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Estimation of linear trend onset in time series

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ABSTRACT

We propose a method to detect the onset of linear trend in a time series and estimate the change point T from the profile of a linear trend test statistic, computed on consecutive overlapping time windows along the time series. We compare our method to two standard methods for trend change detection and evaluate them with Monte Carlo simulations for different time series lengths, autocorrelation strengths, trend slopes and distribution of residuals. The proposed method turns out to estimate T better for small and correlated time series. The methods were also applied to global temperature records suggesting different turning points.

1. Introduction

Structural change detection in time series is an important and difficult issue of increasing interest in many areas of meteorology and earth science [14,15,19,26,42] and applied economics [30,43,32,23,29] among others. The term 'structural change' has several meanings in the time series literature, such as change of the slope of a linear trend, level shift (jump), combination of slope change and level shift and change in characteristics of the underlying system [6,7,39,18,40]. Statistical tests have been developed for detecting structural change in the trend function of a time series, often restricted to a single change, see among others Chu and White [10], Andrews [1], Ploberger and Kramer [37], Bai [2], Perron and Zhu [32] (for multiple structural changes, see Lavielle [22], Bai and Perron [3,4]).

A popular test of general purpose is the CUSUM test for changes at an unknown time point as it does not presume the nature of the change [35,36,13]. Some tests are more elaborate and are designed to disentangle deterministic changes and unit roots, such as the tests in Banerjee et al. [5], Perron [30], Perron [31], Zivot and Andrews [43] and Lee and Strazicich [24].

An important issue is the investigation of slope changes in the presence of correlated stationary and non-stationary noise (see Perron [33], for an extensive discussion and Perron and Yabu [34] among others). In this work, we concentrate on stationary noise with varying strength of autocorrelation. Other related works deal with estimation of the break point from a simple one-break model for a linear trend function that exhibits a change in slope, see Deng and Perron [12] and Perron and Zhu [32] and Chu and White [10].

The aim of this paper is to estimate the time point of a single structural change (breakpoint) from no trend to linear trend. In particular, we are interested in emerging trends of very small size. The existence of small emerging trend is an issue in many applications, e.g., the global temperature records of the recent decades. Through this estimation procedure the existence of the breakpoint is tested as well. In our approach, we compute the statistic of a standard parametric test for linear

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trend on a sliding data window along the time series. The breakpoint is detected from the smoothed profile of the statistic using a two-level cross-checking. Our approach is compared to two known techniques in assessing the onset of a linear trend, the estimation of the breakpoint from the minimization of a cost function of Perron and Zhu [32] and the direct test for changing trend of Chu and White [10].

The remainder of the paper is organized as follows. In Section 2, our proposed method is presented along with a discussion on problematic scenarios and comparison to the other two methods. The simulation setup and results are given in Section 3. In Section 4, the application to the global temperature time series is presented and in Section 5 we draw our conclusions.

2. Estimation of the break point

Assuming a single breakpoint at time T from no trend to linear trend in a time series, Y_t, $t = 1, \ldots, n$, the standard decomposition of Y_t to a systematic part d_t and a stationary random component ε_t (residual) reads

$$Y_t = d_t + \varepsilon_t, \tag{1}$$

where d_t is specified as

$$d_t = \mu_0 + \beta_0 t + \beta B_t \tag{2}$$

and B_t is a dummy variable for the slope change at time T defined by

$$B_t = \begin{cases} 0, & t \leqslant T \\ t - T, & t > T \end{cases} \tag{3}$$

The slope coefficient changes from β_0 to $\beta_0 + \beta$ at the time point T [32], but in our work, we restrict to the case $\beta_0 = 0$. Thus the working null Hypothesis H_0 is that the underlying to the time series process is stationary and without trends, i.e. $\beta = 0$. The alternative Hypothesis H_1, that we are interested in here, is that there is a single but yet unknown breakpoint T, so that β is the coefficient of the linear trend starting at T. H_1 can be postulated as $Y_t = \mu_0 + \beta B_t + \varepsilon_t$, where B_t is as in Eq. (3).

2.1. The method of slope statistic profile (SSP)

We propose a method to detect the single structural break T using a standard parametric linear trend test on sliding data windows along the time series, which we term slope statistic profile (SSP). In the following, we briefly present this parametric linear trend test for a sliding window of length w on the time series Y_t, $t = 1, \ldots, n$. Thus for the first window $[Y_1, \ldots, Y_w]'$ the least square estimator for the trend parameter β is obtained as

$$\hat{\beta} = \frac{\sum_{t=1}^{w} (t - \bar{t}) Y_t}{\sum_{t=1}^{w} (t - \bar{t})^2},$$

where $\bar{t}$ is the mean time. The standard error of $\hat{\beta}$ is estimated from the power spectrum [8]

$$s(\hat{\beta}) = \left[2 \int_0^{0.5} W(f) S(f) df \right]^{1/2}, \tag{4}$$

where $W(f) = \left| \sum_{t=1}^{w} b_t e^{-2\pi i f} \right|^2 t$ with $b_t = \frac{t - \bar{t}}{\sum_{t=1}^{w} (t - \bar{t})^2}$ and $S(f)$ denotes the sample power spectrum of ε_t given as $S(f) = \frac{1}{2\pi} \left(\hat{\gamma}_0 + 2 \sum_{k=1}^{w-1} \hat{\gamma}_k \cos(2\pi f k) \right)$. $\hat{\gamma}_k$ denotes the estimate of the kth order autocovariance of ε_t, given as $\hat{\gamma}_k = \frac{1}{w} \sum_{t=1}^{w-k} \hat{\varepsilon}_{t+k} \hat{\varepsilon}_t$ for $k > 0$, where $\hat{\varepsilon}_t = Y_t - \hat{a} - \hat{\beta} t$ are the estimated residuals ($\hat{a} = \overline{Y} - \hat{\beta} \bar{t}$ and $\overline{Y}$ is the mean of the time series), and $\hat{\gamma}_0 = \frac{1}{w-2} \sum_{t=1}^{w} \hat{\varepsilon}_t^2$ for $k = 0$. In practice, a discrete Fourier spectrum is taken and the integral in Eq. (4) becomes sum at frequencies $f_j = \frac{2\pi j}{w}, j = 0, \ldots, \frac{w}{2}$.

The t-statistic for the parametric linear trend test is $t = \frac{\hat{\beta}}{s(\hat{\beta})} \sim t_{w-2}$, and the null hypothesis of no trend is rejected at the significance level α if $|t| \geqslant t_{w-2,1-\alpha/2}$. In Vafeiadis et al. [41], we showed that this test statistic gives high test power compared to other test statistics for both correlated and white noise residuals.

The t-statistic is computed on overlapping data windows of size w with sliding step one. In this way we obtain the profile of the t-statistic, denoted as $\{\widetilde{U}_i\}$, for $i = 1 + [w/2], \ldots, n - [w/2]$, where $[x]$ is the integer part of x. The form of this profile depends on time series characteristics, i.e. the strength of the autocorrelation, the distribution of the residuals ε_t, and the strength of the linear trend given by β, as well as, the size of the sliding data window w. All these factors are studied using simulations in Section 3.

The profile of the t-statistic $\{\widetilde{U}_i\}$ exhibits small fluctuations (glitches) due to edge effects of the local data windows and therefore we smooth the profile curve using a zero-phase filter of a small order, set to about 5% of w. Such a small filter order removes the glitches in $\{\widetilde{U}_i\}$ but maintains its signature. We denote the smoothed value of $\widetilde{U}_i$ as U_i and refer to as U-profile. In the presentation of the method below, we will assume the situation from no trend to a positive trend, as shown in Fig. 1. Other types of change between no trend and trend can be treated similarly.

