

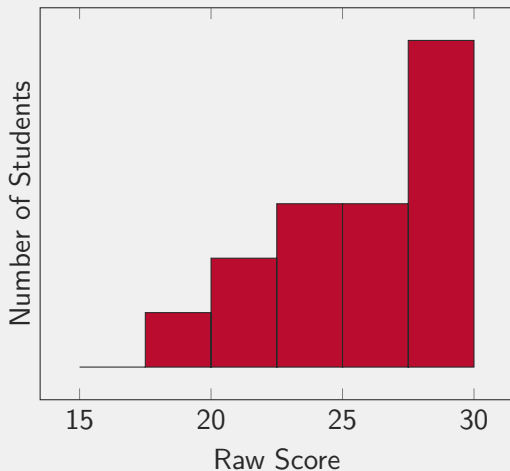


Announcements

- Homework
 - WebWork 6 due tonight at midnight
 - Video HW 3 due Monday night
- Tests are being handed back! We'll talk more about them in a moment!
- Polling: `rembold-class.ddns.net`



Test Discussion



- High: 98%
- Mean: 85%
- Std: 11.1%
- Median: 88%



Problem 1

90%

Catelyn Stark is traveling about the countryside trying to find support for her beleaguered house. Starting from Winterfell, she travels due North 3.5 km before turning and traveling 7 km at an angle of 30° North of West. Finally, she travels the final portion of her journey by traveling a path that takes her 2 km South and 1 km West. How far does Catelyn end up from Winterfell, where she started?



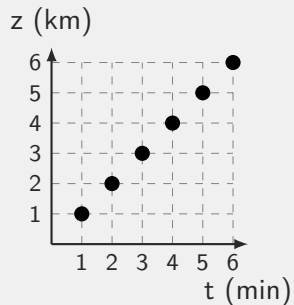
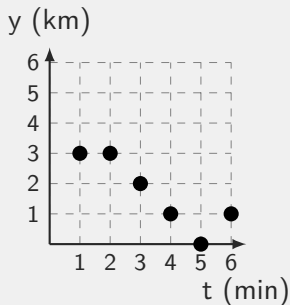
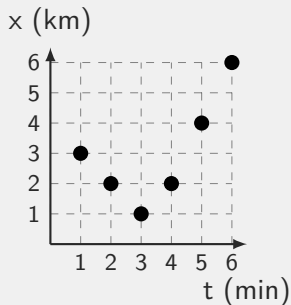
Arya Stark hurls a dagger, which embeds itself in a nearby oak tree. The 500 g dagger embeds itself 5 cm into the oak tree over a span of 0.05 s as it comes to rest. Which of the below values is the best estimate as to the dagger's speed the moment before it struck the tree?



Problem 3

83%

Daenerys Targaryen (65 kg) is out for a ride on her dragon Rhaegal (3000 kg). Their positions in each coordinate are given by the below graphs at different points in time. What is the average momentum of Daenerys and Rhaegal between 2 and 4 minutes?





Problem 4

92%

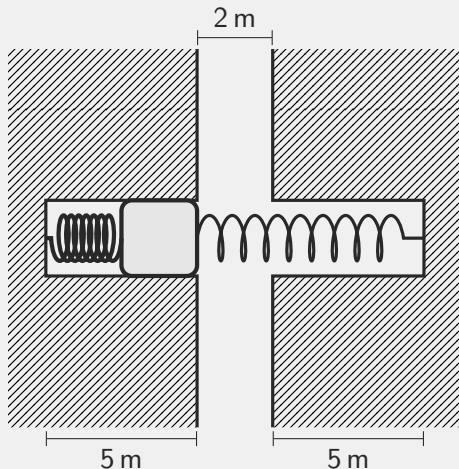
Hodor kindly lifts Bran Stark (45 kg) up off of the ground. Bran starts from rest and is traveling at $\langle 0, 2, 1 \rangle$ m/s 2 s later. If the forces acting on Bran were constant, what net force did Bran feel as he was lifted?



Problem 5

83%

Jon Snow knows nothing about springs, but the Night's Watch are putting in a new doorway through The Wall which utilizes two of them. The idea is that the springs will hold a large block of ice (200 kg) in equilibrium such that it blocks a passage (see left image). Handles on the ice block though will allow it to be forced to the side, allowing individuals to pass by. Finding identical springs proved too difficult in the far North, so the leftmost spring has a spring constant of 500 N/m but the rightmost spring has a spring constant of 400 N/m . Both springs have the same equilibrium length of 5 m however. What force will be necessary to hold the block pushed fully to the left, opening up the 2 m wide passageway? Show all your work for full credit.



