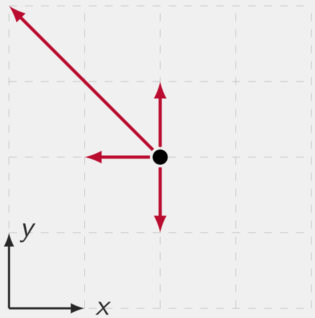
Announcements

- Unless a miracle happened, I'm still grading the Video Assignments. I'm trying to get you feedback though before the next one is due
- WebWorK HW3 is due tonight at midnight
- A reminder that tutors are available from 7:30–9:30pm, Sunday–Thursday, and are meeting in this room
- VHW2 prompt will be posted by this afternoon
    - Due Monday night at midnight

Review Question

Consider the free body diagram to the left. The length of each arrow is drawn to scale for comparison purposes. Given the forces shown, which change in momentum vector below would seem most reasonable?

1. $\langle 2, 1, 0 \rangle \text{kg m/s}$
2. $\langle -3, 0, 0 \rangle \text{kg m/s}$
3. $\langle -6, 4, 0 \rangle \text{kg m/s}$
4. $\langle -3, 3, -1 \rangle \text{kg m/s}$



Upcoming:

Becoming a Precog

Looking to the Future

- Frequently, we know something about the forces and instead want to be able to predict the future motion of the object
- Recasting the Momentum Principle:

$$\vec{\mathbf{p}}_f = \vec{\mathbf{p}}_i + \vec{\mathbf{F}}_{net}\Delta t$$

or

$$\vec{\mathbf{p}}_{future} = \vec{\mathbf{p}}_{now} + \vec{\mathbf{F}}_{net}\Delta t$$

- Can predict the future motion of an object purely based on knowing its momentum now and the forces currently acting on it
  - Nothing else is needed!
  - An object's current momentum holds its entire history (all the individual interactions that got the object to it's current point, speed and momentum)

The Power of Iteration

- Because each future momentum only depends on the current momentum and the net force, we can chain a bunch of them together to work out the path an object would follow
- Akin to interatively using the momentum principle
- Requires only a few things:
  - An initial position
  - An initial momentum
- Each iteration calculate:
  - The current force
  - The new momentum
  - The new position

Example: A Ball in Flight

For objects *near the surface of the Earth*, the force due to gravity can be approximated with

$$\vec{\mathbf{F}}_{gravity} \approx m\vec{\mathbf{g}}$$

where m is the mass of the object and $\vec{\mathbf{g}} = \langle 0, -9.8, 0 \rangle \mathrm{N/kg}$.
Consider then a 0.5 kg ball which is launched from an initial position of

$$\vec{\mathbf{r}}_i = \langle 0, 0, 0 \rangle \,\mathrm{m}$$

with an initial velocity of

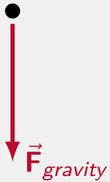
$$\vec{\mathbf{v}}_i = \langle 20, 20, 0 \rangle \,\mathrm{m/s}.$$

Describe the path of the ball and determine the position of the ball 2 s after launch.

Initializing the Problem

- System?
  - The ball
- Surroundings?
  - Air
  - The Earth
- Initial Simplifications?
  - Ball a point mass
  - Air forces negligible

Free Body Diagram:



Iteration Uno

- What is our force?

$$\begin{aligned}\vec{\mathbf{F}}_{net} &= \vec{\mathbf{F}}_{gravity} = mg \\ &= (0.5\,\mathrm{kg})(\langle 0, -9.8, 0\rangle\,\mathrm{N/kg}) \\ &= \langle 0, -4.9, 0\rangle\,\mathrm{N}\end{aligned}$$

- What is our initial momentum?

$$\vec{\mathbf{p}}_i = m\vec{\mathbf{v}}_i = (0.5\,\mathrm{kg})\,\langle 20, 20, 0\rangle = \langle 10, 10, 0\rangle$$

