Announcements

- Homework
  - Online HW6 due tonight!
  - Online HW7 will be due on Friday
    - Will require Glowscript work
    - Make sure you do your work in a public folder so I'll be able to see it!
- Test 1 a week from Friday
  - Looking to get out study materials this weekend
  - Test is written from homework objectives
  - One question from excellent video homework submissions on website!
  - You get a 3x5inch index card on which you can write whatever you want, handwritten, 1 side
  - Calculators allowed (and probably desired)
    - Can't be internet capable. No phones.
- Polling: `rembold-class.ddns.net`

Review Question

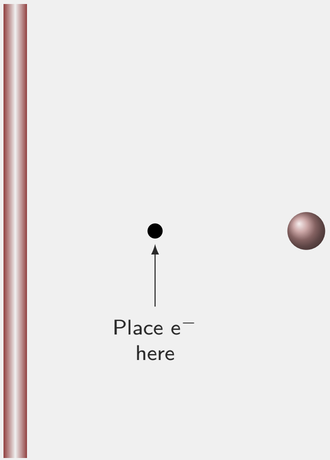
A line charge and a point charge are positioned as seen to the left, both with the same total positive charge. At a point exactly halfway between them, you are going to place an electron. In what direction will the electron move? You can assume the length of the wire is 10 times the distance from the wire to the electron.

A. To the left

B. To the right

C. Upwards

D. No motion (equilibrium)



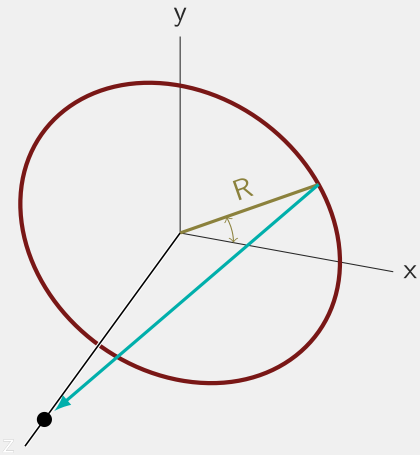
Solution: To the right!

Today's Goals

- Understand the electric field due to a ring of charge on the center axis
  - Refresh our 4 step process
- Learn to computationally calculate electric fields from distributions
  - Take any distribution
  - Break it into a series of point charges
  - Sum over the point charges

- Any position on the ring:

$$\vec{\mathbf{r}}_1 = \langle R\cos\theta, R\sin\theta, 0\rangle$$

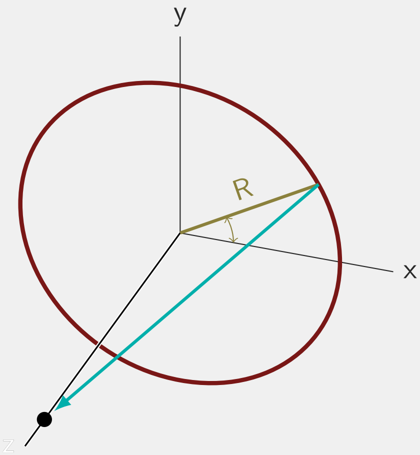


- Any position on the ring:

$$\vec{\mathbf{r}}_1 = \langle R\cos\theta, R\sin\theta, 0\rangle$$

- Observation location on z axis:

$$\vec{\mathbf{r}}_2 = \langle 0, 0, z\rangle$$



Ring Charges

- Any position on the ring:

$$\vec{\mathbf{r}}_1 = \langle R\cos\theta, R\sin\theta, 0\rangle$$

- Observation location on z axis:

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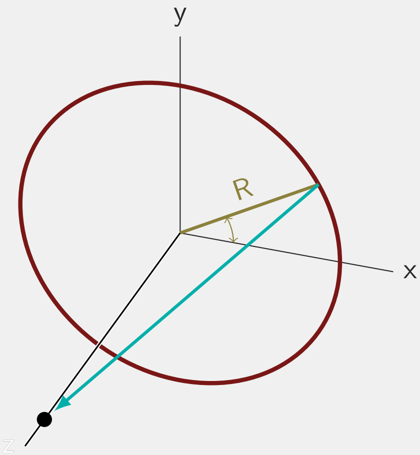
- Displacement vector:

$$\vec{\mathbf{r}} = \langle -R\cos\theta, -R\sin\theta, z\rangle$$



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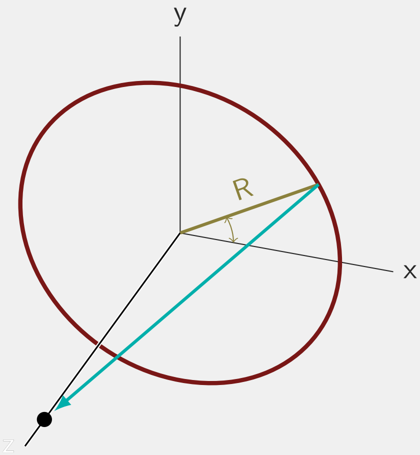
$$\vec{\mathbf{r}} = \langle -R\cos\theta, -R\sin\theta, z\rangle$$

