Announcements

- Homework
  - Video HW 8 due tonight!
  - Webwork due on Wednesday
- Lab this week on Faraday, which we are talking about TODAY
- Test 3 one week from Friday
- No class (SSRD) one week from Wednesday
- Polling: `rembold-class.ddns.net`

Review Question

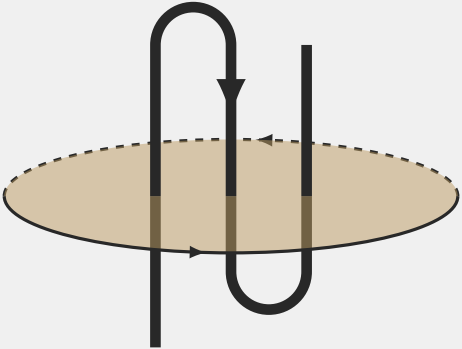
The wire to the right is carrying 5 A of current. What would be the integrated value of $\oint \vec{\mathbf{B}} \cdot d\vec{\ell}$ around the given loop (in the given direction)?

A. $-15\mu_0\,\mathrm{T\,m}$

B. $5\mu_0\,\mathrm{T\,m}$

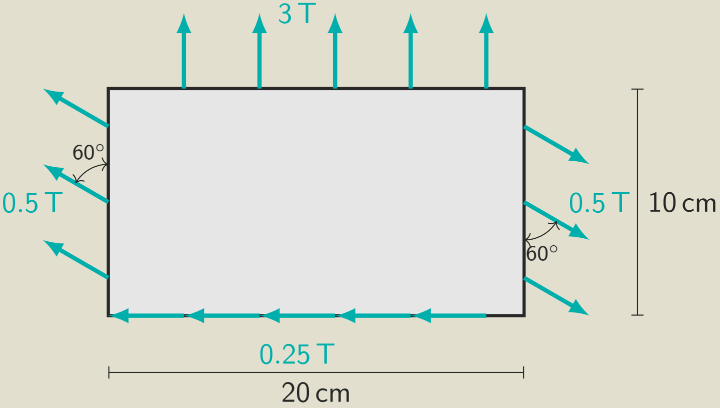
C. $10\mu_0\,\mathrm{T\,m}$

D. $15\mu_0\,\mathrm{T\,m}$



Round and Round

The figure below shows measured values and directions of the magnetic field around a piece of metal. What is the net current flowing through the metal and in what direction is it flowing?



Benefits of Gauss and Ampere Laws

- Gauss's Law really just a more general statement of Coulomb's law

$$\oint \vec{\mathbf{E}} \cdot \hat{\mathbf{n}} dA = \frac{q_{enc}}{\epsilon_0} \quad \Leftrightarrow \quad \vec{\mathbf{E}} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{\mathbf{r}}$$

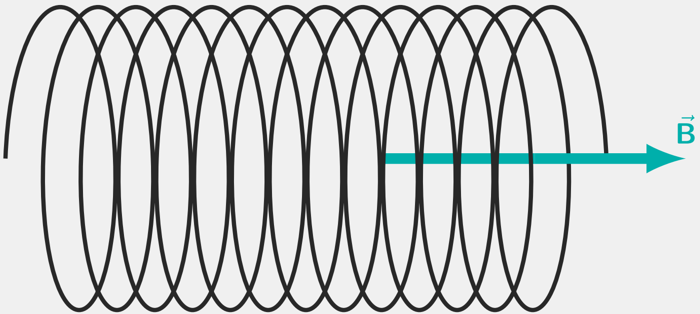
- Ampere's Law really just a more general statement of Biot-Savart law

$$\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 \mathrm{I}_{enc} \quad \Leftrightarrow \quad \vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \frac{q\vec{\mathbf{v}} \times \hat{\mathbf{r}}}{r^2}$$

- Sometimes more complicated to use, sometimes very convenient to use
- Gauss's and Ampere's law have a large benefit in that they still hold true even when moving very rapidly
    - Recall that Coulomb and Biot-Savart break down at very high speeds

