1. Much of this course is going to revolve around vector calculus, so it is worth taking some time to ensure that you are familiar with the different types of integrals that will be showing up. For each of the integrals below, *explain your steps* as you go through computing each integral.

   (a) **Line Integrals**: Frequently occurring in discussions of energy, I shudder at the number of times students have confused these with volume integrals. Determine the work done by the force $\vec{\mathbf{F}} = y^2\hat{\mathbf{x}} - 2x^3\hat{\mathbf{y}}$ along the cubic path $y = x^3$ from (0,0) to (2,8). You only need to worry about 2 dimensions here.

   (b) **Surface Integrals**: Calculating fluxes over a surface will come up repeatedly over this semester. Calculate $\int_S \vec{\mathbf{v}} \cdot d\vec{\boldsymbol{A}}$ where $\vec{\mathbf{v}}(x, y, z) = 6x\hat{\mathbf{y}} + 3y\hat{\mathbf{z}}$ and S is a rectangle lying in the xz plane with diagonal corners given by (-2,0,5) and (3,0,8).

   (c) **Volume Integrals**: These tend to be what people are most familiar with, but we see them all the time in physics when calculating total values (say mass) from a value distribution (mass density). Consider two spheres, one with constant density ρ_0 and one with a density that varies as

   $$\rho(r) = \frac{3\rho_0}{R^2}r^2,$$

   where R is the radius of both spheres. Calculate the mass of each sphere.

2. It is important to always keep in mind what sort of function or operations we are talking about (scalar or vector). Take the function $T(x, y, z)$ which could say describe the temperature at any point in a room. Which of the three operations (divergence, curl and gradient) could be applied to this function? For each operation that can apply:

   - Write out a general formula that would calculate the result.
   - Explain in words how to interpret or physically understand the result.
   - Determine if the result is a scalar or vector value.

3. Similar to the above question but now consider the function $\vec{\mathbf{V}}(x, y, z)$, which could describe the velocity of a fluid at different points in space. Again, which of the three operations (divergence, curl and gradient) could be applied to the function? For each that can,

   - Write out a general formula that would calculate the result.
   - Explain in words how to interpret or physically understand the result.
   - Determine if the result is a scalar or vector value.

4. Griffiths uses some special notion in the form of $\vec{\boldsymbol{\mathscr{r}}}$. Whenever you see this script r, Griffiths is referring to the *separation vector*, or the vector $\vec{\mathbf{r}} - \vec{\mathbf{r}}_s$ where $\vec{\mathbf{r}}_s$ would be the *constant* location of say a source point charge. We rely on these separation vectors extensively when working with Coulomb's law (and also Newton's law of gravity), so it is worth understanding how our vector calculus operators interact with $\vec{\boldsymbol{\mathscr{r}}}$.

   (a) Calculate the below gradients by hand. Remember that $\mathscr{r} = \left|\vec{\boldsymbol{\mathscr{r}}}\right|$ and that it might be easier to calculate these by explicitly writing out the Cartesian coordinates.

      i. $\nabla \mathscr{r}$

      ii. $\nabla \frac{1}{\mathscr{r}}$

   (b) Now calculate the same two gradients in a Jupyter notebook using Sympy. Compare your two answers. Did you get the same values?

5. Visual inspection of vector fields can often times give us immediate knowledge about what values the divergence or curl of that field might take. This is turn can influence what problem-solving methods we might apply, and thus it is important to have a feel for what divergence and curl mean visually on a vector plot. For each of the 4 vector fields sketched below:

   - Which have a non-zero divergence somewhere? If the divergence is non-zero only at discrete points, what are those points?
   - Which have a non-zero curl somewhere? If the curl is non-zero only at some discrete points, what are those points?
   - Provide a brief explanation and rational for your above answers.