Announcements

- Homework
  - The first homework assignment needs to be submitted to WebWorK by midnight tonight for full points!
    - Can submit after that until Friday night, but only for 75% of the points
  - A new homework will be posted after class to be due on Friday
- Things I forgot to mention on Monday
  - Campuswire! If you haven't already, accept the invite! Or email me to ask for one if you lost it.
  - Tutoring! You have so many options!
    - Our embedded tutor: Teddy!
    - Hearth Tutoring: Monday–Thursday, 7:30–9:30pm in the Physics Hearth
- Polling active today: rembold-class.ddns.net
  - Don't need to give your full name, but enough to uniquely identify you please!

Refresher Question

Given the 3 vectors below, does $\vec{\mathbf{A}} - \vec{\mathbf{C}}$ point in the exact same direction as $\vec{\mathbf{A}} + \vec{\mathbf{B}}$?

$$\vec{\mathbf{A}} = \langle 3, 4, 1 \rangle$$
$$\vec{\mathbf{B}} = \langle 6, -1, 5 \rangle$$
$$\vec{\mathbf{C}} = \langle 0, 3, -1 \rangle$$

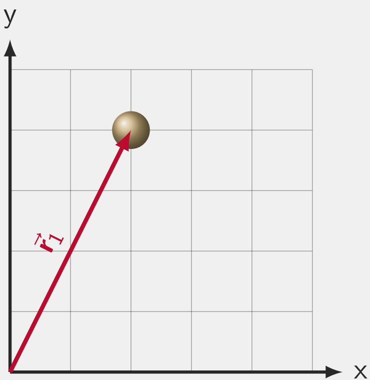
A. Yup!
B. Nope!
C. No, but they have the same magnitude
D. Maybe?

Upcoming:

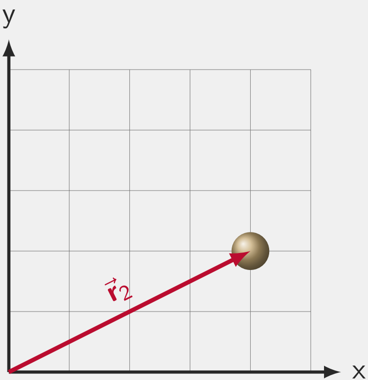
Displacement and Velocity

Moving in Time

- Instead of looking at multiple objects, we can look at the same object at different times
- Because physics is largely concerned with *motion*, this is important to us!



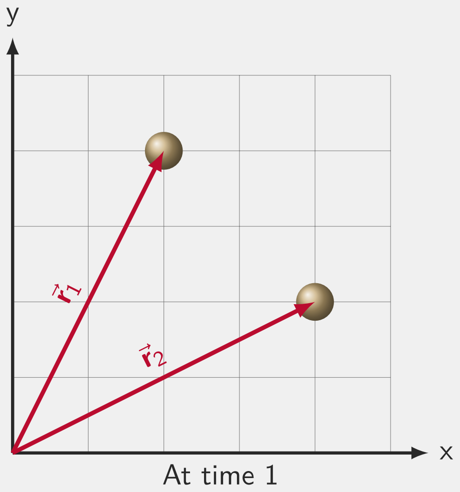
At time 1



At time 2

Vector Differences

- Would be nice to be able to describe motion with vectors



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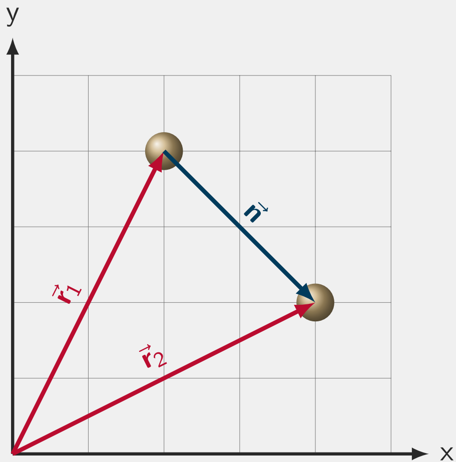

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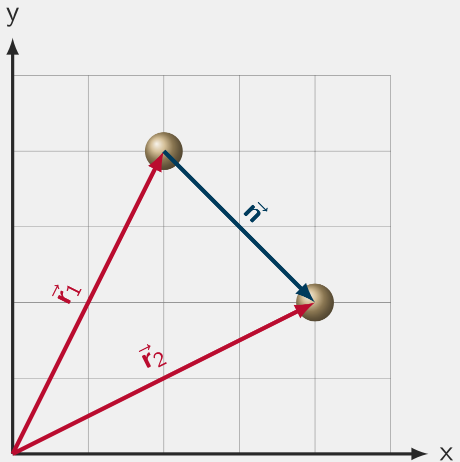

