



# Announcements

- Video HW2 due tonight!
- New WebWork 4 released and due Wednesday at midnight
- Today is the last day to add / drop if you were debating that
- First test on Friday already!
  - I'm posting study materials to the website
  - Just over chapters 1 and 2
  - Study materials include copy of old test and solutions
  - You get a 3x5 inch index card for notes (handwritten) on test day
  - A scientific calculator may be useful on test day, though I have some I can loan.  
(Graphing calcs are fine.)
- Teddy hours:
  - Fridays from 6:30 – 7:30 pm in Hearth
  - Otherwise contact to make other 1-on-1 meeting times
- Polling: `rembold-class.ddns.net`



## Warm Up

What is the correct ordering of the 3 step process to use iterative methods to predict future motion?

A

1. Update  $\vec{p}$  from momentum principle
2. Update net force
3. Update position using  $\vec{v}_{avg}$  approximation

B

1. Update net force
2. Update position using  $\vec{v}_{avg}$  approximation
3. Update  $\vec{p}$  from momentum principle

C

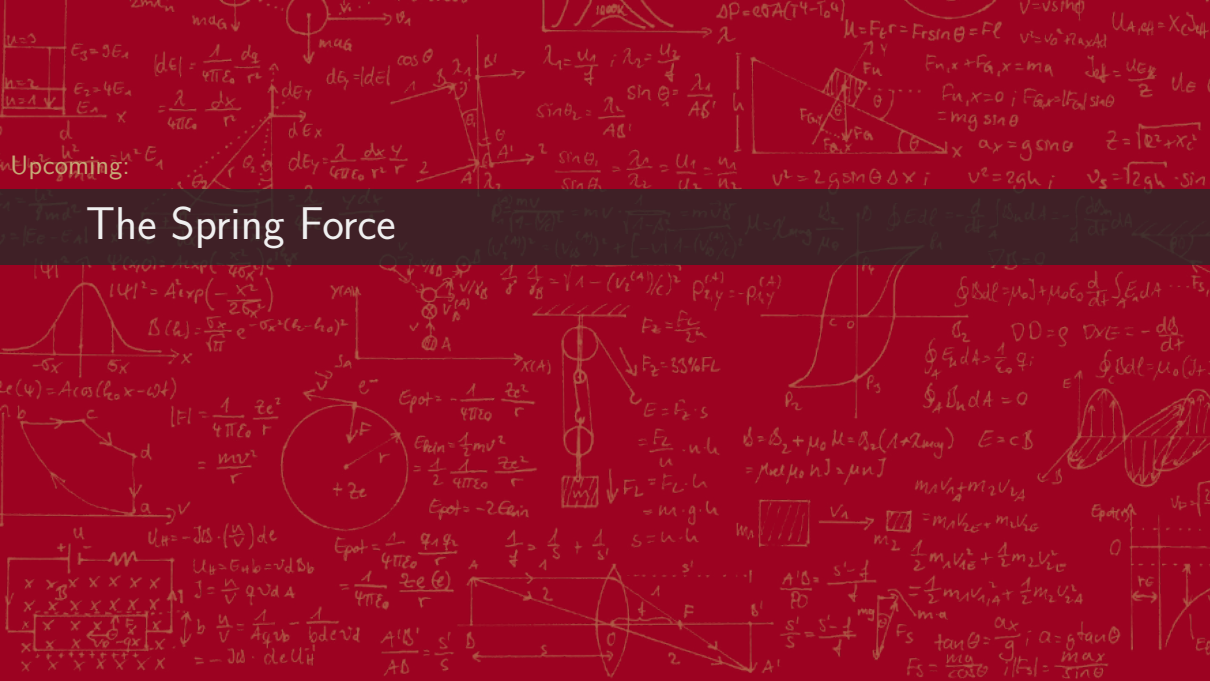
1. Update net force
2. Update  $\vec{p}$  from momentum principle
3. Update position using  $\vec{v}_{avg}$  approximation