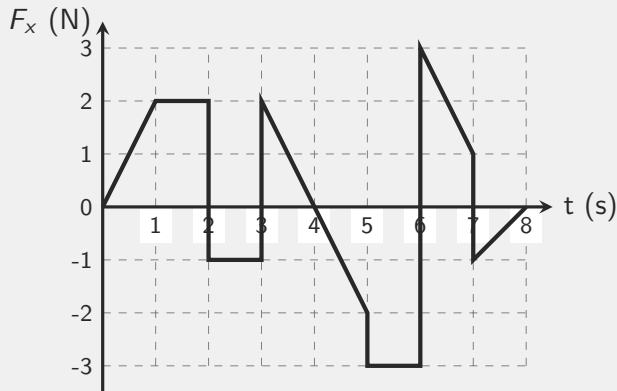


In a pitched battle, Jaime Lannister has his hand cut off. The hand (2 kg) has a velocity of $\langle 2, -0.2, 1 \rangle$ m/s immediately after being severed and is initially at a position of $\langle 0, 1, 0 \rangle$ m. In a bit of dramatic flare, a sudden breeze gusts up as the hand is severed that exerts a time dependent force of $\langle 5 - t, 0, 4 - t \rangle$ N where $t = 0$ the moment the hand is severed. Assuming that Westeros has the same acceleration of gravity as the Earth (9.8 m/s^2), determine the position of the hand in 0.2 s increments until it hits the ground. Show all your work for chances at partial credit.



Extra Credit!

Tyrion Lannister exerts a force in the \hat{x} direction to an initially stationary mug that varies over time as seen below. What direction is the mug traveling afterwards? You must support your answer for credit.

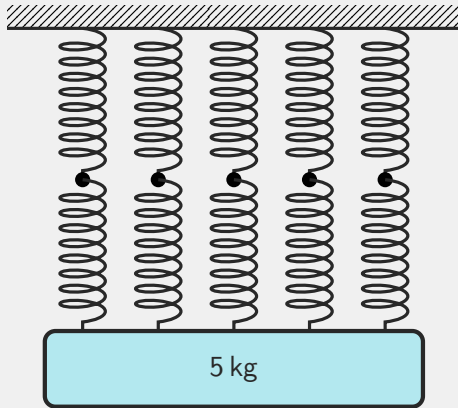




Review Question

A single spring has a spring constant of 50 N/m . When I hang 5 kg from an array of those springs as seen to the right and let it come to rest, how far does the system stretch?

- A) 0.098 m
- B) 0.392 m
- C) 0.981 m
- D) 2.45 m





Spring Loaded Lead

A thin lead rectangular wire with side length of 2 mm is 3 m long. A 10 kg weight is hung from the end and the new length of the wire is carefully measured to be 4.6 mm longer. What is the intermolecular spring constant between lead atoms?

$U=3$
 $U=2$
 $U=1$

$E_3 = 3E_1$
 $E_2 = 4E_1$
 E_1

d

$h^2 = u^2 E_1$

$ma_g \downarrow$
 $ma_g \rightarrow v_1$

$d\vec{r} = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2}$
 $d\vec{r} = \frac{\lambda}{4\pi\epsilon_0} \frac{dx}{r^2}$

$dE_y = \frac{\lambda}{4\pi\epsilon_0} \frac{dx}{r^2} \frac{y}{r}$
 $dE_x = \frac{\lambda}{4\pi\epsilon_0} \frac{dx}{r^2} \frac{x}{r}$

$\Delta P = e\sigma A(T_1 - T_2)$

$U = F_e r = F_r \sin \theta = F_L$
 $v = v_0 \sin \theta$

$F_n \cdot x + F_g \cdot x = ma$
 $F_n \cdot x = 0; F_g \cdot x = F_n \sin \theta = mg \sin \theta$
 $a_x = g \sin \theta$

$z = \sqrt{z^2 + x^2}$
 $v^2 = 2gh$
 $v_s = \sqrt{2gh} \cdot \sin \theta$

Upcoming:

Young's Modulus

$U^2 = A^2 \exp(-\frac{x^2}{2\sigma^2})$
 $B(x) = \frac{\sigma}{\sqrt{\pi}} e^{-\sigma^2(x-x_0)^2}$

$\psi(\psi) = A \cos(k_0 x - \omega t)$

$|F| = \frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$
 $= \frac{mv^2}{r}$

$U_H = -\int \vec{B} \cdot (d\vec{r})$
 $U_H = E_H b = v d B b$

$J = \frac{n}{V} q v d A$
 $b \frac{U}{V} = \frac{1}{A q v b} \int b d v d$
 $= -\int \vec{B} \cdot d\vec{r}$

$E_{pot} = -\frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$
 $E_{kin} = \frac{1}{2} m v^2 = \frac{1}{2} \frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$
 $E_{pot} = -2 E_{kin}$

$E_{pot} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$
 $= \frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$

