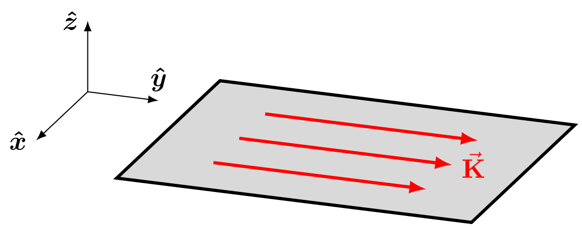
1. Consider an infinite thin sheet with uniform surface current density.

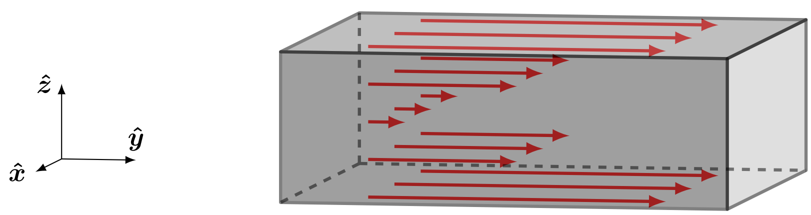


   (a) Use the Biot-Savart law to find $\vec{\mathbf{B}}(x, y, z)$ both above and below the sheet, by integration. This integral can be a bit nasty so feel free to use Sympy, and I'd recommend being very explicit in whether you are assuming you are above or below the sheet when defining your symbols.

   (b) Now solve the problem ($\vec{\mathbf{B}}$ above and below the sheet) using Ampere's Law. Be explicit in what Amperian loop(s) you are using and the direction you are integrating around them. (Yes, I know Griffiths solves this, but you better make sure you can do the same on your own.)

   (c) Now add a second parallel sheet at $z = +a$ with the current running the opposite direction. Use the superposition principle to find $\vec{\mathbf{B}}$ between the two sheets and also outside (above and below) both sheets.

2. Consider a thick *slab* of current:

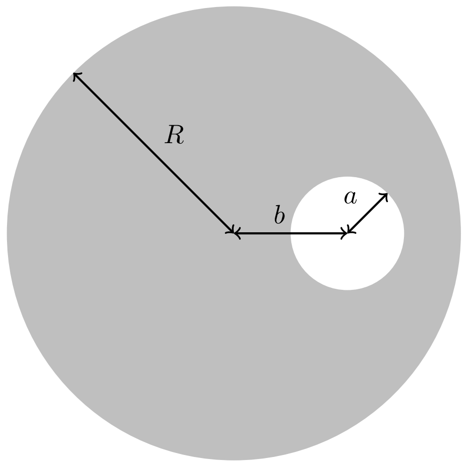


   The slab is infinite in both x and y, but finite in z. The slab has thickness $2h$ and runs from $z = -h$ to $z = h$. The current is flowing in the $+\hat{\boldsymbol{y}}$ direction and is given by

$$\vec{\mathbf{J}} = J_0 z^2 \hat{\boldsymbol{y}}$$

   Find $\vec{\mathbf{B}}$ everywhere in space (above, below, and within the slab).

3. Recall that in many cases a clever use of superposition can make a seemingly complicated problem much more simple. Take the instance where there exists a long (infinite) wire of radius R whose central axis corresponds to the z-axis. The wire carries a uniformly distributed current I_0 pointing in the $+\hat{\boldsymbol{z}}$ direction. A cylindrical hole is drilled out of the conducting wire, parallel to the z-axis. The center of the hole is located at $x = b$ and the radius of the hole is a.



   (a) Determine $\vec{\mathbf{B}}$ *in the hole region.* The fact that the hole is displaced from the center means that your origin for the likely two polar coordinates you'll have is not the same. It might be easiest to shift things in Cartesian. You should find that the magnetic field in the hole is uniform!
   (b) If this is an ordinary wire carrying ordinary household current, and the wire and hole have roughly the dimensions shown above, estimate the strength of the magnetic field in that region. How does it compare to Earth's magnetic field?
   (c) (3 points (bonus)) Make a plot showing the magnitude of the magnetic field *throughout the entire wire* (including hole). Overlay on top of this a quiver plot showing the magnetic field vectors throughout the wire.

4. On the last homework you investigated the movement of a charged particle in constant electric fields and then worked out the magnetic field in the region around a magnetic dipole. In this problem we'll combine the two ideas: taking the magnetic fields you found before to solve for the trajectory of a charged particle in the vicinity of a magnetic dipole.
   (a) Take the same magnetic field from the previous homework where the dipole pointing in the $\hat{\boldsymbol{z}}$ direction and had a magnitude of 10^4. Suppose an alpha particle (2 protons and 2 neutrons) enters the vicinity of the dipole at the point (0,-8,2) traveling at a speed of $100\,\mathrm{m/s}$ in the $+\hat{\boldsymbol{y}}$ direction. Calculate the trajectory of the alpha particle over the next second using a step size of $1 \times 10^{-6}\,\mathrm{s}$. Make two plots of the resulting trajectory:
      i. A 2d plot of the yz plane showing the same streamplot of magnetic fields lines as before with the trajectory of the alpha particle plotted on top
      ii. A 3d plot of just the alpha particle's path over the 1 second.
   (b) By combining two magnetic dipoles, we can created what is known as a magnetic bottle, which is import for many types of containment fields. Take two magnetic dipoles with the same magnitude as above, both pointing in the $+\hat{\boldsymbol{z}}$ direction. Place one at $z = 10$ and the other at $z = -10$. Make a streamplot of the resulting magnetic field in the yz plane.
   (c) Finally, introduce another alpha particle to the scene, starting it at the point (-5,0,0) with an initial velocity of $100\,\mathrm{m/s}$ in the $+\hat{\boldsymbol{z}}$ direction. Numerically calculate the trajectory of the particle over 1 second and create the same two plots as in Part (a). Does the particle seem fully contained?