Announcements

- Homework 1 due on Monday at midnight!
  - We will talk numeric methods of solving systems in CompDay on Monday, but guides are provided on the webpage if you want to tackle the last bits of the springy pendulum earlier
- Bring your laptop on Monday for CompDay
  - Let me know sooner rather than later if you need any help getting Python on your system
- I'm getting invites out to everyone for the overall Virtual Physics Hearth Discord, which is where you can easily find me in my virtual office.

Today's Objectives

- Integrating to solve simple DEs
- Summing forces to find equations of motion
- Working with alternative coordinate systems
  - Converting from one to another
  - Writing Newton's 2nd in them

Q1

You have deduced that a certain equation of motion looks like:

$$\ddot{\vec{r}}(t) = (1\,\mathrm{m/s^2})\hat{\boldsymbol{x}} - (10\,\mathrm{m/s^2})\hat{\boldsymbol{y}}$$

If the particle started at a velocity of (1,0) m/s at the point (0, 100)m, where is the particle located 2 s later?

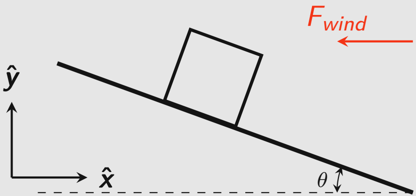
A) (4, 80) m
B) (2, 60) m
C) (2, 80) m
D) (4, 60) m

Solution: A

MECHANICS

Q2

An object sits at rest on an inclined plane. Static friction (μN) and gravity (mg) are present. A wind is also blowing horizontally towards the object and applies a force with magnitude F_{wind}. What vector expression for Newton's second law best applies to this situation?



A) $m\ddot{\vec{r}} = (-F_{wind} - N(\mu \sin\theta + \cos\theta))\,\hat{x} + (-mg + N(\cos\theta + \mu \sin\theta))\,\hat{y}$

B) $m\ddot{\vec{r}} = (-F_{wind} - N(\mu \sin\theta + \sin\theta))\,\hat{x} + (-mg + N(\cos\theta + \mu \cos\theta))\,\hat{y}$

C) $m\ddot{\vec{r}} = (-F_{wind} - N(\mu \cos\theta + \cos\theta))\,\hat{x} + (N(\sin\theta + \mu \sin\theta))\,\hat{y}$

D) $m\ddot{\vec{r}} = (-F_{wind} - N(\mu \cos\theta - \sin\theta))\,\hat{x} + (-mg + N(\cos\theta + \mu \sin\theta))\,\hat{y}$

Solution: D

MECHANICS

Polar Opposites

Q3

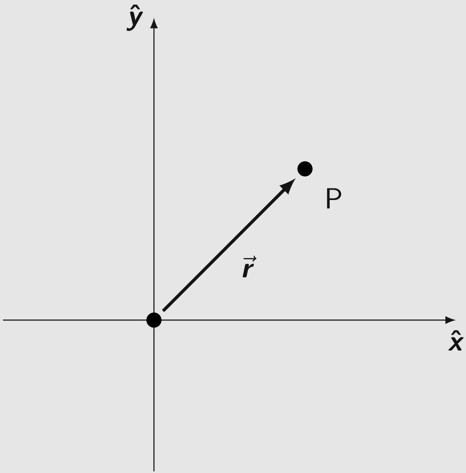
In polar coordinates $(s, \hat{\mathbf{s}}, \phi, \hat{\boldsymbol{\phi}})$, what would be the correct description of the position vector $\vec{\boldsymbol{r}}$ of the point P shown at $(x, y) = (1, 1)$?

A) $\vec{\boldsymbol{r}} = \sqrt{2}\hat{\mathbf{s}}$

B) $\vec{\boldsymbol{r}} = \sqrt{2}\hat{\mathbf{s}} + \frac{\pi}{4}\hat{\boldsymbol{\phi}}$

C) $\vec{\boldsymbol{r}} = \sqrt{2}\hat{\mathbf{s}} - \frac{\pi}{4}\hat{\boldsymbol{\phi}}$

D) $\vec{\boldsymbol{r}} = \frac{\pi}{4}\hat{\boldsymbol{\phi}}$



MECHANICS

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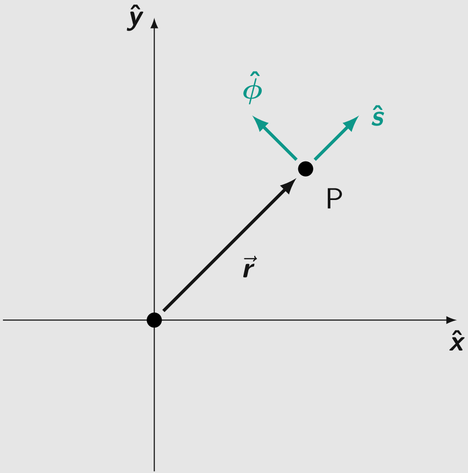
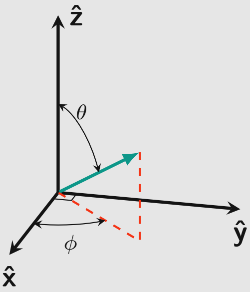


Solution: A

Q4

For something different than what you see in your book or my lecture notes, lets consider spherical coordinates. In any coordinate system, it is important to distinguish between the coefficients being in the new coordinates and the unit vectors themselves being in the new coordinates. What would the position vector look like where the coefficients are given in terms of spherical coordinates (s, ϕ, θ) but the unit vectors are still cartesian?



