a Closed Economy

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July 26, 2020

1 The model

In this closed economy, there are firms, households and a government.

1.1 Firms

There are three types of firms: intermediate consumption and investment goods producers and final investment goods producers. VAT is levied on all types of firms. Intermediate consumption and investment goods are produced in monopolistic competitive sectors. Prices are Rotemberg-sticky. Final investment goods are produced in a perfectly competitive final investment goods sector that uses intermediate inputs supplied by intermediate investment goods producers. Intermediate consumption goods producers use capital goods produced by the final investment goods producers, which serves as a stand-in for VAT in the middle of the supply chain. Final consumption goods are formed simply through aggregation technology.

1.1.1 Intermediate consumption goods producers

There is a unit mass of intermediate consumption goods producers, indexed by $i \in [0,1]$. Each producer is the monopolistic supplier of good i.

Production is subject to a fixed cost and the gross product is produced using a Cobb-Douglas technology.

$$Y_t(i) = \epsilon_t^a K_t^{eff}(i)^{\alpha} \left[\mu^t n_t^c(i)\right]^{1-\alpha} - \mu^t \Phi$$

where $K_t^{eff}(i)$ is capital services used in production, $n_t^c(i)$ is aggregate labour input and Φ is a fixed cost. μ^t represents the labour-augmenting deterministic growth rate in the economy and ϵ_t^a is total factor productivity. On the balanced growth path and in the lfexible price-wage economy, $Y_t(i) = Y_t$, $K_t^{eff}(i) = K_t^{eff}$.

The firm rents capital services K_t^c and hires labour n_t^c to maximize profits intertemporally, taking as given rental rates R_t^k and wages W_t. The profit-maximization problem is equivalent to minimizing costs and then choosing the quantity optimally. The cost-minimization problem is:

$$\min_{K_t^c(i), n_t^c(i)} W_t n_t^c(i) + R_t^k K_t^c(i)$$

subject to the production technology.

Given the solution to the cost-minimization problem, the firm maximizes the present discounted value of its profits by choosing quantities optimally, taking as given its demand function, the marginal costs of production, and the Rotemberg-style price-setting friction. The firm is allowed to adjust its nominal price in each period by paying a quadratic adjustment cost and otherwise indexes its price to an average of steady state and past inflation $\pi_{t+s-1}^{\iota_p}\bar{\pi}^{1-\iota_p}$

$$\max_{\tilde{P}_t(i)} \mathbb{E}_t \sum_{s=0}^{\infty} \frac{\bar{\beta}^s \xi_{t+s} P_t}{\xi_t P_{t+s}} \left[\left(\frac{1}{1+\tau_{t+s}^v} \frac{P_{t+s}(i)}{P_{t+s}} - \frac{MC_{t+s}^c}{P_{t+s}} \right) Y_{t+s}(i) - \frac{\varphi^c}{2} \left(\frac{P_{t+s}(i)}{\pi_{t+s-1}^{\iota_p} \bar{\pi}^{1-\iota_p} P_{t+s-1}(i)} - 1 \right)^2 Y_{t+s} \right]$$

$$\text{s.t.} \quad Y_t(i) = Y_t G'^{-1} \left(\frac{P_t(i)}{P_t} \int_0^1 G' \left(\frac{Y_t(i)}{Y_t}; \tilde{\epsilon}_t^{\lambda,p} \right) \frac{Y_t(i)}{Y_t} di \right)$$

where $\tilde{P}_t(i)$ is the newly set price, MC_{t+s}^c is the nominal marginal cost, φ^c is the Rotemberg price adjustment cost parameter, π_t is inflation defined as $\pi_t = P_t/P_{t-1}$ and $\left[\frac{\bar{\beta}^s \xi_{t+s} P_t}{\xi_t P_{t+s}}\right]$ is the stochastic discount factor.

