A New Method for Estimating Cardiac Transmembrane Potentials from the Body Surface

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***Abstract*.** Transmembrane potentials (TMPs) on the heart surface can be used to calculate body-surface potentials (BSPs) using a bidomain model, in order, for example, to assess the sensitivity of BSPs to TMP changes with pathology. Conversely, TMPs can be estimated from BSPs with inverse methods. In this study, a new inverse approach called regularized waveform identification (RWI) was developed that combines spatial regularization with temporal optimization to estimate TMPs from BSPs with greater accuracy than conventional regularization alone. TMPs were estimated throughout the T wave, using the realistic ventricle-torso model and heart-surface TMPs of the ECGSIM simulation package. We evaluated the sensitivity of our RWI approach to 1, 2, 5 and 10% electrical noise on the body surface. Relative errors (RE) of <15% and correlation coefficients (CC) >0.98 were found. A 10% enlargement of the heart and position errors of ±1cm in all directions yielded REs of <15% and CCs >0.97. Simulation results showed that this approach performed much better than traditional regularization methods alone and is robust in the presence of noise and geometric error. By incorporating temporal information, in the form of the basic TMP wave shape, estimation accuracy was enhanced while maintaining computational simplicity.

Keywords: Bidomain, Electrocardiogram, Inverse Problem, Rank Deficient, Transmembrane Potential

1. Introduction

Diseases such as type 2 diabetes may lead to abnormalities in the transmembrane potentials (TMPs) of myocytes that are manifest as changes in body-surface potentials (BSPs), such as long QT syndrome. Direct inference of TMPs from body surface potentials for use in clinical practice, however, remains a challenge. Several groups have been working on the reconstruction of TMPs from BSPs [Messnarz et al., 2004; Tilg et al., 2002; van Oosterom, 2001]. Nevertheless, limited accuracy and sensitivity to noise are problems that have not been resolved. In this study, a new inverse approach called regularized waveform identification (RWI) was developed. RWI combines spatial regularization with temporal optimization to estimate TMPs from BSPs with greater accuracy than conventional regularization alone.

2. Material and Methods

The objective of the bidomain inverse problem is to estimate heart surface transmembrane potentials, Φ_m, from body surface potentials, V. The TMPs, Φ_m, are defined as the differences between intra- and extra-cellular potentials, i.e., they constitute the source in a bidomain model. Formulations to the inverse problem are based on the forward problem solution A, where A is a transfer matrix, calculated from heart-torso geometries and conductivities, that relates heart surface TMPs to BSPs, [Wang, 2009].

$$V = A\Phi_m \tag{1}$$

To solve the inverse problem, we developed an approach that we call regularized waveform identification (RWI) that builds estimates of Φ_m by combining regularization results with an averaged TMP waveform found via temporal optimization. See Fig. 1.

