

# Announcements

- HW2 due Monday
- Lab 2 write-up due Monday as well
- Quick turn-around before the next lab, as it will also be Monday
  - Simulating odds in R
  - Write-up won't be due till the following Monday though to be clear
- Read Ch 5.1 for Wednesday

## Warm Up

Determine the inner quartile range for the below sequence of numbers.

2, 3, 6, 8, 12, 23, 43, 44, 67

- A) 12
- B) 15
- C) 37
- D) 41

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# Disjoint Outcomes

**Disjoint outcomes:** Can not happen at the same time. Said to be “mutually exclusive”.

- A single coin toss can not be a head and a tail
- A student can not pass and fail a class
- A single card from a deck can not be both Ace and King

Conversely, **non-disjoint outcomes** can happen at the same time.



## Adding joints

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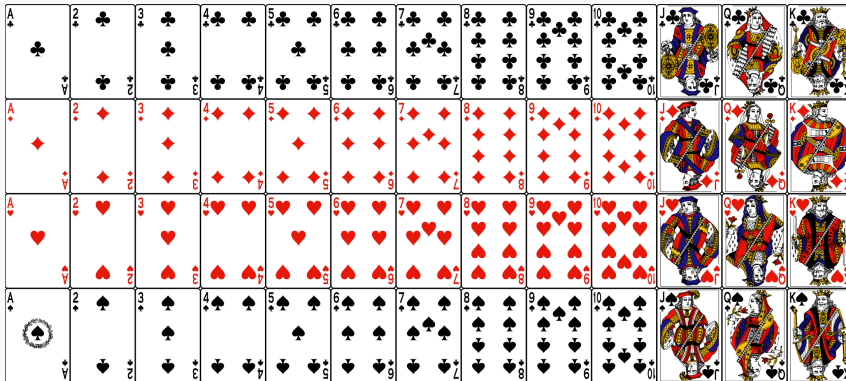
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What are the odds of drawing either a Jack or a 3 from a full deck of cards?

$$P(\text{Jack or } 3) = P(\text{Jack}) + P(3) = \frac{4}{52} + \frac{4}{52} = 0.154$$

# Non-Disjoint Event Probabilities

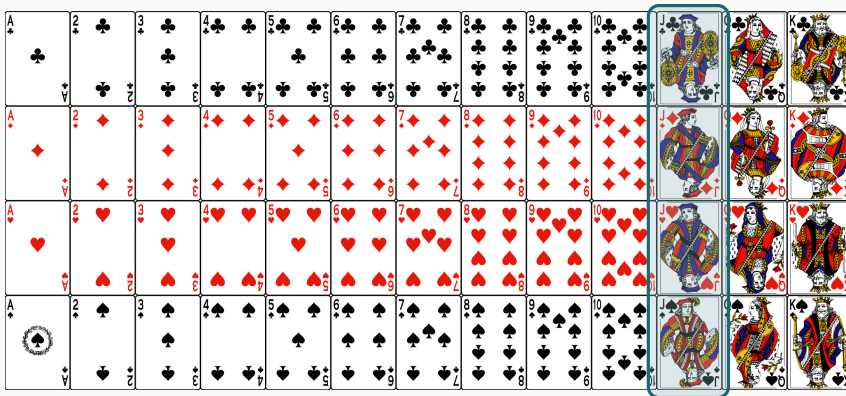
What is the probability of drawing a jack or a red card from a well shuffled full deck?





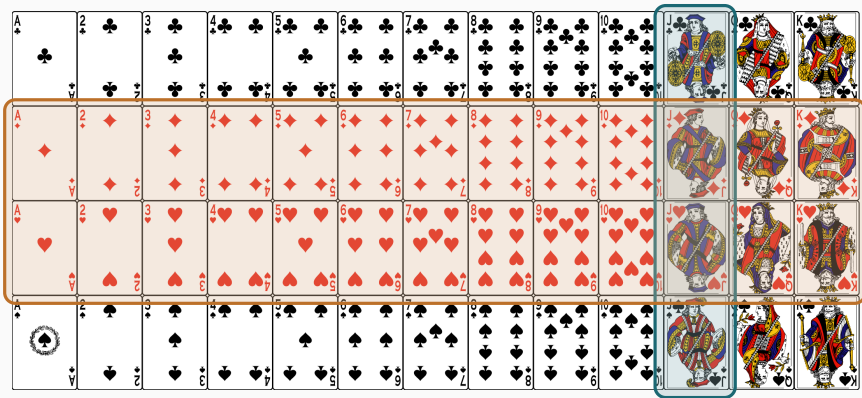
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## General Addition Rule

We need to subtract off some values to keep from double counting.