- What is our desired Δt?
    - Let's say 1 second

- What is our new momentum?

$$\begin{aligned}\vec{\mathbf{p}}_{future} &= \vec{\mathbf{p}}_{now} + \vec{\mathbf{F}}_{net}\Delta t \\ &= \langle 10, 10, 0\rangle + \langle 0, -4.9, 0\rangle\,(1) \\ &= \langle 10, 5.1, 0\rangle\,\mathrm{kg\,m/s}\end{aligned}$$

- What is our new position?

$$\vec{\mathbf{r}}_{future} = \vec{\mathbf{r}}_{now} + \vec{\mathbf{v}}_{avg}\Delta t$$

- How to we get $\vec{\mathbf{v}}_{avg}$?!

Approximating $\vec{\mathbf{v}}_{avg}$

- We can't really know $\vec{\mathbf{v}}_{avg}$, because it relies on future points!
- How to best approximate?
  - Could average $\vec{\mathbf{p}}_{now}$ and $\vec{\mathbf{p}}_{future}$ and divide by the mass
  - More often will just take

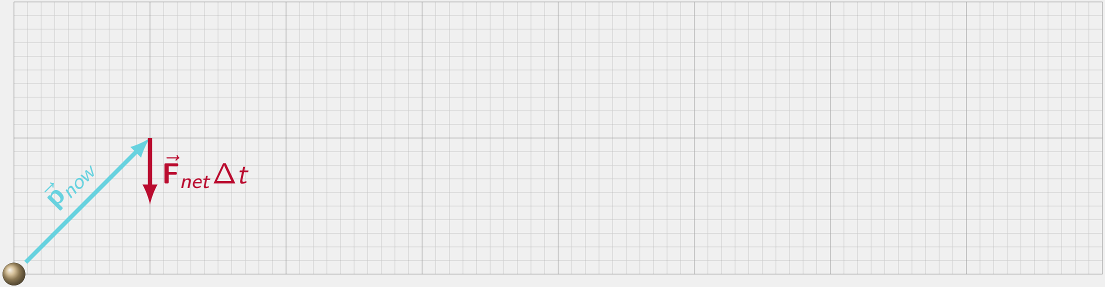
$$\vec{\mathbf{v}}_{avg} \approx \frac{\vec{\mathbf{p}}_{future}}{m}$$

- So we are left with:

$$\begin{aligned}\vec{\mathbf{r}}_{future} &= \vec{\mathbf{r}}_{now} + \frac{\vec{\mathbf{p}}_{future}}{m}\Delta t \\ &= \langle 0, 0, 0\rangle + \frac{\langle 10, 5.1, 0\rangle}{0.5}(1) \\ &= \langle 20, 10.2, 0\rangle \text{ m}\end{aligned}$$

- Can repeat again, using this $\vec{\mathbf{r}}_{future}$ as the next $\vec{\mathbf{r}}_{now}$!

Graphically ($\Delta t = 1\,\text{s}$)



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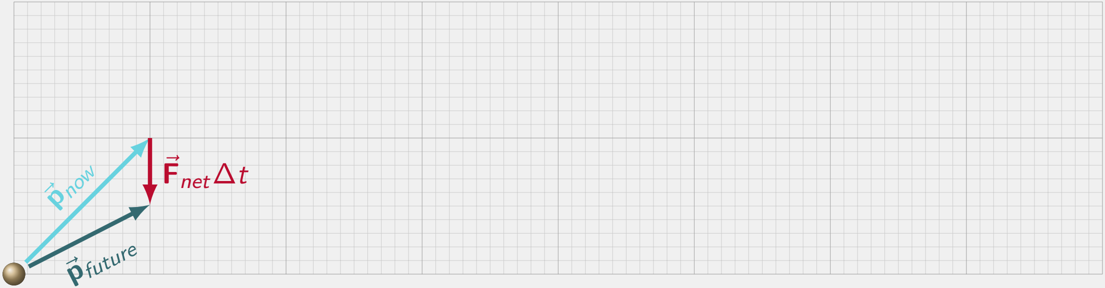