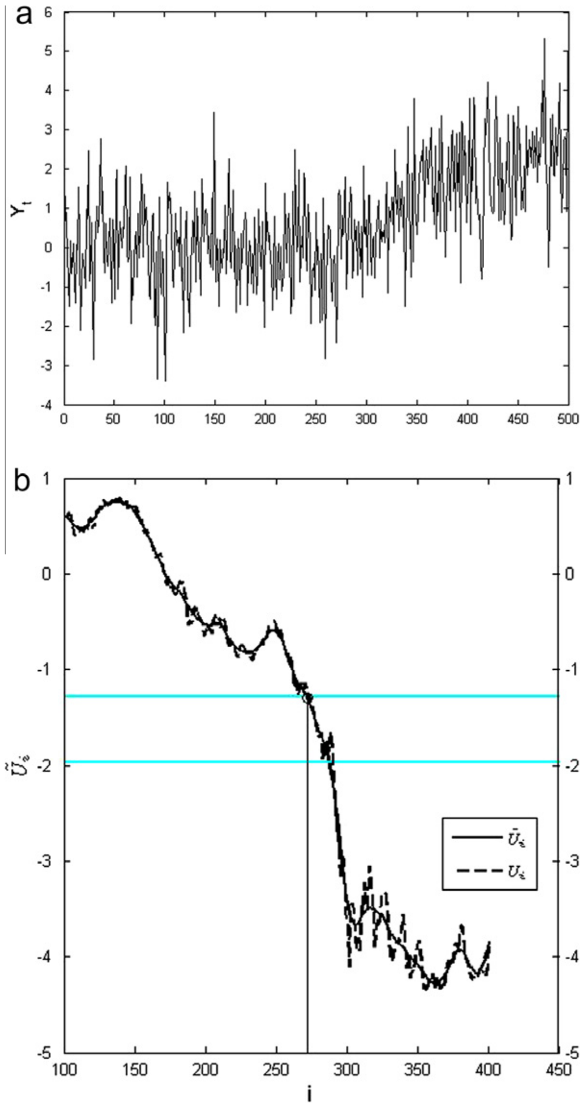

Fig. 1. (a) Time series of length $n = 500$ with an onset of linear trend with coefficient $\beta = 0.01$ at $T = 250$ and residuals ε_t generated by AR(1) with coefficient $\varphi = 0.16$ and normal input white noise. (b) The profile of t-statistic $\{\widetilde{U}_i\}$ using local window of size $w = 200$ and the filtered profile $\{U_i\}$, as denoted in the legend. The horizontal lines denote the thresholds $t_{w-2,0.975}$ and $t_{w-2,0.90}$ and the vertical line the estimated $\widehat{T}$ at time 274.

A first candidate for the breakpoint T is the time point at which the profile crosses the threshold line of rejection of the null hypothesis of no trend at $\pm t_{w-2,1-\alpha/2}$. One could consider one-sided test and use a smaller threshold magnitude $t_{w-2,1-\alpha}$ if there is knowledge for the direction of the linear trend ($-t_{w-2,1-\alpha}$ for positive trend and $t_{w-2,1-\alpha}$ for negative trend). The U-profile most probably does not exhibit a sudden change from small magnitudes, in the absence of trend, to large magnitudes, in the presence of trend, but there is a rather smooth transition due to the use of sliding windows with step one. This consideration comes along with the belief that there are not sudden and abrupt changes in natural variations. Thus a better estimate of T should be searched at times regarding magnitudes of $\{U_i\}$ smaller than $t_{w-2,1-\alpha/2}$. We confine the search of T to a time interval corresponding to the profile segment bounded by $t_{w-2,0.975}$ and $t_{w-2,0.90}$, for the 0.05 and 0.20 significance levels for the two-sided test, respectively, and find the smallest magnitude of $\{U_i\}$ within these bounds. The corresponding time point for this value is the estimate $\widehat{T}$. For the example of Fig. 1, the estimate $\widehat{T} = 274$ of the true breakpoint $T = 250$ corresponds to the profile crossing of the lower bound at $-t_{w-2,0.90} = -1.286$.

This example illustrates a straightforward detection with SSP in a well-behaving situation. More complicated scenarios are presented in Section 2.3, where also the existence of the onset of a linear trend is treated. First, let us review the two methods we compare our method to.

2.2. Other methods for the estimation of the linear trend onset

Here, we briefly present a method for the estimation of T and a test for the existence of a breakpoint. The method of Perron and Zhu [32] that we adapt to our problem setting, denoted hereafter as P–Z method, assumes that there is a break point T and estimates it by minimizing the sum of the squared residuals of the hypothesized model (see Eqs. (1) and (2)) for all possible T values. The estimate $\widehat{T}$ is found as

$$\widehat{T} = \arg\min_{\widehat{T}} Y'(I - P_T)Y,$$

where I is a unit matrix, $Y = [Y_1, \ldots, Y_n]'$ is the initial time series and $P_T = X_T(X_T'X_T)^{-1}X_T'$ is the projection matrix constructed using $X_T = [x(T)_1', \ldots, x(T)_n']$, where $x(T)_t' = [1, t, B_t]$ and B_t is given in Eq. (3). Note that the P–Z method will always provide an estimate $\widehat{T}$ regardless of the existence of a linear trend onset.

To test the existence of a breakpoint T, we consider the asymptotic test of Chu and White [10], denoted hereafter as the C–W test. Chu and White [10] proposed this test for the occurrence or change of linear trend, without requiring prior knowledge about the location of the breakpoint. The null Hypothesis H_0 for the test is that the time series contains only a linear trend without any breakpoint. For the scenario of the onset of a linear trend we consider here, H_0 refers to a stationary process with no trend. The test statistic is

$$T^0(t) = (6\hat{\sigma}_0)^{-1} n^{\frac{3}{2}} \left(\frac{t}{n}\right)^3 |\hat{\beta}_t - \hat{\beta}_n|,$$

where $t = 1, \ldots, n$, meaning that any time t of the series is tested for a possible changing point. We retained the notation T^0 for the function of t as in the presentation in Chu and White [10], though we use T for breakpoint. Actually, the estimated T here is the time t of the maximum T^0. The least squares estimator of the linear trend slope for each t is

$$\hat{\beta}_t = \frac{t\sum_{j=1}^{t} jY_j - \sum_{j=1}^{t} j \sum_{j=1}^{t} Y_j}{(t^4 - t^2)/12},$$

and $\hat{\sigma}_0^2$ is an estimate of the residual variance. Chu and White [10] used the consistent estimator of Newey and West [27] for $\hat{\sigma}_0^2$, given as

$$\hat{\sigma}_{0_1}^2 = \frac{1}{n}\sum_{t=1}^{n}(Y_t - \hat{a}_n - \hat{\beta}_n t)^2 + \frac{2}{n}\sum_{s=1}^{l} w_{sl} \sum_{t=s+1}^{n}(Y_t - \hat{a}_n - \hat{\beta}_n t)(Y_{t-s} - \hat{a}_n - \hat{\beta}_n(t-s)) \tag{5}$$

where $w_{sl} = 1 - s/(l + 1)$ and l is an appropriate truncation lag for the autocorrelation of residuals, and $\hat{a}_j = \overline{Y}_j - \hat{\beta}_j \bar{t}_j$, where $\bar{t}_j = \frac{1}{j}\sum_{t=1}^{j} t$. Chu and White [10] give critical values for T^0 (at the 10%, 5% and 1% levels are 0.708, 0.784 and 0.940, respectively) and compare the performance of the test for various choices of l. In our simulations, we use a large lag $l = 10$ in order to include correlations over many lags, and the test statistic is obtained by replacing $\hat{\sigma}_0$ by $\hat{\sigma}_{0_1}$ in Eq. (5), denoted as T_1^{C-W}.

