a Closed Economy

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July 17, 2020

1 Code description

Click on "config.mod" to select scenarios. Click on "main.mod" to see some deterministic simulation results.

The numbers in the code match the equation numbers in the appendix of DU (2015). The comments largely match the descriptions of the equations in this file.

2 Steady states

Because the model is log-linearized before being put into Dynare, the original steady state values appear as parameters, and the steady state values shown by the "steady" command are zeros. The original steady states are not declared in the parameter block, but values are passed on to them through their dependency on other parameter values and they are subsequently used in the model block. Those relationships are written in Table B.5 in the Appendix of DU (2015). I have only added the VAT collected in the government budget steady state.

3 Linearized equilibrium conditions

Except for the ARRA equations and variables which I have excluded, I have made changes to only two of the equations in DU(2015), which are the equations for deficits and price inflation. And I have created one exogenous process for the evolution of the VAT. I have kept all the equations in the same sequence as those in the appendix of DU(2015), so that they are easy to track.

3.1 Firms

The production function

$$\hat{y}_t = \frac{\bar{y}+\Phi}{\bar{y}}\left[\alpha\hat{k}_t + (1-\alpha)\hat{n}_t + \hat{\epsilon}_t^a\right]$$

The capital-labor-ratio

$$\hat{k}_t = \hat{w}_t - \hat{r}_t^k + \hat{n}_t$$

marginal costs

$$\widehat{mc}_t = (1-\alpha)\hat{w}_t + \alpha\hat{r}_t^k - \hat{\epsilon}_t^a\tilde{\epsilon}_t^a$$

marginal costs in the flexible economy

$$\widehat{mc}_t^f = 0$$

optimal pricing decisions

$$\begin{aligned}\hat{\pi}_t =& \frac{1}{1+\bar{\beta}\mu\iota_p}\iota_p\hat{\pi}_{t-1} + \frac{\bar{\beta}\mu}{1+\beta\mu\iota_p}\,\mathbb{E}_t\,\hat{\pi}_{t+1}\\ &+ \frac{(1-\tau^v)(1-\zeta_p)(1-\bar{\beta}\mu\zeta_p)}{\zeta_p(1+\bar{\beta}\mu\iota_p)}\bar{A}_p(\widehat{mc}_t + \hat{\epsilon}_t^{\lambda,p})\\ &- \frac{(1-\bar{\beta}\mu\zeta_p)(1-\zeta_p)\tau^v}{\zeta_p(1+\bar{\beta}\mu\iota_p)(1+\tau^v)}\widehat{\tau}_t^v\end{aligned}$$

3.2 Households

law of motion for capital

$$\hat{k}_t^p = \left(1-\frac{\bar{x}}{\bar{k}^p}\right)\hat{k}_{t-1}^p + \frac{\bar{x}}{\bar{k}^p}\left(\hat{x}_t + \hat{q}_{t+s}^x\right)$$

capital services

$$\hat{k}_t = \hat{u}_t + \hat{k}_{t-1}^p$$

the static optimality condition

$$\hat{w}_t^h = \nu \hat{n}_t + \frac{\hat{c}_t - (h/\mu)\hat{c}_{t-1}}{1-h/\mu} + \frac{d\tau_t^{n,f}}{1-\bar{\tau}^n}$$

F.O.C for labor

$$\begin{aligned}
\hat{w}_t =& \frac{\hat{w}_{t-1}}{1+\bar{\beta}\mu} + \frac{\bar{\beta}\mu E_t\left[\hat{w}_{t+1}\right]}{1+\bar{\beta}\mu} \\
&+ \frac{\left(1-\zeta_w\bar{\beta}\mu\right)(1-\zeta_w)}{(1+\bar{\beta}\mu)\zeta_w}\bar{A}_w\left[\frac{1}{1-h/\mu}\left[\hat{c}_t - (h/\mu\hat{c}_{t-1})\right] + \nu\hat{n}_t - \hat{w}_t + \frac{d\tau_t^n}{1-\tau_n}\right] \\
&- \frac{1+\bar{\beta}\mu\iota_w}{1+\bar{\beta}\mu}\hat{\pi}_t + \frac{\iota_w}{1+\bar{\beta}\mu}\hat{\pi}_{t-1} + \frac{\bar{\beta}\mu}{1+\bar{\beta}\mu}E_[\hat{\pi}_{t+1}] + \frac{\hat{\epsilon}_t^{\lambda,w}}{1+\bar{\beta}\mu}
\end{aligned}$$