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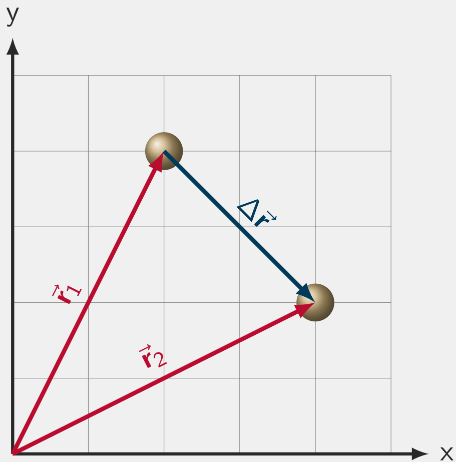
- Rearranging gives:

$$\vec{\mathbf{n}} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1 = \Delta\vec{\mathbf{r}}$$



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Displacements

- We will use the Δ notation throughout the semester to determine a change
- Δ is always the final minus the initial!

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i, \quad \Delta x = x_2 - x_1$$

- Can either be vectors or scalars
- Most frequently will be looking at:

$$\begin{aligned} \Delta\vec{\mathbf{r}} &= \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i \quad \text{change in position} \\ \Delta t &= t_f - t_i \quad \text{change in time} \\ \Delta\vec{\mathbf{v}} &= \vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i \quad \text{change in velocity} \end{aligned}$$

Example!

Suppose you are given three points (say on a map) whose positions can be described with:

$$\vec{\mathbf{r}}_{house} = \langle 30, 10, 10 \rangle \text{ m}$$
$$\vec{\mathbf{r}}_{school} = \langle 40, 100, 0 \rangle \text{ m}$$
$$\vec{\mathbf{r}}_{store} = \langle -50, 0, 0 \rangle \text{ m}$$

Is the store or school a shorter direct walk from your house?

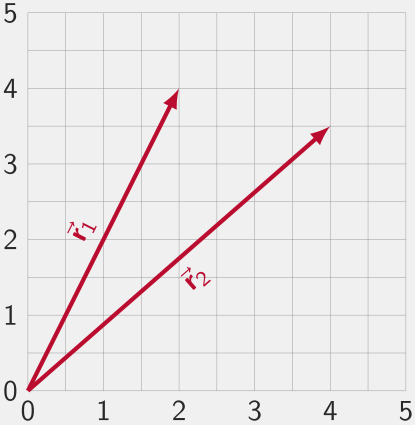
Speed and Velocity

- We already have concepts of both position (a vector) and time (a scalar)
  - Where are you at time 1?
  - Where are you at time 2?
- We can combine these to form an idea of average velocity:

$$\vec{\mathbf{v}}_{avg} = \frac{\Delta \vec{\mathbf{r}}}{\Delta t}$$

- Note that because $\Delta \vec{\mathbf{r}}$ is a vector, then so is the velocity.
- The magnitude of the velocity has a special name, which we call the speed
- Velocity is one of our chief tools when discussing an object's motion

Pointing the Way



$\vec{\mathbf{r}}_1 = \langle 2, 4, 0 \rangle$ m at $t = 0$

$\vec{\mathbf{r}}_2 = \langle 4, 3.5, 0 \rangle$ m at $t = 2$

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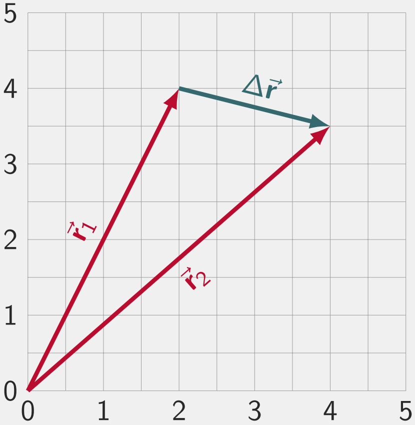

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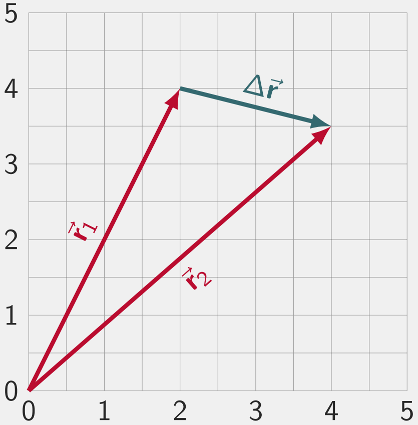

