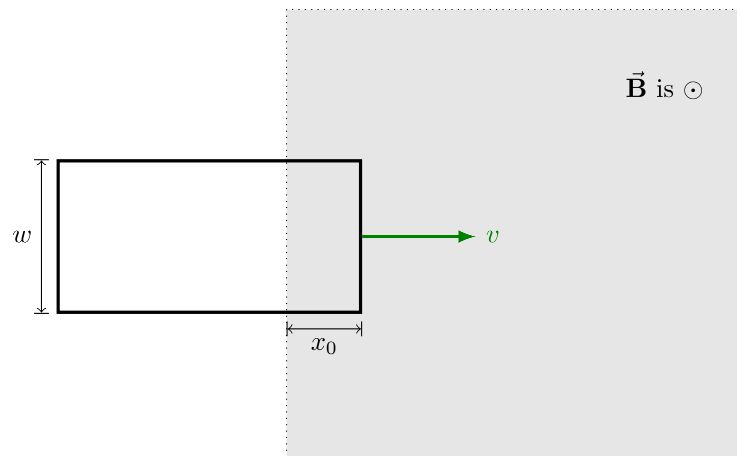
1. Resistors can take a number of forms and have rather different geometries. Imagine a resistor that is composed of two concentric metal spheres which have some poorly conducting material filling the space between them. Let the spheres have radii a and b and the material between them have some conductance σ. The spheres are charged such that they have a potential difference of ΔV.
   (a) What is the current between the two spheres? Is it constant? (*Hint: It might help to initially imagine some charge on the inner sphere so you can use Gauss's Law. Then you can relate that charge to ΔV to get everything in terms of known quantities later.*)
   (b) What does $\boldsymbol{\nabla} \cdot \vec{\mathbf{J}} = \frac{\partial \rho}{\partial t}$ tell you in this case?
   (c) What is the resistance between the two metal shells?

2. Consider a rectangular loop of metal wire moving at constant speed v into a region of uniform magnetic field. The magnetic field is pointing out of the page and is increasing linearly like $B = B_0 + \alpha t$, where B_0 and α are positive constants. At $t = 0$, the loop is a distance x_0 into the field already, as shown.



   (a) Find an expression for the magnitude of the emf around the loop as a function of time during the period the loop is entering the magnetic field.
   (b) Check that your answer to part (a) makes sense by checking the units as well as the special cases when $v = 0$ and when $\alpha = 0$. Does your solution for these special cases hold up with what you'd expect?
   (c) What direction around the loop is the induced current flowing? Explain yourself.

3. A conducting thin disk of radius a and conductivity σ is placed in a uniform magnetic field that varies with time:
$$\vec{\mathbf{B}} = B_0 \sin(\omega t)\hat{\boldsymbol{z}}$$
   (a) Ignoring any effects of extra induced magnetic fields, find the induced electric field in disk.
   (b) Determine the current density in the disk.
   (c) (2 points (bonus)) Make an animation showing the electric field vectors as quivers and the magnitude of the current density as a heat-map over several seconds.

4. Conducting tethers can use the velocity of a spacecraft moving through Earth's magnetic field to generate electrical power. Several experiments have been done with space-based tethers to investigate their feasibility. Examples include a 1996 joint US/Italian mission to deploy a 20 km tether from one of the space shuttles to generate power, or the ESA's YES2 experiment that lowered a 31 km tether as a means of reentry.

   (a) What is a typical low-Earth orbital velocity? Assume you are 350 km above the surface of the Earth in a circular orbit.

   (b) You'll also need to know the magnetic field at your location. You have an expression for the magnetic field due to a dipole, which you can use, but first you need to determine the magnetic dipole moment of the Earth. To do so, look up the local magnetic field strength at some location on Earth here. Using that value and your expression for the magnetic field due to a dipole, you can work backwards to determine the magnetic dipole moment of the Earth.

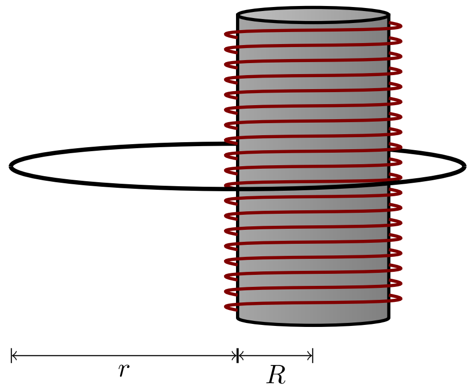
   (c) Assume that the space shuttle is orbiting in a circle about the magnetic equator. In what direction should it deploy the tether to most efficiently create the largest emf possible?

   (d) Take the example where the tether is 20 km long. Given the shuttle's altitude from part (a), and the orientation you arrived at in part (c), determine the potential difference or emf between the end of the tether and the shuttle. (*Hint: The motional emf will separate charges only up to a point. What brings it to a stop and the system into balance?*)

   (e) Great, you have effectively made a battery! To do anything useful with it though, you need to hook it up in a closed circuit. Why can't you just send down another cable and attached it to the end of the tether?

   (f) It turns out that it is the ionized plasma in the upper atmosphere (the aptly named ionosphere) that actually works to complete the circuit! Suppose that the tether itself is comprised of a 1 cm nylon core surrounded by a 3 mm thick copper braid, and then all wrapped in a layer of insulating Teflon. Make your life easy and assume a column of plasma conducts from the end of the tether to the shuttle with a cross sectional area equal to the area of the shuttle, about $200\,\mathrm{m}^2$. What is the conductivity of the ionized plasma? Data taken during the actual experiment indicated that the ionosphere was "somewhat less conductive" than the copper braid. Do your numbers support that?

5. Say you have an infinitely tall solenoid of radius R and with n coils wrapped per unit length. Around it is placed a single loop of wire of radius r which is centered at the left edge of the solenoid.



   (a) Determine the mutual inductance between the loop and the solenoid. Would it be bigger, smaller, or the same if r was twice as big? Explain.

   (b) Suppose that the outer loop was doubled up, so that it wraps around the solenoid twice (with the same r). What is the mutual inductance now?

   (c) If the solenoid was not infinitely long, would the mutual inductance between the solenoid and the single loop get larger, smaller, or stay the same? Explain.

   (d) Find the self-inductance per unit length of the infinite solenoid all by itself. Check that your units work out properly!