Announcements

- Homework
  - Webwork due tonight!
  - New Webwork due on Friday
- Polling: `rembold-class.ddns.net`

Understanding Check

A circle of radius 10 cm is oriented such that a normal to its surface is 130° from the x-axis, 90° from the y-axis, and 50° from the z-axis. An electric field of $\langle 5, 3, 8 \rangle$ V/m permeates the space. What is the electric flux through the circle?

A. 0.061 Vm

B. 0.239 Vm

C. 0.61 Vm

D. 2.91 Vm

The Law of Gauss

- Recall that our objective was to quantify how the electric field passing through a surface relates to the charge inside that surface

Gauss's Law

The electric flux through a surface is related to the amount of charge interior to the surface by

$$\oint \vec{\mathbf{E}} \cdot \hat{\mathbf{n}} dA = \Phi_{el,closed} = \frac{q_{enc}}{\epsilon_0}$$

where q_{enc} is the total enclosed net charge.

- For you math folk, this is directly related to the divergence theorem
- While neat, Gauss's law only really helps us in particular cases
- Notice the circle on the integral sign: it *must* be a *closed* surface!

Actually Using Gauss's Law

- You are always free to choose whatever shape of surface you want, so long as it is closed
- Gauss's Law is always valid, but in many cases it doesn't actually simplify the problem
- For many situations, $\vec{\mathbf{E}} \cdot \hat{\mathbf{n}}$ will be changing over the entire surface, which would force us to actually do the surface integral
  - $\Rightarrow$ not helpful
- But for some, nice, symmetrical situations, we can choose a surface such that $\vec{\mathbf{E}} \parallel \hat{\mathbf{n}}$
  - If E is also constant over that surface:
  - $\Rightarrow \oint \vec{\mathbf{E}} \cdot \hat{\mathbf{n}} dA = |E|A$
  - *That* is easy to work with!

Gauss Plate Charge

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- Can we find the same through Gauss's law?

Nesting Charges

Say we have a situation where a 1 C charge sits at the origin. Surrounding it is a 4 cm thick neutral copper shell.

A. What is the electric field 2 cm from the charge?

B. What is the surface charge density on the interior of the copper shell?

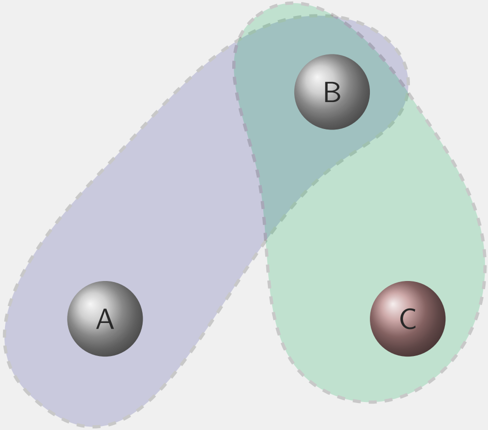
Suppose you have three charges, of which you know the charge of C is 2 C. You also have two Gaussian surfaces which you happen to know the total flux through. The blue surface has a total flux of $\frac{5}{\epsilon_0}$Vm passing through it, while green surface has a total flux of $-\frac{4}{\epsilon_0}$Vm passing through it. What is the charge of charge A?

A. -1 C

B. 3 C

C. 7 C

D. 11 C



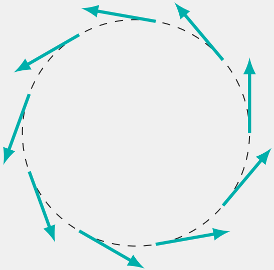
Magnetic Gauss's Law

- Gauss's law can relate the flux of any field to the amount of field causing "stuff" on the interior
- Can talk about magnetic flux just as easily as electric flux
- One huge difference though:
  - Magnetic poles *always* come in pairs
  - Impossible to separate a north or south
  - Will *always* have both a source and a sink of magnetic fields in your Gaussian surface
- Thus, for magnets, Gauss's law says:

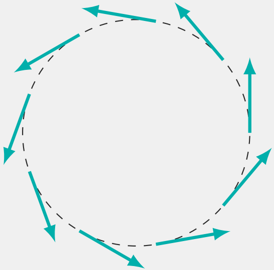
$$\oint \vec{\mathbf{B}} \cdot \hat{\mathbf{n}} dA = 0$$

A New Law of the Land

- How do we work with magnetic fields then?
  - Gauss's law doesn't tell us anything useful
  - Still want a method to predict the "source" of some particular magnetic field
- Sources for magnetic fields are moving charges or currents
  - We'll focus mostly on currents
- How to judge the direction and strength of the current that causes a particular magnetic field?



Enter Ampere's Law



- Right-hand-rule tells us that current must be coming out of the board here
- Can integrate around the loop:

$$\begin{aligned}\oint \vec{\mathbf{B}} \cdot d\vec{\ell} &= B \oint d\ell \\ &= \frac{\mu_0}{4\pi} \frac{2\mathrm{I}}{r} \cdot 2\pi r \\ &= \mu_0 \mathrm{I}\end{aligned}$$

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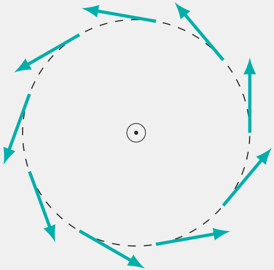

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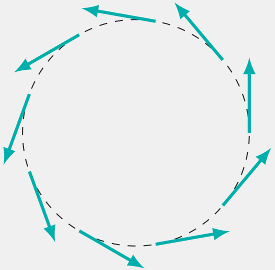

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Ampere's Law

Around any closed loop

$$\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 \mathrm{I}_{enc}$$

where positive or negative currents are determined by right hand rule.