$\frac{1}{s} = \frac{1}{s} + \frac{1}{s'}$

$\frac{A'B'}{AB} = \frac{s'}{s}$

$F_2 = \frac{F_L}{2n}$
 $E = F_2 \cdot s$
 $= \frac{F_L}{n} \cdot u \cdot h$
 $F_L = F_L \cdot h$
 $= m \cdot g \cdot h$
 $s = u \cdot h$

$\frac{1}{s} = \frac{1}{s} + \frac{1}{s'}$

$\frac{A'B'}{AB} = \frac{s'}{s}$

$\frac{A'B'}{AB} = \frac{s' - f}{f}$
 $\frac{s'}{s} = \frac{s' - f}{f}$

Young's Modulus



Stress and Strain

Goal is to eliminate geometry specific to a certain object and leave only material properties.

Strain

- Need to account for length when describing stretchiness
- Accounting for how many springs you have end-to-end

$$\text{strain} = \frac{\Delta L}{L}$$

- A unitless parameter!

Stress

- Need to account for cross-sectional area when talking forces
- Accounting for how many springs you have side-by-side

$$\text{stress} = \frac{F}{A}$$

- Units of Force/Area, which are also units of Pressure (Pa)



Young's Modulus

- Define **Young's Modulus** to be

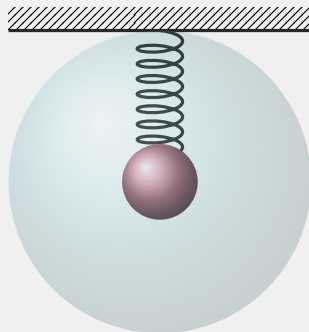
$$Y = \frac{\text{stress}}{\text{strain}} = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$$

- For atomic springs:

$$Y = \frac{(k_{s,i}s/d^2)}{(s/d)} = \frac{k_{s,i}}{d}$$

- With some rearranging:

$$\begin{aligned} F &= Y \cdot A \frac{\Delta L}{L} \\ &= k_{\text{eff}} \Delta L \end{aligned}$$





Young's Modulus

- Define **Young's Modulus** to be

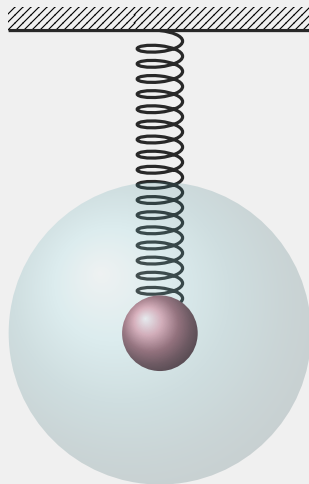
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Young's Modulus

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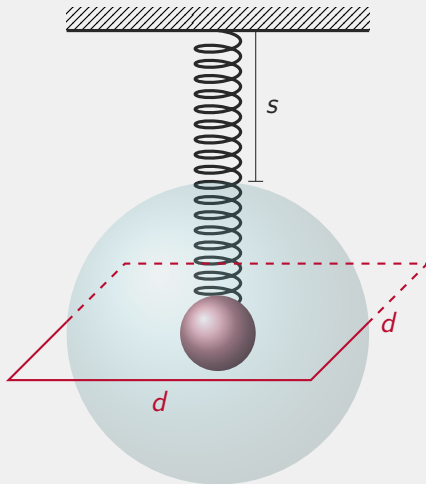
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- With some rearranging:

$$\begin{aligned} F &= Y \cdot A \frac{\Delta L}{L} \\ &= k_{\text{eff}} \Delta L \end{aligned}$$





Example

Silver has a Young's modulus of 83 GPa and has an atomic diameter of about 288 pm. In contrast, Copper has a Young's modulus of 119 GPa and an atomic diameter of about 256 pm. Given an identical wire of both materials, which would stretch more and how much further would it stretch?

$$E_3 = 3E_1$$

$$E_2 = 4E_1$$

$$E_1$$

$$d$$

$$x$$

$$h^2 = u^2 E_1$$

$$|d\vec{E}| = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2}$$

$$= \frac{\lambda}{4\pi\epsilon_0} \frac{dx}{r^2}$$

$$dE_y = \frac{\lambda}{4\pi\epsilon_0} \frac{dx}{r^2} \frac{y}{r}$$

$$dE_x = \frac{\lambda}{4\pi\epsilon_0} \frac{dx}{r^2} \frac{x}{r}$$

$$\cos\theta = \frac{y}{r}$$

$$d\vec{E} = |d\vec{E}| \cos\theta \hat{y}$$

$$d\vec{E} = \frac{\lambda}{4\pi\epsilon_0} \frac{dx}{r^2} \frac{y}{r}$$

$$\lambda_1 = \frac{u_1}{f}; \lambda_2 = \frac{u_2}{f}$$

$$\sin\theta_2 = \frac{\lambda_1}{\lambda_2} \sin\theta_1$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\lambda_2}{\lambda_1} = \frac{u_1}{u_2} = \frac{n_2}{n_1}$$