D

1. Update position using  $\vec{v}_{avg}$  approximation
2. Update net force
3. Update  $\vec{p}$  from momentum principle

Upcoming:

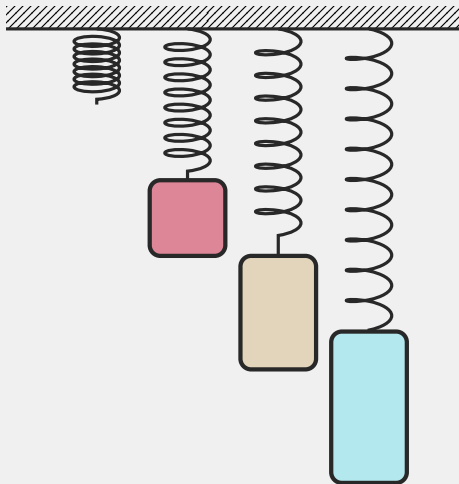
# The Spring Force





# Spring Experimentation

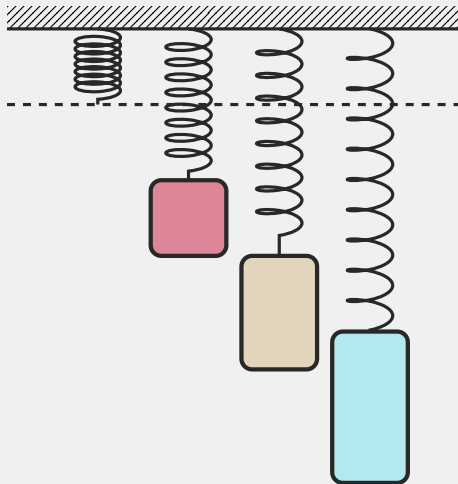
- Springs have a certain “resting” length
- Adding mass or a force stretches or compresses them from that length
- The strength of the force the spring applies in response depends on the distance it is stretched or compressed
- Different springs stretch different amounts





# Spring Experimentation

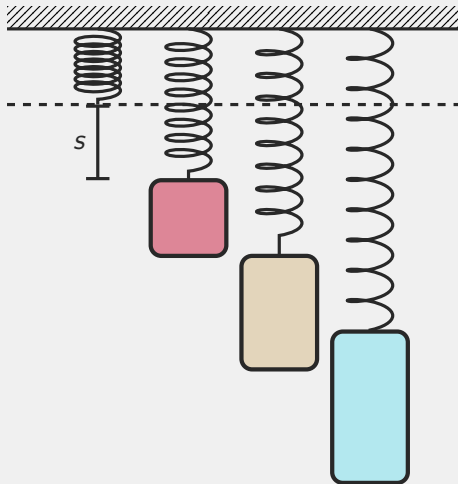
- Springs have a certain “resting” length
- Adding mass or a force stretches or compresses them from that length
- The strength of the force the spring applies in response depends on the distance it is stretched or compressed
- Different springs stretch different amounts





# Spring Experimentation

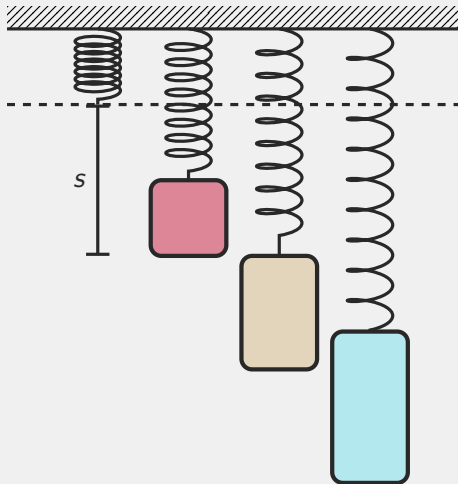
- Springs have a certain “resting” length
- Adding mass or a force stretches or compresses them from that length
- The strength of the force the spring applies in response depends on the distance it is stretched or compressed
- Different springs stretch different amounts