When ε_t is a specific known stationary process, Park and Pantula [28] suggested consistent estimators for σ_0 depending on the error process. In our comparative study, we use an AR(1) error process, $\varepsilon_t = \varphi\varepsilon_{t-1} + a_t$, and the variance σ_0^2 is straightforwardly estimated by

$$\hat{\sigma}_{0_2}^2 = \hat{\sigma}_a^2(1 - \hat{\varphi})^{-2},$$

where $\hat{\varphi}$ is the consistent estimate of φ and $\hat{\sigma}_a^2$ is the innovation variance estimator. The test statistic is obtained by replacing $\hat{\sigma}_0$ by $\hat{\sigma}_{0_2}$ in (5) and is denoted as T_2^{C-W}. When H_0 for no break in trend is rejected the Chu and White test also provides an estimate of the changing point as $\widehat{T} = \arg\max_{1<t<n-1} T^0(t)$.

2.3. Problems in the estimation of the time breakpoint

The detection of a breakpoint T can be a difficult task when the magnitude of the linear trend slope is small in conjunction with a limited observation time (time series length n). Moreover, the estimation of T becomes less accurate when the time series is correlated. Strong correlation causes drifts that may fool the estimation process and adds a large offset in the value of $\widehat{T}$, or even leads to erroneous detection of non-existent trend onset. The local window size w used in our approach may also depend on these factors.

We illustrate some indicative problematic cases below. First, we show the effect of strong correlation on the accuracy of the estimation of T with the methods SSP and P–Z. We consider an AR(1) error process, $\varepsilon_t = \varphi\varepsilon_{t-1} + a_t$, with strong autocorrelation given by $\varphi = 0.9$. In Fig. 2a, it is shown that for very weak linear trend, $\beta = 0.006$, where $n = 1000$, the strong correlation does not allow for a clear trend detection of the true breakpoint $T = 500$ by the t-statistic, and the threshold $-t_{348,0.90} = -1.284(w = 350)$ is crossed by the smoothed U-profile at $\widehat{T} = 652$. We note that there is a smooth decrease of the U-profile for smaller times towards the true $T = 500$, and thus a smaller threshold would be more appropriate for this case. The strong correlation affects also the P–Z method and the minimum of the error function is at $\widehat{T} = 662$, also away from the true breakpoint $T = 500$. When for the same setting there is no trend ($\beta = 0$), the U-profile remains correctly within the

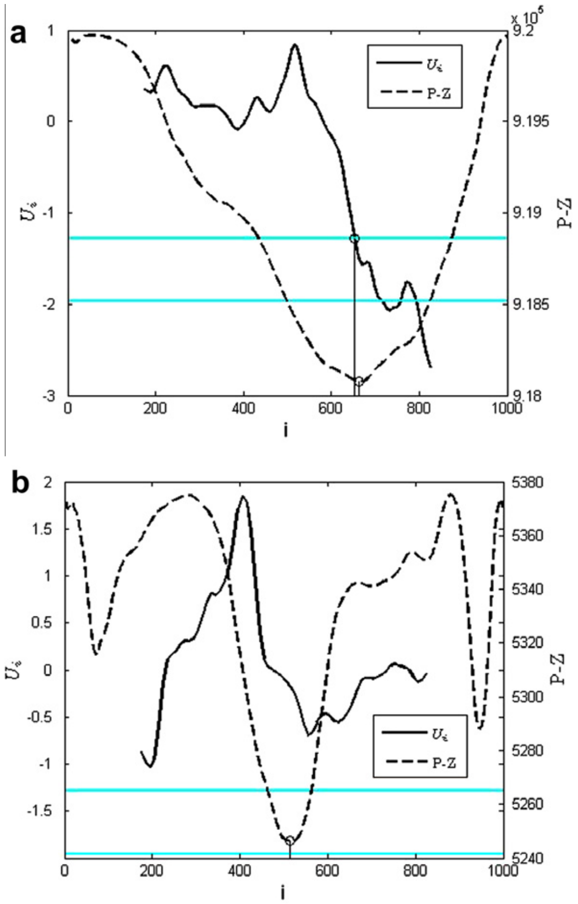

Fig. 2. (a) The profile of the cost function for P–Z and the U-profile for a time series with a random component ε_t generated by AR(1) and normal input white noise. The parameters are $\varphi = 0.9$, $\beta = 0.006$, $n = 1000$, $T = 500$, $w = 350$. (b) Same as for (a) but for $\beta = 0$. The horizontal lines denote the thresholds $t_{w-2,0.975}$ and $t_{w-2,0.90}$and the vertical lines the estimated $\widehat{T}$ at time 652 for SSP and 662 for P–Z in (a), and 521 for P–Z in (b).

significance bounds and no breakpoint is detected, whereas the error function of P–Z obtains a minimum at random (see Fig. 2b). The comparison of the range of values for this case and the case of trend (see the right vertical axes in Fig. 2a and b), regards the minimum as non-significant, but P–Z does not provide a threshold of significance, and the user may be misled to detect the time of the minimum as a breakpoint. To this respect, we suggest that the SSP scheme should be seen both as a test for the existence of a breakpoint and an estimator of the time breakpoint T, in the same way as for the C–W test.

The bounds $t_{w-2,0.975}$ and $t_{w-2,0.90}$ may not always be sufficient to provide a unique solution for $\widehat{T}$ and additional conditions are set to treat specific anomalies of the U-profile. Namely, the U-profile may exhibit multiple slopes and turning points within the two bounds (see Fig. 3a) or cross the two bounds at different time intervals (see Fig. 3b). For the former, we estimate $\widehat{T}$ from the time of the first turning point of the U-profile in the zone formed by the two bounds, in the direction from the largest magnitudes to the smaller ones. This regards the end of the decrease of the significance of the trend slope as detected in the sliding windows. Note that in Fig. 3a, though the $\widetilde{U}$-profile of the t-statistics may cross the $t_{w-2,0.975}$ at later times, the smoothed U-profile crosses the bound at times closer to the true breakpoint $T = 500$ and provides a more stable estimation of $\widehat{T}$. For the latter, the two separate crosses of the bounds would suggest the presence of two breakpoints. However, if we search for a single breakpoint, as in the simulations below, we force the method to choose the crossing that leaves the rest of the profile on the significant level (out of bounds), i.e. the second crossing in the example in Fig. 3b. We note here that though the proposed method is presented for the detection of a single breakpoint it can be extended to search for multiple breakpoints, but this is not treated here.

A small time window w may produce more glitches in the $\widetilde{U}$-profile and possibly several crossings of the U-profile and thus more breakpoint candidates that would complicate the search process. On the other hand, a large w would limit the interval domain for $\widehat{T}$ defined as $[w/2 + 1, n - w/2]$. Thus, a trade-off value for w should be sought and we consider this issue in the simulations below.

Let us now examine the simple setting where the null Hypothesis H_0 of no trend is true. For this H_0, we consider only SSP and C–W, as P–Z does not provide a test statistic. As shown in the original paper [10], the C–W test has satisfactory empirical