F.O.C for consumption

$$\begin{aligned}
\hat{c}_t =& \frac{1}{1+h/\mu}E_t\left[\hat{c}_{t+1}\right] + \frac{h/\mu}{1+h/\mu}\hat{c}_{t-1} \\
&+ \frac{1-h/\mu}{\sigma[1+h/\mu]}E_t[\hat{\xi}_{t+1} - \hat{\xi}_t] \\
&- \frac{[\sigma-1][\bar{w}\bar{n}/\bar{c}]}{\sigma[1+h/\mu]}\frac{1}{1+\lambda_w}\frac{1-\tau^n}{1+\tau^c}(E_t[\hat{n}_{t+1}] - \hat{n}_t)
\end{aligned}$$

F.O.C for bond holdings

$$E_t[\hat{\xi}_{t+1} - \hat{\xi}_t] = -\hat{q}_t^b - \hat{R}_t + E_t[\hat{\pi}_{t-1}]$$

F.O.C for private capital

$$\begin{aligned}
\hat{Q}_t =& -\hat{q}_t^b - (\hat{R}_t - E_t[\hat{\pi}_{t+1}]) + \frac{1}{\bar{r}^k(1-\tau^k) + \delta\tau^k + 1 - \delta} \\
&\times\left[(\bar{r}^k(1-\tau^k) + \delta\tau^k)\hat{q}_t^k - (\bar{r}^k - \delta)d\tau_{t+1}^k \right. \\
&\left. + \bar{r}^k(1-\tau^k)E_t(\hat{r}_{t+1}^k) + (1-\delta)E_t(\hat{Q}_{t+1})\right]
\end{aligned}$$

F.O.C for private investment

$$\hat{x}_t = \frac{1}{1+\bar{\beta}\mu}\left[\hat{x}_{t-1} + \bar{\beta}\mu E_t(\hat{x}_{t+1}) + \frac{1}{\mu^2 S''(\mu)}[\hat{Q}_t + \hat{q}_t^x]\right]$$

F.O.C for capital utilization

$$\hat{u}_t = \frac{a'(1)}{a''(1)}\hat{r}^k_t \equiv \frac{1-\psi_u}{\psi_u}\hat{r}^k_t$$

change in profits

$$\frac{d\Pi^p_t}{\bar{y}} = \frac{1}{1+\lambda_p}\hat{y}_t - \widehat{mc}_t$$

3.3 The government

$$\begin{aligned}
&\frac{1}{\bar{R}}\frac{\bar{b}}{\bar{y}}[\hat{b}_t - \hat{R}_t - \hat{q}^b_t] + \bar{\tau}^n\frac{\bar{w}\bar{n}}{\bar{c}}\frac{\bar{c}}{\bar{y}}\frac{d\tau^n_t}{\tau^n} + \frac{\bar{c}}{\bar{y}}\frac{d\tau^v_t}{(1+\tau^v)^2} + \bar{\tau}^k\frac{\bar{k}}{\bar{y}}[\bar{r}^k - \delta]\frac{d\tau^k_t}{\tau^k} \\
&+ \frac{\bar{\tau}^v}{1+\bar{\tau}^v}\frac{\bar{c}}{\bar{y}}\hat{c}_t + \bar{\tau}^n\frac{\bar{w}\bar{n}}{\bar{c}}\frac{\bar{c}}{\bar{y}}(\hat{w}_t + \hat{n}_t) + \bar{\tau}^k\frac{\bar{k}}{\bar{y}}\left[\bar{r}^k\hat{r}^k_t + (\bar{r}^k_t - \delta)\hat{k}^p_{t-1}\right] \\
=&\hat{g}_t + \frac{\bar{s}}{\bar{y}}\hat{s}_t + \frac{\bar{b}}{\mu\bar{y}}\frac{\hat{b}_{t-1} - \hat{\pi}_t}{\bar{\pi}}
\end{aligned}$$

3.4 The monetary policy rule

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)[\psi_1\hat{\pi}_t + \psi_2(\hat{y}_t - \hat{y}^f_t)] + \psi_3\Delta(\hat{y}_t - \hat{y}^f_t) + \hat{\epsilon}^r_t$$

3.5 Market clearing

$$\hat{y}_t = \frac{\bar{c}}{\bar{y}}\hat{c}_t + \frac{\bar{x}}{\bar{y}}\hat{x}_t + \hat{g}_t + \frac{\bar{r}^k\bar{k}}{\bar{y}}\hat{u}_t$$

