Announcements

- Written homework 4 will be posted today!
  - In general, make sure you submit the correct assignments to the correct stops in Gradescope!
- Lab 4 write-up due tonight!
- I'm adding a Multiple Regression Lab on Wednesday, so bring your laptops
- Test 1 a week from this Friday
  - I'll aim to get study materials up by the weekend
  - If you haven't done them, you do have the practice problems from each chapter
- Read Ch 2.1 and 2.2 for Friday

Warm Up

For a given beauty score, are male professors evaluated higher, lower, or about the same as female professors?

	Estimate	Std. Error	t value	Pr($>|t|$)
(Intercept)	4.09	0.04	107.85	0.00
beauty	0.14	0.03	4.44	0.00
gender.male	0.17	0.05	3.38	0.00

$R^2_{adj} = 0.057$

A) higher

B) lower

C) the same

D) it is impossible to tell from this information

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Model Assumptions

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n$$

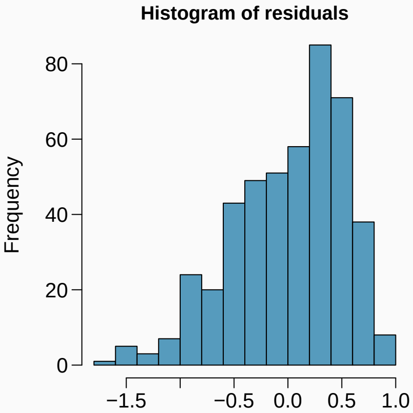
The least squares model depends on the following conditions:

- □ Residuals are nearly normal (unimodal and symmetric)
- □ Residuals have constant variability
- □ Residuals are independent
- □ Each variable is linearly related to the response

We generally use graphical methods to check these.

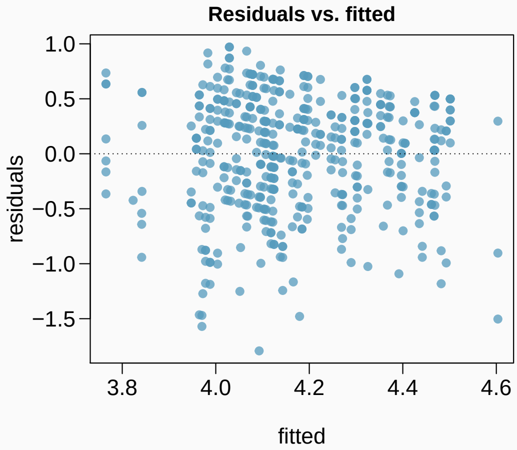
Condition 1: Nearly normal residuals

- Use histogram of residuals
- Mostly concerned with being unimodal and no odd outliers

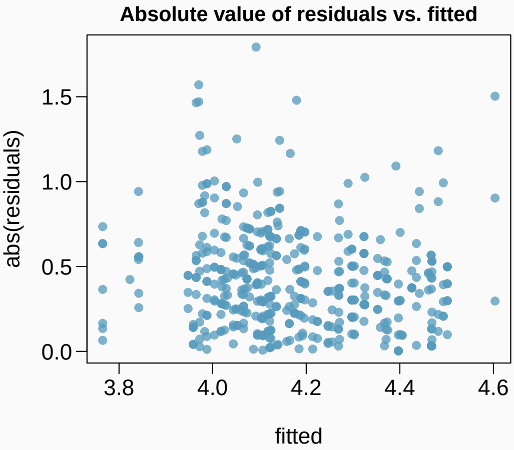


Condition 2: Constant Residual Variability

Use a scatterplot of residuals and/or absolute value of residuals vs predicted values.



Residuals vs. fitted



Absolute value of residuals vs. fitted

Condition 2: Why vs Predicted?

- When we did simple linear regression, we checked the constant variance using a plot of ***residuals vs x***.
- With multiple regression, we check constant variance using a plot of ***residuals vs predicted***.

Why the difference?

