# Announcements

- CompDay 5 due tonight
- HW6 due on Monday
- Midterm getting handed out on Monday
- Trying really hard to get grade reports posted over the weekend
- Responses: `rembold-class.ddns.net`

# Today's Objectives

- Understand how the Lagrangian formulation relates to Newton
- Be able to identify how many Lagrange equations will be necessary for a variety of problems
- Be able to compute kinetic and potential energies in other coordinate systems
- Be capable of reducing a Lagrange equation to a corresponding differential equation

# Q1

The Lagrangian is defined as

$$\mathcal{L} = T - U$$

What best describes why we subtract $U$ instead of adding it?

A) Because the difference between the kinetic and potential energy is the amount of energy available to a system

B) Because potential energy is the negative of the work done on a particle

C) Because forces point “downhill” on potential energy curves

D) Because when energy is conserved, $\Delta T = -\Delta U$

# Q2

How many Lagrange equation are necessary to completely describe the paths of 3 unconstrained particles in 2D space?

A) 1

B) 5

C) 6

D) 9

# Q3

One of the beautiful things about the Lagrangian Equations (and the Hamilton Principle in general) is that they look the same in any coordinate system. You still need to be able to write the kinetic and potential energies in whatever coordinate system you choose. As an example, take the coordinate system $u, v$ where:

$$x = v^2 \cos(u), \qquad y = v^2 \sin(u)$$

What would the kinetic energy of a particle of mass $m$ look like in this 2D coordinate system?

A) $\frac{1}{2} m v^2 \left(v^2 \dot{u}^2 + 4\dot{v}^2\right)$

B) $\frac{1}{2} m \left(\dot{v}^2 + \dot{u}^2\right)$

C) $\frac{1}{2} m \left(v^2 \dot{u}^2 - 4\dot{v}^2 + 4v^2 \dot{u}\dot{v} \cos(u) \sin(u)\right)$

D) $\frac{1}{2} m v^2 \left(v^2 \dot{u}^2 + 4\dot{v}^2 + 4v^2 \dot{u}\dot{v} \cos(u) \sin(u)\right)$

# Q4

The kinetic and potential energy of our previous particle are given by:

$$T = \frac{1}{2} m v^2 \left( v^2 \dot{u}^2 + 4 \dot{v}^2 \right) \quad \text{and} \quad U = m g v^2 \sin(u)$$

What would the differential equation from the Lagrangian equation in the $u$ direction look like?

A) $\ddot{u} - \dfrac{4 \dot{u} \dot{v}}{u} + \dfrac{g \cos(u)}{uv} = 0$

B) $\ddot{u} + \dfrac{4 \dot{u} \dot{v}}{v} + \dfrac{g \cos(u)}{v^2} = 0$

C) $\ddot{u} - \dfrac{4 \dot{u} \dot{v}}{v} - \dfrac{g \cos(u)}{uv} = 0$

D) $\ddot{u} + \dfrac{4 \dot{u} \dot{v}}{u} - \dfrac{g \cos(u)}{v^2} = 0$

# Q5

Suppose we have a particle constrained to move along the surface of a sphere (like yourself for instance). How many Lagrange equations would be need to describe the motion of that particle?

A) 1

B) 2

C) 3

D) 4