3.6 Exogenous processes

technology

$$\hat{\epsilon}^a_t = \rho_a\hat{\epsilon}^a_{t-1} + u^a_t$$

monetary policy

$$\hat{\epsilon}^r_t = \rho_r\hat{\epsilon}^r_{t-1} + u^r_t$$

total government purchase

$$\hat{g}_t = \hat{g}_{t-1} + \tilde{u}_t^g$$

transfer

$$\hat{s}_t = \tilde{u}_t^s$$

tax

$$\hat{\epsilon}_t^\tau = \rho_\tau \hat{\epsilon}_{t-1}^\tau + u_t^\tau$$

price mark-up shock

$$\hat{\tilde{\epsilon}}_t^{\lambda,p} = \rho_{\lambda,p} \hat{\tilde{\epsilon}}_{t-1}^{\lambda,p} + u_t^{\lambda,p} - \theta_{\lambda,p} u_{t-1}^{\lambda,p}$$

wage mark-up shock

$$\hat{\tilde{\epsilon}}_t^{\lambda,w} = \rho_{\lambda,w} \hat{\tilde{\epsilon}}_{t-1}^{\lambda,w} + u_t^{\lambda,w} - \theta_{\lambda,w} u_{t-1}^{\lambda,w}$$

government bond wedge

$$\hat{q}_t^b = \rho_b \hat{q}_{t-1}^b + u_t^b$$

private bond wedge

$$\hat{q}_t^k = \rho_k \hat{q}_{t-1}^k + u_t^k$$

relative price of investment

$$\hat{q}_t^x = \rho_b \hat{q}_{t-1}^x + u_t^x$$

4 Explanation for extended parts

This section explains my extensions. While it is easy to include the VAT in the government budget, it is not so straightforward in the price dynamics, which I write down here.

4.1 VAT in the firms' problem

4.1.1 The profit-maximization problem

Uhlig's appendix includes an optimal relative price determination equation (given after D.12). If VAT is included, this equation becomes

$$P_t^\star(i) = \arg\max_{\tilde{P}_t(i)} \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \frac{\bar{\beta}^s \xi_{t+s} P_t}{\xi_t P_{t+s}} \left[\frac{\tilde{P}_t(i)}{1+\tau_{t+s}^v} \left(\prod_{l=1}^{s} \pi_{t+l-1}^{\iota_p} \bar{\pi}^{1-\iota_p} \right) - MC_{t+s}(i) \right] Y_{t+s}(i)$$

First, we find out the derivative before taking the sum: (From here until log-linearization, η_p and λ_p are variables with their time indices omitted temporarily.)

$$\begin{aligned}
&\frac{d\left(Y_{t+s}(i)\left[\frac{P_{t+s}(i)}{1+\tau_{t+s}^v} - MC_{t+s}(i)\right]\right)}{d\tilde{P}_t(i)} \\
=&\frac{1}{1+\tau_{t+s}^v}\frac{d\left[\tilde{P}_t(i)\chi_{t,t+s}Y_{t+s}(i)\right]}{d\tilde{P}_t(i)} - \frac{d\left[MC_{t+s}Y_{t+s}(i)\right]}{d\tilde{P}_t(i)} \\
=&\frac{1}{1+\tau_{t+s}^v}\chi_{t,t+s}Y_{t+s}(i) + \frac{1}{1+\tau_{t+s}^v}\chi_{t,t+s}\tilde{P}_t(i)\frac{dY_{t+s}(i)}{d\tilde{P}_t(i)} - MC_{t+s}(i)\left[-\eta_p\frac{Y_{t+s}(i)}{P_{t+s}(i)}\right] \\
=&\frac{1}{1+\tau_{t+s}^v}\chi_{t,t+s}Y_{t+s}(i) + \frac{1}{1+\tau_{t+s}^v}\chi_{t,t+s}\tilde{P}_t(i)\frac{dY_{t+s}(i)}{\frac{1}{\chi_{t,t+s}}dP_{t+s}(i)} - MC_{t+s}(i)\left[-\eta_p\frac{Y_{t+s}(i)}{P_{t+s}(i)}\right] \\
=&\frac{1}{1+\tau_{t+s}^v}\chi_{t,t+s}Y_{t+s}(i) + \frac{1}{1+\tau_{t+s}^v}\chi_{t,t+s}P_{t+s}(i)\left[-\eta_p\frac{Y_{t+s}(i)}{P_{t+s}(i)}\right] + MC_{t+s}(i)\left[\eta_p\frac{Y_{t+s}(i)}{P_{t+s}(i)}\right] \\
=&\frac{1}{1+\tau_{t+s}^v}\chi_{t,t+s}Y_{t+s}(i) + \frac{1}{1+\tau_{t+s}^v}\left[-\eta_p\chi_{t,t+s}Y_{t+s}(i)\right] + MC_{t+s}(i)\left[\eta_p\frac{Y_{t+s}(i)}{P_{t+s}(i)}\right] \\
=&y_{t+s}(i)Y_{t+s}\left(\frac{1}{1+\tau_{t+s}^v}\chi_{t,t+s}\left[1-\eta_p\left(y_{t+s}(i)\right)\right] + \eta_p\frac{MC_{t+s}(i)}{P_t(i)}\right)
\end{aligned}$$