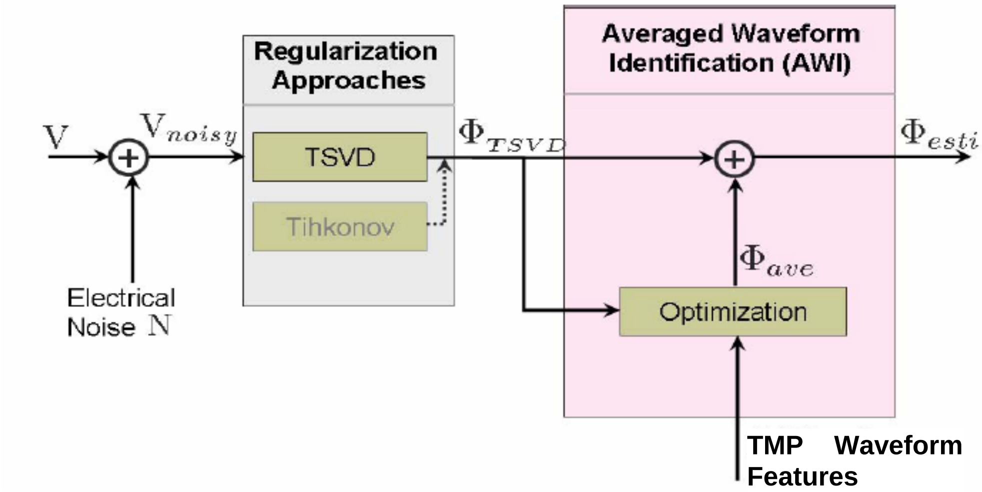

Figure 1. *RWI inverse-problem solution that combines 1) Conventional spatial regularization with 2) Identification of the averaged TMP waveform via temporal optimization.*

To evaluate our RWI approach, we used the heart-torso models from the ECGSIM package [van Oosterom and Oostendorp, 2004]. TMP estimates found with noise added to body-surface potentials and for errors in heart size and position were compared to the TMPs in the ECGSIM package.

2.1 Regularization

The least-squares-error solution for Φ_m in Eq. 1 at a given instant in the cardiac cycle is ill-posed. Regularization allows useful solutions for Φ_m. In the scheme depicted in Fig. 1, several regularization methods were tested. We found that regularization via truncated singular value decomposition (TSVD) was more effective than zero- or second-order Tikhonov regularization. The transfer matrix A has one or more near-zero singular values, which contribute to its ill-conditioned property. TSVD solves this problem by removing the small singular values from A, resulting in a rank-deficient matrix A_k. Source potentials were estimated with the pseudo-inverse of A_k, i.e.

$$\Phi_{TSVD} = (A_k^t A_k)^{-1} A_k^t V \tag{2}$$

The above solution is the minimum-norm least-squares solution [Hansen, 1997], as

$$\min \|\Phi_m\|^2 \text{ subject to } \|A_k \Phi_m - V\|^2 .$$

2.2 Averaged Waveform Identification (AWI)

As we saw the transfer matrix A of the bidomain model is not full-rank, so that the inverse problem is a rank-deficient one. Further, a uniform strength double-layer source will not produce an external field [van Oosterom, 2003], i.e.

$$Ae = A\begin{bmatrix}1\\1\\\vdots\\1\end{bmatrix} = [0] \tag{3}$$

Where, e is a unit vector, and $[0]$ is a zero vector. We then get:

$$\frac{1}{M} A \begin{bmatrix} 11\cdots1 \\ 11\cdots1 \\ \vdots \\ 11\cdots1 \end{bmatrix} \Phi_{esti} = A \begin{bmatrix} \phi_{ave} \\ \phi_{ave} \\ \vdots \\ \phi_{ave} \end{bmatrix} = A\Phi_{ave} = [0] \tag{4}$$

in which, Φ_{esti} represents the ideal estimation of Φ_m. The matrix Φ_{ave} has identical vectors, ϕ_{ave}, which represent the spatial average of Φ_{esti} and contain the temporal information of the TMPs. ϕ_{ave} is the averaged waveform and Φ_{ave} is the averaged waveform matrix. Since Φ_{esti} satisfies Eq. (1),

$$V = A\Phi_{esti} = A(\Phi_{esti} - \Phi_{ave}) = A\Phi_{dev} \tag{5}$$

so that

$$\|\Phi_{dev}\|^2 \leq \|\Phi_{esti}\|^2 \tag{6}$$

As we mentioned, the regularization result Φ_{TSVD} was found by following the criteria of minimizing the residual error in Eq. 1 and the energy level measured by $\|\Phi_m\|^2$. It is reasonable, therefore, to approximate Φ_{dev} with Φ_{TSVD}. It then follows that the final estimate is

$$\Phi_{esti} = \Phi_{ave} + \Phi_{TSVD} \tag{7}$$

To identify the averaged waveform ϕ_{ave}, we use a general waveform generated from the dominant T-wave [Oosteriom, 2003], and a repolarization timing parameter ρ_{ave} that is defined as the instant when the steepest potential descent occurs. It was determined with an unconstrained nonlinear optimization approach performed with the LMA algorithm in MATLAB®. Based on the TMP repolarization waveform, the following three criteria were used to formulate the optimization objective function.