# Spring Experimentation

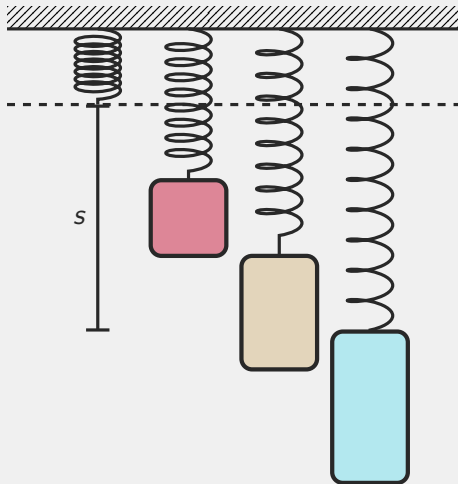
- Springs have a certain “resting” length
- Adding mass or a force stretches or compresses them from that length
- The strength of the force the spring applies in response depends on the distance it is stretched or compressed
- Different springs stretch different amounts





# Spring Experimentation

- Springs have a certain “resting” length
- Adding mass or a force stretches or compresses them from that length
- The strength of the force the spring applies in response depends on the distance it is stretched or compressed
- Different springs stretch different amounts

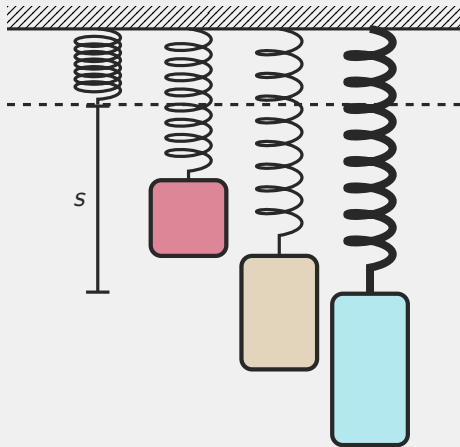






# Spring Experimentation

- Springs have a certain “resting” length
- Adding mass or a force stretches or compresses them from that length
- The strength of the force the spring applies in response depends on the distance it is stretched or compressed
- Different springs stretch different amounts





# Strong Spring, Weak Spring

- Stiffer springs result in a greater force for less displacement
- Combining both spring stiffness and displacement, we arrive at

$$|\vec{\mathbf{F}}_{spring}| = k_s |s|$$

where

$$s = L - L_0$$

- $L$  is the current spring length whereas  $L_0$  is the spring length when relaxed



## Simple Example

We'll hang a weight on the spring in the front of the room. By measuring the spring's displacement, we want to determine the spring constant of the spring.



# Simple Example

We'll hang a weight on the spring in the front of the room. By measuring the spring's displacement, we want to determine the spring constant of the spring.

- System: The mass/hanger



# Simple Example

We'll hang a weight on the spring in the front of the room. By measuring the spring's displacement, we want to determine the spring constant of the spring.

- System: The mass/hanger
- Surroundings: The air, Earth, and spring



# Simple Example

We'll hang a weight on the spring in the front of the room. By measuring the spring's displacement, we want to determine the spring constant of the spring.

- System: The mass/hanger
- Surroundings: The air, Earth, and spring
- $\Delta \vec{p} = 0 \quad \Rightarrow \quad \vec{F}_{net} = 0$



# Simple Example

We'll hang a weight on the spring in the front of the room. By measuring the spring's displacement, we want to determine the spring constant of the spring.

- System: The mass/hanger
- Surroundings: The air, Earth, and spring
- $\Delta \vec{p} = 0 \quad \Rightarrow \quad \vec{F}_{net} = 0$
- $\Rightarrow \left| \vec{F}_{spring} \right| = \left| \vec{F}_{gravity} \right|$



# Simple Example

We'll hang a weight on the spring in the front of the room. By measuring the spring's displacement, we want to determine the spring constant of the spring.

