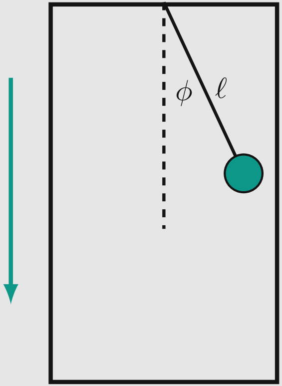
Announcements

- Midterm due Monday at Midnight
- HW6 also technically due then if you haven't already turned it in
- CompDay 7 on Monday on using Sympy to help with Lagrangian problems
  - Both CD6 and 7 due the following Wednesday
- Homework 7 will be posted on Monday
- Responses: `rembold-class.ddns.net`

Today's Objectives

- To practice general Lagragian problems
- To understand how to identify conserved values
- To understand how to interpret equilibrium points and stability from Lagrangian differential equations

Q1



The box to the left has a pendulum attached to the ceiling. Box and pendulum are dropped in a freefall and we want to describe the resulting motion of the pendulum. What would the expression for the kinetic energy look like?

A) $\frac{1}{2}m\left(\ell^2\dot{\phi}^2 - \frac{1}{2}gt^2\right)$

B) $\frac{1}{2}m\left(\ell^2\dot{\phi}^2 - g^2t^2 + 2gt\ell\dot{\phi}\cos(\phi)\right)$

C) $\frac{1}{2}m\left(\ell^2\dot{\phi}^2 + g^2t^2 - 2gt\ell\dot{\phi}\sin(\phi)\right)$

D) $\frac{1}{2}m\left(\ell^2\dot{\phi}^2 - g^2t^2 - 2gt\ell\dot{\phi}\cos(\phi)\right)$

MECHANICS

Lagrangian Examples

Q2

Taking the same problem as before, finish determining the Lagrangian and determine the corresponding equation of motion. Which equation below best describes $\ddot{\phi}$?

A) $\ddot{\phi} = -\dfrac{g}{\ell^2}$

B) $\ddot{\phi} = 0$

C) $\ddot{\phi} = -\dfrac{g}{\ell}\phi$

D) None of the above

Q3

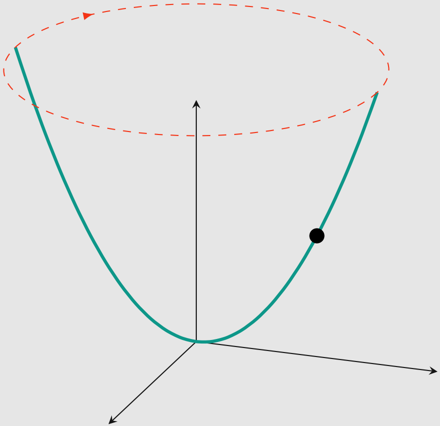
Consider the situation to the right, where a bead is free to slide along a wire that has been bent into the shape of a parabola. The wire is then spun about the z-axis at speed ω. How many generalized coordinates do you need?

A) 1

B) 2

C) 3

D) 4



Q4

Let's let our generalize coordinate be ρ, the distance from the z-axis. The parabola is described by the expression:

$$z = 2\rho^2$$

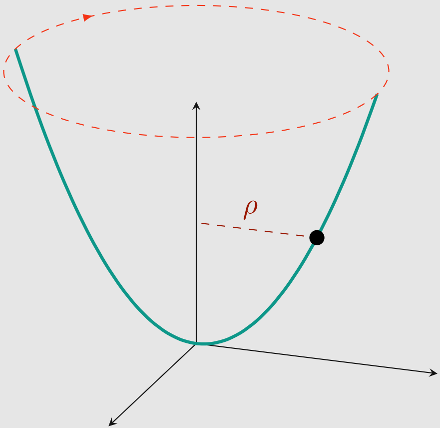
What is the Lagrangian of the system?

A) $\frac{1}{2}m(\rho^2 + 4\omega^2\dot{\rho}^2 + 4g\rho^2)$

B) $\frac{1}{2}m(\omega^2\rho^2 + 16\rho^2\dot{\rho}^2 + \dot{\rho}^2 - 4g\rho^2)$

C) $\frac{1}{2}m(\rho^2 + 4\omega^2\dot{\rho}^2 + \dot{\rho}^2 - 4g\rho^2)$

D) $\frac{1}{2}m(\omega^2\rho^2 + 16\rho^2\dot{\rho}^2 + 4g\rho^2)$



Q5

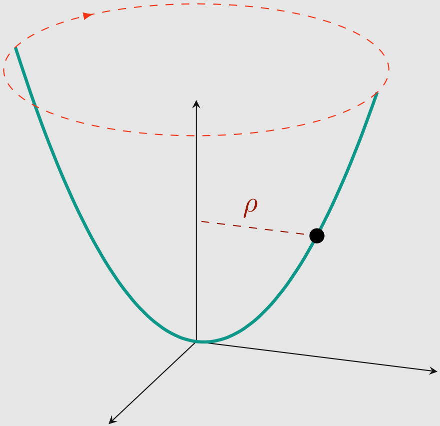
Solving the Lagrangian equations for $\ddot{\rho}$, what do you get?

A) $\ddot{\rho} = \dfrac{(-4g + \omega^2 - 32\dot{\rho}^2)\rho}{32\rho^2 + 1}$

B) $\ddot{\rho} = \dfrac{(-4g + \omega^2 - 16\dot{\rho}^2)\rho}{16\rho^2 + 1}$

C) $\ddot{\rho} = \dfrac{(4g + \omega^2 - 32\dot{\rho}^2)\rho}{16\rho^2 + 1}$

D) $\ddot{\rho} = \dfrac{(4g + \omega^2 - 16\dot{\rho}^2)\rho}{\rho^2 - 1}$



Q6

Where will equilibrium positions exist for our particle on the wire?

A) At $\rho = 0$ and when $\omega = 2\sqrt{g}$

B) At $\rho = 0$

C) At $\rho = 0$ and at $\rho = \pm 1/4$

D) There are no equilibrium positions