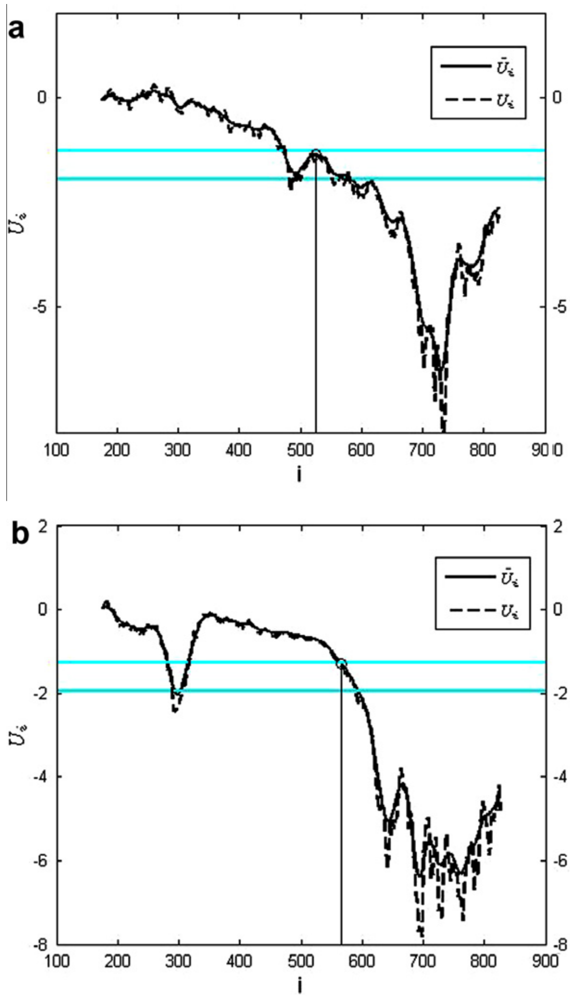

Fig. 3. The profile of $\widetilde{U}$ and U for two time series in (a) and (b), both generated by ε_t being AR(1) and with normal input white noise and the parameters $\varphi = 0.4$, $\beta = 0.01$, $n = 1000$, $T = 500$, $w = 350$. The horizontal lines denote the thresholds $t_{w-2,0.975}$ and $t_{w-2,0.90}$ and the vertical lines the estimated $\widehat{T}$ from SSP at time 526 in (a) and 574 in (b).

size when the residuals are from white noise or correlated noise and the global trend is quite large ($\beta_0 = 0.2$ in Eq. (2)). Our simulations for $\beta_0 = 0$ also confirmed this (see Section 3). On the other hand, the SSP method, as presented above, does not perform as well in the presence of significant autocorrelation, exhibiting significant linear trends at random time intervals, which depending on the autocorrelation strength can be of size w or larger. This results in consecutive values of the $\widetilde{U}$-profile being out of the bounds of slope significance and indicates false detection of a breakpoint. To disregard these false rejections, we should require a minimum proportion of the consecutive values of the U-profile (as percentage of n) to be outside the bounds when we search for a breakpoint T. We investigated the minimum proportion that accounts for a proper significance of the test (rejection of H_0 of no trend at 5%) as a function of the autocorrelation strength and the time series length n. We considered residuals ε_t given by AR(1), so that the autocorrelation strength is determined by the model coefficient φ. The results showed that for smaller n and larger φ, the criterion has to be more strict (larger percentage) in order to attain the correct significance level of 5%. In particular, we observed that the minimum proportion of consecutive points out of bounds, P_{Uk}, has a rather exponential dependence on n and φ, as shown in Fig. 4 for residuals generated by AR(1) and normal input white noise. For a range of n and φ values, we obtained by nonlinear fitting the form P_U^g (g for Gaussian)

$$P_U^g(n,\varphi) = 0.3578e^{-0.0071n+1.1296\varphi}. \tag{6}$$

This exponential function approximates P_U^g quite well, giving coefficient of determination $R^2 = 0.9443$. For example, when $n = 200$ and $\varphi = 0.078$, the fit gives P_U^g that corresponds exactly to the 5% significance level, while for $\varphi = 0.59$ the fit underestimates the correct P_U^g giving significance level at 3.6%; for $n = 400$ and $\varphi = 0.41$, the fit gives P_U^g corresponding to the significance level 3.8, and for $\varphi = 0.77$, the fit gives 6.5. Generally, the rejection rates using the minimum percentage P_U^g from the fit in Eq. (6) are within the range of ±2 of the 5% significance level.

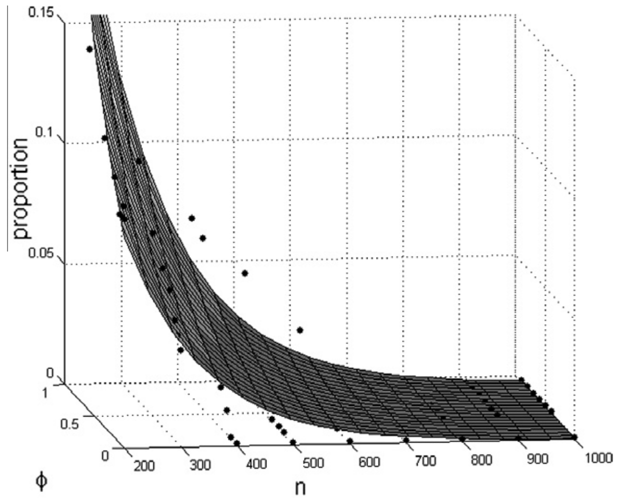

Fig. 4. Minimum percentage P_U^g of consecutive points of the *U*-profile outside the $\pm t_{w-1,1-\alpha/2}$ bounds that are required in SSP in order to have a rejection rate at 5% displayed as a function of the autocorrelation strength φ and time series length *n* (black dots). The gray-shaded surface is the graph of the exponential fit in Eq. (6). The residuals ε_t are generated by AR(1) and normal input white noise, and *w* was 40% of *n*.

We also found that the minimum proportion P_U as a function of φ and n has quite the same form when the input noise follows uniform or exponential distribution, being $P_U^u(n,\varphi) = 0.3366e^{-0.0062n+1.0238\varphi}$ and $P_U^e(n,\varphi) = 0.2777e^{-0.0067n+1.422\varphi}$, respectively (*u* for uniform and *e* for exponential distribution). These fits give $R^2 = 0.9526$ and $R^2 = 0.9421$, respectively, whereas the fit of Eq. (6) on the data from uniform and exponential noise is $R^2 = 0.9239$ and $R^2 = 0.9286$, i.e., a relative decrease of goodness-of-fit at the level of 1.7% and 1.2%, respectively. This result allows us to consider the expression in Eq. (6) for defining the smallest proportion of consecutive points out of bounds P_U^g for any type of noise distribution. We add the criterion of minimum percentage P_U^g in the SSP method in order to render the correct significance of the test. In the simulations as well as in the application below, the coefficient φ is first found by fitting an AR(1) model to the time series and then P_U^g is computed by Eq. (6).

3. Monte Carlo simulation

3.1. Simulation setup

We generate Monte Carlo realizations for different stochastic processes according to the model in Eqs. (1)–(3). We vary the factors involved as follows: time series length $n = 200, 500, 1000$, breakpoint *T* and moving window *w* given as percentages of *n*, 35%, 50%, 65% for *T* and 10%, 20%, 30%, 40% for *w*, and coefficient of the linear trend $\beta = 0.0, 0.006, 0.008, 0.01$. The residuals ε_t are generated by an AR(1) process, $\varepsilon_t = \varphi\varepsilon_{t-1} + a_t$, where a_t follows normal, uniform and exponential distribution, and the parameter φ (equal to first lag autocorrelation of residuals) varies as $\varphi = 0.0, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$, where $\varphi = 0.0$ denotes white noise residuals ε_t. These values of φ regard the longest time series ($n = 000$) and, in order the results to be comparable for different *n*, the $\varphi > 0$ values are adjusted accordingly for the smaller time series, i.e., $\varphi = 0.16, 0.25, 0.36, 0.49, 0.64, 0.81$ for $n = 500$ and $\varphi = 0.01, 0.031, 0.078, 0.168, 0.328, 0.59$ for $n = 200$. The combinations of all the values of n, β, T, w, φ and the distribution types of input noise give a total of $3 \times 4 \times 3 \times 4 \times 7 \times 3 = 3024$ cases for SSP, and 756 for P–Z and the two variants of the C–W test (T_1^{C-W} and T_2^{C-W}), as there is no parameter *w*. For each case, $M = 1000$ Monte Carlo realizations are generated.

For the tests T_1^{C-W} and T_2^{C-W}, critical values are provided in Chu and White [10] and Park and Pantula [28] for only some of the cases considered in our study. Therefore we obtained empirical critical values at the 5% significance level from 10,000 replications for each case under the null hypothesis of no trend.