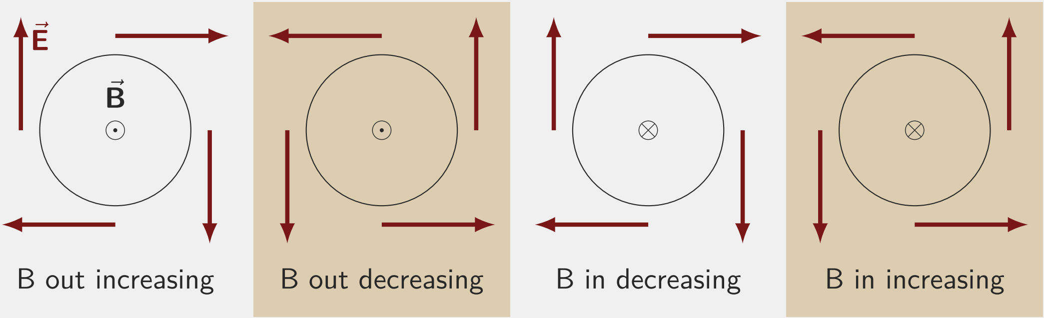
Something New

- Thus far treated electric and magnetic fields mostly separately
  - Only meshing thus far was in how observers moving at different speeds would observe the same scene
- There is a more directly relationship between the two
  - Not one that we could have predicted from what we know thus far
  - Was originally observed experimentally



A Slippery Slope

- A constant current $\Rightarrow$ a constant magnetic field $\Rightarrow$ BORING
- But a changing current $\Rightarrow$ a changing magnetic field $\Rightarrow\ldots$



Getting the Direction Correct

- Direction of the electric field is going to determine the direction of the currents that are formed
- Getting the direction correct then is *important*!
- Based on the change in the magnetic field over time
  - Increasing $\vec{\mathbf{B}}$? Thumb in the other direction and fingers give electric field direction
  - Decreasing $\vec{\mathbf{B}}$? Thumb in same direction and fingers give electric field direction

The curling electric field will always oppose changes in the magnetic field.

A Curly $\mathcal{E}$

- Recall that we proved earlier this semester that coulomb forces can not create curly electric fields
  - The whole $\oint \vec{\mathbf{E}} \cdot d\ell = 0$ thing, or $\Delta V = 0$ around a loop
- We'll call these electric fields due to non-coulombic forces then: E_{NC}
- Can calculate a net emf as we go around the loop:

$$\mathcal{E} = \oint \vec{\mathbf{E}}_{NC} \cdot d\vec{\ell} = E_{NC}(2\pi r)$$

- Note that the loop *must* contain the changing magnetic field within it for this to work! Otherwise you are back to having the total $\Delta V = 0$ around the loop.

Enter Faraday's Law

- We want to relate the change in magnetic field somehow to this $\mathcal{E}$ that can drive a current in our loop

Faraday's Law

Faraday's Law relates the curling electric field to the change in magnetic flux through a surface:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\ell} = -\frac{\mathrm{d}}{\mathrm{d}t} \int \vec{\mathbf{B}} \cdot \hat{\mathbf{n}} dA$$

$$\mathcal{E} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$$

where Φ_B is the magnetic flux.

- Note that we made this the generic $\vec{\mathbf{E}}$ (not $\vec{\mathbf{E}}_{NC}$), since the coulombic part would integrate to zero anyway

Methods of Changing Flux

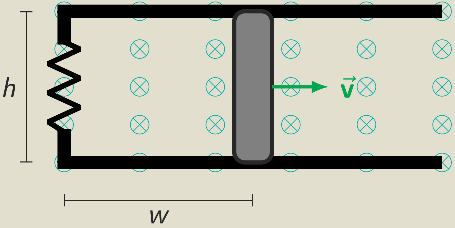
- So we will only get curling electric fields (and currents) when there is a changing magnetic flux through the loop
- If $\vec{\mathbf{B}}$ is constant across the surface:

$$\Phi_B = BA\cos\theta$$

- Gives us three things we can vary to get a changing flux:
  - Change the magnetic field itself
    - This is what we've been thinking of thus far
  - Change the area of the loop
  - Change the angle between the normal of the surface and the magnetic field

Relationship to Motional Emfs

Let's use Faraday's law to determine the emf created in the below circuit:



Suppose a wind turbine is rotating the loop shown to below at $10\,\mathrm{rad/s}$. A magnetic field of $2\,\mathrm{T}$ points upwards. What is the emf in the loop? What is the peak voltage in the loop?