- $\Phi_{esti} \leq 10$ mV, (TMP voltage is less than 10 mV);
- $\Phi_{esti} \geq -90$ mV, (TMP voltage is greater than -90 mV);
- $\Phi_{esti}(t_i) \geq \Phi_{esti}(t_j)\ \ (t_i \leq t_j)$, (TMP is monotonically decreasing during repolarization).

3. Results

Our inverse approach was tested by reconstructing TMPs with BSPs calculated from the reference TMPs using Eq. 1. The process is depicted in Fig. 2. Note that the regularization result Φ_{TSVD} does not show the basic TMP shape, but by adding Φ_{ave} to it, we closely approximate the reference TMPs.

We evaluated the sensitivity of our RWI approach to electrical noise on the body surface and to geometrical errors in the heart model as shown in Table 1. As is common in tests of this sort, 1, 2, 5 and 10% Gaussian noise was added to the BSPs. Relative errors of <15% and correlation coefficients >0.98 were found as shown in Table 1. We enlarged the heart size by 10% and shifted it by ±1cm in the X, Y and Z directions to simulate possible errors in heart models. REs of <15% and CCs >0.97 were found as shown in Table 2.

***Table 1**. Relative Error (RE) and Correlation Coefficient (CC) of Transmembrane Potentials with Noise*

Noise	0%	1%	2%	5%	10%
RE	0.03±0	0.12±0.0008	0.14±0.0016	0.13±0.0017	0.15±0.0037
CC	0.999±0	0.985±0.0001	0.983±0.0002	0.983±0.0002	0.980±0.0006