- System: The mass/hanger
- Surroundings: The air, Earth, and spring
- $\Delta \vec{p} = 0 \Rightarrow \vec{F}_{net} = 0$
- $\Rightarrow |\vec{F}_{spring}| = |\vec{F}_{gravity}|$
- $k_s = \frac{mg}{s}$





# Vectorizing Springs

- Want to be able to understand springs in all three dimensions



# Vectorizing Springs

- Want to be able to understand springs in all three dimensions
- Define the length vector to point from the fixed end of the spring to the free end.

$$\vec{\mathbf{L}} = |\vec{\mathbf{L}}| \hat{\mathbf{L}}$$



# Vectorizing Springs

- Want to be able to understand springs in all three dimensions
- Define the length vector to point from the fixed end of the spring to the free end.

$$\vec{\mathbf{L}} = |\vec{\mathbf{L}}| \hat{\mathbf{L}}$$

- The spring stretch is still just a scalar:

$$s = |\vec{\mathbf{L}}| - L_0$$



# Vectorizing Springs

- Want to be able to understand springs in all three dimensions
- Define the length vector to point from the fixed end of the spring to the free end.

$$\vec{\mathbf{L}} = |\vec{\mathbf{L}}| \hat{\mathbf{L}}$$

- The spring stretch is still just a scalar:

$$s = |\vec{\mathbf{L}}| - L_0$$

- $s$  being positive means a stretched spring, force should be toward fixed end, or in  $-\hat{\mathbf{L}}$  direction



# Vectorizing Springs

- Want to be able to understand springs in all three dimensions
- Define the length vector to point from the fixed end of the spring to the free end.

$$\vec{\mathbf{L}} = |\vec{\mathbf{L}}| \hat{\mathbf{L}}$$

- The spring stretch is still just a scalar:

$$s = |\vec{\mathbf{L}}| - L_0$$

- $s$  being positive means a stretched spring, force should be toward fixed end, or in  $-\hat{\mathbf{L}}$  direction
- $s$  being negative means compressed spring, force should be away from fixed end, or in  $\hat{\mathbf{L}}$  direction



# Vectorizing Springs

- Want to be able to understand springs in all three dimensions
- Define the length vector to point from the fixed end of the spring to the free end.

$$\vec{\mathbf{L}} = |\vec{\mathbf{L}}| \hat{\mathbf{L}}$$

- The spring stretch is still just a scalar:

$$s = |\vec{\mathbf{L}}| - L_0$$

- $s$  being positive means a stretched spring, force should be toward fixed end, or in  $-\hat{\mathbf{L}}$  direction
- $s$  being negative means compressed spring, force should be away from fixed end, or in  $\hat{\mathbf{L}}$  direction

## Spring Force

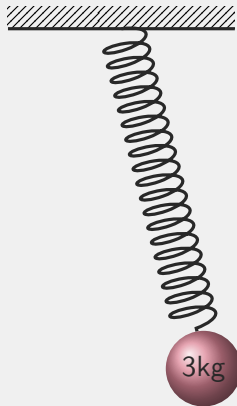
$$\vec{\mathbf{F}}_{spring} = -k_s s \hat{\mathbf{L}}$$



## Understanding Check

The system to the right has a 3 kg mass attached to the end of a spring with spring constant 10 N/m. The fixed end of the spring is located at  $\vec{r} = \langle 0, 0, 0 \rangle$  m while the mass end of the spring is as  $\vec{r} = \langle 1, -2, 2 \rangle$  m. If the spring has a relaxed length of 1 m, what is the spring force currently acting on the mass?