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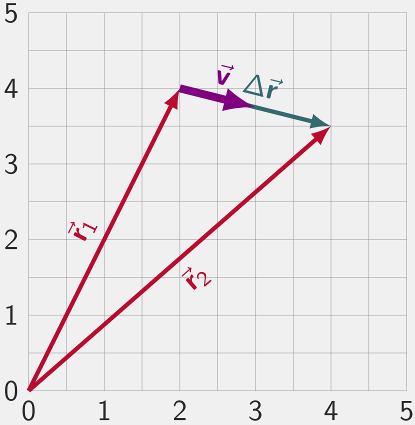

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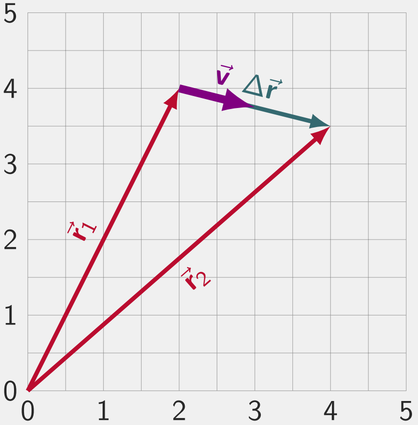

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$$|\vec{\mathbf{v}}| = \sqrt{1^2 + (-0.25)^2 + 0^2} = 1.031\,\text{m/s}$$

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- Hey! That's a derivative!

$$\vec{\mathbf{v}} = \frac{\mathrm{d}\vec{\mathbf{r}}}{\mathrm{d}t} = \left\langle \frac{\mathrm{d}r_x}{\mathrm{d}t}, \frac{\mathrm{d}r_y}{\mathrm{d}t}, \frac{\mathrm{d}r_z}{\mathrm{d}t} \right\rangle$$

Average vs Instantaneous

Suppose an apple is tossed straight up in the air and has it's position at any point in time given by:

$$\vec{\mathbf{r}}(t) = \left\langle 0, 10t - 4.9t^2, 0 \right\rangle \text{ m}$$

Find the average velocity between $t = 1$ and $t = 2$ seconds. Find the instantaneous velocity at $t = 1.5$ seconds.

Hitting the Throttle. . .

- Velocity is a pretty good measure of an object's motion
- Interactions though we said resulted in changes in motion
- So the strength of an interaction is more related to $\delta\vec{\mathbf{v}}$
- Acceleration is the amount of change in the velocity over a certain time interval
  - Is also a vector
  - Direction relative to velocity determines speeding up, slowing down, or turning
- Defined as

$$\vec{\mathbf{a}} = \frac{\mathrm{d}\vec{\mathbf{v}}}{\mathrm{d}t} \quad \text{or} \quad \vec{\mathbf{a}}_{avg} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t}$$

- Units of $\mathrm{m/s^2}$

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- Mass also seems to play a role
    - The greater the mass, the greater the interaction

Enter Momentum

- Since the strength of interactions seems to depend on both mass and velocity positively, let's consider a new quantity:

$$\vec{\mathbf{p}} \approx m\vec{\mathbf{v}}$$

- This combined quantity is called the momentum!
  - Has units of kg m/s
  - Is still a vector!
  - Warning: If you go really fast, this scaling changes a bit from the above
- The strength of an interaction then is going to be related to changes in an object's momentum!

Practice Time!

Suppose you bring a go-cart ($m = 100\,\mathrm{kg}$) and a semi-truck ($m = 2000\,\mathrm{kg}$) to a stop by pressing against them. Both initially have a velocity of $\langle 10, 0, 15 \rangle\,\mathrm{m/s}$ and you manage to stop them in $3\,\mathrm{s}$.

- At what speed are both objects traveling initially?
- What is the final momentum of both the go-cart and the semi-truck?
- What is the change in momentum of the go-cart?
- What is the $\Delta\vec{\mathbf{p}}$ of the semi-truck?
- How does the direction of $\Delta\vec{\mathbf{p}}$ compare for the go-cart and semi-truck?
- How does the magnitude of $\Delta\vec{\mathbf{p}}$ compare?
- Stopping which vehicle demanded the greater interaction?