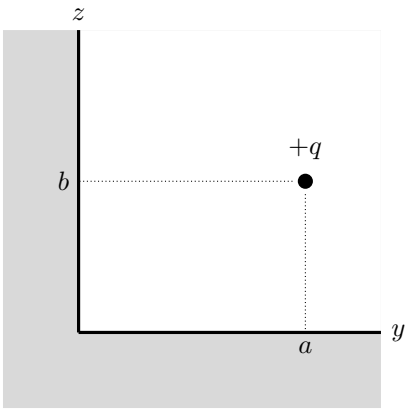
1. Suppose you have a one dimensional problem where no charges are present in the region and the potential is known at two points:

$$V(x = -1) = 3\,\mathrm{V}$$
$$V(x = 1) = 8\,\mathrm{V}$$

   Solve the Laplace equation to determine V at all points between -1 and 1. Show all your work and confirm that your solution both satisfies the Laplace equation and has the correct boundary conditions.

2. Suppose you have a situation with a small charge of $+q$ located at $(0, a, b)$ meters. Everywhere that $y < 0$ or $z < 0$ is an infinite conductor which is grounded such that $V = 0$.



   (a) Determine an expression for the potential at any point in space. Verify that your potential is 0 on the xy and xz plane.

   (b) Find the surface charge distribution on the conductor lying in the xy plane. This should be a function of x and y.

   (c) Taking the parameters of

$$q = 1\,\mu\mathrm{C}$$
$$a = 3\,\mathrm{m}$$
$$b = 2\,\mathrm{m}$$

      i. Make a contour plot of the potential in all regions where it is not equal to 0. You can just show a single slice at $x = 0$.

      ii. Plot a heatmap of the surface charge density on the xy plane. Be sure to let x extend both negative and positive here.

3. In this problem you'll explore computationally solving Laplace's equation using the Method of Relaxation. We'll start here in just one dimension, using the exact same parameters as in Problem 1. As such, you should know exactly what you should get, and can thus check the validity of your solver. I'll leave much of the details of the code up to you, but I'll go ahead and provide you some pseudocode to guide you along the general process:

   - Cut up the space of interest into equal size bins of some size a.
   - Set the boundary conditions at the edge points
   - Assign some "reasonable" guess to all the values between the endpoints
   - Set yourself a goal for the desired accuracy and a max number of iterations (in case you don't reach your goal you don't want your program running forever)
   - For each iteration:
     1. For each value **not** on the boundary set it equal to the average of the two neighboring points
     2. Compute the error (for instance the difference between the previous iterations points and this iterations points)
     3. Compare the error to your goal to see if you need to keep iterating
   - Plot the results and compare to your actual solution from Problem 1.

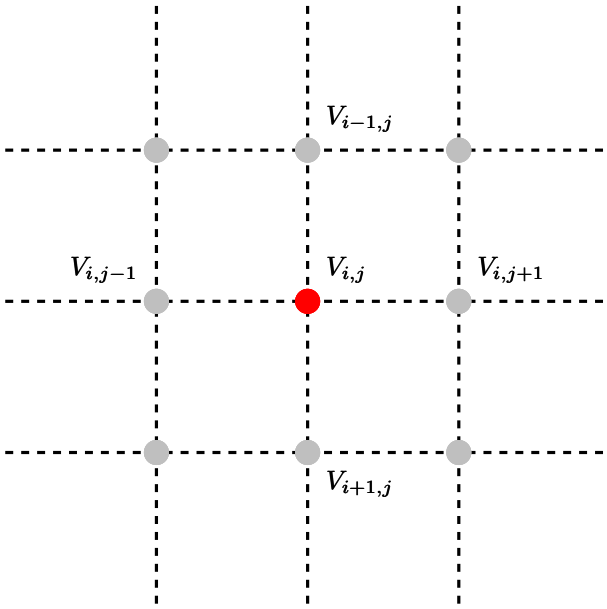
   Use your solver to answer the following questions:

   (a) Plot 4 images of your potential at different iterations as it relaxes from your original guess toward the actual value.

   (b) Make a plot of how many iterations it takes to converge to acceptable levels of error (0.1, 0.01, 0.001 etc).

   (c) Attempt to increase/reduce the number of iterations needed to achieve acceptable error levels by using better or worse initial guesses. What kinds of initial guess converge quickest? What kinds of initial guess are slow to converge?

4. In most cases you'd never need to computationaly use the Method of Relaxation to find solutions to a 1D Laplace equation. However, in high dimensions this can be an excellent (or the *only*) method to come to grips with a solution. As such, the goal in this problem is to expand your process from Problem 3 to work for a two dimensional problem. Most of the steps are the same, but the main difference will be in how you compute the new average from neighboring cells. In 1D we really only had the option of the cells to the left or right. In two dimensions we have some options:



We could consider all 8 neighbors, but in this problem lets only consider the orthagonal neighbors to count towards the average (not the diagonals). As such, each time you relax, the new value of each cell in the array will look like

$$V_{i,j} = \frac{1}{4}\left(V_{i-1,j} + V_{i,j-1} + V_{i,j+1} + V_{i+1,j}\right)$$

Use you new and improved method of relaxation code to answer the following questions.

(a) Taking your mesh to be a square, set one boundary to have a voltage of 10 V and the other 3 boundaries to be at 0 V. Make reasonable assumptions about the interior and then relax to find a solution accurate to within 0.1%. (I'm leaving the dimensions of the square up to you here. You may need to shrink it if the calculations are taking too long. Sizes between 25 and 100 have seemed ok on my end.) Make a 3D surface and contour plot of your result. (You can put them on the same graph if you want to be fancy.)

(b) You can keep the square mesh, but now set one of the sides to have half the points at 10 V and the other half at −10 V. Keep the other three sides at 0 V. Create new 3D surface and contour plots, and comment about the existence of any local maxes or mins. Do they go away with tight error tolerances?

(c) Set the outer boundary to whatever potentials you like, but add a hole to your mesh by declaring some small region in the interior to be a fixed potential as well (also up to you). Again plot a 3D surface and contour of the resulting potential. This part mimics have a 2D cavity on the interior of your system, and can not really be solved analytically.