To assess the accuracy of the estimation of the breakpoint *T* independently of the time series length *n*, we use MSE/n^2, where $MSE = \frac{1}{M}\sum_{i=1}^{M}(\widehat{T}_i - T)^2$ is the mean-squared error across all *M* realizations. To allow for fair comparison of SSP to P–Z, whenever an estimate $\widehat{T}$ from P–Z is out of the interval $[w/2+1, n-w/2]$ it is set to the corresponding limit of the interval.

3.2. Simulation results

3.2.1. Independent residuals

First, we present simulation results with independent residuals. As shown in Table 1, P–Z and SSP methods have similar accuracy in estimating correctly the breakpoint, whereas C–W gives much larger MSE/n^2, often by an order of magnitude.

Table 1
MSE/n^2 for the methods P–Z, T_1^{C-W}, T_2^{C-W} and SSP, for β = 0.01, 0.008 and 0.006, and for ε_t generated by normal and uniform white noise: (a) n = 500, w = 200 and T = 175, 250 and 325, and (b) n = 200, w = 80 and T = 70, 100 and 130. The smallest MSE/n^2 of the four methods for each case is highlighted.

		$\varepsilon_t \sim N(0,1)$				$\varepsilon_t \sim U[-1/2, 1/2]$			
T	β	P–Z	T_1^{C-W}	T_2^{C-W}	SSP	P–Z	T_1^{C-W}	T_2^{C-W}	SSP
(a) n = 500, w = 200									
175	*0.01*	0.0029	0.1056	0.1056	**0.0029**	**0.0001**	0.0985	0.0985	0.0013
	0.008	0.0055	0.1135	0.1111	**0.0052**	**0.0002**	0.0999	0.0999	0.0013
	0.006	**0.0120**	0.1425	0.1359	0.0177	**0.0004**	0.1017	0.1017	0.0015
250	*0.01*	**0.0024**	0.0466	0.0466	0.0030	**0.0001**	0.0440	0.0440	0.0014
	0.008	**0.0044**	0.0482	0.0482	0.0055	**0.0002**	0.0445	0.0445	0.0014
	0.006	**0.0091**	0.0514	0.0511	0.0110	**0.0004**	0.0448	0.0448	0.0012
325	*0.01*	**0.0025**	0.0157	0.0157	0.0033	**0.0002**	0.0140	0.0140	0.0017
	0.008	**0.0069**	0.0167	0.0167	0.0077	**0.0002**	0.0140	0.0140	0.0022
	0.006	0.0106	0.0196	0.0185	**0.0060**	**0.0005**	0.0142	0.0142	0.0022
(b) n = 200, w = 80									
70	*0.01*	0.0521	0.5264	0.4195	**0.0388**	0.0038	0.1307	0.1114	**0.0032**
	0.008	0.0609	0.7581	0.6268	**0.0427**	0.0070	0.1663	0.1208	**0.0036**
	0.006	0.0693	1.0795	0.9344	**0.0452**	0.0159	0.2542	0.1707	**0.0070**
100	*0.01*	0.0427	0.2135	0.1831	**0.0401**	**0.0031**	0.0485	0.0483	0.0036
	0.008	0.0367	0.3174	0.2609	**0.0194**	0.0058	0.0511	0.0503	**0.0049**
	0.006	0.0386	0.4871	0.4144	**0.0214**	0.0115	0.0594	0.0557	**0.0070**
130	*0.01*	0.0313	0.1578	0.1586	**0.0122**	**0.0057**	0.0158	0.0125	0.0077
	0.008	0.0388	0.2379	0.2143	**0.0121**	**0.0059**	0.0178	0.0173	0.0073
	0.006	0.0394	0.3778	0.3432	**0.0167**	0.0774	0.0202	**0.0191**	0.0226

When ε_t is generated from normal white noise, SSP seems to be more accurate than P–Z when the true breakpoint is near the edges of the series and the linear trend slope is of small size (see highlighted values in Table 1). We note that a larger sliding window w makes SSP more accurate and the results in Table 1 are for w being 40% of n. Only for a small fraction of the 1000 realizations SSP could not detect a breakpoint (the t-statistic profile does not stay long enough out of the significance bounds, according to the criterion of smallest proportion P_U^g). The percentage of these cases increased however with the decrease of time series length, reaching 35% for n = 200.

When n decreases, MSE/n^2 increases for all approaches, and as Table 1b shows, SSP gives the best estimation of the breakpoint for most of the combinations of T, β and types of ε_t. When ε_t is generated from uniform white noise, all methods produce very small MSE/n^2 for all β and T, but this is basically because the amplitude of uniform noise is smaller than for the normal noise. For this type of noise, SSP performs worse than P–Z when n = 500, and the same as or better than P–Z, depending on β and T, when n = 200.

The C–W test has small power, compared to the SSP and P–Z methods for all cases discussed above. C–W seems to increase its power when the breakpoint occurs late in the time series but not in any competitive way, apart from a single case (n = 200; T = 130, β = 0.006 and uniform independent residuals). For the T_1^{C-W} and T_2^{C-W} tests, we computed the empirical critical values for the rejection of H_0 at 5% significance level and the simulation results are shown in Appendix A.

3.2.2. Autocorrelated residuals

When the residuals ε_t are autocorrelated, the degree of correlation, monitored in simulations with the coefficient φ of the AR(1) residual process, combined with the time series length n and the trend slope β, has a major effect on the efficiency of all approaches. The autocorrelation of residuals may mask the trend, so that the onset of trend is undetected, with a probability that increases with the strength of the autocorrelation. For SSP this means that the H_0 of β = 0 not rejected because the t-statistic profile may not exceed for long enough the significance bounds, according to the P_U criterion and then SSP fails to give estimation of T. In order to establish fair comparison between SSP and P–Z, results of P–Z are considered only for the cases for which SSP detected trend onset.

We observe that in the presence of trend, the use of the P_U criterion does not affect the accuracy in the estimation of T when the occurrence of trend is confirmed, but it does decrease the number of cases a trend is found, at a rate that increases with the autocorrelation strength of the residuals. The lack of trend detection holds equally for both SSP and C–W (for the latter see also Chu and White [10]). The results on MSE/n^2 with and without the P_U criterion for one simulation setup, presented in Table 2, show that MSE/n^2 improves slightly with the P_U criterion, but the number of rejections of H_0 decreases dramatically for large φ. The results on other combinations of the parameters of the simulation setup gave similar results. The simulation results for SSP presented below are with the use of the P_U criterion.

For large sample size (n = 1000) and when autocorrelation gets stronger, MSE/n^2 increases for both SSP and P–Z methods with an abrupt increase for P–Z method for very large φ and a steady increase with φ for SSP. As shown in Fig. 5a, P–Z is more accurate than SSP for small to moderate φ and less accurate for large φ, and this feature persists for varying β. Similar results are obtained when ε_t is from an AR(1) process with exponential input white noise while for uniform input noise both methods provide much better estimations of the breakpoint T, but with the same differences in their accuracy.

Table 2
MSE/n^2 and number of rejections of H_0 for the methods SSP and P–Z with and without the use of P_U criterion for β = 0.01, n = 500, w = 200 and T = 250 and for ε_t generated by normal white noise. The results on P–Z regard only the cases H_0 was rejected when applying SSP from a total of M = 1000 Monte Carlo realizations.

