Announcements

- Video Homework 5 due tonight
  - I'm planning to score and get feedback to you on them tomorrow morning.
- Test on Wednesday (Ch 3–5)
  - Study materials available online!
  - You get a new notecard
  - I'll have my box of calculators, but email me beforehand if you know you'll want to use one, just so I'm aware please.
- No new assignment is due until a week from Wednesday. Enjoy your midsemester break (once you finish studying and taking the test)!

Review Question

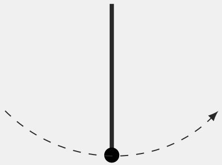
Jill of the Jungle swings across a ravine on a vine. At the bottom of the swing, how does the magnitude of the force by the vine on Jill compare to the force of the Earth on Jill?

A) $F_{vine} > F_{Earth}$

B) $F_{vine} = F_{Earth}$

C) $F_{vine} < F_{Earth}$

D) Not enough information to tell



Energy Basics

- Energy is really tough to specifically define
- "Discovered" years and years after Newton's time
- Comprised of many different forms
  - Rest
  - Kinetic
  - Potential
  - Thermal
  - Chemical
  - Nuclear, etc
- Can be transformed back and forth between types
- Unit: Joules (J)

The Energy Principle

- Unlike momentum, energy is a scalar
- Can not be destroyed, only transformed (Energy Conservation)
- Want to keep track of how much energy is entering or leaving system
- Time for our new fundamental principle!

The Energy Principle

$$\Delta E_{system} = \text{ Energy inputs from surroundings}$$

Why Energy?

Pros

- Only a number! No vectors
- Can encompass many different types of reactions
- Simple conservation principle

Cons

- More abstract that momentum
- Can be difficult or impossible to do the accounting in some instances
- Doesn't give information on directionality

A Single Particle

- Everything initially will be looking at point masses
  - More complicated objects will introduce their own forms of energy
- A single particle has energy of
$$E = \gamma mc^2$$
where m is the mass and c is the speed of light
- Here,
$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$
where v is the particle velocity and c still the speed of light
- Units: $\mathrm{kg}(\mathrm{m}/\mathrm{s})^2 = \mathrm{Nm} = \mathrm{J}$

Break it Down

- Oftentimes useful to break up into the non-moving and moving portions of energy
- Rest Energy is when the object is at ... rest
  - Here $\vec{\mathbf{v}} = 0$ and so $\gamma = 1$
  - Thus

$$E_{rest} = mc^2$$

  - Look at that (probably) familiar equation!
- Kinetic Energy is the energy of the objects motion
  - Can get via subtraction:

$$E_{kinetic} = K = \gamma mc^2 - mc^2$$

- Can always combine the two to write

$$E = mc^2 + K$$

Energy Example

An bumblebee is moving at $2\,\mathrm{m/s}$ in the positive x-direction. If this bumblebee has a mass of $5\,\mathrm{g}$, determine its total energy. Approximately what fraction of this energy comes from its movement?

Low Speed Approximations

- Low speed was easy with momentum, because if $\vec{\mathbf{v}}$ was 0 then the entire thing was zero
- A little trickier with kinetic energy because we don't want to include the rest energy
- Need to be more precise in our approximation:

$$E = \frac{mc^2}{\sqrt{1-(v/c)^2}} = mc^2\left[1 + \frac{1}{2}\left(\frac{v}{c}\right)^2 + \frac{3}{8}\left(\frac{v}{c}\right)^4 + \frac{5}{16}\left(\frac{v}{c}\right)^6 + \cdots\right]$$

- We don't want all those terms, so we only keep the first two

$$E = mc^2 + \frac{1}{2}mv^2$$

- The kinetic bit is thus just

$$K \approx \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

Rocket Energetics

Suppose we want a rocket in deep space to accelerate from rest to 500 km/s. The rocket has a mass of 10 000 kg.

- What is the rockets rest energy?
- How much energy needs to be added to get the rocket to the desired speed? Use the low speed approximation.
- If you used the full energy expression, how close would the needed energy be to your above calculation?

Understanding Check

What is the total energy of a proton ($m = 1.67 \times 10^{-27}$ kg) moving at 98% the speed of light ($c = 3 \times 10^{8}$ m/s)?

A) 72×10^{-12} J

B) 150×10^{-12} J

C) 222×10^{-12} J

D) 755×10^{-12} J

High Speed Momentum

- Technically, $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$ only holds true for slower moving objects
- At high speeds, $\Delta\vec{\mathbf{p}}$ was no longer proportional to the strength of interaction
- Required a revamp in the model:
  - Any remake still needs to simplify to $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$ at low speeds
  - Needs to keep $\Delta\vec{\mathbf{p}}$ proportional to the interaction strength at *all* speeds
- Ended up with

$$\vec{\mathbf{p}} = \gamma m\vec{\mathbf{v}} \quad \text{where} \quad \gamma = \frac{1}{\sqrt{1 - \left(\frac{|\vec{\mathbf{v}}|}{c}\right)^2}}$$

- c is the speed of light ($\approx 3 \times 10^8$ m/s)

Total Energy to Momentum

- Since the full forms of both momentum and energy involve γ, we can combine them to get
$$E^2 - (pc)^2 = (mc^2)^2$$
which is true in all reference frames at all speeds!
- Also gives us a neat way to find the momentum of massless objects (like photons for light!)

Electron Fun

What is the magnitude of the momentum of an electron ($m = 9.1 \times 10^{-31}$ kg) which has a total energy of 10 times its rest energy?

Practice Time!

A large box is at rest when 100 J of energy is added to the system. All of it goes to kinetic energy, and afterwards the box is moving with velocity $\langle 10, 0, 5 \rangle$ m/s.

- What is the mass of the box?
- What is the rest energy of the box?