s}$

$F_2 = 33\% FL$

$E = F_2 \cdot s$

$= \frac{F_L}{u} \cdot u \cdot h$

$= F_L \cdot h$

$= m \cdot g \cdot h$

$s = u \cdot h$

$\frac{1}{\gamma} = \frac{1}{s} + \frac{1}{s'}$

$\frac{A'B'}{AB} = \frac{s'}{s}$

$\oint \vec{B} \cdot d\vec{r} = \mu_0 I + \mu_0 \epsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A}$

$\oint \vec{E} \cdot d\vec{A} = \frac{1}{\epsilon_0} q$

$\oint \vec{B} \cdot d\vec{A} = 0$

$\vec{B} = B_z + \mu_0 \vec{u} = B_z(1 + \lambda_{mag})$

$E = c \vec{B}$

$\vec{B} = B_z + \mu_0 \vec{u} = B_z(1 + \lambda_{mag})$

$E = c \vec{B}$





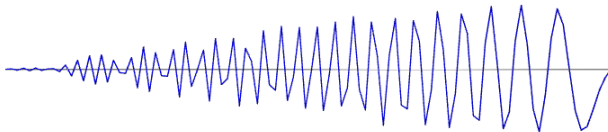
# Sound Propagation

- One side of a solid object doesn't react instantly to what is happening on the other side
- Information has to get transmitted through the atomic springs
- Stiffer springs oscillate quicker
- Materials with stiffer atomic springs “react” faster to effects on the far side
- Sound is a principle example of these displacements

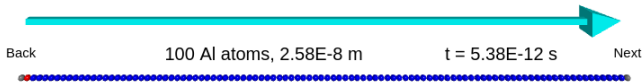


# Spring and Glowscript Demo

The measured value for the speed of sound in aluminum is 4800 m/s.  
Click anywhere to proceed, or click the atom below Back.



dist. =  $2.53\text{E-}8$  m  
 $v = 4704$  m/s



$$E_3 = 3E_1$$

$$E_2 = 4E_1$$

$$E_1$$

$$d$$

$$x$$

$$h^2 = u^2 E_1$$

$$|d\vec{E}| = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2}$$

$$= \frac{\lambda}{4\pi\epsilon_0} \frac{dx}{r^2}$$

$$dE_y = \frac{\lambda}{4\pi\epsilon_0} \frac{dx}{r^2} \frac{y}{r}$$

$$dE_x = \frac{\lambda}{4\pi\epsilon_0} \frac{dx}{r^2} \frac{x}{r}$$

$$d\vec{E} = |d\vec{E}| \cos\theta$$

$$d\vec{E} = |d\vec{E}| \sin\theta$$

$$\lambda_1 = \frac{u_1}{f}$$

$$\lambda_2 = \frac{u_2}{f}$$

$$\sin\theta_2 = \frac{\lambda_1}{\lambda_2}$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\lambda_2}{\lambda_1} = \frac{u_1}{u_2} = \frac{v_1}{v_2}$$

$$U = F_e r = F_r \sin\theta = F_L$$

$$v = v_0 \sin\theta$$

$$F_n x + F_g x = ma$$

$$F_n x = 0; F_g x = F_n \sin\theta$$

$$= mg \sin\theta$$

$$a_x = g \sin\theta$$

$$v^2 = 2g \sin\theta \Delta x$$

$$v^2 = 2gh$$

$$v_s = \sqrt{2gh} \sin\theta$$

$$U_A, \phi_A = X_C \omega$$

$$U_E = \frac{U_C}{2}$$

$$z = \sqrt{z^2 + x^2}$$

Upcoming:

# Fluid Contact Forces

$$|U|^2 = A^2 \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

$$B(x) = \frac{\sigma}{\sqrt{\pi}} e^{-\sigma^2(x-x_0)^2}$$

$$e(\psi) = A \cos(\psi_0 x - \omega t)$$

$$|F| = \frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$$

$$= \frac{mv^2}{r}$$

$$E_{pot} = -\frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$$

$$E_{kin} = \frac{1}{2} mv^2$$

$$= \frac{1}{2} \frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$$

$$E_{pot} = -2E_{kin}$$

$$E_{pot} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$$

$$\frac{1}{s} = \frac{1}{s} + \frac{1}{s'}$$

$$F_2 = \frac{F_L}{2n}$$

$$E = F_2 \cdot s$$

$$= \frac{F_L}{n} \cdot u \cdot h$$

$$F_L = F_L \cdot h$$

$$= m \cdot g \cdot h$$

$$s = u \cdot h$$

$$b = b_2 + \mu_0 u = b_2 (1 + \mu_0 u)$$

$$E = c b$$

$$= \mu_0 c b_2 h \approx \mu_0 J$$

$$m_1 v_{1x} + m_2 v_{2x} = m_1 v_{1x}' + m_2 v_{2x}'$$

$$\frac{1}{2} m_1 v_{1x}^2 + \frac{1}{2} m_2 v_{2x}^2 = \frac{1}{2} m_1 v_{1x}'^2 + \frac{1}{2} m_2 v_{2x}'^2$$