φ	Without P_U criterion			With P_U criterion		
	SSP	P–Z	Rejections	SSP	P–Z	Rejections
0.16	0.0045	0.0038	1000	0.0045	0.0038	1000
0.25	0.0061	0.0050	1000	0.0060	0.0049	995
0.36	0.0096	0.0074	1000	0.0089	0.0072	977
0.49	0.0157	0.0133	997	0.0131	0.0128	913
0.64	0.0247	0.0232	971	0.0213	0.0228	709
0.81	0.0291	0.0441	797	0.0235	0.0441	343

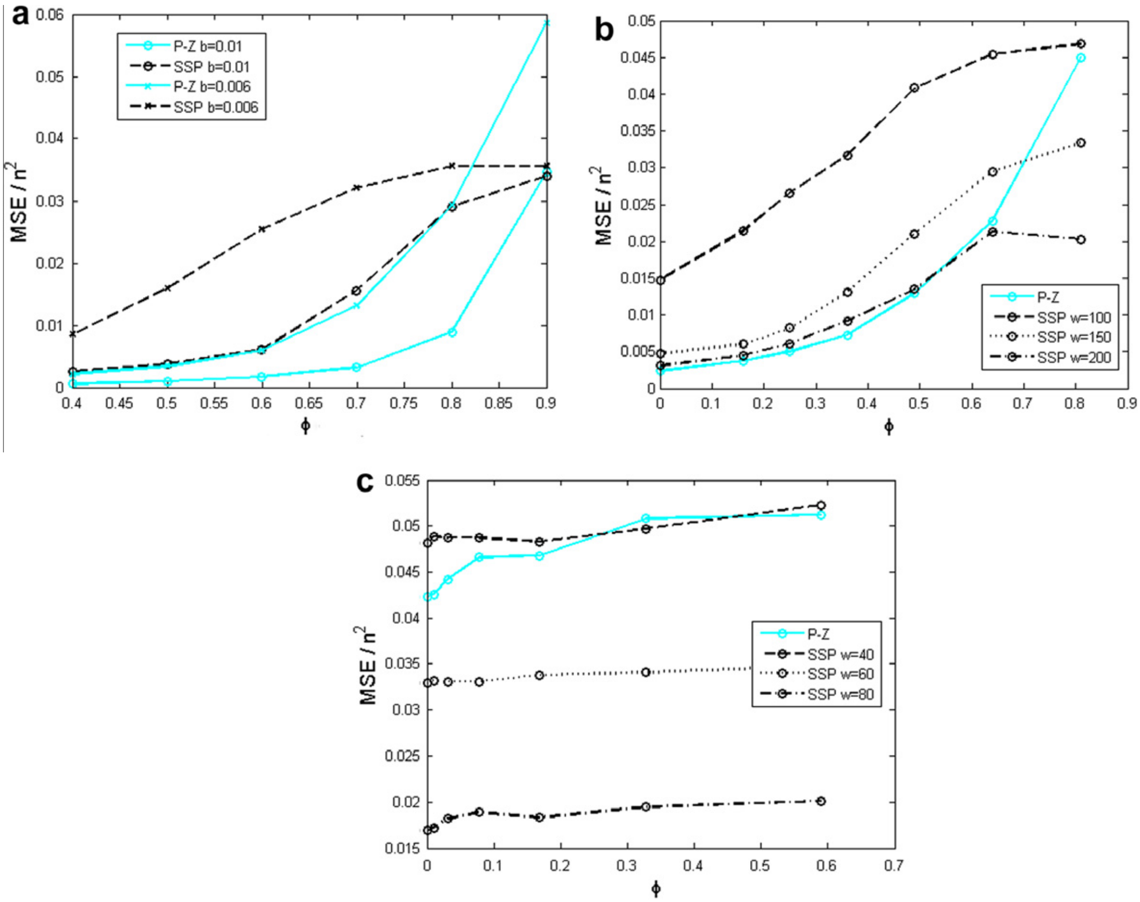

Fig. 5. MSE/n^2 as a function of φ for P–Z and SSP and ε_t generated by AR(1) and normal input white noise: (a) β = 0.01 and 0.006, n = 1000, T = 500 and w = 350, (b) β = 0.01, n = 500, T = 250, w = 100, 150 and 200, and (c) β = 0.01, n = 200, T = 100, w = 40, 60 and 80.

As shown in Fig. 5b and c, a larger w improves the accuracy of SSP to a better level than for P–Z when n gets small. In particular, for n = 500 both approaches (and using $w \geqslant 150$ for SSP) have similar power until a moderate autocorrelation strength (φ = 0.64), but for larger φ SSP remains at about the same level of accuracy while P–Z looses the accuracy dramatically (see Fig. 5b). When n = 200, MSE/n^2 does not change with φ for both P–Z and SSP, but SSP can further decrease the level of MSE/n^2 by increasing w (see Fig. 5c). Similar results were obtained for exponential and uniform input noise. For example, for uniform noise and n = 500, both methods obtain very small MSE/n^2 also for the strongest autocorrelation, e.g. for φ = 0.81 MSE/n^2 is 0.0070 for P–Z and 0.0069 for SSP. However, when n = 200, SSP gives smaller MSE/n^2 only when autocorrelation gets stronger, contrary to the normal noise, where smaller MSE/n^2 is obtained for any φ (see Fig. 5c). Regarding the C–W test, again it has small power compared to SSP and P–Z as for independent residuals, e.g., for the case of Fig. 5b, MSE/n^2 ranges from 0.0477 to 0.1205 for T_1^{C-W} and from 0.0477 to 0.1262 for T_2^{C-W}, both much larger than the errors for SSP and P–Z.

The performance of SSP heavily depends on the sliding window size w and our simulations suggest $w > 30$ of n, so the results for varying T and β presented below are all for $w = 40$ of n.

In our investigation for the detection of T at different locations and varying trend slope β we found again that C–W gives the worst estimation of T and performs somehow better when T occurs late in the time series and the autocorrelation is not strong. SSP and P–Z compete each other with P–Z winning in accuracy only when T occurs early in the time series and the autocorrelation is weak (see Fig. 6). The accuracy of SSP decreases slowly with the decrease of β and the increase of φ, whereas P–Z is much more sensitive to both β and φ, giving larger MSE/n^2 than SSP for all but early occurrence of T. For smaller n, such as $n = 200$ shown in Fig. 6c, P–Z looses accuracy and performs worse than SSP even for small φ and for all locations of T. The results on the three methods using exponential and uniform input white noise were qualitatively the same.

The simulation results showed that the proposed SSP method cannot reach the accuracy of P–Z in the estimation of the breakpoint T when the time series is long and has only weak autocorrelation, and this holds for any location of T in the time series and any input noise distribution for the AR(1) process of the residuals ε_t. However, provided that a long enough sliding window size w is used, larger than 30% of n, SSP improves over P–Z as the time series gets shorter, the autocorrelation stronger and the trend slope weaker. Fig. 7 shows the accuracy on the estimation of T of both methods as a function of n for weak and strong autocorrelation values ($\varphi = 0.4$ and $\varphi = 0.9$ for $n = 1000$ and adjusting φ for smaller n). For weak autocorrelation, there is a cross-over at $n = 500$, SSP giving better estimation of T than P–Z for $n < 500$ and worse for $n > 500$ (Fig. 7a), whereas for strong autocorrelation SSP outperforms P–Z for any n (Fig. 7b). Note that autocorrelation and time series length have opposite effect on the estimation accuracy of T, and therefore for strong autocorrelation in Fig. 7b, MSE/n^2 does not reduce with n.

We considered also other systems for correlated residuals ε_t, namely the autoregressive moving average system ARMA(1,1) ($\varepsilon_t = \varphi\varepsilon_{t-1} + a_t - \theta a_{t-1}$) and the nonlinear autoregressive system NAR(1) ($\varepsilon_t = 7(|\varepsilon_{t-1}|/(|\varepsilon_{t-1}| + 2))$), see Lee et al. [25], Brooks [11]. For ARMA(1,1), we found that SSP gave better estimation of T for small and moderate time series length, as found for AR(1), while for NAR(1) better estimation was found only for small time series (see Table 3).