The result is the **General Addition Rule**:

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The probability of drawing a Jack or a red card?

$$P(\text{Jack or Red}) = \frac{4}{52} + \frac{26}{52} - \frac{2}{52} = 0.538$$

## Practice

What is the probability that a randomly sampled student thinks marijuana should be legalized or they agree with their parents political views?

- A)  $\frac{40 + 36 - 78}{165}$
- B)  $\frac{114 + 118 - 78}{165}$
- C)  $\frac{78}{165}$
- D)  $\frac{78}{188}$

| <i>Legalize MJ</i> | <i>Share Parents' Politics</i> |            | <i>Total</i> |
|--------------------|--------------------------------|------------|--------------|
|                    | <i>No</i>                      | <i>Yes</i> |              |
| <i>No</i>          | 11                             | 40         | 51           |
| <i>Yes</i>         | 36                             | 78         | 114          |
| <i>Total</i>       | 47                             | 118        | 165          |

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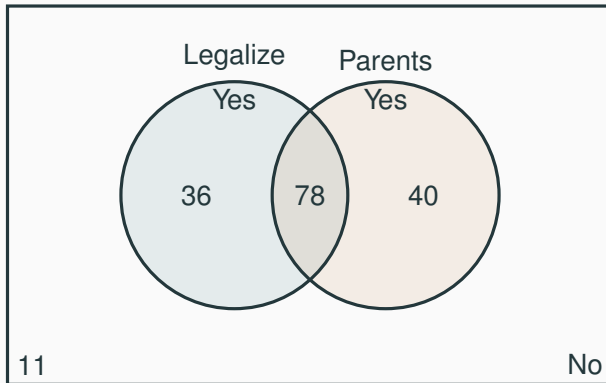
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## Venn and the Art of Motorcycle Riding

Using Venn diagrams can also be an effective way to convey information about disjoint (on non-disjoint) data.



# Distributing Probabilities

A **probability distribution** lists all possible events and the probabilities with which they occur.

- The probability distribution for the gender of one child:

| Event       | Male | Female |
|-------------|------|--------|
| Probability | 0.5  | 0.5    |

- Probability distributions have rules:
  1. Events listed must be disjoint
  2. Each probability must be between 0 and 1
  3. Adding all probabilities must total to 1



## Skittles Odds

Given the image to the right, let's construct a probability distribution for the colors of original skittles in my bag.

| Color  | Probability |
|--------|-------------|
| Red    |             |
| Green  |             |
| Orange |             |
| Purple |             |
| Yellow |             |



## Skittles Odds

Given the image to the right, let's construct a probability distribution for the colors of original skittles in my bag.

| Color  | Probability |
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| Red    | 0.368       |
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| Orange |             |
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| Orange | 0.158       |
| Purple | 0.158       |
| Yellow | 0.053       |



## Bestowing Complements

A **sample space** is the collection of all possible outcomes of a trial.

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These two events are complementary.

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- Looking at all the colors, the event: “Draw a red skittle” is complementary to the event “Draw anything but a red skittle”.

In a survey, 52% of respondents said they are Democrats. What is the probability that a randomly selected respondent from this sample is a Republican?

- A) 0.48
- B) more than 0.48
- C) less than 0.48
- D) can not calculate using information given

# Independence

Two processes are *independent* if knowing the outcome of one provides no useful information about the outcome of the other.

- Knowing that a coin landed on heads on the first flip does not provide any useful information determining what the coin will land on in the second toss
- Knowing someone's eye color does not provide any useful information about their age
- But drawing an ace from a deck of cards does effect the odds of drawing a jack on the second draw (assuming you don't replace the ace)

## Multiplication Rule

If  $A$  and  $B$  represent events from two independent processes, then

$$P(A \text{ and } B) = P(A) \times P(B)$$

Or if you have multiple independent events,

$$P(A \text{ and } B \text{ and } C) = P(A) \times P(B) \times P(C)$$

## Example

In my bag of skittles from earlier, assume I draw three skittles, replacing each back into the bag after each draw. What is the probability that I draw a red, green, and an orange skittle?

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$$\begin{aligned}P(\text{red and green and orange}) &= P(\text{red}) \times P(\text{green}) \times P(\text{orange}) \\&= 0.368 \times 0.263 \times 0.158 \\&= 0.015\end{aligned}$$