$$= \frac{1}{2} m_1 v_{1x}^2 + \frac{1}{2} m_2 v_{2x}^2$$

$$\frac{A'B'}{AB} = \frac{s' - f}{s}$$

$$\frac{s'}{s} = \frac{s' - f}{f}$$

$$F_s = \frac{mg}{\cos\theta}$$

$$F_s = \frac{mg}{\sin\theta}$$

$$E_{pot} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

$$U_H = E_H b = v_d B b$$

$$J = \frac{n}{V} q v_d A$$

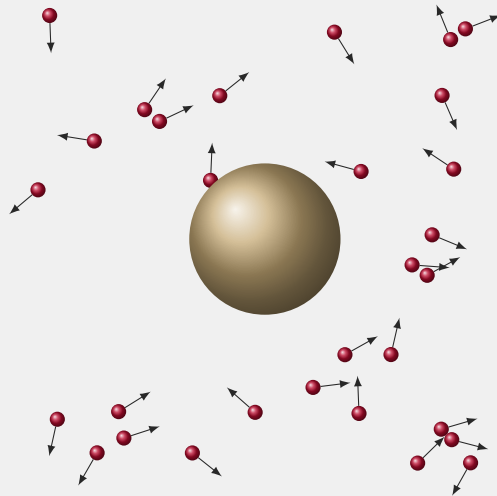
$$b \frac{u}{V} = \frac{1}{A q v_d} - \frac{1}{b d e v_d}$$

$$= -J d \cdot dE_H$$



# Bouncy Forces

- Contact Forces can take different forms
- Approximated solids with springs because of crystal-like structure
- Fluids (including gases) operate in a more free fashion
- Apply a contact force by bombarding a surface
  - Atoms/Molecules bounce back  $\Rightarrow$  change in momentum





# Under Pressure

dum-dum-dum-da-da-dum-dum

- Total force applied depends on the area being struck by the atoms
- Define pressure as

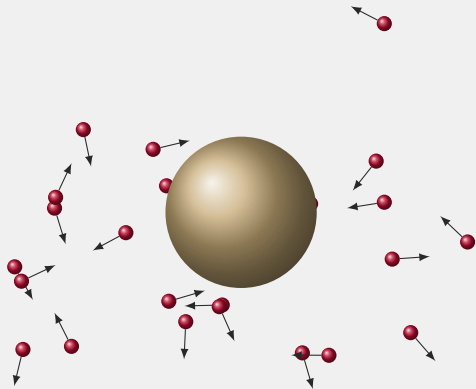
$$P = \frac{F}{A}$$

where  $A$  is the area being bombarded

- Standard unit is a  $\text{N}/\text{m}^2$  or a Pascal (Pa)
- Can determine force due to pressure by multiplying the pressure by the area



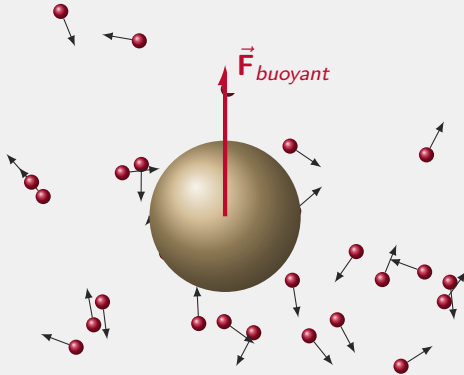
# You Buoy Me Up



- In practice, gravity pushes more things towards the bottom
- More atoms  $\Rightarrow$  more bombardments  $\Rightarrow$  more pressure  $\Rightarrow$  greater force
- Bottom of object feels a greater force due to pressure than top
- Gives a net push upwards, called the buoyant force
- Magnitude of buoyant force equals the *weight* ( $mg$ ) of the displaced fluid



# You Buoy Me Up



- In practice, gravity pushes more things towards the bottom
- More atoms  $\Rightarrow$  more bombardments  $\Rightarrow$  more pressure  $\Rightarrow$  greater force
- Bottom of object feels a greater force due to pressure than top
- Gives a net push upwards, called the buoyant force
- Magnitude of buoyant force equals the *weight* ( $mg$ ) of the displaced fluid



# I'm on a BOAT

Suppose we have the (somewhat boring) rectangular boat below. The boat has a total mass of 800 kg and has a 2 m by 1 m bottom cross-section and sidewalls 50 cm high. Assuming we place the boat in water with a density of  $1000 \text{ kg/m}^3$ , will the boat stay dry or fill with water?

