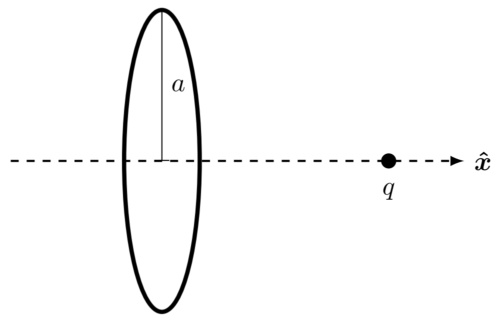
1. While we will spend a large amount of time working with source distributions and the fields they create, we are usually ultimately interested in the effect those fields will have on the motion of other charges. Here we will look at a ring of charge to develop an approximate solution for the motion of a test charge near the ring. You may have to dust off a little knowledge of differential equations or mechanics for this problem!

   Consider a thin ring at the origin with positive charge Q and radius a which has its center axis pointing along the $\hat{\boldsymbol{x}}$ direction.

   A charged ring with these parameters will produce an electric field along the axis of:

   $$E_x = \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2 + a^2)^{3/2}}$$

   (a) Write down the differential equation that describes the motion of a particle with negative charge $(-q)$ and mass m that is carefully positioned on the x-axis. (*Note: The particle has sign of the charge opposite to the ring, so q is the magnitude of the charged particle.*)

   (b) What kind of motion would you expect for a charge in this position? Why? Does this differential equation describe that sort of motion? (*Hint: Can this differential equation even be solved in a closed form?*)

   (c) Consider the situation where the particle is very close to the large ring ($x \ll a$). Determine the approximate form of the differential equation in this case, keeping only terms that depend linearly on x. (*Doing this is called linearizing the differential equation and will get us a solvable equation.*)

   (d) Solve the differential equation for the case where the particle starts from rest at a distance x_0 from the ring.

   (e) To get the behavior of the test particle at further distances from the ring, we need to resort to numerical methods. The notebook here will get you started with the basics of finding the motion of a particle numerically if you haven't seen it before. Numerically calculate the motion of the particle over 0.2 seconds when $x_0 = 0.1\,\text{m}$ and when $x_0 = 0.5\,\text{m}$. Make plots comparing these numeric solutions with your approximate solution from part (d) (1 plot for each initial condition). For your other parameters, you can use:

   $$Q = 1 \times 10^{-3}\,\text{C}$$
   $$q = -1 \times 10^{-6}\,\text{C}$$
   $$a = 1\,\text{m}$$
   $$m = 1 \times 10^{-3}\,\text{kg}$$

2. Much of this class will focus on learning alternative methods of finding electric fields. However, sometimes you just need to do the direct integration. This problem is one which could be solved using Gauss's Law (which is **much** easier), but it is excellent practice in how to setup a more complicated direct integration problem. The integration here can be tricky, so feel free to look up the form in a table or use Sympy to help you out, just show (in the case of Sympy) or cite (in the case of an integration table) your work.
   (a) Find the electric field a distance z above the center of a spherical shell of radius R which carries a uniform charge density σ. Do this by direct integration, and just treat the case when $z > R$ (so outside the sphere). Be careful when evaluating your square roots, $\sqrt{R^2 - 2Rz + z^2} = (z - R)$ when $z > R$. Express your answer in terms of the total charge on the sphere q.
   (b) Check your answer using a units check and your knowledge from Intro Physics. Briefly discuss what the solution should look like outside the sphere.

3. Spherical charge distributions tend to be special in how they let us approach them and in the general simplicity of their solutions. Here consider a sphere of radius R, centered at the origin, with a radially symmetric charge distribution $\rho(r)$.
   (a) What $\rho(r)$ is required for the electric field *inside the sphere* to have the form $E(r) = cr^n$ where c and n are constants?
      i. The case of $n = -2$ is special. How so?
      ii. Some values of n are physically unrealistic because they would lead to an infinite amount of charge on the sphere. What values of n are allowed?
   (b) What $\rho(r)$ is required for the radial electric field inside the sphere to have a constant magnitude? Is this distribution physically realizable? Why or why not?
   (c) For each realizable (or physical) charge distribution, what does the electric field look like outside the sphere?

4. The Dirac Delta Function is an important tool when describing models utilizing point distributions (eg. point charges or point masses). It can also be used when talking about tightly constrained distributions (eg. charge on a ring or thin shell).

   The linear charge density for a series of charges on the x-axis is given by:

$$\lambda(x) = \sum_{n=0}^{10} q_0 n^2 \delta\left(x - \frac{n}{10}\right)$$

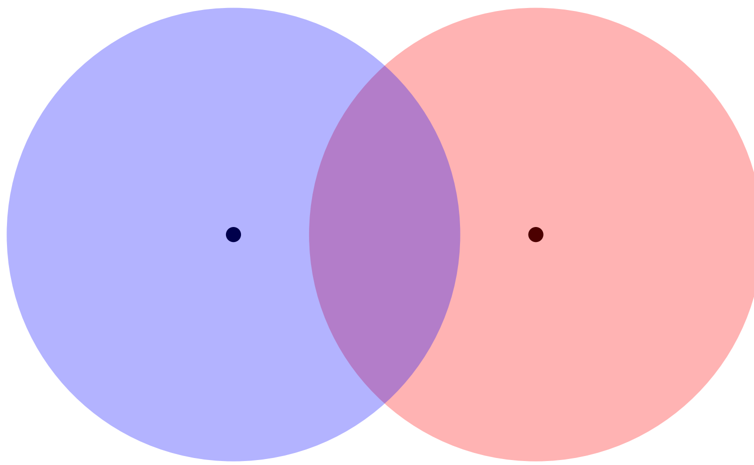
   (a) Write a sentence or two describing the units of each term in the equation (including the delta function!)
   (b) What is the *total* charge on the x-axis?
   (c) Sketch a diagram showing the location and relative size (charge) of the point charges comprising this distribution.

5. Sometimes when solving E&M problems it can help to consider an arbitrary location. Consider how choosing an arbitrary point in the overlapping region (part c) might help you in this problem.

   (a) For a cloud of charge (radius, R) with a uniform charge density (ρ_0), determine the electric field inside and outside the cloud.

   (b) Graph the electric field as a function of distance from the center of the cloud in Jupyter. Choose nice values of ρ_0 and R for your plot.

   (c) Now consider two oppositely charged clouds, both with radii R and uniform charge densities. They overlap as shown with their centers separated by distance d. Find the electric field in the overlapping region. (*Hint: How might Gauss's Law and superposition help you here?*)



   (d) Sketch (by hand, unless you really want to use Python) the approximate electric fields throughout both charges, including in the overlapping region.

   (e) In the limit that d becomes very small compared to R, discuss in words and sketch of what the resulting total charge distribution in space looks like.