Name: ______________________________

A MONSTROUS FINAL

Please answer the following questions in the space provided. Show *and explain* your work for full credit and any chance at partial credit!

Old Classics
$\Delta\vec{\mathbf{p}} = \vec{\mathbf{F}}_{net}\Delta t$
$\Delta K + \Delta U = W + Q$
$\Delta\vec{\mathbf{L}} = \vec{\boldsymbol{\tau}}_{net}\Delta t$
$F_{\perp} = \frac{mv^2}{r}$

Useful Constants
$g = 9.8\,\mathrm{m/s^2}$
$\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C^2/Nm^2}$
$\mu_0 = 4\pi \times 10^{-7}$
$\frac{1}{4\pi\epsilon_0} = 9 \times 10^{9}\,\mathrm{Nm^2/C^2}$
$\frac{\mu_0}{4\pi} = 1 \times 10^{-7}\,\mathrm{T\,s/C}$
$c = 3 \times 10^{8}\,\mathrm{m/s}$
$q_e = 1.6 \times 10^{-19}\,\mathrm{C}$
$m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$

p	n	µ	m	c	k	M	G
10^{-12}	10^{-9}	10^{-6}	10^{-3}	10^{-2}	10^{3}	10^{6}	10^{9}

(6) 1. In the movie Aliens, the xenomorphs actually possess a small amount of charge. The tracker device picks up the electric fields created by this charge to pinpoint a xenomorph's location. Suppose the individual holding the tracker is standing at the point $\langle 1, 2, 0\rangle$ m. A xenomorph with a charge of 500 mC is located at $\langle 5, 5, 0\rangle$ m and a xenomorph with a charge of 250 mC is located at $\langle -5, -6, 0\rangle$ m. What is the electric field at the tracker's location if you approximate the xenomorphs as point charges? Report your answer in vector form.

$obs = \langle 1,2,0\rangle$

$p_1 = \langle 5,5,0\rangle,\ q_1 = 500\times10^{-3} \Rightarrow \vec{r}_1 = obs - p_1 = \langle -4,-3,0\rangle \quad |\vec{r}_1| = 5$

$p_2 = \langle -5,-6,0\rangle,\ q_2 = 250\times10^{-3} \Rightarrow \vec{r}_2 = obs - p_2 = \langle 6,8,0\rangle \quad |\vec{r}_2| = 10$

$$\vec{E}_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1}{|r_1|^2}\hat{r}_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1}{|r_1|^3}\vec{r}_1 = \frac{(9\times10^9)(500\times10^{-3})}{5^3}\langle -4,-3,0\rangle$$

$$= \langle -1.44\times10^{8}, -1.08\times10^{8}, 0\rangle \text{ N/C}$$

$$\vec{E}_2 = \frac{1}{4\pi\epsilon_0}\frac{q_2}{|r_2|^2}\hat{r}_2 = \frac{1}{4\pi\epsilon_0}\frac{q_2}{|r_2|^3}\vec{r}_2 = \frac{(9\times10^9)(250\times10^{-3})}{10^3}\langle 6,8,0\rangle$$

$$= \langle 1.35\times10^{7}, 1.8\times10^{7}, 0\rangle \text{ N/C}$$

$$\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2 = \langle -1.305\times10^{8}, -9\times10^{7}, 0\rangle \text{ N/C}$$

(4) 2. It is a little known fact that Cthulhu is capable of creating and warping electric and magnetic fields about himself. Suppose an electron is moving in the positive $\hat{\mathbf{x}}$ direction at 10 m/s. Cthulhu has created and warped the electric and magnetic fields at this point in space so that there is a magnetic field pointing in the $-\hat{\mathbf{y}}$ direction with a magnitude of 100 mT and an electric field equal to $\langle 1, 0, 1\rangle$ N/C. What is the speed of the electron 1 µs later?