Then the F.O.C of the sum is given by:

$$\begin{aligned}
&\mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \frac{\bar{\beta}^s \xi_{t+s} P_t}{\xi_t P_{t+s}} \left[\frac{\tilde{P}_t(i)}{1+\tau_{t+s}^v} \left(\prod_{l=1}^{s} \pi_{t+l-1}^{\iota_p} \bar{\pi}^{1-\iota_p} \right) - MC_{t+s}(i) \right] Y_{t+s}(i) \\
=&\mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \frac{\bar{\beta}^s \xi_{t+s} P_t}{\xi_t P_{t+s}} y_{t+s}(i)Y_{t+s}\left(\frac{1}{1+\tau_{t+s}^v}\chi_{t,t+s}\left[1-\eta_p\left(y_{t+s}(i)\right)\right] + \eta_p\frac{MC_{t+s}(i)}{P_t(i)}\right) \\
=&0
\end{aligned}$$

Aggregate output Y_{t+s} can be detrended by the growth rate μ and be removed from the equation.

As we have the relationship between markup and the elasticity of substitution:

$$\eta_p = \frac{1}{\lambda_p} + 1$$

Substituting and rearranging,

$$\begin{aligned}
&\mathbb{E}_t \sum_{s=0}^{\infty} \frac{\left(\mu\bar{\beta}\zeta_p\right)^s \xi_{t+s} P_t}{\xi_t P_{t+s}} y_{t+s}(i) \left[\frac{1}{1+\tau_{t+s}^v}\chi_{t,t+s}\left(-\frac{1}{\lambda_p}\right) + \eta_p \frac{MC_{t+s}(i)}{P_{t+s}}\frac{P_{t+s}}{P_t}\frac{P_t}{P_t(i)}\right] = 0 \\
\Rightarrow &\mathbb{E}_t \sum_{s=0}^{\infty} \frac{\left(\mu\bar{\beta}\zeta_p\right)^s \xi_{t+s}}{\xi_t} y_{t+s}(i) \left[\frac{1}{\left(1+\tau_{t+s}^v\right)\prod_{l=1}^{s}\pi_{t+l}}\chi_{t,t+s}\left(\frac{1}{\lambda_p}\right) p_t^{\star}(i) - \eta_p mc_{t+s}(i)\right] = 0 \\
\Rightarrow &\mathbb{E}_t \sum_{s=0}^{\infty} \frac{\left(\mu\bar{\beta}\zeta_p\right)^s \xi_{t+s}}{\xi_t}\left(\frac{1}{\lambda_p}\right) y_{t+s}(i) \left[\frac{1}{\left(1+\tau_{t+s}^v\right)\prod_{l=1}^{s}\pi_{t+l}}\chi_{t,t+s} p_t^{\star}(i) - (1+\lambda_p) mc_{t+s}(i)\right] = 0 \\
\Rightarrow &p_t^{\star}(i) = \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \frac{\left(\mu\bar{\beta}\zeta_p\right)^s \xi_{t+s}}{\xi_t} y_{t+s}(i)\left[\eta_p mc_{t+s}(i)\right]}{\mathbb{E}_t \sum_{s=0}^{\infty} \frac{\left(\mu\bar{\beta}\zeta_p\right)^s \xi_{t+s}}{\xi_t} y_{t+s}(i)\left[\frac{1}{\left(1+\tau_{t+s}^v\right)\prod_{l=1}^{s}\pi_{t+l}}\chi_{t,t+s}\frac{1}{\lambda_p}\right]} \\
\Rightarrow &p_t^{\star}(i) = \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \frac{\left(\mu\bar{\beta}\zeta_p\right)^s \xi_{t+s}}{\xi_t\lambda_p} y_{t+s}(i)\left[\frac{\eta_p}{\eta_p-1} mc_{t+s}(i)\right]}{\mathbb{E}_t \sum_{s=0}^{\infty} \frac{\left(\mu\bar{\beta}\zeta_p\right)^s \xi_{t+s}}{\xi_t\lambda_p} y_{t+s}(i)\left[\frac{\chi_{t,t+s}}{\left(1+\tau_{t+s}^v\right)\prod_{l=1}^{s}\pi_{t+l}}\right]}
\end{aligned}$$

The last two expressions are just for one to know the optimal relative price, and not directly useful for deriving the Phillips curve.

Marginal cost is the same across all agents.

