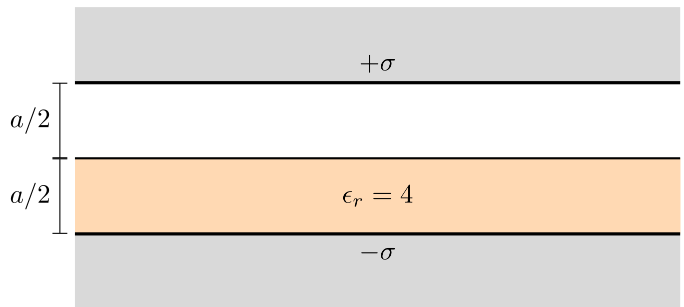
1. You have a large (infinite in the x and y directions) parallel-plate capacitor comprised of two big metal sheets. The upper one has a free charge density of $+\sigma$, and the lower one a free charge density of $-\sigma$. The space between the sheets is half filled with a linear dielectric with a dielectric constant of $\epsilon_r = 4$. The top region is just a vacuum.



   (a) Find $\vec{\mathbf{D}}$,$\vec{\mathbf{E}}$, and $\vec{\mathbf{P}}$ (direction and magnitude, giving all of them in both regions between the plates)

   (b) Find the amount and location of bound charge (surface or volume) everywhere

   (c) Using the bound charges above and any free charge, go back and compute $\vec{\mathbf{E}}$ in both regions directly using Gauss's Law. You should confirm your answers from earlier.

   (d) Assume the bottom plate is grounded and the top plate has a potential of 100 V. The two plates are separated by a total distance of 1 mm. Determine the potential everywhere between the two plates and plot it in Jupyter.

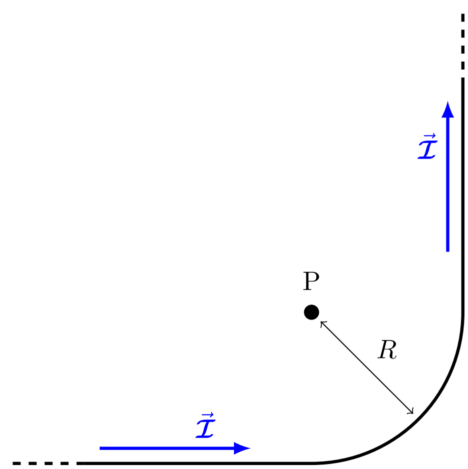
2. Griffiths works out in Example 5.2 the general solution to motion of a particle in "crossed E and B fields" ($\vec{\mathbf{E}}$ pointing in the $\hat{\boldsymbol{z}}$ direction, $\vec{\mathbf{B}}$ pointing in the $\hat{\boldsymbol{x}}$ direction). Make sure you can follow along with his work, as this problem will build off his results.

   (a) Suppose the particle starts at the origin at $t = 0$, with a given velocity of $\vec{\mathbf{v}}(t = 0) = v_0\hat{\boldsymbol{y}}$. Use Griffiths formal results (Equation 5.6) to find the special initial speed, v_0, which results in simple straight-line constant-speed motion. Verify this answer makes sense by utilizing Phys 222 style right-hand rule arguments and the Lorentz force law, $\vec{\boldsymbol{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}})$.

   (b) Now suppose your particle starts at the origin with $\vec{\mathbf{v}}(t = 0) = v_1\hat{\boldsymbol{y}}$, where v_1 is exactly **half** of the v_0 value you found above. Find and plot the resulting trajectory of the particle. Is the kinetic energy of the particle constant with time? Is this consistent with energy conservation?

3. We will be working with "current densities" for much of the rest of the term. Here are some simple examples to get your feet wet.
   (a) A solid cylindrical straight wire (radius a) has a current $\vec{\mathcal{I}}$ flowing down it. If that current is uniformly distributed *only* over the outer surface of the wire (none is flowing through the volume of the wire, just the surface), what is the magnitude of the surface current density, $\vec{\mathbf{K}}$.
   (b) Suppose that current does flow throughout the volume of the wire, but in such a way that the volume current density $\vec{\mathbf{J}}$ grows quadratically with distance from the central axis ($J = As^2$ where A is some constant). What is the formula for $\vec{\mathbf{J}}$ everywhere in the wire in terms of $\vec{\mathcal{I}}$?
   (c) A DVD has been rubbed so that it has a fixed, constant, uniform surface electric charge density σ everywhere on its top surface. It is spinning at angular velocity ω about its center (which is at the origin). What is the magnitude of surface current density $\vec{\mathbf{K}}$ at a distance r from the center?

4. An infinitely long wire has been bent into a right angle turn, as shown. The "curved part" where it bends is a perfect quarter circle, with radius R. Point P is exactly at the center of that quarter circle. A steady current $\vec{\mathcal{I}}$ flows through the wire.



Use direct integration of Biot-Savart to find $\vec{\mathbf{B}}$ at point P (Make sure to specify both magnitude and direction).

5. In this problem we want to model the movement of a charged particle in a magnetic field. For any magnetic field that isn't extremely simple, this can become rather complex, so we'll look to find solutions to the motion numerically. Here we'll start with a very simple case to make sure things are working correctly and then move to visualizing a more complex situation. I'm providing you with a bit of a notebook to get you jump started if you want help with part (a).

   (a) Take the case where a proton is moving in the x direction in the vicinity of a constant magnetic field.

$$\begin{aligned} m_p &= 1.67 \times 10^{-27}\,\text{kg} \\ q_p &= 1.6022 \times 10^{-19}\,\text{C} \\ \vec{\mathbf{B}} &= (1 \times 10^{-4}\,\text{T})\hat{\boldsymbol{y}} \\ \vec{\mathbf{v}}_0 &= (1 \times 10^{3}\,\text{m/s})\hat{\boldsymbol{x}} \end{aligned}$$

   Use Euler's method to determine the position of the particle over the first 0.005 s as it travels. Plot the results on the 3D plot.

   (b) Repeat part (a) but this time let the proton also be moving slightly in the $\hat{\boldsymbol{y}}$ direction.

$$\vec{\mathbf{v}}_0 = (1 \times 10^{3}\,\text{m/s})(\hat{\boldsymbol{x}} + \hat{\boldsymbol{y}})$$

   (c) Both of these were boring constant magnetic fields, with behavior you could have determined analytically. Suppose instead we take the case where a charged proton is in the vicinity of a bar magnet (a magnetic dipole). You'll numerically solve for this motion in the future, but for now we want to just focus on calculating and visualizing the magnetic field around the bar magnet.

   The magnetic field due to a dipole is given by Griffiths in Equation 5.89. You'll need to translate this into a python function which can return to you the x, y and z components of the magnetic field at any point in space. Recall that such functions as `np.dot` exist and that you can represent vectors as 1d arrays.

      i. Take the case where the dipole moment points in the $\hat{\boldsymbol{z}}$ direction and has a magnitude of $1 \times 10^4\,\text{Am}^2$. Make a **streamplot** of the yz cross section of the magnetic field in the vicinity of this dipole. For a huge dipole like this one you can let y and z vary from say -10 to 10 meters.

      ii. Now take the case where the dipole is tilted 30° from the z axis towards the positive y axis (same magnitude). Again, make a streamplot of the yz cross section of the magnetic field in the vicinity of this dipole. Does it look like you'd expect?

   Don't lose this code, as we'll make use of it in the future to find the trajectory of a charged particle near this dipole.