$q_e = -1.6\times10^{-19}$

$\vec{v} = \langle 10,0,0\rangle$ m/s

$\vec{B} = \langle 0, -100\times10^{-3}, 0\rangle$ T

$\vec{E} = \langle 1,0,1\rangle$ N/C

$\vec{v}\times\vec{B} = \langle 0,0,-10(100\times10^{-3})\rangle$

by rhr

$$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$$

$$= (-1.6\times10^{-19})\left(\langle 1,0,1\rangle + \langle 0,0,-1\rangle\right)$$

$$= (-1.6\times10^{-19})\langle 1,0,0\rangle$$

$$= -1.6\times10^{-19}\,\hat{x} \text{ N}$$

$$\Delta\vec{p} = \vec{F}\Delta t$$

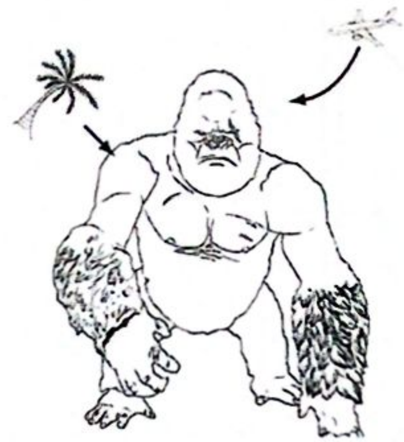
$$m_e(\vec{v}_f - \vec{v}_i) = \vec{F}\Delta t$$

$$\vec{v}_f = \vec{F}\frac{\Delta t}{m_e} + \vec{v}_i$$

$$= \frac{(-1.6\times10^{-19})(1\times10^{-6})}{9.11\times10^{-31}} + 10$$

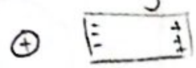
$$= -175{,}621 \text{ m/s } \hat{x}$$

(3) 3. King Kong has been shuffling his feet as he walks and acquired a net negative charge. He noticed that all manner of material starts sticking and being attracted to him, including such things as neutral airplanes made of conducting metal and neutral trees made of insulating wood. Explain why neutral objects of both types are attracted to Kong, and comment on the strength of the attraction.



Charged objects will polarize nearby neutral objects

- For conductors, this totally separates the charges, as they are free to move



- For insulators, small dipoles are created in each atom, since the charges are much more tightly held

The charges separate more in the conductor, so it should have the greater force disparity and hence the stronger attraction

In either case, the result is that the attracted charges are closer, and since $F \propto \frac{1}{r^2}$ this means the net force will be attractive

4. The appearance of the T-Rex is Jurassic Park is prefaced with water ripples in a cup. Suppose that Ian Malcom observes a wave in the cup and describes it with the following equation:

$$D = (1\,\text{cm})\cos\left(\frac{2\pi}{4}t + \frac{2\pi}{0.5}y\right)$$

(1) (a) What is the maximum height of the water waves?

A. 0.25 cm B. 0.5 cm C. 1 cm D. 4 cm

Corresponds to the amplitude

(2) (b) What is the speed at which the water wave is moving?

A. 1 cm/s B. 1.57 cm/s C. 12.5 cm/s D. 25.13 cm/s

$T = 4s$
$\lambda = 0.5m$ $v = \frac{\lambda}{T} = \frac{0.5}{4} = 0.125\ m/s$

(1) (c) In what direction is the water wave moving?

A. Positive x direction

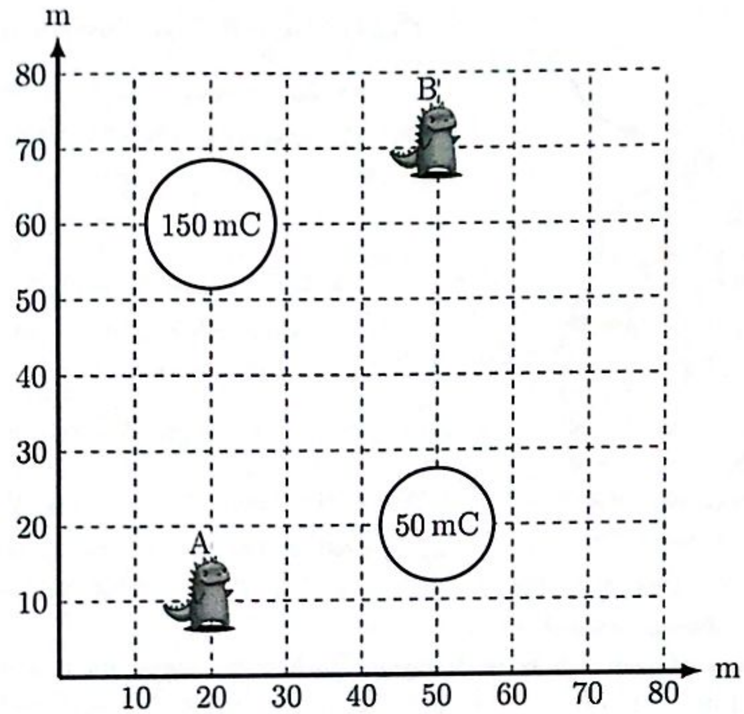
B. Positive y direction

C. Negative y direction

D. Negative z direction

since $y, + \Rightarrow$ neg dir

(6) 5. Surprisingly, Godzilla hunts by detecting minor fluctuations in electric potential as he moves around slightly charged prey (accurately portrayed here as circles…). In the image below, what is the change in electric potential as Godzilla moves from point A to point B?