According to DU page XXXI and SW page 18, the term $\frac{\xi_{t+s}y_{t+s}(i)}{\xi_t\lambda_p}$ can be cancelled out to a first order (why?). Taking the equation given earlier,

$$\begin{aligned}
&\mathbb{E}_t \sum_{s=0}^{\infty} \left(\mu\bar{\beta}\zeta_p\right)^s \left[\frac{1}{\left(1+\tau_{t+s}^v\right)\prod_{l=1}^{s}\pi_{t+l}}\chi_{t,t+s} p_t^{\star}(i) - (1+\lambda_{p,t+s}) mc_{t+s}(i)\right] = 0 \\
\Rightarrow &\mathbb{E}_t \sum_{s=0}^{\infty} \left(\mu\bar{\beta}\zeta_p\right)^s \left[\frac{1}{1+\tau^v} d\left(\frac{\prod_{l=1}^{s}\pi_{t+l-1}^{\iota_p}\bar{\pi}^{1-\iota_p}}{\prod_{l=1}^{s}\pi_{t+l}}\right) + \frac{1}{1+\tau^v} d\left(p_t^{\star}(i)\right) + d\left(\frac{1}{1+\tau_{t+s}^v}\right)\right. \\
&\left. - \overline{mc} d\left(\lambda_{p,t+s}\right) - (1+\lambda_p) d(mc_{t+s})\right] = 0
\end{aligned}$$

Like SW, minus the same expression for time $t+1$ which is multiplied by $\mu\bar{\beta}\zeta_p$. (Some terms can be cancelled and some terms can be separated from $\sum_{s=0}^{\infty}\left(\mu\bar{\beta}\zeta_p\right)^s$)

$$
\begin{aligned}
&\frac{1}{1-\mu\bar{\beta}\zeta_p}\frac{1}{1+\tau^v}d\left(p_t^\star(i)\right)-\frac{\mu\bar{\beta}\zeta_p}{1-\mu\bar{\beta}\zeta_p}\frac{1}{1+\tau^v}d\left(p_{t+1}^\star(i)\right)\\
&+\frac{\mu\bar{\beta}\zeta_p}{1-\mu\bar{\beta}\zeta_p}\frac{1}{1+\tau^v}\left[\iota_p\frac{\bar{\pi}^{\iota_p-1}\bar{\pi}^{1-\iota_p}}{\bar{\pi}}d(\pi_t)-\frac{\bar{\pi}^{\iota_p}\bar{\pi}^{1-\iota_p}}{(\bar{\pi})^2}d(\pi_{t+1})\right]\\
&-d\left(\frac{1}{1+\tau_t^v}\right)-\overline{mc}d\left(\lambda_{p,t}\right)-(1+\lambda_p)d(mc_t)\\
=&0
\end{aligned}
$$

in terms of deviations from steady state:

$$
\begin{aligned}
&\frac{1}{1-\mu\bar{\beta}\zeta_p}\frac{1}{1+\tau^v}\widehat{p}_t^\star(i)-\frac{\mu\bar{\beta}\zeta_p}{1-\mu\bar{\beta}\zeta_p}\frac{1}{1+\tau^v}\,\mathbb{E}_t\,\widehat{p}_{t+1}^\star(i)\\
&+\frac{\mu\bar{\beta}\zeta_p}{1-\mu\bar{\beta}\zeta_p}\frac{1}{1+\tau^v}\left(\iota_p\widehat{\pi}_t-\mathbb{E}_t\,\widehat{\pi}_{t+1}\right)\\
&+\frac{\tau^v}{(1+\tau^v)^2}\widehat{\tau}_t^v-\overline{mc}\bar{\lambda}_p\widehat{\lambda}_{p,t}-(1+\bar{\lambda}_p)\overline{mc}\widehat{mc}_t\\
=&0
\end{aligned}
$$

Take the linearized evolution of the price index:

$$
\hat{p}_t^\star=\frac{\zeta_p}{1-\zeta_p}\left(\hat{\pi}_t-\iota_p\hat{\pi}_{t-1}\right)
$$

Substituting for $\widehat{p}_t^\star$ and $\widehat{p}_{t+1}^\star$:

$$
\begin{aligned}
&\frac{1}{1-\mu\bar{\beta}\zeta_p}\frac{1}{1+\tau^v}\frac{\zeta_p}{1-\zeta_p}\left(\hat{\pi}_t-\iota_p\hat{\pi}_{t-1}\right)-\frac{\mu\bar{\beta}\zeta_p}{1-\mu\bar{\beta}\zeta_p}\frac{1}{1+\tau^v}\frac{\zeta_p}{1-\zeta_p}\left(\mathbb{E}_t\,\hat{\pi}_{t+1}-\iota_p\hat{\pi}_t\right)\\
&+\frac{\mu\bar{\beta}\zeta_p}{1-\mu\bar{\beta}\zeta_p}\frac{1}{1+\tau^v}\left(\iota_p\widehat{\pi}_t-\mathbb{E}_t\,\widehat{\pi}_{t+1}\right)+\frac{\tau^v}{(1+\tau^v)^2}\widehat{\tau}_t^v-\overline{mc}\bar{\lambda}_p\widehat{\lambda}_{p,t}-(1+\bar{\lambda}_p)\overline{mc}\widehat{mc}_t\\
=&0
\end{aligned}
$$

