Announcements

- Homework
  - Homework 7 and Lab due on Monday
- Test a week from today!
  - Over Chapters 2,3,4
  - I'm trying to get study materials posted today or tomorrow
  - I'm also trying hard to have grading done by that point
- Final Project Info sent out today!
  - I'm getting potential datasets uploaded over the weekend.
  - You are always free to find your own as well. Just make sure they have enough variables to give your options for multiple regression.
- Polling: `rembold-class.ddns.net`

ANOVA in R

Run the ANOVA test on a linear fit to the data:

```
anova(lm(aldrin$aldrin ~ aldrin$depth))
```

```
Response: aldrin$aldrin
             Df Sum Sq Mean Sq F value   Pr(>F)
aldrin$depth  2 16.961  8.4803  6.1338 0.006367 **
Residuals    27 37.329  1.3826
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Checking Conditions: (1) independence

Does this condition appear to be satisfied?

In this study the we have no reason to believe that the aldrin concentration won't be independent of each other.

Checking Conditions: (2) approximately normal

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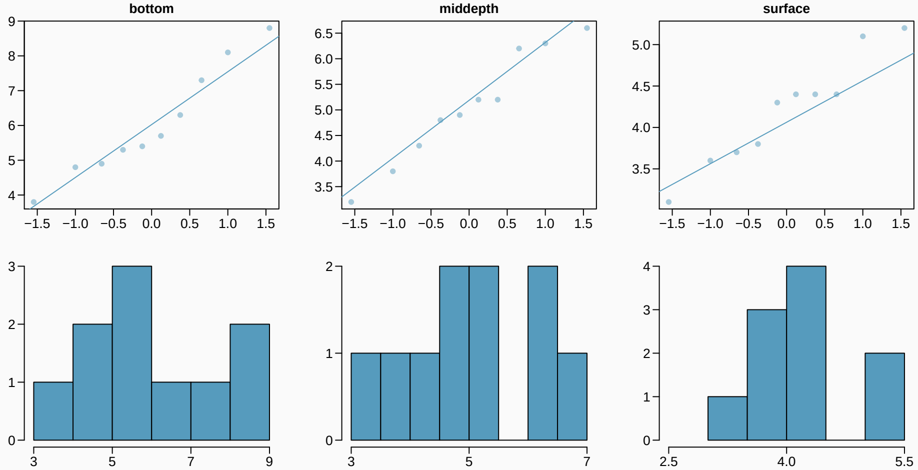

Checking Conditions: (3) constant variance

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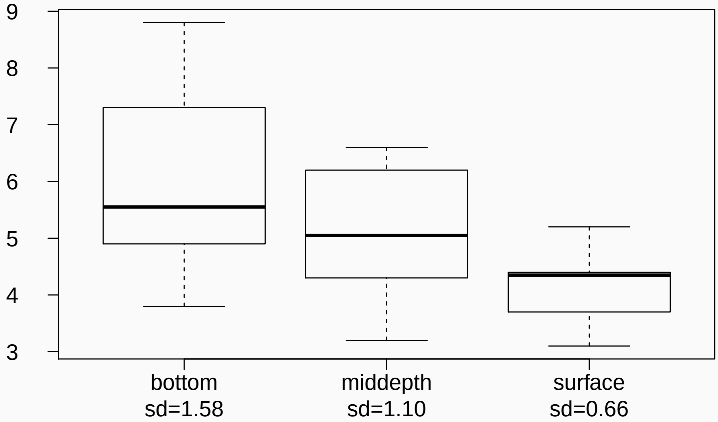

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- When we run too many tests, the Type 1 Error rate increases.
- This issue is resolved by using a modified significance level.

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  where K is the number of comparisons being considered.
- If there are k groups, then usually all possible pairs are compared and $K = \frac{k(k-1)}{2}$.

Understanding Check!

In the aldrin data set depth has 3 levels: bottom, mid-depth, and surface. If $\alpha = 0.05$, what should be the modified significance level for two sample t tests for determining which pairs of groups have significantly different means?

A) $\alpha^* = 0.05$

B) $\alpha^* = 0.05/2 = 0.025$

C) $\alpha^* = 0.05/3 = 0.0167$

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Which means differ?

If the ANOVA assumption of equal variability across groups is satisfied, we can use the data from all groups to estimate variability:

- Estimate any within-group standard deviation with $\sqrt{MSE}$, which is s_{pooled}
- Use the error degrees of freedom, $n - k$, for t-distributions

Difference in two means: after ANOVA

$$SE = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \approx \sqrt{\frac{MSE}{n_1} + \frac{MSE}{n_2}}$$

Bottom and Mid Check

Is there a difference between the average aldrin concentration at the bottom and at mid depth?

	n	mean	sd
bottom	10	6.04	1.58
middepth	10	5.05	1.10
surface	10	4.2	0.66
overall	30	5.1	1.37

