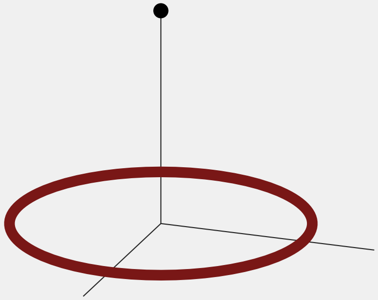
Announcements

- Happy Valentine's Day!
- Homework
  - Online HW7 is due tonight
  - VHW4 due on Monday
    - Last of the test content
- I'll be posting an old test and solutions to the webpage to help you study
- Test 1 next Friday!
  - Chapters 13-15
  - You get a 3x5in index card, one sided and handwritten
  - Will have a few minutes after receiving the test to discuss with neighbors (no writing during this time)
- Physics Tea today at 3pm!
- Polling: `rembold-class.ddns.net`

Review Question!

Suppose you have a ring charge lying on the surface of the ground such that it is in the xz plane. It has a charge of 20 nC and a radius of 5 cm. 10 cm directly above the center of the ring, a small droplet of oil floats motionless. If the oil has a charge of 10 nC, what is the mass of the oil drop?

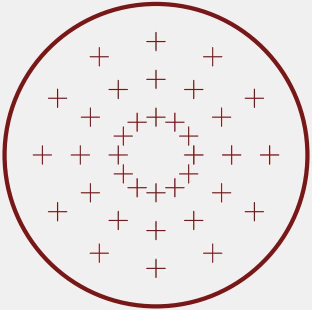
A) 0.129 g
B) 3.10 μg
C) 0.013 g
D) 1.28 g



Solution: 0.013 g

Understanding a Charged Plane

- One of the last distributions we want to understand is a charged plane
- Charge is spread out over an *area*, not a distance as in the line or ring charges
- Will look at circular planes, since we already have the machinery and since we'll normally be less interested about the edge anyway



Adding Rings

- We create a circular plane by adding together a bunch of rings of different radius!

$$\Delta E = E_{ring} = \frac{1}{4\pi\epsilon_0}\frac{\Delta q z}{(r^2+z^2)^{3/2}}\left\langle 0,0,1\right\rangle$$

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- Δq is how much of the total charge is on a single ring:

$$\frac{\Delta q}{\text{area of ring}} = \frac{Q}{\text{area of disk}} \quad \Rightarrow \quad \Delta q = Q\frac{2\pi r\Delta r}{\pi R^2}$$

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- Written as infinitesimally thin rings:

$$dE_z = \frac{1}{2\epsilon_0}\frac{Q}{\pi R^2}\frac{z}{(r^2+z^2)^{3/2}} r\,dr$$

Integrate and Approximate!

- Integrating gives us:

$$E_z = \frac{(Q/A)}{2\epsilon_0}\left[1 - \frac{z}{(R^2 + z^2)^{1/2}}\right]$$

- If we are close to the surface of the plane and far from the edges:

$$E_z \approx \frac{(Q/A)}{2\epsilon_0}\left[1 - \frac{z}{R}\right] \approx\approx \frac{(Q/A)}{2\epsilon_0}$$

  - This has zero distance dependence!
  - Basically a constant!

Charge Densities

Linear Charge Density

Can define linear charge density as

$$\lambda = \frac{Q}{L}$$

which has units of C/m.

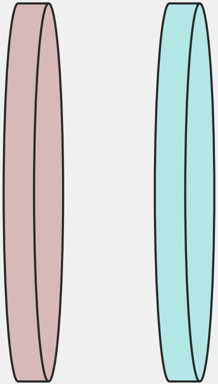
Surface Charge Density

Can define surface charge density as

$$\sigma = \frac{Q}{A}$$

which has units of C/m^2

Capacitor Magic



- A common charge distribution we'll come across is that of a capacitor
- Consists of 2 equally and oppositely charged plates a small distance apart
- We'll want to know the electric field between the plates

$$
\begin{aligned}
E_{net} &= E_{pp} + E_{np} \\
&\approx \frac{\sigma_+}{2\epsilon_0} + \frac{\sigma_-}{2\epsilon_0} \\
&\approx \frac{\sigma}{\epsilon_0}
\end{aligned}
$$

A circular capacitor has plates which measure 10 cm in diameter. If the plates are charged to $\pm 20\,\mathrm{nC}$, what is the magnitude of the electric field between the plates?

