Announcements

- CompDay 11 due tonight
- Homework 10 due on Monday
- Don't forget to be thinking/reading through your Final Chapter!
- Will be starting Chapter 12 for Friday
- Responses: `rembold-class.ddns.net`

Today's Objectives

- So we can calculate principal axes. But how do they help us in practice?
- Understand the origin and use of the Euler Equations
- Be able to understand stability in the context of rigid bodies rotating around an axis
- Practice flipping between understanding results in the body frame and in the space frame

Q1

We saw last class that for a particular mass distribution we could write the inertial tensor in the principal axes frame as:

$$\vec{\mathbf{I}}_e = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \end{bmatrix} \quad \text{and} \quad \hat{\mathbf{e}}_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \quad \hat{\mathbf{e}}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \hat{\mathbf{e}}_3 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$$

Suppose you are rotating the mass distribution about the axis $\vec{\omega} = \sqrt{2}\hat{\boldsymbol{x}}$. What is the angular momentum of the system as measured in the principal axes frame?

A) $\vec{\mathbf{L}} = \begin{bmatrix} 2 \\ 0 \\ 6 \end{bmatrix}$ B) $\vec{\mathbf{L}} = \begin{bmatrix} 2\sqrt{2} \\ 0 \\ 0 \end{bmatrix}$ C) $\vec{\mathbf{L}} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$ D) $\vec{\mathbf{L}} = \begin{bmatrix} 2\sqrt{2} \\ 4\sqrt{2} \\ 0 \end{bmatrix}$

Q2

For a rigid body moving through space at a constant non-zero speed and rotating about its center of mass at a constant non-zero rate about a constant axis, is the principal axes coordinate system an inertial coordinate system?

A) Yes!

B) No!

C) Sometimes!

D) I have no clue what is happening!

Q3

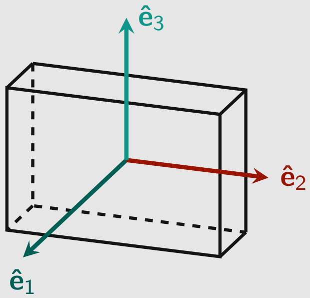
The box to the below right has a uniform density and different dimensions on each length, such that I have sketched in the principal axes. Were I to try to spin the box about any of those axes, I could never get it perfect, only close to that axis. About which axis will the rotation of the box be unstable, such that it's rotation vector would change wildly?

A) $\hat{\mathbf{e}}_1$

B) $\hat{\mathbf{e}}_2$

C) $\hat{\mathbf{e}}_3$

D) None of them will be unstable



Q4

Suppose you have an object with its inertial tensor in the body frame given by:

$$\vec{\mathbf{I}} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

The object is rotated such that $\vec{\omega} = 10\hat{\mathbf{e}}_3$. If there is even the slightest deviation from this, the result is that the angular momentum precesses about $\hat{\mathbf{e}}_3$. For this system and rotation, how long would it take one precession to complete?

A) 0.5 s

B) 1.25 s

C) 5 s

D) It depends on the amount of deviation!

MECHANICS

It's the Principal of the Thing