1. In electromagnetism, developing a deeper understanding of vector calculus can assist in developing a deeper understanding of the physical systems. Sometimes basic proofs can pave the way for us to simplify problems, and knowing the basics behind those proofs can be useful. Here we'll look at a few very basic proofs for developing an intuition about gradients, curls and divergences.

   (a) One of the things implied by the gradient theorem is that any closed loop integral of a gradient must be equal to 0, $\oint \boldsymbol{\nabla} T \cdot d\vec{\boldsymbol{\ell}} = 0$. Convince yourself of this if you can't see how. Use this, along with Stokes theorem, $\int_S (\boldsymbol{\nabla} \times \vec{\mathbf{v}}) \cdot d\vec{\mathbf{A}} = \oint_C \vec{\mathbf{v}} \cdot d\vec{\boldsymbol{\ell}}$, to show that

   $$\boldsymbol{\nabla} \times \boldsymbol{\nabla} T(x, y, z) = 0$$

   What essential argument must be made to ensure this result generalizes and applies to any situation?

   (b) What does your result from (a) tell you about the swirly-ness of the gradient of a temperature field over any surface?

   (c) Similarly, over a closed surface Stokes theorem tells us that $\oint_S (\boldsymbol{\nabla} \times \vec{\mathbf{v}}) \cdot d\vec{\mathbf{A}} = 0$. Again, make sure you understand this if it isn't clear. Now use this result in conjunction with the divergence theorem, $\int_V (\boldsymbol{\nabla} \cdot \vec{\mathbf{v}}) dV = \oint_S \vec{\mathbf{v}} \cdot d\vec{\mathbf{A}}$, to show that

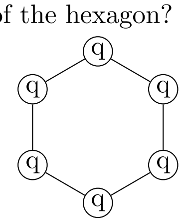
   $$\boldsymbol{\nabla} \cdot (\boldsymbol{\nabla} \times \vec{\mathbf{v}}) = 0$$

   Again, what is the essential argument that must be made to ensure this generalizes and is applicable in all instances?

   (d) Did you find these useful for better understanding the relationships between vector operations and the various fundamental theorems? It's ok if the answer is no, I'm just gathering some feedback!

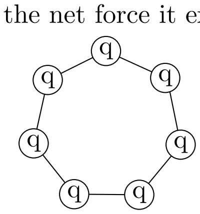
2. The concept of superposition is critically important to the study of electrodynamics, and will be hugely useful to us now when studying electrostatics. For this problem, before working through the mathematical details, think about how superposition can help you in this instance.

   (a) Let's place 6 equal charges q at the vertices of a hexagon with edge length ℓ. What is the net force on a test charge Q placed at the center of the hexagon?

   (b) Say now we remove the top charge, leaving 5 charges at the other vertices. What is the net force on the charge now? Explain your reasoning and back it up with math.

   (c) Now let's place 7 equal charges q at the vertices of a heptagon (still with edge length ℓ). If we place our test charge Q in the center, what is the net force it experiences?

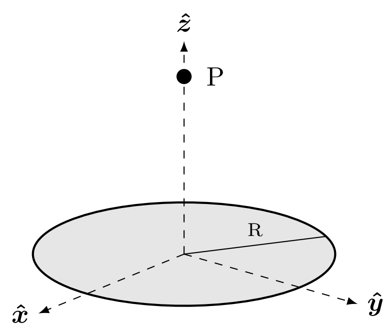
   (d) Again, lets remove the top charge, leaving the other 6 where they were at. What is the net force on our test charge now? Explain yourself and back it up with the math.

   (e) How is your reasoning in parts b and d similar?

3. Physics is both a mathematical and visual science. It is thus important that you can sketch and plot figures of various types. Early in this class, understanding and plotting electric fields will be crucial. Plot the below fields over a reasonable span. *For each case, try to describe a physical situation where that field would be applicable.*
   (a) $\vec{\mathbf{v}}(x,y) = \vec{\mathbf{r}}$ where $\vec{\mathbf{r}}$ is the normal $\vec{\mathbf{r}}$ in polar coordinates
   (b) $\vec{\mathbf{v}}(x,y) = \frac{x}{\left(\sqrt{x^2+y^2}\right)^3}\hat{\boldsymbol{x}} + \frac{y}{\left(\sqrt{x^2+y^2}\right)^3}\hat{\boldsymbol{y}}$
   (c) $\vec{\mathbf{v}}(x,y) = \hat{\boldsymbol{\phi}}$ where $\hat{\boldsymbol{\phi}}$ is the polar coordinate

4. Often times in this class you'll produce some new formula describing a situation which you might not have any existing intuition for. So you should always conclude by asking yourself: *Does this make sense given the physics I know?* Here we'll practice some ways of checking yourself.

   Consider a thin disk of radius R with a uniform charge density $-\sigma$.



   (a) Find the electric field at point P, which is a distance z above the center of the disk, by integrating across the disk surface. (Yes, this particular field is well known but it is good practice.)
   (b) If you were very far from the disk, what would you expect the field to look like using intuition from Intro Physics? Explicitly check that the limiting form of your solution at very large z. (By limiting form here we don't just mean "it goes to zero". The question is *how* does the function behave at these limits? Like $\frac{1}{z}$? Like e^{-z}?)
   (c) If you were very close to the disk, what might you expect the field to look like (again, using your knowledge from Intro Phys)? Explicitly check the limiting form of the solution at very small z.
   (d) Create a plot using Matplotlib showing your solution, your small z approximation and your large z approximation. Be sure your range includes both positive and negative z values!

5. When working through some physics problems, you will typically find yourself in a situation where a strict analytical solution to your problem proves impossible either due to transcendental equations, non-integrable forms, or other problematic situations. In these situations, it can be useful to pause a moment and consider under what conditions you are really interested in solving your problem. Sometime these conditions can provide you with limitations or simplifications that can let you get very close to your original goal. Here I'll tell you what simplification you might want to make, but in the future, always keep in mind and ask yourself: *What assumptions or approximations can I make here and why?*

   Here we have two charges of identical mass m, one with a charge of q and one with a charge of $5q$. They both hang from strings of length ℓ from a common point. Here you can assume that the electric force on each mass is small compared to the gravitational force on each.

   (a) Find an approximate expression for the angle θ that each charge makes with respect to the vertical.

   (b) Describe how the above assumption played out in your calculations. What did it simplify and why was that important?

   (c) Show that the units for your solution do indeed work out.

   (d) Show that the limiting behavior for large masses, large string lengths, and small charges are all physically reasonable and what you'd expect.