Multiply both sides by $(1-\mu\bar{\beta}\zeta_p)(1+\tau^v)(1-\zeta_p)$:

$$
\begin{aligned}
&\zeta_p\left(\hat{\pi}_t-\iota_p\hat{\pi}_{t-1}\right)-(\mu\bar{\beta}\zeta_p)\zeta_p\left(\mathbb{E}_t\,\hat{\pi}_{t+1}-\iota_p\hat{\pi}_t\right)+(1-\zeta_p)(\mu\bar{\beta}\zeta_p)\left(\iota_p\widehat{\pi}_t-\mathbb{E}_t\,\widehat{\pi}_{t+1}\right)\\
&+(1-\mu\bar{\beta}\zeta_p)(1+\tau^v)(1-\zeta_p)\left[\frac{\tau^v}{(1+\tau^v)^2}\widehat{\tau}_t^v-\overline{mc}\bar{\lambda}_p\widehat{\lambda}_{p,t}-(1+\bar{\lambda}_p)\overline{mc}\widehat{mc}_t\right]\\
=&0
\end{aligned}
$$

Collecting terms:

$$
\begin{aligned}
&\zeta_p(1+\mu\bar{\beta}\iota_p)\hat{\pi}_t - \mu\bar{\beta}\zeta_p\,\mathbb{E}_t\,\hat{\pi}_{t+1} - \zeta_p\iota_p\hat{\pi}_{t-1} \\
&+(1-\mu\bar{\beta}\zeta_p)(1+\tau^v)(1-\zeta_p)\left[\frac{\tau^v}{(1+\tau^v)^2}\widehat{\tau}_t^v - \overline{mc}\bar{\lambda}_p\widehat{\lambda}_{p,t} - (1+\bar{\lambda}_p)\overline{mc}\widehat{mc}_t\right] \\
=&0
\end{aligned}
$$

Solving for $\hat{\pi}_t$:

$$
\begin{aligned}
\hat{\pi}_t =& \frac{1}{1+\bar{\beta}\mu\iota_p}\iota_p\hat{\pi}_{t-1} + \frac{\bar{\beta}\mu}{1+\beta\mu\iota_p}\,\mathbb{E}_t\,\hat{\pi}_{t+1} \\
&+\frac{(1-\tau^v)(1-\zeta_p)(1-\bar{\beta}\mu\zeta_p)}{\zeta_p(1+\bar{\beta}\mu\iota_p)}\bar{A}_p(\widehat{mc}_t + \hat{\epsilon}_t^{\lambda,p}) \\
&-\frac{(1-\bar{\beta}\mu\zeta_p)(1-\zeta_p)\tau^v}{\zeta_p(1+\bar{\beta}\mu\iota_p)(1+\tau^v)}\widehat{\tau}_t^v
\end{aligned}
$$

Some definitions which are employed above:

Define $y(i) = Y(i)/Y$ as the relative output. For the Kimball aggregator, the solution to the cost minimization problem is

$$
P(i) = \frac{\Lambda}{Y}G'(y(i))
$$

To obtain the price elasticity of demand

$$
\eta(y(i)) = -\frac{\frac{dY(i)}{Y(i)}}{\frac{dP(i)}{P(i)}}
$$

We use

$$
\frac{dY(i)}{Y(i)} = \frac{Y\,dy(i)}{Y(i)} = \frac{dy(i)}{y(i)}
$$

$$
\frac{dP(i)}{P(i)} = d\ln P(i) = d\ln\left[\frac{\Lambda}{Y}G'(y(i))\right] = \frac{G''(y(i))}{G'(y(i))}dy(i)
$$

Therefore

$$
\eta(y(i)) = -\frac{1}{y(i)}\frac{G'(y(i))}{G''(y(i))}
$$

A_p is given by

$$A_p(y_t(i)) \equiv \frac{\lambda^p(y_t(i))}{2 + \frac{G'''(y_t(i))}{G''(y_t(i))} y_t(i)}$$

On the balanced growth path

$$A_p(y) = \frac{1 + G''(y)/G'(y)}{2 + G'''(y)/G''(y)} = \frac{1}{\lambda^p(y)\hat{\eta}_p(y) + 1}$$

$\lambda^p(y)$ denotes the markup on the balanced growth path. In the Dixit-Stiglitz case, $\hat{\eta}_p(y) = 0$ and $A = 1$.