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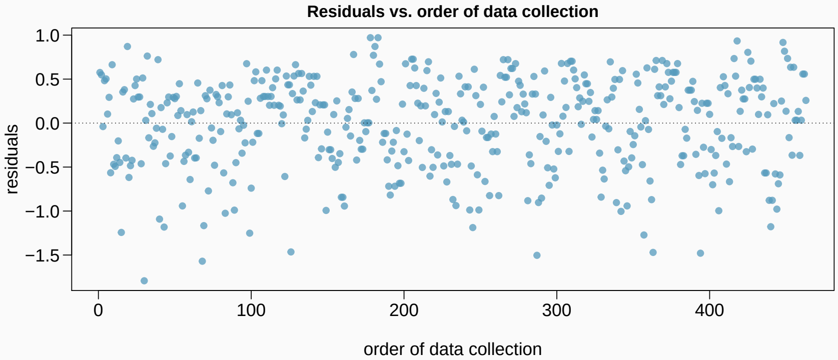
- When we did simple linear regression, we checked the constant variance using a plot of ***residuals vs x***.
- With multiple regression, we check constant variance using a plot of ***residuals vs predicted***.

Why the difference?

- In multiple regression there are many explanatory variables, so a plot of residuals vs one of them wouldn't give a full picture.

Condition 3: Independent Residuals

Scatterplot of residuals vs order of data collection.



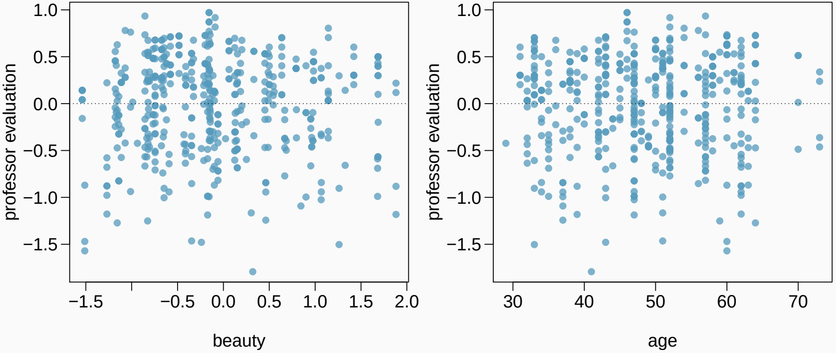
Helps check against related effects or time-of-day effects.

Condition 3: More on Independent Residuals

- Checking for independent residuals allows us to indirectly check for independent observations
- If observations and residuals are independent, we would not expect to see any trend in the residuals vs order of data scatterplot
- This condition is often violated with time-series data. Such data require more advanced time series regression techniques

Condition 4: Linear Relationships

Scatterplot of residuals vs ***each*** numerical explanatory variable



Upon Failure

If your multiple regression model fails one of the diagnostics:

- See if you can fix it!
  - Did you add an extra explanatory variable that it turns out was non-linear?
  - Can you try a different model that might better match your data?
- In not, report the model of note its shortcomings
  - Failings can indicate important parts of the data as well!

Conditional Probability

Relapse Example

Researchers randomly assigned 72 chronic users of cocaine into three groups: desipramine (antidepressant), lithium (standard cocaine treatment), and placebo. Results of the study are summarized below.

	relapse	no relapse	total
desipramine	10	14	24
lithium	18	6	24
placebo	20	4	24
total	48	24	72

Source: `http://www.oswego.edu/~srp/stats/2_way_tbl_1.htm`

Marginal Probability

What is the probability that a patient relapsed?

	relapse	no relapse	total
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$$\text{P(relapsed)} = \frac{48}{72} \approx 0.67$$

Joint Probability

What is the probability that the patient received the antidepressant (desipramine) and relapsed?