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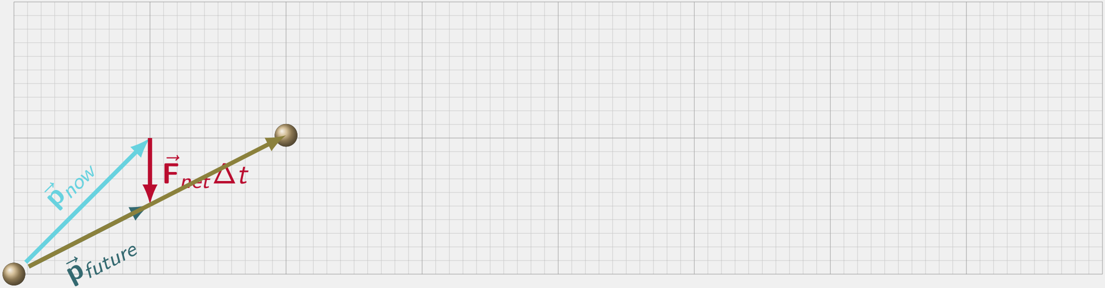

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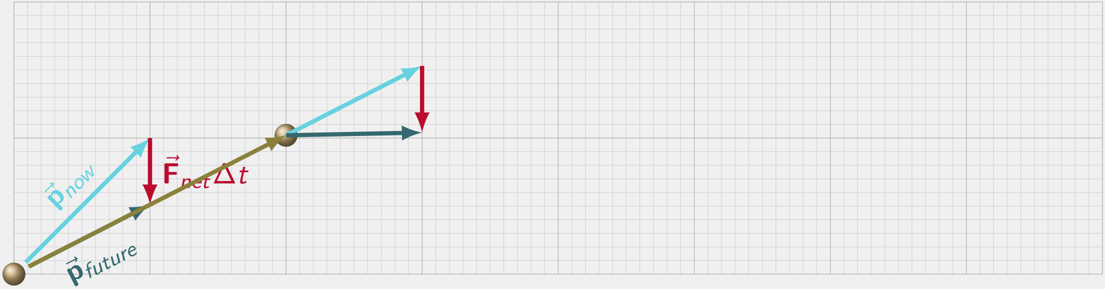

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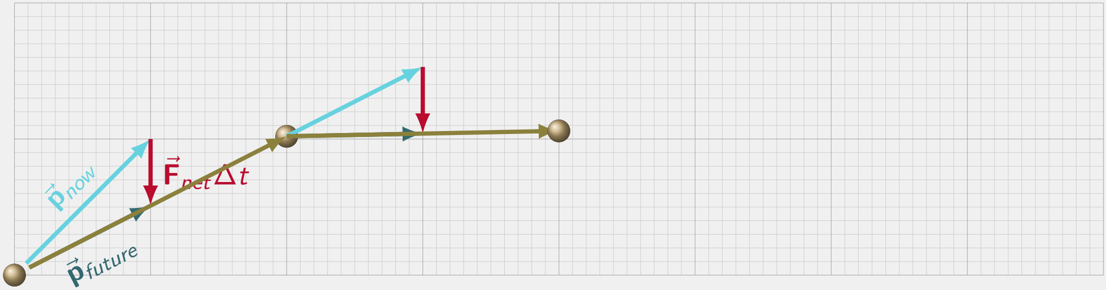