1.1.2 Intermediate investment goods producers

The intermediate investment goods producer has a similar production technology as the intermediate consumption goods producer.

$$Y_t^k(i) = \epsilon_t^a K_t^k(i)^\alpha \left[\mu^t n_t^k(i)\right]^{1-\alpha} - \mu^t \Phi$$

and a similar cost-minimization problem:

$$\min_{K_t^k(i), n_t^k(i)} W_t n_t^k(i) + R_t^k K_t^k(i)$$

But the profit-maximization problem differs in that the tax is excluded:

$$\max_{\tilde{P}_t^k(i)} \mathbb{E}_t \sum_{s=0}^{\infty} \frac{\bar{\beta}^s \xi_{t+s} P_t}{\xi_t P_{t+s}} \Bigg(\left[\frac{P_{t+s}^k(i)}{P_{t+s}} (1+\tau_{t+s}) - \frac{P_{t+s}^k(i)}{P_{t+s}} \tau_{t+s} - MC_{t+s}^k \right] Y_{t+s}^k(i) \\ - \frac{\varphi^k}{2} \left(\frac{P_{t+s}^k(i)}{\pi_{t+s-1}^{\iota_p} \bar{\pi}^{1-\iota_p} P_{t+s-1}^k(i)} - 1 \right)^2 Y_{t+s}^k \Bigg)$$

The firm charges $\tilde{P}_{t+s}^k(i)\,(1+\tau_{t+s})$ to the final investment goods producers, but earns only the revenue of $\tilde{P}_{t+s}^k(i)$ per unit of output.

1.1.3 Final investment goods producers

The representative final goods producer maximizes profits by choosing intermediate inputs $Y_t^k(i)$, $i \in [0,1]$, subject to a production technology with time-varying elasticity of substitution.

$$\max_{Y_t^k, Y_t^k(i)} (1+\tau_t) P_t^k Y_t^k - \tau_t P_t^k Y_t^k - \int_0^1 P_t^k(i)\,(1+\tau_t)\, Y_t^k(i) di \quad \text{s.t.} \quad \int_0^1 G\left(\frac{Y_t^k(i)}{Y_t^k}; \tilde{\epsilon}_t^{\lambda,p} \right) di = 1$$

2 Derivations

2.1 VAT in the intermediate firms' problem

The derivative with respect to $P_t(i)$:

$$\frac{d\left[\left(\left[\frac{P_t(i)}{(1+\tau_t^v)P_t} - \frac{MC_t^c}{P_t}\right] Y_t(i)\right) - \frac{\varphi^c}{2}\left(\frac{P_t(i)}{\pi_{t-1}^{\iota_p}\bar{\pi}^{1-\iota_p}P_{t-1}(i)} - 1\right)^2 Y_t - \frac{\bar{\beta}\xi_{t+1}}{\xi_t\pi_{t+1}}\frac{\varphi^c}{2}\left(\frac{P_{t+1}(i)}{\pi_t^{\iota_p}\bar{\pi}^{1-\iota_p}P_t(i)} - 1\right)^2 Y_{t+1}\right]}{dP_t(i)}$$

$$\begin{aligned}
=&\frac{1}{(1+\tau_t^v)P_t}\frac{d[P_t(i)Y_t(i)]}{dP_t(i)} - \frac{d\left[\frac{MC_t^c}{P_t}Y_t(i)\right]}{dP_t(i)} - \varphi Y_t \frac{d\left[\frac{P_t(i)}{\pi_{t-1}^{\iota_p}\bar{\pi}^{1-\iota_p}P_{t-1}(i)} - 1\right]}{dP_t(i)} - \varphi\frac{\bar{\beta}\xi_{t+1}}{\xi_t\pi_{t+1}}Y_{t+1}\frac{d\left[\frac{P_{t+1}(i)}{\pi_t^{\iota_p}\bar{\pi}^{1-\iota_p}P_t(i)} - 1\right]}{dP_t(i)}\\
=&\frac{1}{(1+\tau_t^v)P_t}Y_t(i) + \frac{1}{(1+\tau_t^v)P_t}P_t(i)\frac{dY_t(i)}{dP_t(i)} - \frac{MC_t^c}{P_t}\left[-\eta_p\frac{Y_t(i)}{P_t(i)}\right]\\
&- \varphi Y_t\frac{1}{\pi_{t-1}^{\iota_p}\bar{\pi}^{1-\iota_p}P_{t-1}(i)} + \varphi\frac{\bar{\beta}\xi_{t+1}}{\xi_t\pi_{t+1}}Y_{t+1}\frac{P_{t+1}(i)}{\pi_t^{\iota_p}\bar{\pi}^{1-\iota_p}P_t^2(i)}\\
=&\frac{1}{(1+\tau_t^v)P_t}Y_t(i) + \frac{1}{(1+\tau_t^v)P_t}P_t(i)\left[-\eta_p\frac{Y_t(i)}{P_t(i)}\right] + \frac{MC_t^c}{P_t}\left[\eta_p\frac{Y_t(i)}{P_t(i)}\right]\\
&- \varphi Y_t\frac{1}{\pi_{t-1}^{\iota_p}\bar{\pi}^{1-\iota_p}P_{t-1}(i)} + \varphi\frac{\bar{\beta}\xi_{t+1}}{\xi_t\pi_{t+1}}Y_{t+1}\frac{P_{t+1}(i)}{\pi_t^{\iota_p}\bar{\pi}^{1-\iota_p}P_t^2(i)}
\end{aligned}$$