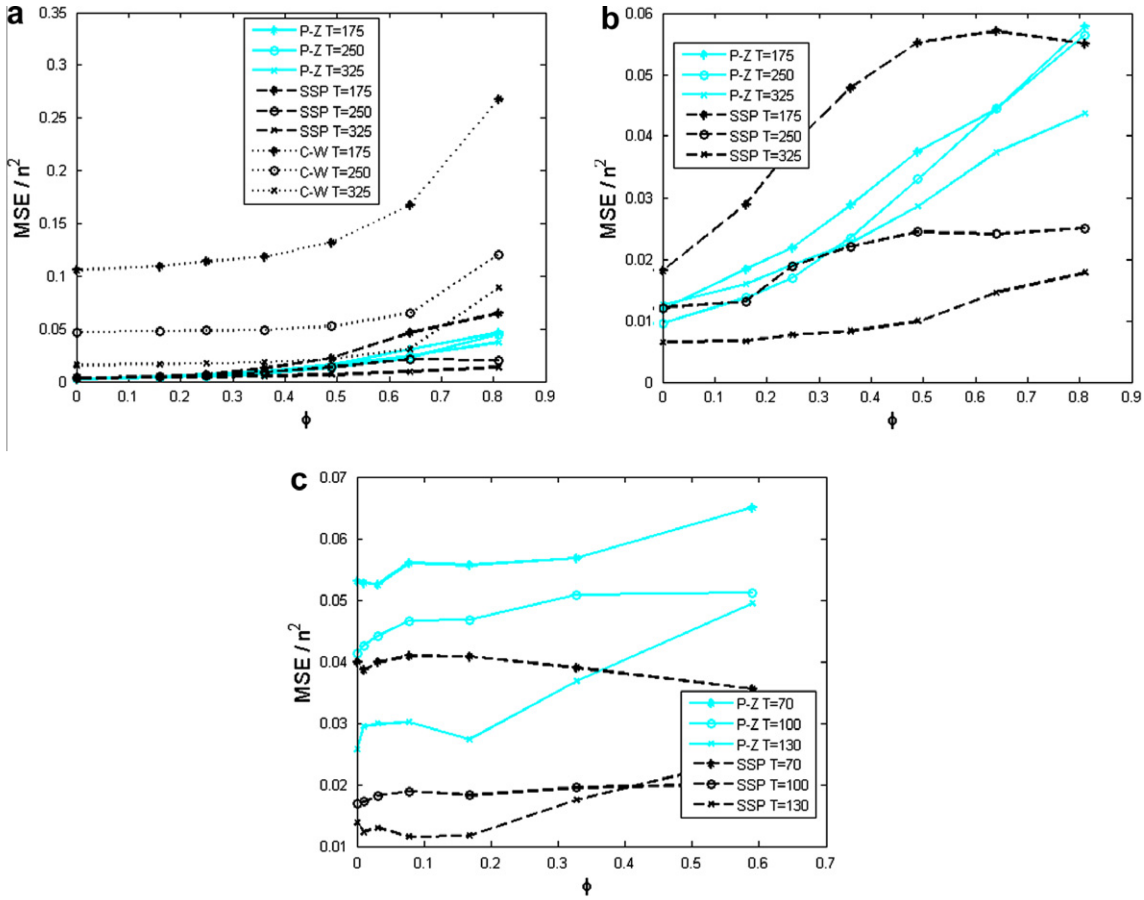

Fig. 6. (a) MSE/n^2 as a function of φ for P–Z, SSP and C–W and ε_t generated by AR(1) and normal input white noise, where $n = 500$, $\beta = 0.01$ and $T = 175$, 250 and 325, as shown in the legend. (b) Same as in (a) but for $\beta = 0.006$ and only for P–Z and SSP. (c) Same as (a) but for $n = 200$ and $T = 70$, 100 and 130 and only for P–Z and SSP.

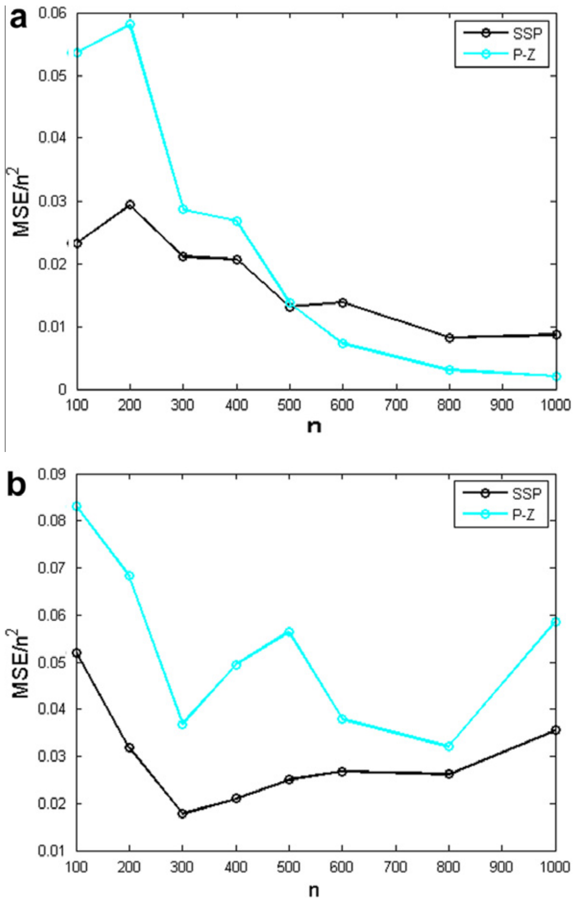

Fig. 7. MSE/n^2 as a function of n for P–Z and SSP and ε_t generated by AR(1) and normal input white noise, $\beta = 0.006$, $T = n/2$ and $w = 30$ of n. In (a) the φ parameter is 0.4 for $n = 1000$ and it is adjusted for smaller n, and in (b) φ is 0.9 for $n = 1000$.

Table 3
MSE/n^2 for the methods SSP and P–Z with $\beta = 0.006$, $T = n/2$, $w = 30$ of n and for ε_t generated by ARMA(1,1) and NAR(1) systems and normal input white noise, for different time series lengths. The smallest $MSE/n2$ of the two methods for each case of the time series models tested here is highlighted.

n	ARMA(1,1) $\phi = 0.9$, $\theta = -0.5$		ARMA(1,1) $\phi = 0.9$, $\theta = 0.5$		NAR(1)	
	SSP	P–Z	SSP	P–Z	SSP	P–Z
200	**0.0193**	0.0406	**0.0185**	0.0413	**0.0287**	0.0521
500	**0.0247**	0.0348	**0.0239**	0.0332	0.0117	**0.0095**
1000	0.0872	**0.0521**	0.0805	**0.0257**	0.0011	**0.0006**

Thus in more realistic data conditions of relatively small time series containing significant correlation, SSP can detect better than P–Z the onset of a relatively small linear trend. SSP turns out to be capable of distinguishing a small exogenous linear trend from a trend formed by the correlated process up to the limit of small β and large φ for all tested trend slopes β, positions of T and distribution of input white noise to the residual process. Our simulation results showed that the C–W test has no power to detect the onset of a weak linear trend and produced worse results than both SSP and P–Z.