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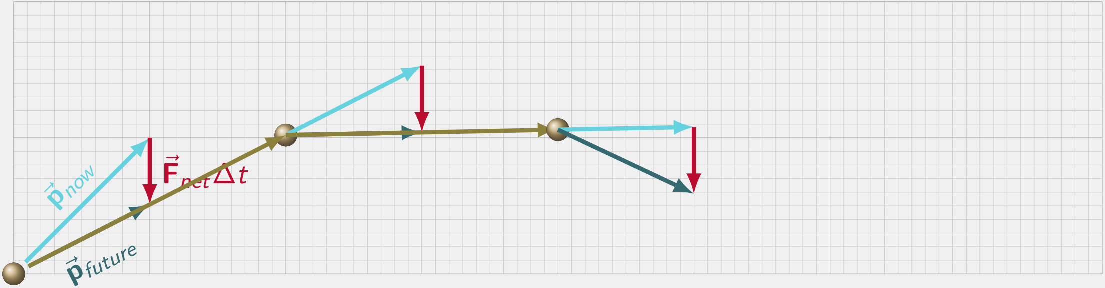

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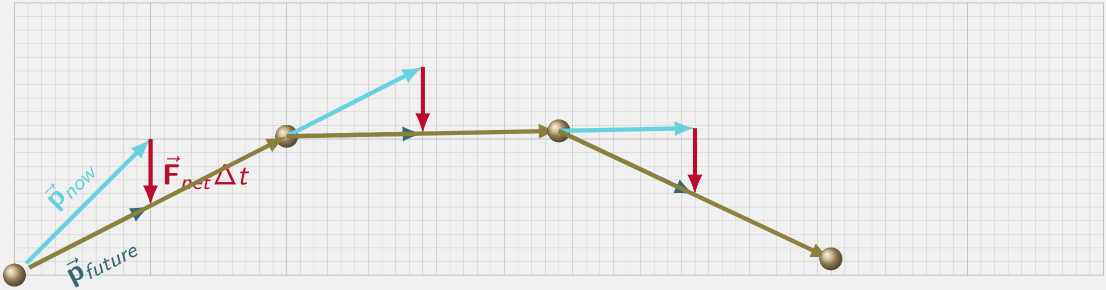
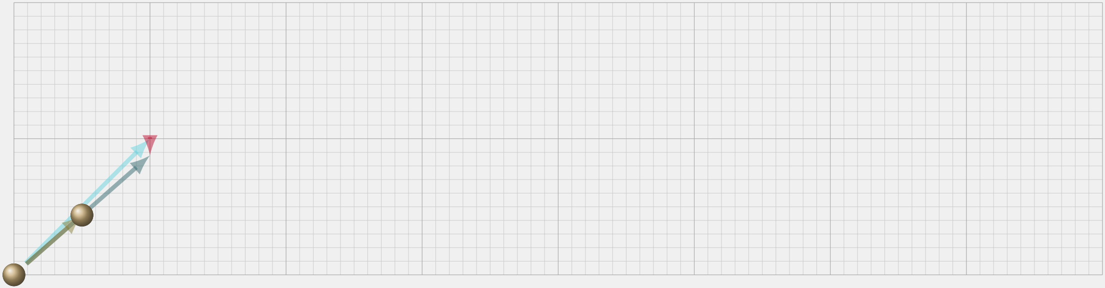


Graphically ($\Delta t = 0.25\,\mathrm{s}$)



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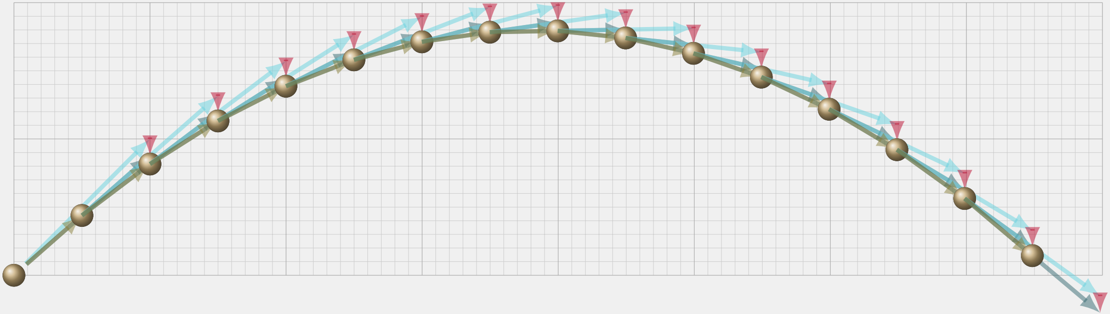