Interpretations of A: The degree to which $\hat{p}_t^\star$ responds to current and future values of the deviations in the marginal cost is increasing in A, which inturn depends on the properties of $G(\cdot)$. (In the model of Eichenbaum and Fisher (2004), there is the relationship that $\hat{p}_t = AE_{t-\tau}\hat{mc}_t$.)

A rise in marginal cost induces a firm to increase its price. A higher value of $\hat{\eta}_t^p(y(i))$ means that, for any given rise in its price, the more elastic is the demand curve for the firm's good. So relative to $\hat{\eta}_t^p(y(i)) = 0$, the firm will raise its price by less. Inflation will respond by less to movements in marginal cost.

4.2 VAT in the government budget constraint

The government budget constraint before linearization (given in DU D.44):

$$\frac{b_t}{R_t^{gov}} + \frac{\tau_t^v}{1+\tau_t^v} c_t + \tau_t^n n_t w_t + \tau_t^k k_t^s r_t^k - \tau_t^k \left[a(u_t) + \delta\right] \frac{k_{t-1}^p}{\mu} = \bar{y} g_t + s_t + \frac{b_{t-1}}{\mu \pi_t}$$

where k_t^s is the return on capital. The GBC can be more clearly written as

$$\frac{b_t}{R_t^{gov}} + \frac{\tau_t^v}{1+\tau_t^v} c_t + \tau_t^n n_t w_t - \tau_t^k \left[u_t r_t^k - a(u_t) + \delta\right] \frac{k_{t-1}^p}{\mu} = \bar{y} g_t + s_t + \frac{b_{t-1}}{\mu \pi_t}$$

On DU page XVII, there is $a'(1) = \bar{r}^k$ and $a(1) = 0$.

Finding out the steady state:

$$\frac{\bar{b}}{\bar{R}} + \frac{\bar{\tau}^v}{1+\bar{\tau}^v} \bar{c} + \bar{\tau}^n \bar{w} \bar{n} - \bar{\tau}^k \left(\bar{r}^k - \delta\right) \bar{k} = \bar{y} \bar{g} + \bar{s} + \frac{\bar{b}}{\mu \bar{\pi}}$$

Log-linearizing,

$$
\begin{aligned}
&\frac{\bar{b}\left(1+\hat{b}_t\right)}{\bar{R}\left(1+\hat{R}_t+\hat{q}_t^b\right)}+\frac{\bar{\tau}^v\left(1+\hat{\tau}_t^v\right)}{1+\bar{\tau}^v\left(1+\hat{\tau}_t^v\right)}\bar{c}\left(1+\hat{c}_t\right)+\bar{\tau}^n\bar{w}\bar{n}\left(1+\hat{w}\right)\left(1+\hat{n}_t\right)\left(1+\hat{\tau}_t^n\right)\\
&-\bar{\tau}^k\left(1+\hat{\tau}_t^k\right)\left[\left(1+\hat{u}_t\right)\bar{r}^k\left(1+\hat{r}_t^k\right)-\delta\right]\bar{k}\left(1+\hat{k}_{t-1}^p\right)\\
=&\bar{y}\bar{g}\left(1+\hat{g}_t\right)+\bar{s}\left(1+\hat{s}_t\right)+\frac{\bar{b}}{\mu\bar{\pi}\left(1+\hat{\pi}_t\right)}\left(1+\hat{b}_{t-1}\right)\\
\Rightarrow&\frac{1}{\bar{R}}\frac{\bar{b}}{\bar{y}}[\hat{b}_t-\hat{R}_t-\hat{q}_t^b]+\bar{\tau}^n\frac{\bar{w}\bar{n}}{\bar{c}}\frac{\bar{c}}{\bar{y}}\frac{d\tau_t^n}{\tau^n}+\frac{\bar{c}}{\bar{y}}\frac{d\tau_t^v}{(1+\tau^v)^2}+\bar{\tau}^k\frac{\bar{k}}{\bar{y}}[\bar{r}^k-\delta]\frac{d\tau_t^k}{\tau^k}\\
&+\frac{\bar{\tau}^v}{1+\bar{\tau}^v}\frac{\bar{c}}{\bar{y}}\hat{c}_t+\bar{\tau}^n\frac{\bar{w}\bar{n}}{\bar{c}}\frac{\bar{c}}{\bar{y}}\left(\hat{w}_t+\hat{n}_t\right)+\bar{\tau}^k\frac{\bar{k}}{\bar{y}}\left[\bar{r}^k\hat{r}_t^k+(\bar{r}_t^k-\delta)\hat{k}_{t-1}^p\right]\\
=&\hat{g}_t+\frac{\bar{s}}{\bar{y}}\hat{s}_t+\frac{\bar{b}}{\mu\bar{y}}\frac{\hat{b}_{t-1}-\hat{\pi}_t}{\bar{\pi}}
\end{aligned}
$$