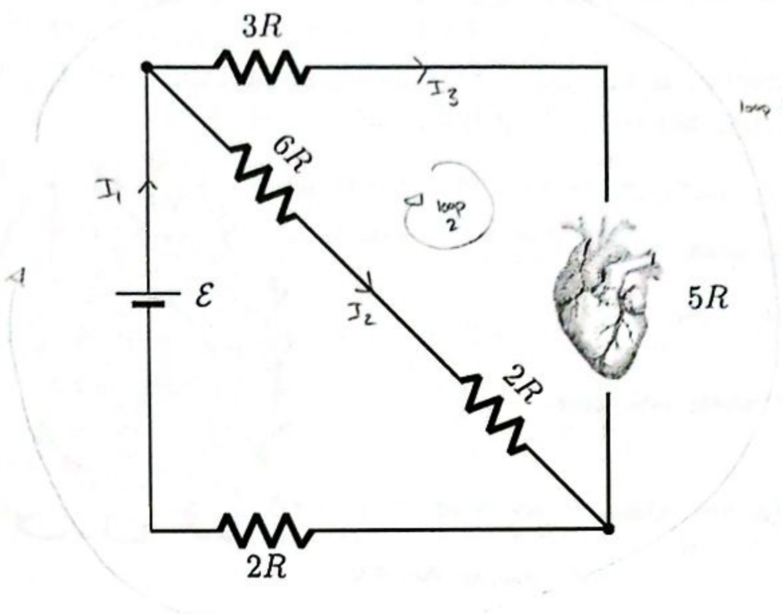


$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2} = (9\times10^9)\left(\frac{150\times10^{-3}}{50\text{m}} + \frac{50\times10^{-3}}{\sqrt{10^2+30^2}}\right) = 4.123\times10^7\ \text{V}$$

$$V_B = \text{''} + \text{''} = (9\times10^9)\left(\frac{150\times10^{-3}}{\sqrt{10^2+30^2}} + \frac{50\times10^{-3}}{50\text{ m}}\right) = 5.169\times10^7\ \text{V}$$

$$V_B - V_A = 1.046\times10^7\ \text{V}$$

(5) 6. Victor Frankenstein wishes to bring his monster to life, but to do so requires a current of 500 A across the heart. Given the circuit below, what emf does Victor require to ensure the needed current across the heart? You can take R to be $100\,\mathrm{m\Omega}$.



$I_1 = I_2 + I_3$

loop 1: $\mathcal{E} - 3RI_3 - 5RI_3 - 2RI_1 = 0 \Rightarrow \mathcal{E} = 2RI_1 + 8RI_3$

loop 2: $-3RI_3 - 5RI_3 + 2RI_2 + 6RI_2 = 0 \Rightarrow I_2 = I_3$

$\Rightarrow I_1 = I_3 + I_3 = 2I_3$

We need $I_3 = 500\,\mathrm{A}$

$\Rightarrow I_1 = 2(500) = 1000\,\mathrm{A}$

$\Rightarrow \mathcal{E} = 2(100\times10^{-3})(1000) + 8(100\times10^{-3})(500)$

$= 600\,\mathrm{V}$

7. Say Sully and Mike open their portal door into a region where a changing electric field is flowing directly out of the door (out of the page). In a twist on the normal door shape, this door is perfectly round, with a radius of 1 m (they opened a door to the Shire apparently). The electric field strength coming out of the door is dropping by 100 V/m every 1 ms.

(2) (a) In what direction is a magnetic field flowing around the door? Clockwise or counter-clockwise. Explain yourself for full points.

