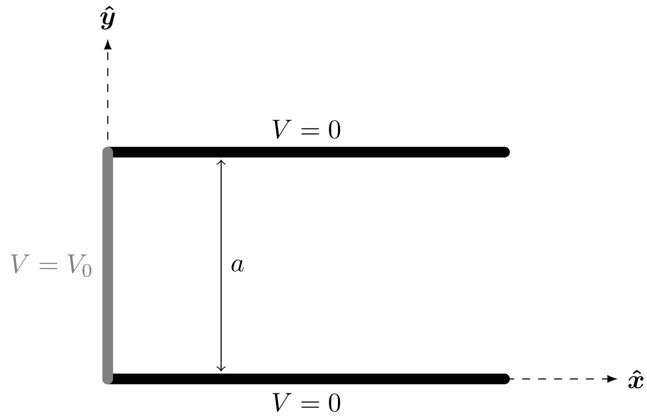
1. Griffiths works out an example for the potential inside a region with the boundary conditions shown below



and finds solutions of the form:

$$V(x, y) = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V_0}{\pi n} \sin\left(\frac{n\pi y}{a}\right) e^{-\frac{n\pi x}{a}}$$

This is going to be a common theme when solving partial differential equations, and the resulting equation is hard to get a feel for or visualize. So it is worth drawing on the power of computers to help us visualize these types of situations. For this problem take $V_0 = 10\,\mathrm{V}$ and $a = 1\,\mathrm{m}$.

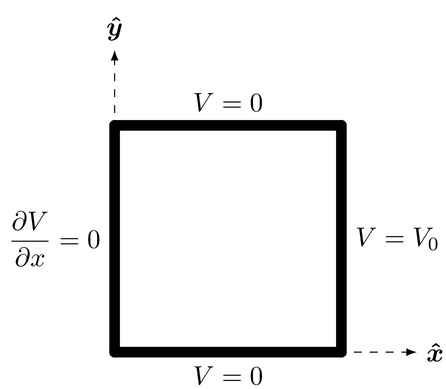
(a) Plot the approximate solution as a 3D surface plot for just the first term (i.e. $n = 1$).

(b) Plot the approximate solutionas a 3D surface plot using the first 5 terms. What do you notice about the boundary where $V = V_0$?

(c) Plot the approximate solution such that the boundary where $V = V_0$ looks very close to constant. How many terms did you need? If you hadn't already done it for part 2, you are going to want a method for automatically adding up the needed terms so you don't have to copy and paste the same code maybe 100's of times!

(d) Given this potential, sketch (by hand) what the electric field looks like.

2. A square rectangular pipe with side length a runs parallel to the z-axis (from $-\infty$ to ∞). The 4 sides are maintained with the boundary conditions given in the figure (each of the 4 sides is insulated from the others at the corners).



(a) Find the potential $V(x, y, z)$ at all points inside the pipe.

(b) State in words what the boundary on the left wall means. What does it tell you? Is the left wall a conductor? An insulator?

(c) Plot the potential as a function of x and y assuming that $V_0 = 15\,\mathrm{V}$ and $a = 1\,\mathrm{m}$.

(d) Plot the electric field vectors as a function of x and y (same assumptions as part c).

(e) Find and plot the charge density σ on the bottom conducting wall ($y = 0$). For plotting you can make the same assumptions as part c.

3. Say you have a cubical box (sides of length a) made of 6 metal plates that are insulated from each other at the corners. The left wall is located at $x = -a/2$ and the right wall at $x = a/2$. Both left and right walls are held at a constant potential $V = V_0$. All four other walls are grounded. *Note that we've set the geometry such that the cube runs from $y = 0$ to $y = a$, from $z = 0$ to $z = a$, but from $x = -a/2$ to $x = a/2$. This should actually help some with the math.*

(a) Find the potential $V(x, y, z)$ everywhere inside the box.

(b) Is the potential at the center 0?

(c) Is the electric field at the center 0? Why or why not?

(d) What are 2 ways you could try to check your solution for the potential? Check your solution using these methods and comment if these gave your greater or less confidence in your results.

4. We have a sphere of known radius R upon which we have glued charges to the outside surface such that the electric potential at the surface is given by

$$V_0 = k\cos(3\theta)$$

where k is some constant.

Your goal in this problem will be to determine the potential inside and outside the sphere (assuming there are no other charges other than those on the surface), as well as determining the charge density on the surface of the sphere. I try to walk you through things in the different parts.

(a) Rewrite the potential at the surface in terms of Legendre polynomials. You'll likely need to check into some trig identities to get the potential initially looking more like a Legendre polynomial. To figure out what combination of polynomials you need, you can either use guess-and-check algebra sorts of methods or take advantage of Fourier's trick.

(b) Using this boundary condition and the knowledge that V should be finite inside the sphere, find the electric potential, $V(r, \theta)$ inside the sphere. You don't need to re-derive the general solution, you can just start with

$$V(r, \theta) = \sum_l \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta)$$

(c) Using the same boundary condition and the knowledge that V should vanish far from the sphere, find the electric potential, $V(r, \theta)$, outside the sphere.

(d) Show explicitly that your solutions to parts b and c match at the surface of the sphere.

(e) Take the "normal" derivative of each of your solutions and use their difference to determine the surface charge density on the sphere:

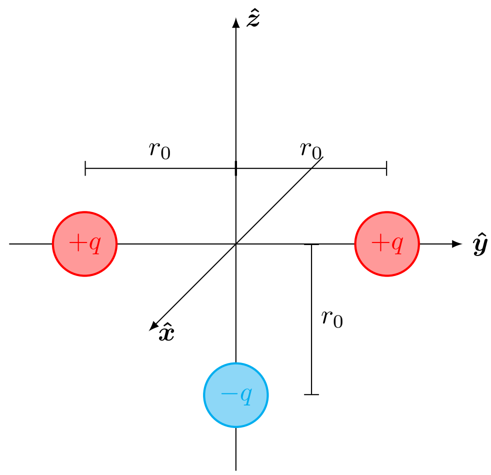
$$\left(\frac{\partial V_{out}}{\partial r} - \frac{\partial V_{in}}{\partial r} \right) = -\frac{\sigma}{\epsilon_0}$$

(f) Plot the surface charge density as a function of θ.

(g) (3 points (bonus)) Use matplotlib's `basemap` package to plot a filled contour plot of the surface charge density over the surface of a sphere. This might help you get started or serve as a template.

5. The multipole expansion is a very powerful approximation that arises in a number of different kinds of field theories. The beauty of it is that it can provide a simple approximate form for the field for from the sources that produce the field. Often times, this can be helpful when you only care about the dominant contributions since others will only provide small correction factors.

   In this problem you'll investigate the multipole expansion for the charge configuration shown below:

   (a) For the three charges shown above, determine the approximate potential at a distance far from the origin of coordinates. Keep only the two lowest non-vanishing orders of the expansion. (*Note that each change is a distance* r_0 *from the origin.*)

   (b) Using your above answer, find the approximate electric field produced by this system of charges far from the origin. Express your answer in spherical coordinates.

   (c) (2 points (bonus)) Plot the electric field over a reasonable range *that showcases the dipole contribution*. You can use either a quiver plot or a streamplot for this.