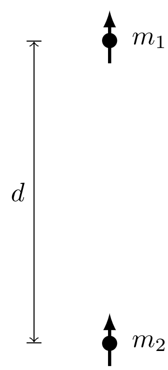
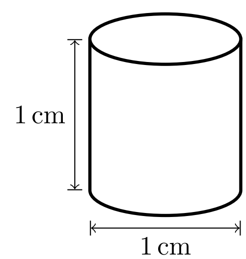
1. When playing with toy magnets they often seem to feel a force much more suddenly than might be implied by a $1/r^2$ force law. Probably because the force between two magnetic dipoles does not follow a $1/r^2$ relation!

   (a) Consider two small cylindrical magnets (which you can treat as prefect dipoles for the first part of this problem). The magnetics have dipole moments m_1 and m_2 and are separated by a distance d in the vertical direction. As drawn, with both magnetic dipoles facing in the same direction, calculate the force between the magnets. Does the sign of your resulting expression agree with what you'd expect?





   (b) You can make a crude estimation for the magnetic moment of a little toy magnet. Assume the atomic dipole moment of an iron atom is due to a single unpaired electron spin. The magnetic dipole moment of a single electron spin is

$$m_e = \left(\frac{e}{2M_e}\right)\frac{\hbar}{2}$$

   where M_e is the *mass* of the electron and e the charge. What is the total magnetic dipole moment of the simple cylindrical iron magnet with dimensions shown above? (*You'll need to look up density and atomic masses probably.*)

   (c) Say you were to stack two of the magnets such that they repelled one another. How high above the second magnet would you expect the first magnet to float? Does this seem reasonable?

2. An infinitely long cylinder (radius R), oriented with its axis along the $\hat{\boldsymbol{z}}$ direction, carries a "baked-in" magnetization of

$$\vec{\mathbf{M}} = ks\hat{\boldsymbol{z}}$$

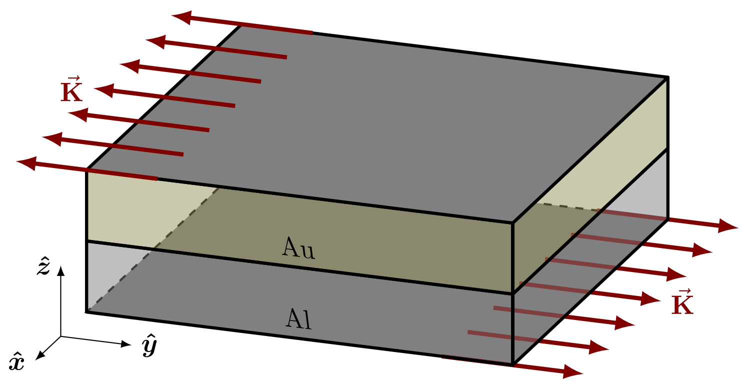
where k is a constant and s the normal cylindrical coordinate.

(a) Calculate all bound currents that might be present.

(b) Using those bound currents and normal Ampere's Law, calculate the magnetic field inside and outside the cylinder.

(c) Using the auxiliary form of Ampere's Law ($\oint \vec{\mathbf{H}} \cdot d\vec{\boldsymbol{\ell}} = \vec{\boldsymbol{\mathcal{I}}}_{enc,free}$), calculate the same magnetic fields inside and outside the cylinder. Do they agree with your calculations from part (b)?

3. Two infinite sheets have some surface current density $\vec{\mathbf{K}} = K_0(\pm\hat{\boldsymbol{y}})$ pointing in opposite directions. Between the two sheets (and insulated from them) lies an infinite slab of gold and an infinite slab of aluminum.



(a) Determine the auxiliary field $\vec{\mathbf{H}}$ everywhere in space. (*Hint: This should feel **real** familiar to you...*)

(b) Determine the magnitude and direction of the magnetic field $\vec{\mathbf{B}}$ in all 4 regions (above both plates, in gold, in aluminum, and below both plates).

(c) What is the net bound surface current density at the interface between the gold and aluminum? In what direction is it flowing?

4. If $\vec{\mathbf{J}}_{free} = 0$, then we arrive upon a situation where

$$\nabla \times \vec{\mathbf{H}} = 0$$

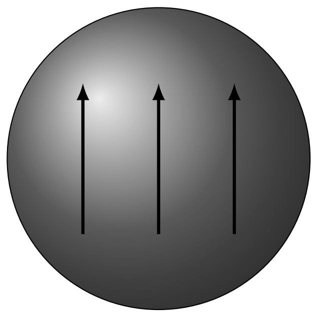
But this is the same condition we had back with electrostatics! So we can go ahead and write out a *scalar* potential W:

$$\vec{\mathbf{H}} = -\nabla W$$

which we can then use to write

$$\nabla^2 W = \nabla \cdot \vec{\mathbf{M}}$$

But this is just Poisson's/Laplace's equation, which we've already spent a chapter learning to solve! Let's use that machinery (separation of variables) to solve for the case of a uniformily magnetized sphere.



$\vec{\mathbf{M}} = M_0\hat{\boldsymbol{z}}$

(a) What is $\nabla \cdot \vec{\mathbf{M}}$ inside and outside the sphere?

(b) Write out general solutions to Laplace's equation for W_{in} and W_{out}. Can you immediately rule out some constants due to physical/realistic requirements?

(c) When solving Laplace's equation, we need to know boundary conditions in order to solve for the arbitrary constants. One of these requires that

$$W_{in}(R, \theta) = W_{out}(R, \theta)$$

The other relates the slopes of W at the surface (*Hint: check out Eq 6.24*). Use the boundary conditions to solve for all your arbitrary constants and write out the final expressions for W_{in} and W_{out}.

(d) Take the case of the potential interior to the sphere. Now that you've solved for the potential, you can work back to find $\vec{\mathbf{H}}$ and from there you can find $\vec{\mathbf{B}}$. Do so and confirm that you get:

$$\vec{\mathbf{B}} = \frac{2}{3}\mu_0\vec{\mathbf{M}}$$

(*Hint: It might be useful to convert back to Cartesian*)

(e) Ideally we'd like to visualize our solution. But solving for $\vec{\mathbf{H}}_{out}$ in Cartesian coordinates gets really ugly looking. For visualization purposes though, we don't really need an expression, so we would be fine finding $\vec{\mathbf{H}}$ numerically. If you construct a matrix of all the potentials W at different points in space, you can then use `np.gradient` to numerically calculate a corresponding gradient field. Use this to create a plot which shows a heatmap of the potential W and overtop that has a streamplot showing the path of the magnetic field lines. Take the radius of the sphere and M_0 to be 1 for plotting purposes.