Announcements

- Homework 6 posted (now in its entirety!)
  - I *REALLY* suggest you start this one early. Separation of variable problems are not short and take *time*.
- Exam 1 a week from Friday: *in class*
- Have read Ch 3.4 on the Multipole expansion by Friday

Q1

Given $\nabla^2 V = 0$ in Cartesian coordinates, we separated $V(x,y,z) = X(x)Y(y)Z(z)$. Will a similar approach work for spherical coordinates? I.e. can we separate $V(r,\theta,\phi) = R(r)\Theta(\theta)\Phi(\phi)$?

A. Sure

B. Not quite, the angular bits can not be isolated from one another

C. No, because in spherical coordinates the Laplace equation has lots of cross terms in it

D. I think so, but I'm not sure why

Q1

Given $\nabla^2 V = 0$ in Cartesian coordinates, we separated $V(x, y, z) = X(x)Y(y)Z(z)$. Will a similar approach work for spherical coordinates? I.e. can we separate $V(r, \theta, \phi) = R(r)\Theta(\theta)\Phi(\phi)$?

A. Sure

B. Not quite, the angular bits can not be isolated from one another

C. No, because in spherical coordinates the Laplace equation has lots of cross terms in it

D. I think so, but I'm not sure why

Q2

The general solution for the electric potential in spherical coordinates with azimuthal symmetry is:

$$V(r,\theta) = \sum_{\ell=0}^{\infty}(A_\ell r^\ell + \frac{B_\ell}{r^{\ell+1}})P_\ell(\cos\theta)$$

Consider a metal sphere at some constant potential. Which terms in the sum vanish outside the sphere?

A. All the A_ℓ's
B. All the A_ℓ's except A_0
C. All the B_ℓ's
D. All the B_ℓ's except B_0

Q2

The general solution for the electric potential in spherical coordinates with azimuthal symmetry is:

$$V(r,\theta) = \sum_{\ell=0}^{\infty}(A_\ell r^\ell + \frac{B_\ell}{r^{\ell+1}})P_\ell(\cos\theta)$$

Consider a metal sphere at some constant potential. Which terms in the sum vanish outside the sphere?

A. All the A_ℓ's

B. All the A_ℓ's except A_0

C. All the B_ℓ's

D. All the B_ℓ's except B_0

Q3

$$V(r,\theta) = \sum_{\ell=0}^{\infty}(A_\ell r^\ell + \frac{B_\ell}{r^{\ell+1}})P_\ell(\cos\theta)$$

Now say that V everywhere on a spherical shell is a constant V_0, and there are no charges inside the sphere. Which terms do you expect to appear when solving for the potential inside the sphere?

A. Many A_ℓ terms, but no B_ℓ's

B. Many B_ℓ terms, but no A_ℓ's

C. Just A_0

D. Just B_0

Q3

$$V(r,\theta) = \sum_{\ell=0}^{\infty}(A_\ell r^\ell + \frac{B_\ell}{r^{\ell+1}})P_\ell(\cos\theta)$$

Now say that V everywhere on a spherical shell is a constant V_0, and there are no charges inside the sphere. Which terms do you expect to appear when solving for the potential inside the sphere?

A. Many A_ℓ terms, but no B_ℓ's

B. Many B_ℓ terms, but no A_ℓ's

C. Just A_0

D. Just B_0

Q4

Given an initial condition of

$$V_0 = \sum_{\ell=0}^{\infty} C_\ell P_\ell(\cos\theta)$$

we want to solve for C_ℓ. We can do so by multiplying both sides by what and then integrating?

A. $P_m(\cos\theta)$

B. $P_m(\sin\theta)$

C. $P_m(\cos\theta)\sin\theta$

D. $P_m(\sin\theta)\cos\theta$

Q4

Given an initial condition of

$$V_0 = \sum_{\ell=0}^{\infty} C_\ell P_\ell(\cos\theta)$$

we want to solve for C_ℓ. We can do so by multiplying both sides by what and then integrating?

A. $P_m(\cos\theta)$

B. $P_m(\sin\theta)$

C. $P_m(\cos\theta)\sin\theta$

D. $P_m(\sin\theta)\cos\theta$

Q5

Suppose V on a spherical shell of radius R is:

$$V(R, \theta) = V_0(1 + \cos^2\theta)$$

Which terms do you expect to appear when solving for the potential inside the shell? (Again, there are no other charges inside the shell.)

A. Many A_ℓ terms, but no B_ℓ's
B. Many B_ℓ terms, but no A_ℓ's
C. Just A_0 and A_2
D. Just B_0 and B_2

Q5

Suppose V on a spherical shell of radius R is:

$$V(R, \theta) = V_0(1 + \cos^2\theta)$$

Which terms do you expect to appear when solving for the potential inside the shell? (Again, there are no other charges inside the shell.)

A. Many A_ℓ terms, but no B_ℓ's
B. Many B_ℓ terms, but no A_ℓ's
C. Just A_0 and A_2
D. Just B_0 and B_2

Example

Let's work out the full solution for the potential on the inside of the spherical shell if the potential on the shell is given by:

$$V(R,\theta) = V_0(1+\cos^2\theta)$$

given the general form of the potential:

$$V(r,\theta) = \sum_{\ell=0}^{\infty}(A_\ell r^\ell + \frac{B_\ell}{r^{\ell+1}})P_\ell(\cos\theta)$$

Q6

Suppose V on a spherical shell of radius R is:

$$V(R,\theta) = V_0(1+\cos^2\theta)$$

What would be the value of the B_2 term when solving for the potential outside the shell?

A. $\frac{4RV_0}{3}$

B. $\frac{2V_0}{3}$

C. $\frac{2R^3V_0}{3}$

D. $\frac{4V_0}{3}$

Q6

Suppose V on a spherical shell of radius R is:

$$V(R,\theta) = V_0(1+\cos^2\theta)$$

What would be the value of the B_2 term when solving for the potential outside the shell?

A. $\dfrac{4RV_0}{3}$

B. $\dfrac{2V_0}{3}$

C. $\dfrac{2R^3V_0}{3}$

D. $\dfrac{4V_0}{3}$