Announcements

- Homework 11 posted and due on Monday
- Be reading/thinking/working on your final chapter projects
  - Due to me by 7am Monday morning
  - Failure to have them to me by this point will mean you forsake the peer evaluation part of the grade
- I'm prioritizing getting solutions posted, and then I'll return to grading. That gives you a chance to check what you've turned in even if I haven't gotten it graded yet.
- Responses: `rembold-class.ddns.net`

Today's Objectives

- Making sure we are comfortable with the ideas of finding normal modes and frequencies
- Using normal modes and frequencies to construct general solutions
- Finding normal modes and frequencies of non-linear systems

Q1

The normal modes and frequencies can be combined to form the general solution. How many arbitrary constants should you expect in your general solution that you would have to solve for with initial conditions?

A) 0, we already accounted for those in the normal modes

B) 2

C) 3

D) 4

Q2

The system to the right consists of a mass free to slide horizontally which is attached to the wall via a spring. Attached to that mass is a pendulum. The corresponding Lagrangian is given by:

$$\mathcal{L} = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\left(\dot{x}^2 + \ell^2\dot{\phi}^2 + 2\dot{x}\dot{\phi}\ell\cos(\phi)\right) - \frac{1}{2}kx^2 + mg\ell\cos(\phi)$$

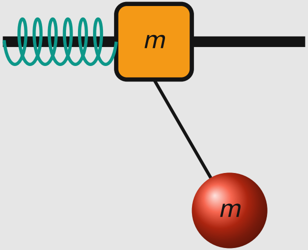
What expression best describes an expression for $\ddot{\phi}$?

A) $m\ell^2\ddot{\phi} = mg\ell\sin(\phi) - m\cos(\phi)\ddot{x}$

B) $m\ell^2\ddot{\phi} = -mg\ell\sin(\phi) - m\ell\cos(\phi)\ddot{x}$

C) $m\ell^2\ddot{\phi} = -mg\ell\sin(\phi) + m\ell\cos(\phi)\ddot{x}$

D) $m\ell^2\ddot{\phi} = mg\ell\cos(\phi) + m\ell\sin(\phi)\ddot{x}$



Q3

Given the same system, assuming that both masses are near equilibrium, what could the $\vec{\mathbf{M}}$ matrix look like?

$$\mathcal{L} = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\left(\dot{x}^2 + \ell^2\dot{\phi}^2 + 2\dot{x}\dot{\phi}\ell\cos(\phi)\right) - \frac{1}{2}kx^2 + mg\ell\cos(\phi)$$

Assume that you are setting up the derivatives as the vector: $\begin{bmatrix} \ddot{x} \\ \ddot{\phi} \end{bmatrix}$

A) $\begin{bmatrix} 2k & m\ell \\ m\ell & m\ell^2 \end{bmatrix}$

B) $\begin{bmatrix} 2m & -m\ell \\ -m\ell & m\ell^2 \end{bmatrix}$

C) $\begin{bmatrix} 2m & m\ell \\ m\ell & m\ell^2 \end{bmatrix}$

D) $\begin{bmatrix} 2k\ell & m\ell \\ m\ell & m^2 \end{bmatrix}$

Q4

Which of the following would thus be one of the normal frequencies for this system if $m = \ell = 1$ and $k = 10$?

A) 2.58 rad/s

B) 25.82 rad/s

C) 3.80 rad/s

D) 5.08 rad/s