Because all the firms face the same problem, they choose the same price and produce the same quantity. In other words, $P_t(i) = P_t$, $Y_t(i) = Y_t$ $\forall i$. We also have the relationship between markup and the elasticity of substitution: $\eta_p = \frac{1}{\lambda_p} + 1$. Therefore F.O.C w.r.t $P_t(i)$:

$$
\begin{aligned}
&\left[\frac{1}{\left(1+\tau_t^v\right) P_t}\left(1-\eta_p\right)+\eta_p \frac{MC_t^c}{P_t^2}-\varphi \frac{1}{\pi_{t-1}^{\iota_p} \bar{\pi}^{1-\iota_p} P_{t-1}}\right] Y_t+\varphi \frac{\bar{\beta} \xi_{t+1}}{\xi_t \pi_{t+1}} \frac{\pi_{t+1}}{\pi_t^{\iota_p} \bar{\pi}^{1-\iota_p} P_t} Y_{t+1}=0 \\
\Rightarrow &\left[\frac{1}{\left(1+\tau_t^v\right) P_t}\left(1-\eta_p\right)+\eta_p \frac{MC_t^c}{P_t^2}-\varphi \frac{1}{\pi_{t-1}^{\iota_p} \bar{\pi}^{1-\iota_p} P_{t-1}}\right] Y_t+\varphi \frac{\bar{\beta} \xi_{t+1}}{\xi_t} \frac{1}{\pi_t^{\iota_p} \bar{\pi}^{1-\iota_p} P_t} Y_{t+1}=0 \\
\Rightarrow &\left[\frac{1}{\left(1+\tau_t^v\right)}\left(1-\eta_p\right)+\eta_p mc_t^c-\varphi \frac{\pi_t}{\pi_{t-1}^{\iota_p} \bar{\pi}^{1-\iota_p}}\right] Y_t+\varphi \frac{\bar{\beta} \xi_{t+1}}{\xi_t} \frac{1}{\pi_t^{\iota_p} \bar{\pi}^{1-\iota_p}} Y_{t+1}=0 \\
\Rightarrow &\left(1-\bar{\eta}_p\right) d\left(\frac{1}{1+\tau_t^v}\right)+\frac{1}{1+\tau^v} d\left(1-\eta_p\right)+\overline{mc}^c d\left(\eta_p\right)+\bar{\eta}_p d\left(mc_t^c\right) \\
&-\frac{\varphi \bar{Y}}{\bar{\pi}} d\left(\pi_t\right)-\varphi \bar{Y} \bar{\pi}^{\iota_p} d\left(\frac{1}{\pi_{t-1}^{\iota_p}}\right)+\frac{\varphi \bar{\beta} \bar{Y}}{\bar{\pi}^{1-\iota_p}} d\left(\frac{1}{\pi_t^{\iota_p}}\right)+\frac{\varphi \bar{\beta}}{\bar{\pi}} d\left(Y_{t+1}\right)+\frac{\varphi \bar{\beta} \bar{Y}}{\bar{\pi} \bar{\xi}} d\left(\xi_{t+1}\right)+\frac{\varphi \bar{\beta} \bar{Y} \bar{\xi}}{\bar{\pi}} d\left(\frac{1}{\xi_t}\right)=0
\end{aligned}
$$