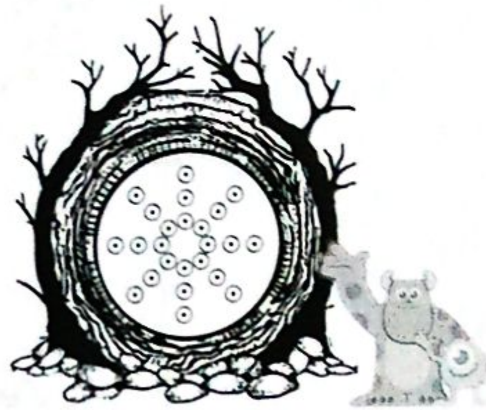
Area stays the same

E field decreases

$\Rightarrow \Phi_E$ decreasing

$\Rightarrow \frac{d\Phi_E}{dt}$ points into page

$\Rightarrow \vec{B}$ pointing clockwise by RHR and Maxwell-Ampere



(4) (b) What is the magnitude of the created magnetic field at the location of the door-frame?

By Maxwell-Ampere:

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \left(\cancel{I_{enc}}^{\,0} + \varepsilon_0 \frac{d\Phi_E}{dt} \right)$$

$$B \cdot 2\pi r = \mu_0 \varepsilon_0 \frac{d}{dt}(E \cdot A)$$

$$= \mu_0 \varepsilon_0 \pi r^2 \frac{dE}{dt}$$

$$B = \frac{\mu_0 \varepsilon_0 r}{2} \frac{dE}{dt} = \frac{(4\pi\times10^{-7})(8.85\times10^{-12})(1\,m)}{2} \cdot \frac{100}{1\times10^{-3}}$$

$$= 5.56\times10^{-13}\,T$$

(3) 8. Imagine that the Ghostbuster's Proton Blaster guns utilize a form of electromagnetic radiation to channel and accelerate the protons toward the Stay Puft Marshmallow Man. You use a compass to determine that the magnetic field of the radiation is $\langle 10, 0, 0\rangle$ mT and the radiation is propagating in the positive $\hat{\mathbf{y}}$ direction. Determine the magnitude and direction of the corresponding electric field.

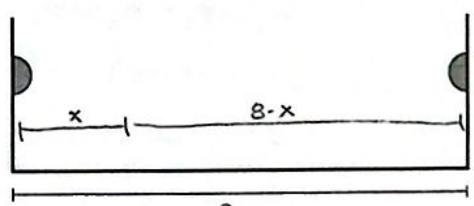
We know that $|\vec{E}| = c|\vec{B}| \Rightarrow |\vec{E}| = (3\times10^8)(10\times10^{-3}) = 3\times10^6$ V/m

Also $\vec{E}\times\vec{B}$ points in direction of propagation

$\Rightarrow$ E must point in $+\hat{z}$ direction

$\vec{E} = \langle 0, 0, 3\times10^6\rangle$ V/m

(4) 9. You are blaring out some sweet beats from MonsterCat on your synchronized speakers. Your room is 8 m across and one speaker is located on each side of the room as shown. The conditions in the room are such that the speed of sound is 343 m/s. Find one location in the room where you can't hear the bass (125 Hz), and one location where you basically only hear the base (where it is really loud). Measure your distances from the left side of the room.



Can't hear $\Rightarrow$ destructive interference

loud $\Rightarrow$ constructive interference

$\lambda f = v \Rightarrow \lambda = \frac{v}{f} = \frac{343}{125} = 2.744$ m

$$k\Delta x + \cancel{\Delta\phi_0}^{\,0} = \begin{cases} \pm 2\pi m \\ \pm 2\pi(m+1/2) \end{cases}$$

Quite spot:

$\frac{2\pi}{\lambda}(8-2x) = 2\pi(m+1/2)$

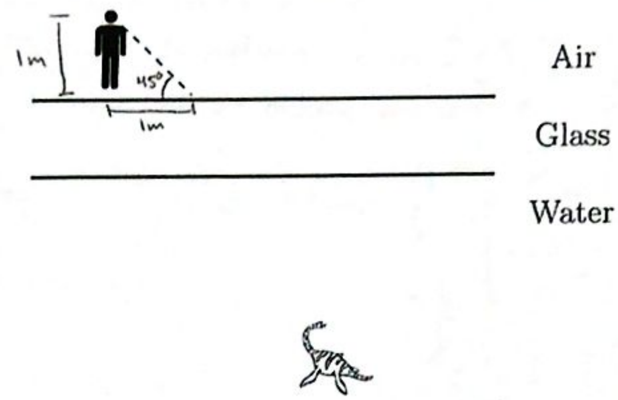
$m=0 \Rightarrow x = 3.314$ m

Loud spot:

$\frac{2\pi}{\lambda}(8-2x) = 2\pi m$

$m=0 \Rightarrow x = 4$ m (which makes a lot of sense)

(5) 10. You are sightseeing in a fancy glass bottomed boat on Lake Loch Ness. For security reasons, the glass on the bottom of the boat is 3 m thick and has an index of refraction of 1.8. The lake has a constant depth of 30 m and water has an index of refraction of 1.33. Standing a height of 1 m above the glass surface, you happen to catch sight of the Loch Ness Monster! The image of Nessie appears to be 1 m in front of you. How far away from your feet is Nessie?