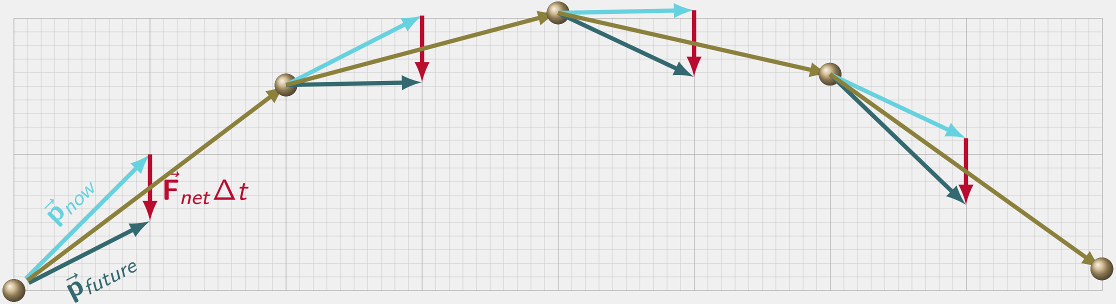
Wait! Why?!

- Why did just changing our time interval seemingly change the physics of our situation?!
  - After all, the ball did fly further when we used a smaller time interval
- Back to the

$$\vec{\mathbf{v}}_{avg} \approx \frac{\vec{\mathbf{p}}_{future}}{m}$$

- If Δt gets too large, this breaks down
  - $\vec{\mathbf{p}}_{future}$ too far in the future to be a decent predictor for the average velocity
  - Still the best option for instances with changing force
- For constant force problems averaging the now and future momenta and dividing by the mass will give the best approximation

Graphically ($\Delta t = 1\,\mathrm{s}$, Averaging Momenta)



Why all this effort?

- You may ask why we are championing this method
- Most Intro books teach how to find solutions to this problem analytically
  - Result in the classic kinematic equations
  - Give an official range of 81.72 meters
- So why do this iteration when an "exact" solution exists?

Analytic vs Iterative (Computational)

- Analytic solutions are those where an equation exists that will give us a solution to our problem
  - Nice for reuse. Tweak some parameters? Easy, just plug the new ones into the equation
  - Nice for people desperate for equations. You know who you are...
- Iterative or Computational methods don't yield a nice equation, instead they yield the solution directly
  - If you change a parameter, you need to rerun the entire iteration
  - Rely more directly on fundamental principles

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Important!

Most real world problems do not have analytic solutions. They simply do not exist.

Changing Forces

- Iterative methods also work in cases where the force is changing
- Just choose Δt to be small enough that the force is approximately constant on each interval!
- One common changing type of force is that of a spring, which we will look at more in depth on Monday!

Practice Time!

You are standing 5 meters from a wall which has a window located 3 meters up the wall. The window is 1 meter tall. You want to toss a 2 kg ball to your friend who is standing in the open window. You give the ball an initial velocity of $\langle 6, 10, 0 \rangle$ m/s. For convenience with the numbers, just let $\vec{\mathbf{g}} = \langle 0, -10, 0 \rangle$ m/s^2.

- Using the iterative method with time intervals of 0.5 s, determine if your ball will make it through the window.
- What method did you use to estimate $\vec{\mathbf{v}}_{avg}$? How would your answer change if you'd used the other method? Which is (more) accurate?
- How would your answer change if there was also a 30 N wind blowing toward the building?