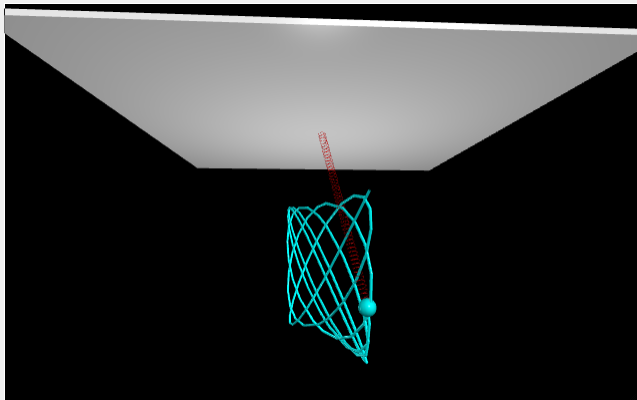
- A)  $\frac{1}{3} \langle 20, -40, 40 \rangle$  N
- B)  $\langle 20, -40, 40 \rangle$  N
- C)  $\langle 6.6, 13, -13 \rangle$  N
- D)  $\frac{1}{3} \langle -20, 40, -40 \rangle$  N





# Iterating!

This new step in making sure to calculate the new force is really all that you need to add to your iteration method!





# Fundamental Interactions



# The Four Horsemen

- All known forces in the universe can be boiled down to about 4 fundamental forces
  - Gravitational
  - Electromagnetic
  - Weak Force
  - Strong Force
- In increasing order of strength
- The last 3 have been combined to form the Standard Model
- Often times we'll look at macro effects from these forces, but they provide a good starting location

$$E_3 = 3E_1$$

$$E_2 = 4E_1$$

$$E_1$$

$$d$$

$$x$$

$$h^2 = u^2 + v^2$$

$$u = 3$$

$$h = 2$$

$$h = A \sqrt{v}$$

$$|d\mathbf{E}| = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2}$$

$$= \frac{\lambda}{4\pi\epsilon_0} \frac{dx}{r^2}$$

$$dE_y = \frac{\lambda}{4\pi\epsilon_0} \frac{dx}{r^2} \frac{y}{r}$$

$$= \frac{\lambda}{4\pi\epsilon_0} \frac{y dx}{r^3}$$

$$d\mathbf{E} = |d\mathbf{E}| \cos\theta$$

$$dE_y = |d\mathbf{E}| \cos\theta$$

$$dE_x = |d\mathbf{E}| \sin\theta$$

$$dE_y = \frac{\lambda}{4\pi\epsilon_0} \frac{y dx}{r^3}$$

$$dE_x = \frac{\lambda}{4\pi\epsilon_0} \frac{x dx}{r^3}$$

$$\lambda_1 = \frac{u_1}{f}$$

$$\lambda_2 = \frac{u_2}{f}$$

$$\sin\theta_2 = \frac{\lambda_1}{\lambda_2}$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\lambda_1}{\lambda_2} = \frac{u_1}{u_2} = \frac{v_1}{v_2}$$

$$\Delta P = e\sigma A(T_1 - T_2)$$

$$U = F_e r = F_r \sin\theta = F_L$$

$$v = v_0 \sin\theta$$

$$F_n \cdot x + F_g \cdot x = m a$$

$$F_n \cdot x = 0; F_g \cdot x = F_g \sin\theta$$

$$= m g \sin\theta$$

$$a_x = g \sin\theta$$

$$z = \sqrt{r^2 + x^2}$$

$$v^2 = 2 g \sin\theta \Delta x$$

$$v^2 = 2 g h$$

$$v_s = \sqrt{2 g h} \cdot \sin\theta$$

$$U_{A, \text{eff}} = X \cos\theta$$

$$U_E = \frac{U_g}{2}$$

# Gravity

$$|U|^2 = A^2 \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

$$B(x) = \frac{\sqrt{x}}{\sqrt{\pi}} e^{-\sigma^2(x - x_0)^2}$$

$$U(\psi) = A \cos(k_0 x - \omega t)$$

$$|F| = \frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$$

$$= \frac{mv^2}{r}$$

$$U_H = -\int \mathbf{B} \cdot \left(\frac{d\mathbf{U}}{d\mathbf{U}}\right) d\mathbf{U}$$