$$\Delta P = e\sigma A(T_1 - T_2)$$

$$U = F_e r = F_r \sin\theta = F_L$$

$$v = v_0 \sin\theta$$

$$U_A, \phi = X \sin\theta$$

$$F_n x + F_g x = ma$$

$$F_n x = 0; F_g x = F_g \sin\theta = mg \sin\theta$$

$$a_x = g \sin\theta$$

$$z = \sqrt{z^2 + x^2}$$

$$v^2 = 2g \sin\theta \Delta x$$

$$v^2 = 2gh$$

$$v_s = \sqrt{2gh} \sin\theta$$

Relations to Macroscopic Forces

$$|U|^2 = A^2 \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

$$B(x) = \frac{\sqrt{x}}{\sqrt{\pi}} e^{-\sigma^2(x-x_0)^2}$$

$$E(\psi) = A \cos(k_0 x - \omega t)$$

$$|F| = \frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$$

$$= \frac{mv^2}{r}$$

$$E_{pot} = -\frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$$

$$E_{kin} = \frac{1}{2} mv^2 = \frac{1}{2} \frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$$

$$E_{pot} = -2E_{kin}$$

$$E_{pot} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$$

$$\frac{1}{s} = \frac{1}{s'} + \frac{1}{s''}$$

$$\frac{A'B'}{AB} = \frac{s'}{s}$$

$$F_2 = \frac{F_L}{2}$$

$$E = F_2 \cdot s$$

$$= \frac{F_L}{2} \cdot u \cdot h$$

$$F_L = F_L \cdot h$$

$$= m \cdot g \cdot h$$

$$s = u \cdot h$$

$$b = b_2 + \mu_0 u = b_2 (1 + \mu_0 u)$$

$$= \mu_0 u \mu_0 h J = \mu_0 J$$

$$m_1 v_{1x} + m_2 v_{2x} = m_1 v_{1x}' + m_2 v_{2x}'$$

$$\frac{1}{2} m_1 v_{1x}^2 + \frac{1}{2} m_2 v_{2x}^2 = \frac{1}{2} m_1 v_{1x}'^2 + \frac{1}{2} m_2 v_{2x}'^2$$

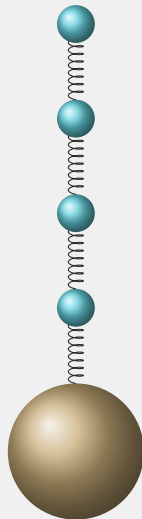
$$= \frac{1}{2} m_1 v_{1x}^2 + \frac{1}{2} m_2 v_{2x}^2$$

$$F_s = \frac{ax}{\cos\theta}; F_s = \frac{mg}{\sin\theta}$$



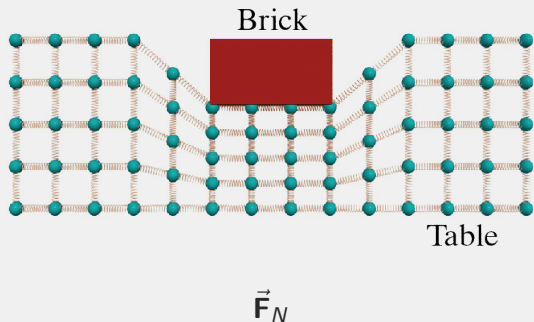
Tension

- Force exerted by wires or strings
- Propagates up the sequence of atomic springs
- If atomic masses small compared to the end mass, then force equal along entire length





Normal Forces

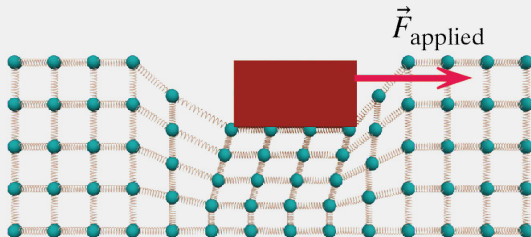


- When a solid object is placed atop another, they influence the atomic springs
- “Table” compresses until spring forces counteract gravity
- Force due to compression of the table atoms
- Points perpendicular to the surface
 - Hence called the normal force



Friction

- Movement of brick moving across table forces down new atoms
- Can also visualize the sideways atomic springs pushing back
- “Depth” that it has sunk plays a role
 - Related to the normal force in some fashion
- Springs left behind will bounce back and jiggle, raising temperature
- Force is parallel to the surface!



$$|\vec{F}_{friction}| \approx \mu |\vec{F}_N|$$