Solution to the Final Project of Topics in Advanced Macroeconomics

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1. **the Bellman Equation:**

$$[r + d(h)]W(h,x) = \max_{\lambda}\{x - e(\lambda) + \delta[W(h,b) - W(h,x)]$$
$$+\lambda \int_x^1 \max\{0, W(h,y) - W(h,x)\}dF(y)\} + \alpha[W(h + \hat{h}, x) - W(h,x)]$$

where x is the current wage rate and y is the wage rate of the new offer. The upper limit of the integral is 1 because of the characteristic of the beta distribution.

We assume that in any instant the possibility of two events happening together is small, so that we only consider these events as happening in different instances.

When x = b, the value function is for the umemployed.

2. **steady state population distribution $g(h)$ by health status**

Since we assume that the probability distribution is stationary and even the whole population does not change, we have 6 equations which could help us pin down the value of η, as well as the population distribution for different health states $g(h)$:

$$\eta \cdot g(0) + (1 - \alpha) \cdot g(0) = g(0)$$

$$\alpha \cdot g(0) + (1-\alpha) \cdot g(0.2) = g(0.2)$$
$$\alpha \cdot g(0.2) + (1-\alpha) \cdot g(0.4) = g(0.4)$$
$$\alpha \cdot g(0.4) + (1-\alpha) \cdot g(0.6) = g(0.6)$$
$$\alpha \cdot g(0.6) + (1-\alpha) \cdot g(0.8) = g(0.8)$$
$$g(0) + g(0.2) + g(0.4) + g(0.6) + g(0.8) = 1$$

We are able to obtain

$$g(0) = g(0.2) = g(0.4) = g(0.6) = g(0.8) = 0.2$$

$$\eta = 0.2 \cdot \alpha = 0.2 \times 0.075 = 0.015$$

The above results will be useful in the simulation part.

equilibrium unemployment

- For $h = 0$

$$\delta(g(h) - u(h)) + \eta = (\lambda(h) + d(h) + \alpha)u(h)$$

(LHS = inflow to unemployment, RHS = outflow from unemployment)

$$\Rightarrow u(h = 0) = \frac{\delta g(h) + \eta}{\delta + \alpha + \lambda(h) + d(h)}$$

- For $h = 0.2, 0.4, h = 0.6$, and $h = 0.8$

$$\delta \cdot (g(h) - u(h)) + \alpha \cdot u(h - \hat{h}) = (\lambda(h) + d(h) + \alpha) \cdot u(h)$$

(LHS = inflow to unemployment, RHS = outflow from unemployment)

(Because this is a continuous time model, we can ignore the cases when two events happen simultaneously–for instance, $\alpha(1-\lambda)$. But we could sum up the probability of both events if they happen to two different groups of people)

$$\Rightarrow u(h = 0.2, 0.4, 0.6, 0.8) = \frac{\delta g(h) + \alpha \cdot u(h - \hat{h})}{\delta + \alpha + \lambda(h) + d(h)}$$

In the above equation(s) we can plug in $u(h - \hat{h})$ and iterate in sequence to compute for expressions of $u(h)$ for different levels of h.

The results will be useful when we simulate the steady state unemployment rate. And they will also be useful in the following equilibrium distribution:

equilibrium distribution of workers by firm type

$$[\delta + \lambda \cdot \overline{F}(w)](1-u)L(w) = u\lambda F(w)$$

Plugging in the expression(s) for $u(h)$ yields

- For $h = 0$

$$[\delta+\lambda\cdot\overline{F}(w)]\left(1-\frac{\delta g(h)+\eta}{\delta+\alpha+\lambda(h)+d(h)}\right)L(w) = \frac{\delta g(h)+\eta}{\delta+\alpha+\lambda(h)+d(h)}\lambda F(w)$$

- For $h = 0.2, h = 0.4, h = 0.6, h = 0.8$

$$[\delta+\lambda\cdot\overline{F}(w)]\left(1-\frac{\delta g(h)+\alpha\cdot u(h-\hat{h})}{\delta+\alpha+\lambda(h)+d(h)}\right)L(w) = \frac{\delta g(h)+\alpha\cdot u(h-\hat{h})}{\delta+\alpha+\lambda(h)+d(h)}\lambda F(w)$$

These two equations will later be used to derive the observed wage distribution by health status in question 5.

3.4.5. **An explanation for the codes and graphs**:

The codes are in the matlab file "search VFI.m". The value functions and policy functions across different health states are shown in different colors. I have left out the cases when $h = 0, h = 0.2$ because these two value functions do not converge, which would ruin the graph if I keep them.

In solving the model, I write the Bellman equation in matrix form. The columns denote different values of wage x, and the rows denote different values of search effort λ. I write a loop which runs through different values of health h. I am doing so because each value function with state variable $\hat{h}$ depends on another value function with a different state variable $h + \hat{h}$. And because only $V(h = 1) = 0$ is known, I am looping from the last health state $h = 1$ to the first health state $h = 0$.

In the simulation part, I am using:

- λ as an expression for the job finding rate. I am taking average over λ corresponding to each h.
- $\lambda\overline{F}(w)$ as an expression for the job transition rate. I dot multiply them and then take the average.
- $L(w) = \dfrac{u \cdot \lambda \cdot F(w)}{(1-u) \cdot (\delta + \lambda \cdot (1 - F(w)))}$ to obtain the observed CDF in numerical form. This equation is a transformation of the equation on slides p14, chap 8.

Results:

Table 1: simulation results

	job finding rate	unemployment rate	job transition rate
h = 0	0	0.20	0
h = 0.2	0.32	0.03	0.16
h = 0.4	0.09	0.03	0.05
h = 0.6	0.05	0.02	0.03
h = 0.8	0.02	0.01	0.01

(The graphs are shown on the next page.)

Graphs:

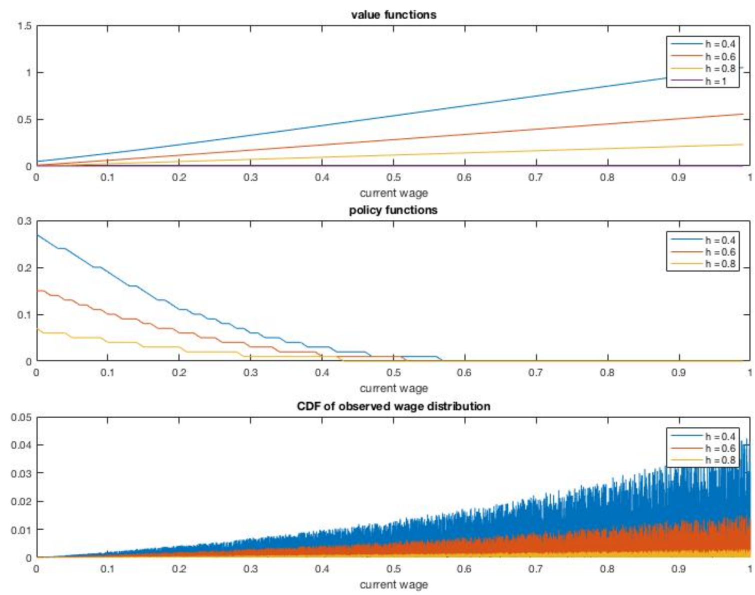

Figure 1: value functions, policy functions and CDF of observed wage distribution from file "searchVFI.m"