$$U_H = E_H b = v d B b$$

$$J = \frac{n}{V} q v d A$$

$$b \frac{U}{V} = \frac{1}{A q v b} \int b d v d$$

$$= -\int \mathbf{B} \cdot d\mathbf{U}_H$$

$$E_{\text{pot}} = -\frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$$

$$E_{\text{kin}} = \frac{1}{2} m v^2$$

$$= \frac{1}{2} \frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$$

$$E_{\text{pot}} = -2 E_{\text{kin}}$$

$$E_{\text{pot}} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$$

$$\frac{A'B'}{AB} = \frac{s'}{s}$$

$$F_2 = \frac{F_L}{2n}$$

$$E = F_2 \cdot s$$

$$= \frac{F_L}{n} \cdot u \cdot h$$

$$F_L = F_L \cdot h$$

$$= m \cdot g \cdot h$$

$$s = u \cdot h$$

$$b = b_2 + \mu_0 U = b_2(1 + \lambda_{\text{mag}})$$

$$E = c B$$

$$= \mu_0 c^2 h J = \mu_0 J$$

$$m_1 v_{1A} + m_2 v_{1B}$$

$$= m_1 v_{2C} + m_2 v_{2D}$$

$$\frac{1}{2} m_1 v_{1A}^2 + \frac{1}{2} m_2 v_{1B}^2$$

$$= \frac{1}{2} m_1 v_{2C}^2 + \frac{1}{2} m_2 v_{2D}^2$$

$$m a$$

$$\tan\theta = \frac{a_x}{g}; a = g \tan\theta$$

$$F_s = \frac{m g}{\cos\theta}; |F_s| = \frac{m g}{\sin\theta}$$

$$E_{\text{pot}} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$$

$$E_{\text{pot}} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$$



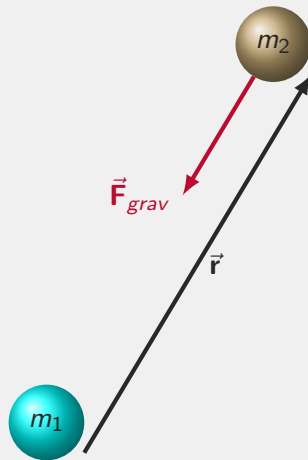
- We've seen that near the surface of the Earth the force of gravity can be approximated as

$$\vec{\mathbf{F}}_{gravity} \approx m\vec{\mathbf{g}}$$

where  $\vec{\mathbf{g}} = \langle 0, -9.8, 0 \rangle \text{ N/kg}$

- More accurately, the force of gravity is defined as

$$\vec{\mathbf{F}}_{\text{grav on 2 by 1}} = -G \frac{m_1 m_2}{|\vec{\mathbf{r}}|^2} \hat{\mathbf{r}}$$





# Break it Down

$$\vec{F}_{\text{grav on 2 by 1}} = -G \frac{m_1 m_2}{|\vec{r}|^2} \hat{r}$$

- $m_1$  and  $m_2$  are the masses of the two objects
- $|\vec{r}|$  is the distance between the objects
- $\hat{r}$  is a direction pointing from the mass to the object it is interacting with
  - Points *to* the object of interest
- $G$  is a measure of the strength of gravity

$$G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$$



## Dragging ya down

Let's calculate the force of the Earth on me, a 75 kg individual. For reference, the Earth has a mass of  $5.972 \times 10^{24}$  kg and an average radius of 6371 km. Compare this force to our earlier approximation.



## Practice Time!

Suppose an astronaut aboard the International Space Station is playing with a new spring. Curious about the spring constant, the astronaut suspends a 500 g mass from the end of the spring, causing the spring to stretch 5 cm from its relaxed length and then remain motionless. The ISS orbits about 400 km above the surface of the Earth. The Earth has a mass of  $5.972 \times 10^{24}$  kg and an average radius of 6371 km.

- What is the spring constant?
- Suppose the astronaut stretched the spring 1 cm further and then let it go from rest. Where is the mass located 0.1 seconds later?