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
depth	2	16.96	8.48	6.13	0.0063
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$$T_{df_E} = \frac{(\bar{x}_{bottom} - \bar{x}_{middepth})}{\sqrt{\frac{MSE}{n_{bottom}} + \frac{MSE}{n_{middepth}}}}$$

$$T_{27} = \frac{(6.04 - 5.05)}{\sqrt{\frac{1.38}{10} + \frac{1.38}{10}}} = \frac{0.99}{0.53} = 1.87$$

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Fail to reject H_0, the data do not provide convincing evidence of a difference between the average aldrin concentrations at bottom and mid depth.

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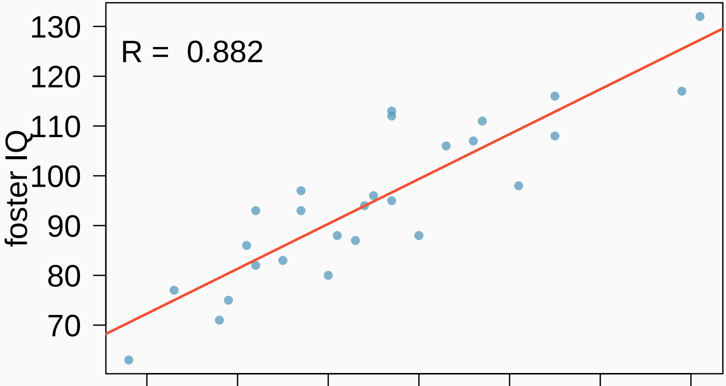
Reject H_0, the data provide convincing evidence of a difference between the average aldrin concentrations at bottom and surface.

Back to Linear Regression!

Nature or nurture?

In 1966 Cyril Burt published a paper called “The genetic determination of differences in intelligence: A study of monozygotic twins reared apart?” The data consist of IQ scores for [an assumed random sample of] 27 identical twins, one raised by foster parents, the other by the biological parents.



Testing for the slope

Assuming that these 27 twins comprise a representative sample of all twins separated at birth, we would like to test if these data provide convincing evidence that the IQ of the biological twin is a significant predictor of IQ of the foster twin. What are the appropriate hypotheses?

(a) $H_0 : b_0 = 0$; $H_A : b_0 \neq 0$

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Testing for the slope (cont.)

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	9.2076	9.2999	0.99	0.3316
bioIQ	0.9014	0.0963	9.36	0.0000

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  Remember: We lose 1 degree of freedom for each parameter we estimate, and in simple linear regression we estimate 2 parameters, β_0 *and* β_1.

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$$\begin{aligned} T &= \frac{0.9014 - 0}{0.0963} = 9.36 \\ df &= 27 - 2 = 25 \\ p - value &= P(|T| > 9.36) < 0.01 \end{aligned}$$

Confidence interval for the slope

Remember that a confidence interval is calculated as *point estimate* $\pm$ *ME* and the degrees of freedom associated with the slope in a simple linear regression is $n - 2$. Which of the below is the correct 95% confidence interval for the slope parameter? Note that the model is based on observations from 27 twins.

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- The regression output gives b_1, SE_{b_1}, and ***two-tailed*** p-value for the t-test for the slope where the null value is 0.
- We rarely do inference on the intercept, so we'll be focusing on the estimates and inference for the slope.

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- The ultimate goal is to have independent observations.