5 Calibration and estimation

5.1 Estimation

In this section I will describe the measurement equations. Ideally I should follow SW (2007) to collect nominal data and deflate it using the GDP deflator. Unfortunately most of the UK data I could find is already in real terms, either by chained volume measure or by current price measure. Johannes Pfeifer, one of the developers of Dynare, warns that there could be errors when using the chained price method, as relative prices changes create distortions if goods (for instance, consumption goods and investment goods) are not measured in uniform units. Therefore I stick to the current price measure instead.

The method for writing the measurement equations is similar to that of the main text of SW (2007) and DU (2015). There is also a more detailed technical appendix written by other authors. The dynare guide for measurement equations written by Johannes Pfeifer is also very helpful.

The log deviations are scaled by a factor of 100. Therefore they do not need to be divided by the percentage sign.

There is a large excel spreadsheet titled "a millennium of macroeconomic data for the UK" on the website of Bank of England. Especially there are a few worksheets with quarterly series.

I am using "A18. Population in the UK and Ireland, 000s, 1086-2016"-"United Kingdom of Great Britain and N.Ireland" for the population.

I am using "Q2. Quarterly GDP, Industrial Production, and Expenditure Components"-"Expenditure components, £mn, GDP at market prices, current prices" to compute the per capita real output growth:

$$100\left[\log\left(\frac{GDP_t}{POP_t}\right)-\log\left(\frac{GDP_{t-1}}{POP_{t-1}}\right)\right]=100(\hat{y}_t-\hat{y}_{t-1})+100(\mu-1)$$

I am using "Q2. Quarterly GDP, Industrial Production, and Expenditure Components"-"Expenditure components, £mn, household consumption, current prices" to compute the growth in per capita real consumption:

$$100\left[\log\left(\frac{CONS_t}{POP_t}\right)-\log\left(\frac{CONS_{t-1}}{POP_{t-1}}\right)\right]=100(\hat{c}_t-\hat{c}_{t-1})+100(\mu-1)$$

I am using "Q2. Quarterly GDP, Industrial Production, and Expenditure Components"-"Expenditure components, £mn, gross fixed capital formation" to compute the growth in per capita real investment:

$$100\left[\log\left(\frac{INV_t}{POP_t}\right)-\log\left(\frac{INV_{t-1}}{POP_{t-1}}\right)\right]=100(\hat{x}_t-\hat{x}_{t-1})+100(\mu-1)$$

I am using "Q1. Qrtly headline series"-"Spliced Average Weekly Earnings series, 1919-2015" to compute the growth in per capita real wages:

$$100\left[\log\left(W_t\right)-\log\left(W_{t-1}\right)\right]=100(\hat{w}_t-\hat{w}_{t-1})+100(\mu-1)$$

In the above equations, we have taken log differences. Although the data is all measured at current market price, the nominal effect will be canceled out.

We are interested in the average hours worked over the whole population. I obtained "Average actual weekly hours of work, Total (millions), Seasonally Adjusted" from ONS to compute the per Capita hours worked (We normalize the data so that $\bar{n}=0$):

$$100\log\left(\frac{HOURS_t}{POP_t}\right)=100\hat{n}_t+\bar{\bar{n}}^{obs}$$

The measurement equations for the inflation rate and the interest rate, in contrast, are not demeaned because we are interested in the steady state inflation rate and interest rate.

I am using "Q1. Qrtly headline series"-"GDP deflator (market prices), 2013=1" to compute inflation:

$$100 \log \left(\frac{GDPP_t}{GDPP_{t-1}} \right) = 100\hat{\pi}_t + 100(\bar{\pi} - 1)$$

The log of gross inflation can be viewed as net inflation too.

I obtained Bank of England base rate from its homepage and converted it into quarterly rates. As the base rate moves more frequently than quarterly, I only take the values observed at the end of one quarter and ignore the changes within the quarter:

$$100BR_t/4 = 100\hat{R}_t + 100(\bar{R} - 1)$$

I am using "Q1. Qrtly headline series"-"Spliced interpolated monthly corporate bonds spreads 1854-2016"-"Quarterly average of monthly series" as the returns on UK corporate bond. (Another way is to take the corporate bond yields and minus the treasury bond yield.)

$$100CBSPREAD_t/4 = 100\hat{q}_t^k + \bar{\bar{q}}^{obs}$$