A) $\vec{r} = \sin\theta\cos\phi\,\hat{\mathbf{x}} + \sin\theta\cos\phi\,\hat{\mathbf{y}} + s\cos\theta\,\hat{\mathbf{z}}$

B) $\vec{r} = s\sin\theta\sin\phi\,\hat{\mathbf{x}} + s\sin\theta\cos\phi\,\hat{\mathbf{y}} + s\cos\theta\,\hat{\mathbf{z}}$

C) $\vec{r} = s\sin\theta\cos\phi\,\hat{\mathbf{x}} + s\sin\theta\sin\phi\,\hat{\mathbf{y}} + s\cos\theta\,\hat{\mathbf{z}}$

D) $\vec{r} = s\cos\theta\cos\phi\,\hat{\mathbf{x}} + s\cos\theta\sin\phi\,\hat{\mathbf{y}} + s\sin\theta\,\hat{\mathbf{z}}$

Solution: C

MECHANICS

Q5

The previous meant you could express the cartesian coefficients in terms of the new coordinates, but what about the unit vectors? If you can write down the position, one nice method to get the unit vectors is to realize that

$$\hat{\mathbf{s}} = \frac{\frac{\mathrm{d}\vec{\boldsymbol{r}}}{\mathrm{d}s}}{\left\|\frac{\mathrm{d}\vec{\boldsymbol{r}}}{\mathrm{d}s}\right\|}, \quad \hat{\boldsymbol{\phi}} = \frac{\frac{\mathrm{d}\vec{\boldsymbol{r}}}{\mathrm{d}\phi}}{\left\|\frac{\mathrm{d}\vec{\boldsymbol{r}}}{\mathrm{d}\phi}\right\|}, \quad \hat{\boldsymbol{\theta}} = \frac{\frac{\mathrm{d}\vec{\boldsymbol{r}}}{\mathrm{d}\theta}}{\left\|\frac{\mathrm{d}\vec{\boldsymbol{r}}}{\mathrm{d}\theta}\right\|}, \qquad \text{etc}$$

As such, how can the unit vector $\hat{\boldsymbol{\phi}}$ be expressed in terms of the cartesian unit vectors (and otherwise spherical coordinates)?

A) $\hat{\boldsymbol{\phi}} = \sin\theta\cos\phi\hat{\mathbf{x}} + \sin\theta\sin\phi\hat{\mathbf{y}} + \cos\theta\hat{\mathbf{z}}$

B) $\hat{\boldsymbol{\phi}} = \cos\theta\cos\phi\hat{\mathbf{x}} + \cos\theta\sin\phi\hat{\mathbf{y}} - \sin\theta\hat{\mathbf{z}}$

C) $\hat{\boldsymbol{\phi}} = -s\sin\theta\sin\phi\hat{\mathbf{x}} + s\sin\theta\cos\phi\hat{\mathbf{y}}$

D) $\hat{\boldsymbol{\phi}} = -\sin\phi\hat{\mathbf{x}} + \cos\phi\hat{\mathbf{y}}$

Solution: D

Q6

Once you found all the new unit vectors, you could rewrite all the cartesian vectors in terms of the new, then plug all your substitutions in to the original position vector and simplify. If you do that here, you get

$$\vec{\boldsymbol{r}} = s\hat{\mathbf{s}}$$

which shouldn't be surprising considering the polar case earlier. To find Newton's second law, we need to take derivatives though. What would the first derivative look like?

A) $\dot{\vec{\boldsymbol{r}}} = \dot{s}\hat{\mathbf{s}} + s\dot{\theta}\hat{\boldsymbol{\theta}} + s\dot{\phi}\sin\theta\hat{\boldsymbol{\phi}}$

B) $\dot{\vec{\boldsymbol{r}}} = \dot{s}\hat{\mathbf{s}}$

C) $\dot{\vec{\boldsymbol{r}}} = \dot{s}\hat{\mathbf{s}} + s\dot{\theta}\cos\theta\cos\phi\hat{\boldsymbol{\theta}}$

D) $\dot{\vec{\boldsymbol{r}}} = \dot{s}\hat{\mathbf{s}} + s\dot{\theta}\hat{\boldsymbol{\phi}} + s\dot{\theta}\cos\phi\hat{\boldsymbol{\theta}}$

Solution: A

MECHANICS

Polar Opposites

An observation...

You'd need one *more* derivative to get the necessary acceleration! Doing so gives you:

$$\ddot{\vec{r}} = \left(\ddot{s} - s\dot{\theta}^2 - s\dot{\phi}^2 \sin^2\theta\right)\hat{\mathbf{s}} + \left(s\ddot{\theta} + 2\dot{s}\dot{\theta} - s\dot{\phi}^2 \sin\theta\cos\theta\right)\hat{\boldsymbol{\theta}} + \left(s\ddot{\phi}\sin\theta + 2s\dot{\theta}\dot{\phi}\cos\theta + 2\dot{s}\dot{\phi}\sin\theta\right)\hat{\boldsymbol{\phi}}$$

And this is why Newton's 2nd law can get really ugly in different coordinate systems...