Questions are from Taylor, Chapter 6. Please show all your work and write legibly for full credit!

- **6.4**: Deriving Snell's Law. The two sections of travel will be through different media, and thus have different indexes of refraction (n). You might want to remember your properties of trig and triangles to get things in terms of the classic θ angles. You don't have to solve the Euler-Lagrange equation here, as you can write out the distances/times and take the derivatives directly.
- **Straight Lines in Curved Coordinates**: This will spring off of the polar problem that we saw in class. The goal is to show that the shortest path between two given points in a plane is a straight line, but where we are using polar coordinates. The starting and ending points will be given by:

$$r(0) = 3, \qquad r(\pi/2) = 1$$

  (a) Write out your expression for ds in terms of polar coordinates. The equations are a bit easier if you start with considering things in terms of $\phi(r)$, so go ahead and factor out the necessary term (this is the opposite of which term we factored out in class).

  (b) Solve the Euler-Lagrange equation for $\phi(r)$. An important thing will happen here, where one side of your expression should equal 0. Looking at your expression, realize that this is akin to saying that the derivative of something is equal to 0, which means that that something must be a constant! Let your constant be k here, and go ahead and solve for $\phi'(r)$ in terms of r and k.

  (c) To determine $\phi(r)$, integration of both sides will be necessary. The integral on the dr side will be MUCH nicer if you make the substitution that:

$$\frac{k}{r} = \cos(u)$$

  Remember that on the $d\phi$ side you still go from some initial point ϕ_0 to some final ϕ. What is $\phi(r)$?

  (d) Rewrite $\phi(r)$ to be $r(\phi)$ as we normally tend to think of polar coordinates. You have two constants still in your expression: k and ϕ_0. Use your starting and ending points to solve for those constants.

  (e) Make a polar plot of your final $r(\phi)$ with your constants plugged in. It can be necessary sometimes to set the r limits of your polar plot to get something that makes sense, so see my updated section in the Matplotlib guide if things look bizarre. Do you get the predicted straight line that passes through the two desired points?

- **6.23**: This is actually a really neat problem walking you through a real-life situation when you'd want to minimize a travel time. A few recommendations:
  - When assuming that ϕ is small, you need only keep ϕ terms of first order or less (throw away ϕ^2 terms and higher). Keep y'^2 terms though.
  - You may want to use Sympy for the algebra, though it can be done by hand.

  In addition to answering the question, plot the path of the airplane! **Bonus points if you include the wind shear somehow on your plot as well!**