- Magnitude of $\vec{\mathbf{r}}$:

$$|\vec{\mathbf{r}}| = \sqrt{R^2 + z^2}$$



Ring Charge Continued

- Unit vector:

$$\hat{\mathbf{r}} = \frac{\langle -R\cos\theta, -R\sin\theta, z\rangle}{\sqrt{R^2+z^2}}$$

Ring Charge Continued

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$$\hat{\mathbf{r}} = \frac{\langle -R\cos\theta, -R\sin\theta, z\rangle}{\sqrt{R^2+z^2}}$$

- Efield due to small point of charge:

$$\Delta E = \frac{1}{4\pi\epsilon_0}\frac{\Delta Q}{|\vec{\mathbf{r}}|^2}\hat{\mathbf{r}} = \frac{1}{4\pi\epsilon_0}\frac{\Delta Q}{R^2+z^2}\cdot\frac{\langle -R\cos\theta, -R\sin\theta, z\rangle}{\sqrt{R^2+z^2}}$$

Ring Charge Continued

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- Little charge ΔQ:

$$\frac{\Delta Q}{R\Delta\theta} = \frac{Q}{2\pi R} \quad \Rightarrow \quad \Delta Q = \frac{Q}{2\pi}\Delta\theta$$

Ring Charge Continued

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- Analytic Solution:

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{2\pi}\frac{1}{(R^2+z^2)^{3/2}}\int_0^{2\pi}\langle -R\cos\theta, -R\sin\theta, z\rangle\, d\theta$$

Understanding Check

The x and y components of the electric field along the z axis will sum to zero. What will the final expression for the electric field in the z direction be?

A. $\dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{2\pi}\dfrac{1}{(R^2+z^2)^{3/2}}$

B. $\dfrac{1}{4\pi\epsilon_0}\dfrac{Qz}{(R^2+z^2)^{3/2}}$

C. $\dfrac{1}{4\pi\epsilon_0}\dfrac{Qz}{2\pi}\dfrac{1}{(R^2+z^2)^{3/2}}$

D. $\dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{4\pi}\dfrac{z^2}{(R^2+z^2)^{3/2}}$

Solution: B

Computing Electric Fields Computationally

- Already know how to implement the core parts:
  - Create a charge
  - Determine displacement vector
  - Calculate electric field due to that charge
- Main stumbling points will likely be in setting up the charges and summing over everything

Setting up the charges

- In the past we'd name a charge so we could refer to its location later

```
Fred = sphere(pos=vec(1,2,3), radius=1, color=color.cyan)
```

- We don't want to come up with 20 different names, let alone more. . .
- Instead, we'll add things to a list, which we can read back from later

```
list_of_charges=[]
i = 0
while i < 10:
   list_of_charges.append( sphere(pos=vec(i,0,0)) )
   i = i + 1
```

1. Create a blank list of charges
2. Use a loop to choose positions you want the charges to be
3. Create a charge at each position and add it to the end of the charge list

Figuring out the splitting

- Related to creating the charges, you'll probably need to figure out how break them into the desired number of pieces
- Say you have a line charge with a length of 1 m and a charge of 10 mC that you want to break up into 10 parts. You might do something like:

```
1  Q = 10e-3
2  L = 1
3  N = 10
4
5  list_of_charges=[]
6  y = -L/2
7  while y <= L/2:
8      list_of_charges.append( sphere( pos=vec(0,y,0),
9                      radius=L/(2*N),
10                     color=color.red,
11                     q=Q/N) )
12     y = y + L/N
```

Line 6: This starts you at -L/2, and increases by L/N each time until you hit L/2

Adding it All Up

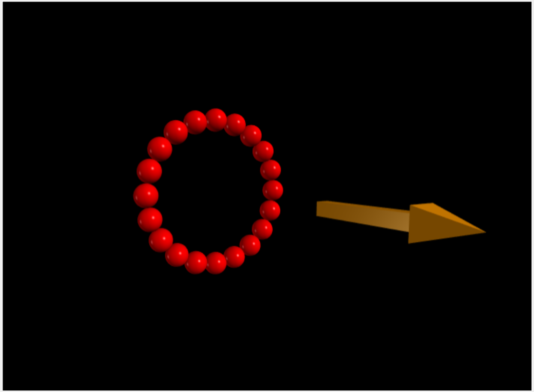
- So we have the setup, now we need to calculate the electric field contribution from each charge and add them all up
- We need to loop *back* through our list of charges, calculating the electric field due to that charge and adding it to our total electric field

```
obsloc = vec(3,0,0)
Efield = vec(0,0,0)
for charge in list_of_charges:
   r = obsloc - charge.pos
   dE = oofpez*charge.q/(r.mag**2)*r.hat
   Efield = Efield + dE
```

- Each time we loop through, the next object in the `list_of_charges` list gets given the name of `charge` and then we calculate our field and add it to a running total

All Together Now!

- Let's return to our ring charge distribution
- Say we have a ring:
  - 5 cm in diameter
  - with 10 μC of charge
  - Break it into 20 chunks
  - Want to know electric field at $\langle 10, 0, 0 \rangle$ cm



Click image to link to code