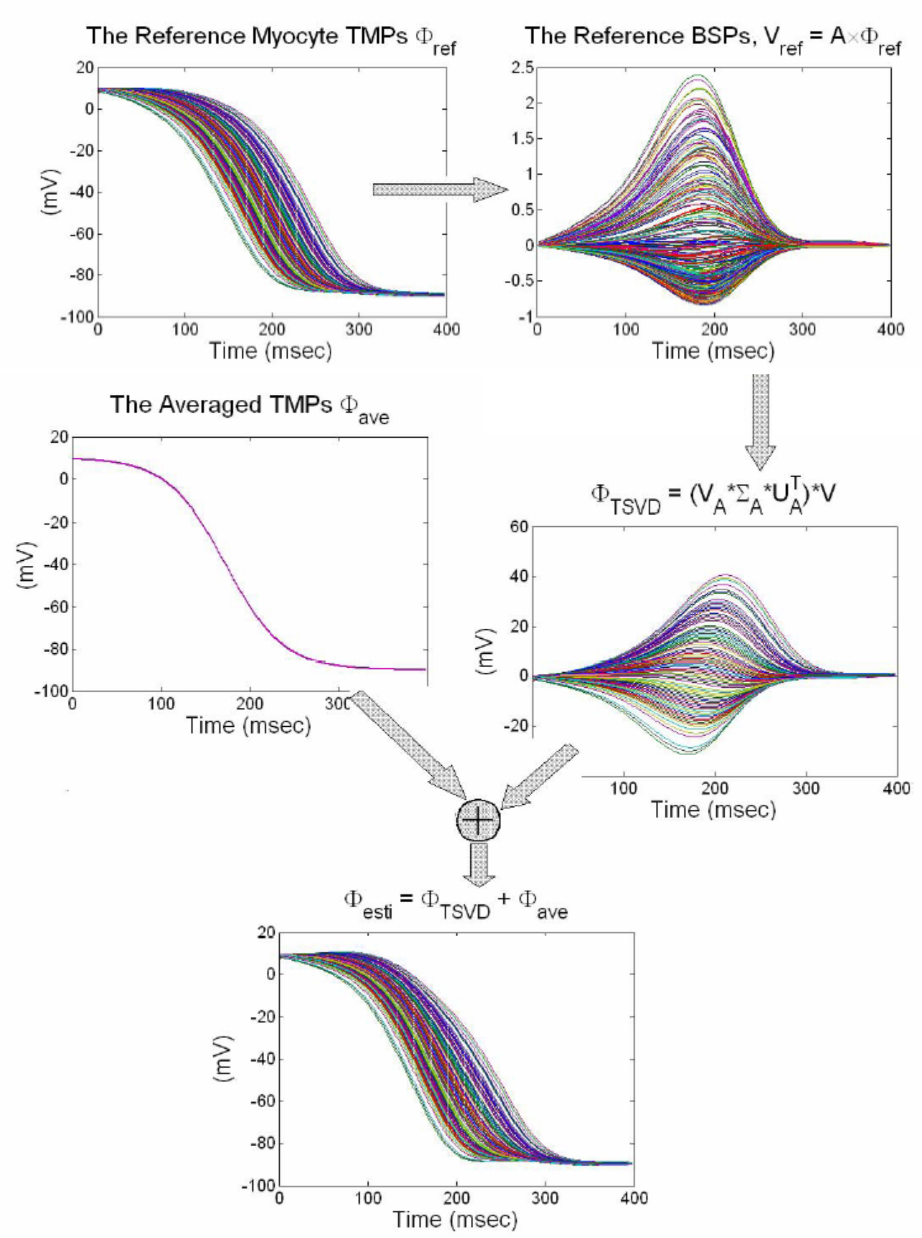

Figure 2*. The RWI inverse approach with no noise. The regularization result found using singular value truncation was added to average TMP waveform found via optimization. As shown in Table 1, the relative error for the estimated TMPs was 2.7%; the correlation coefficient was 0.999.*

Table 2*. RE and CC) of Transmembrane Potentials with Geometric Errors in Heart Size and Location.*

Geometrical Error	Size + 10%	X (cm)		Y (cm)		Z (cm)	
		1	-1	1	-1	1	-1
RE	0.125	0.131	0.117	0.113	0.116	0.112	0.152
CC	0.982	0.982	0.984	0.986	0.985	0.986	0.974

4. Conclusions

In this study, a new inverse approach, regularized waveform identification, was developed to reconstruct myocyte TMPs from BSPs. Simulation results showed that this approach performed much better than traditional regularization methods alone. By incorporating temporal information into the process, in the form of the general TMP wave shape, estimation accuracy was enhanced while maintaining computational simplicity.

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References

Hansen PC, Rank-deficient and Discrete Ill-posed Problems, SIAM, 1997, 45-69.

Messnarz B, Tilg B, Modre R, Fischer G, and Hanser F, A new spatiotemporal regularization approach for reconstruction of cardiac transmembrane potential patterns, *IEEE Transactions On Biomedical Engineering*, 51: 273-281, 2004.

Tilg B, Fischer G, Modre R, Hanser F, Messnarz B, Schocke M, Kremser C, Berger T, Hintringer F, and Roithinger FX, Model-based imaging of cardiac electrical excitation in humans, *IEEE Transactions On Medical Imaging*, 21: 1031-1039, 2002.

van Oosterom A and Oostendorp TF, ECGSIM; An interactive tool for studying the genesis of QRST waveforms, *Heart*, 90:165-168, 2004. (**http://www.ecgsim.org/**)

van Oosterom A, The dominant T wave and its significance, *J of Cardiovascular Electrophysiology*, 14: 180-187, 2003.

van Oosterom A, Genesis of the T wave as based on an equivalent surface source model, *J of Electrocardiol.*, 34: 217-227, 2001.

Wang S, Electrocardiographic Consequences of Electrical and Anatomical Remodeling in Diabetic and Obese Humans, PhD dissertation, Washington University in St. Louis, 2009.