Announcements

- CompDay 1 due tonight
- Homework 2 posted and due Monday
- I'm aiming to have scored and have feedback to you on HW1 by Friday
- Read through Ch 3.2 by Friday
- Responses: `rembold-class.ddns.net`

MECHANICS

Such a Drag * Drag

Today's Objectives

- Writing sum of forces into velocity based differential equations
- Solving different types of simple differential equations
- Having some intuition about how quadratic drag behavesA
- Understanding how switching to complex numbers can help us with systems of equations

Q1

Friction could also be described as a force that opposes the direction of motion, except it is constant (assuming the normal force does not change). Consider a situation where a puck sits upon a table and is free to move about in the x and y plane. Friction, quadratic drag, gravity and the normal force are the forces present. What expression best describes the equation of motion in the $\hat{\boldsymbol{x}}$ direction?

A) $m\dot{v}_x = -\mu N v_x \sqrt{v_x^2 + v_y^2} - c\sqrt{v_x^2 + v_y^2}\, v_x$

B) $m\dot{v}_x = \mu N \sqrt{v_x^2 + v_y^2} - c\sqrt{v_x^2 + v_y^2}$

C) $m\dot{v}_x = -\dfrac{\mu N v_y}{\sqrt{v_x^2 + v_y^2}} - c\sqrt{v_x^2 + v_y^2}\, v_y$

D) $m\dot{v}_x = -\dfrac{\mu N v_x}{\sqrt{v_x^2 + v_y^2}} - c\sqrt{v_x^2 + v_y^2}\, v_x$

Q2

An object undergoes a drag force such that its equations of motion take the form:

$$\ddot{x} = -D\sqrt{v_x}$$

where D is a constant. If the object started with a speed of v_0, what expression describes the object's speed as a function of time?

A) $\dfrac{(2\sqrt{v_0} - Dt)^2}{4}$

B) $\dfrac{-Dv_0^2}{2}$

C) $\dfrac{(v_0 + Dt)^2}{2}$

D) $\dfrac{(2\sqrt{v_0} - Dt)}{2}$

Q3

For linear drag, we saw that for two free falling objects with the same density, the larger object would hit the ground first assuming they started at a very high height. Suppose here you have two large cannonballs, each made of steel, such that quadratic drag will dominate. Ball A has a radius of twice Ball B. If they both fall far enough to reach terminal velocity, how much faster is Ball A traveling than Ball B when they hit the ground?

A) The same speed

B) $\sqrt{2}$ times as fast

C) 2 times as fast

D) 4 times as fast

Q4

Say you are high up in a helicopter and have a small cannonball you plan to toss out the window. In which situation would you be able to analytically solve for the path of the cannonball on its way to the Earth? You can imagine that the vertical direction is the positive $\hat{z}$ direction with $\hat{x}$ and $\hat{y}$ pointing north and west, respectively.

A) Toss it out purely horizontally to the Northwest

B) Toss it out slightly downward but due East

C) Toss it out purely horizontally to the West

D) None of the above

Q5

Suppose you have the coupled differential equations:

$$\dot{x} = -\frac{x}{2} + y$$
$$\dot{y} = -x - \frac{y}{2}$$

and want to combine both into a complex differential equation to better solve your problem. If $\eta = x + iy$, the equivalent complex differential equation would be:

A) $\dot{\eta} = -(\frac{1}{2} + i)\eta$

B) $\dot{\eta} = -\frac{(1+i)}{2}\eta$

C) $\dot{\eta} = (i+2)\eta$

D) One does not exist for this set of coupled equations