A. $36\,\mathrm{kN/C}$

B. $72\,\mathrm{kN/C}$

C. $144\,\mathrm{kN/C}$

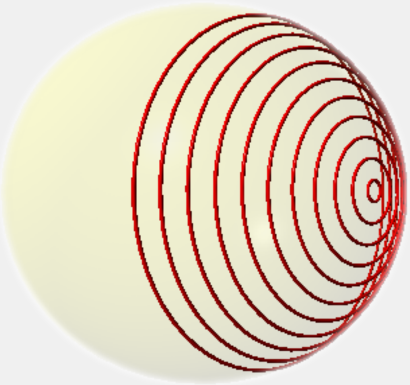
D. $288\,\mathrm{kN/C}$

Solution: $288\,\mathrm{kN/C}$

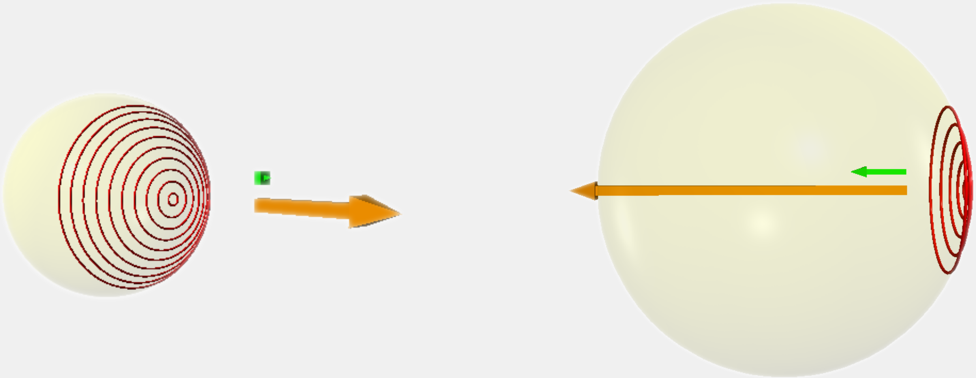
A Shell of its former self...

- We can also use ring charges to build up the electric field due to a hollow charged sphere
- Looking at a hollow shell first
  - Spherically symmetric so only need to look in one direction
  - Can create from a range of different rings with different radii
- Results:

$$E_{out} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$
$$E_{in} = 0 \text{ everywhere!}$$



Numerical Demo



Solid Spheres

- We can "create" a solid sphere by adding up a bunch of spherical shells!
- We already know the electric field on the outside of a solid sphere:

$$E_{out} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

- What about on the inside though?
  - Shells beyond the desired radius contribute nothing
  - Only shells interior to the radius will contribute like a point charge
  - Question then becomes how much charge is interior to the radius?

$$q = Q\left(\frac{\text{volume interior}}{\text{total volume}}\right) = Q\left(\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3}\right)$$

  - The electric field inside is thus just

$$E_{in} = \frac{1}{4\pi\epsilon_0}\left(\frac{r^3}{R^3}\right)\frac{Q}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{Qr}{R^3}$$

Rehash

- Same plan to determine the electric field due to any distribution
  - Break it into pieces and determine the geometery
  - Determine the electric field due to any individual piece of charge
  - Add up all the contributions
  - Check yo'self
- Distributions we've discussed:
  - Line Charges
  - Ring charges
  - Charged planes
  - Charged Shells
  - Charged Spheres