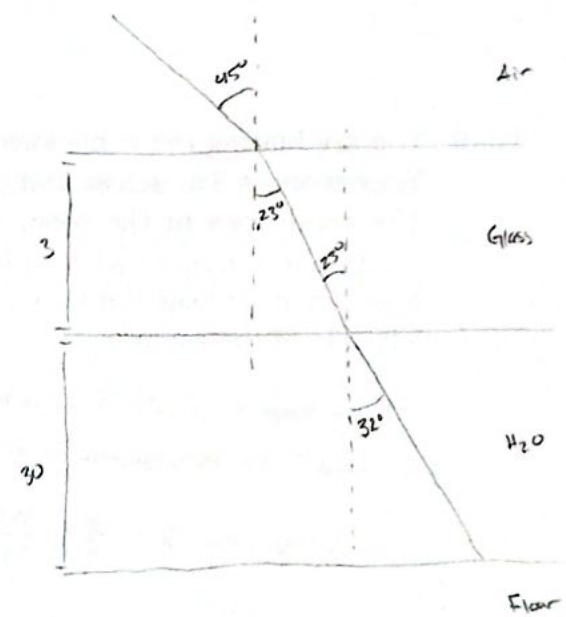


Air → Glass:

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$1 \sin(45°) = 1.8 \sin(\theta_2)$

$\Rightarrow \theta_2 = 23.1311°$



Glass → H_2O:

$n_2 \sin\theta_2 = n_3 \sin\theta_3$

$1.8 \sin(23.1311°) = 1.33 \sin\theta_3$

$\theta_3 = 32.1176°$

$x_1 = 1m$ $\quad x_2 = 3\tan\theta_2 = 1.28$

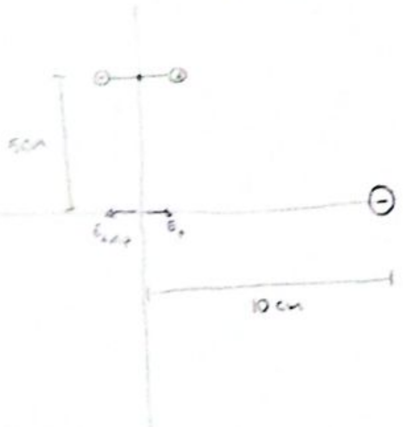
$x_3 = 30\tan\theta_3 = 18.83$

$x_{tot} = 21.1134\ m$

$y_{tot} = 33\ m$

tot dist = 39.176 meters

2 (bonus)) 11. A very un-monstrous dipole sits centered at the position $\langle 0, 5, 0 \rangle$ cm. A positive charge of $-15\,\mathrm{nC}$ is located at the point $\langle 10, 0, 0 \rangle$ cm. What dipole moment is necessary so that the electric field at the origin is zero? The dipole can have any orientation you need, and give youranswer in vector form.



Want $E_{point} = E_{\perp, dipole} \Rightarrow \frac{1}{4\pi\epsilon_0} \frac{q_0}{(10\times10^{-2})^2} = \frac{1}{4\pi\epsilon_0} \frac{p}{(5\times10^{-2})^3}$

$\Rightarrow |p| = 1.875 \times 10^{-10}$

Points $- \rightarrow +$ so $\vec{p} = \langle 1.875\times10^{-10}, 0, 0 \rangle$ Cm

1 (bonus)) 12. Draw me a picture of your favorite topic from class this semester!

1 (bonus)) 13. Complete the short conceptual survey on the next page to help out our department!

You all have been amazing this semester and I'll be very sad to see you go! To all you seniors, best of luck in your next step and feel free to stay in touch! And to everyone else, I will hopefully see you around next year! Grab a monster cookie on your way out and best of luck in any last finals you still need to take! Have a great summer!