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	relapse	relapse	total
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P(relapsed and desipramine) $= \frac{10}{72} \approx 0.14$

Conditional Probability

Conditional Probability

The conditional probability of the outcome of interest A given condition B is calculated as

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

$$P(\text{relapse}|\text{desi}) = \frac{P(\text{relapse and desi})}{P(\text{desi})}$$

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$$\begin{aligned} P(\text{relapse}|\text{desi}) &= \frac{P(\text{relapse and desi})}{P(\text{desi})} \\ &= \frac{10/72}{24/72} \\ &= 0.42 = \frac{10}{24} \end{aligned}$$

	relapse	no relapse	total
desipramine	*10*	14	*24*
lithium	18	6	24
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Understanding Check

What is the probability that the patient received lithium given that the patient relapsed?

	relapse	no relapse	total
desipramine	10	14	24
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total	48	24	72

A) 0.25

B) 0.375

C) 0.75

D) 0.9

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Conditional Probabilities and Independence

Generally, if $P(A|B) = P(A)$, then the events A and B are said to be independent.

- Conceptually: Giving B doesn't tell us anything about A.
- Mathematically: We know that if A and B are independent, then $P(A \text{ and } B) = P(A) \times P(B)$. Thus:

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{P(A) \times P(B)}{P(B)} = P(A)$$

Breast Cancer Screening

- American Cancer Society estimates that about 1.7% of women have breast cancer.

  Source: `http://www.cancer.org/cancer/cancerbasics/cancer-prevalence`

- Susan G. Komen For The Cure Foundation states that mammography correctly identifies about 78% of women who truly have breast cancer.

  Source: `http://ww5.komen.org/BreastCancer/AccuracyofMammograms.html`

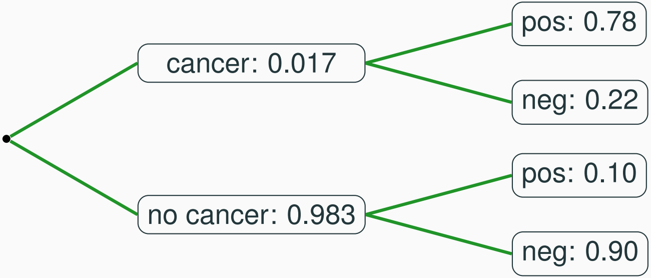
- An article published in 2003 suggests that up to 10% of all mammograms result in false positives for patients who do not have cancer.

  Source: `http://www.ncbi.nlm.nih.gov/pmc/articles/PMC1360940`

Note: These percentages are approximate and very difficult to estimate.

Inverting Probabilities

If a mammogram yields a positive result, what is the probability that a patient actually has cancer?



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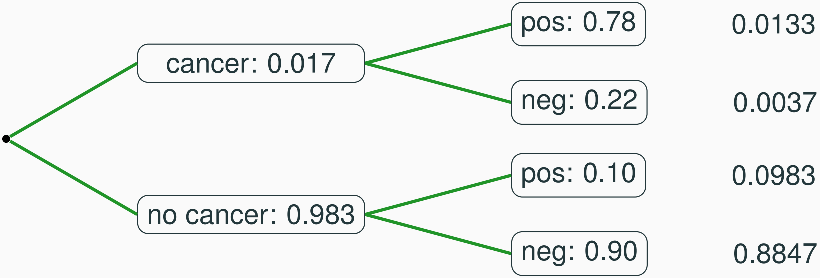

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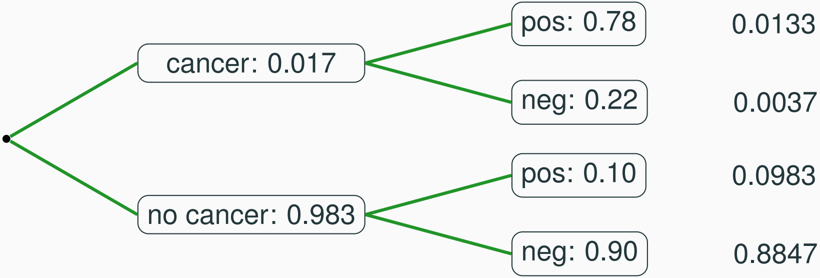

cancer: 0.017

pos: 0.78 0.0133

neg: 0.22 0.0037

no cancer: 0.983

pos: 0.10 0.0983

neg: 0.90 0.8847

$$P(C|+) = \frac{P(C \text{ and } +)}{P(+)} = \frac{0.0133}{0.0133 + 0.0983} = 0.12$$