4. Application

We applied both SSP and P–Z methods to two instrumental records of the mean annual surface temperature anomaly of the Northern Hemisphere (NH), the TaveNH2v time series from 1856 to 2005 and the Crutem3NH time series from 1850 to 2006, registered at the Climatic Research Unit (CRU) of the University of East Anglia [9]. The temperatures data are updated continuously on a monthly basis and also can be found at http://www.cru.uea.ac.uk/cru/data/:/tem2/ for TaveNH2v and

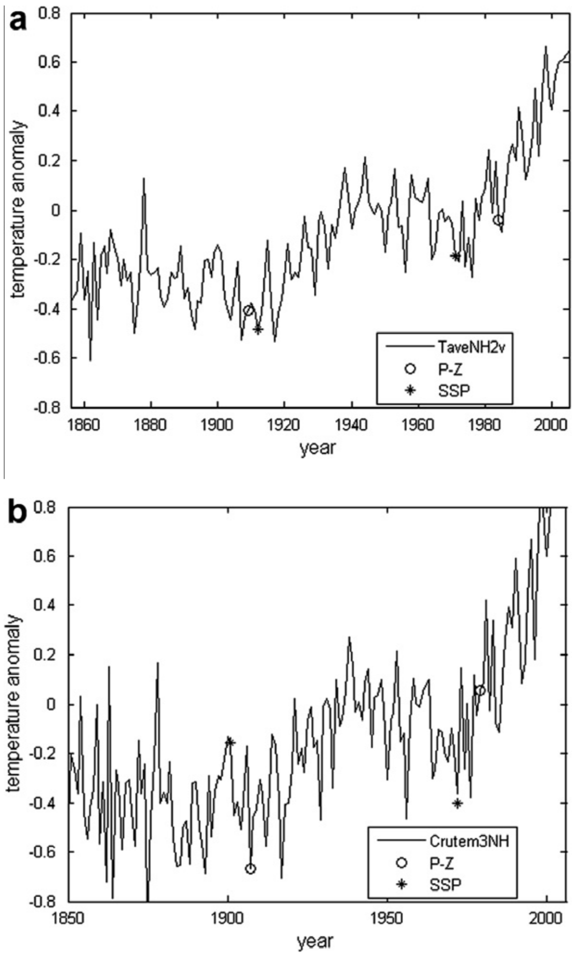

Fig. 8. The TaveNH2v time series in (a) and the Crutem3NH time series in (b). The estimated breakpoints by SSP and P–Z are marked on the time series as given in the legends.

/temperature/ for Crutem3NH. The two time series are shown in Fig. 8. The length of the two time series is 150 and 157, respectively, and therefore we set $w = 50$, i.e. about 35% of n. Both time series have strong autocorrelation that can also be seen from the coefficient of the fitted AR(1), $\varphi = 0.7133$ for Crutem3NH and $\varphi = 0.8121$ for TaveNH2v. For such large φ, the minimum proportion of points out of bounds P_U should be 26% (26 consecutive sliding windows) for Crutem3NH and 31% (29 consecutive sliding windows) for TaveNH2v, according to Eq. (6). However, both time series do not seem to exhibit a monotonic trend after a trend onset, but they rather involve multiple changing points, and therefore we would not expect to find significant t-statistics for that many consecutive sliding windows as required by the P_U criterion. Indeed the use of P_U criterion indicates only one linear trend onset for Crutem3NH at the year 1900 and none for TaveNH2v. Thus we proceed to estimate linear trend onset without the use of P_U criterion. Fig. 9a and b shows for both time series the smoothed t-statistic profile for SSP and the cost function profile for P–Z, respectively. Both methods identify two linear trend onsets. For the TaveNH2v series, P–Z estimates the breakpoints at 1909 and 1984 and SSP at 1912 and 1971, indicating a small delay in the estimation of the second breakpoint with P–Z. Similarly, for the Crutem3NH series, P–Z estimates the years 1907 and 1979 as possible breakpoints and SSP the years 1900 and 1972.

The overall results suggest that both SSP and P–Z identify with small deviations two onsets of upward trends in the instrumental global temperature records. These results agree also with relevant reports in the literature. It is generally accepted that the trend of global-mean surface temperature is not monotonic over the 20th-century, rising from the start of the century to the 1940s, then falling slightly during the middle part of the century, and rising again rapidly from the mid-1970s onwards [38]. Goossens and Berger [17] observed an abrupt increase at 1900 ± 25 years in the series of global annual average of NH, which is similar to TaveNH2v [21]. Ivanov and Entimov [20] indicate that the cooling trend of NH stops at 1963, and the temperature time series locks up into a new regime of linear warming period around 1970, while Gil-Alana [16] suggests 1964 as the last break year of the NH time series. We note that the second breakpoint of SSP is close to the reported estimates, whereas P–Z suggests a later date.

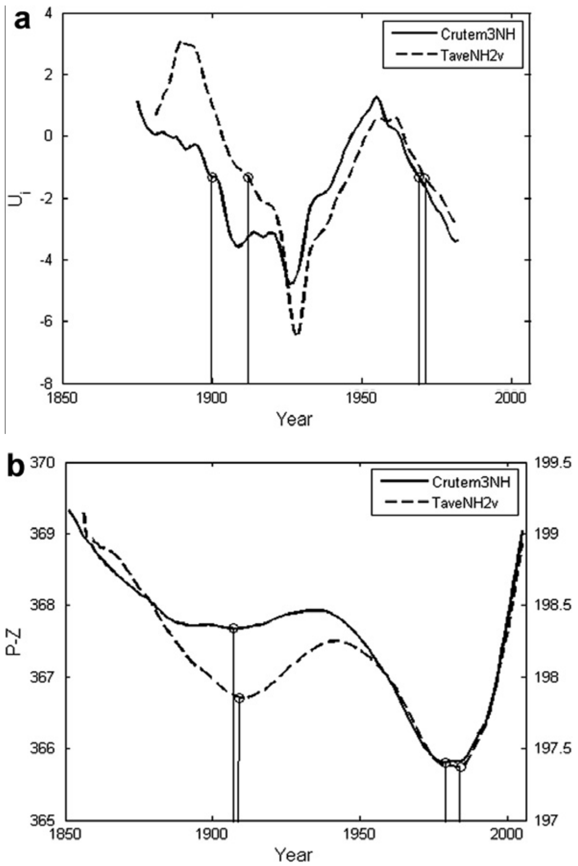

Fig. 9. The profiles for the SSP method in (a) and the P–Z method in (b) for the TaveNH2v and the Crutem3NH time series, as given in the legends. The estimated breakpoints are marked with open circles and vertical lines.

5. Conclusion

The proposed method of slope statistic profile (SSP) estimates the linear trend onset on a time series making use of a linear trend test statistic on sliding windows along the time series. The rationale is that if a trend persists for a part of the time series then the corresponding part of the statistic profile is out of the significance bounds. Thus the time of the onset of linear trend T is searched at the part of the smoothed slope statistic profile being within the bounds of significance levels 0.05 and 0.20. Prior to the estimation of the time location, the existence of T is examined requiring a proportion of the smoothed slope statistic profile be out of bounds, and this proportion is determined by the autocorrelation at lag one $r(1)$ and length n of the time series.

The proposed method is compared to the method of Perron and Zhu (P–Z) that minimizes a cost function of a variable breakpoint T for an assumed trend model, and the test of Chu and White for the occurrence of T. For this we made Monte Carlo simulations for different settings, regarding n, $r(1)$, the time location of T, the strength of linear trend slope β, and the residual distribution. Moreover, for SSP the sliding data window size w is a free parameter that was investigated. The computational study showed that w should be long enough (larger than 30% of n), so that estimation of other spurious onsets occurring at small time scales is avoided. SSP and P–Z outperform C–W in almost all settings and P–Z attains better accuracy in estimating T when there is no correlation in the data, but its accuracy decreases with the increase of the strength of autocorrelation and decrease of n. On the other hand, SSP is less affected by these two factors, and turns out to outperform P–Z for strongly autocorrelated or small time series. Also, SSP tends to perform better than P–Z when β is small. Regarding the location of the breakpoint T, SSP estimates it better when it occurs over the middle of the time series.

The results on the simulations, using also different systems for the correlated residuals, suggest that SSP is particularly useful for many real world problems regarding the onset of weak trends in short and correlated data. To illustrate the practical relevance of SSP, we applied SSP and P–Z on two time series of the mean annual surface temperature of the Northern Hemisphere. For both series, the two methods detected two breakpoints of temperature rise at close dates, but with SSP being closer to the time suggested in other elaborated studies on this problem.

We note that SSP is primarily developed to detect and estimate the time location of the onset of a linear trend, but it can easily be extended to other scenarios, such as a change of trend, e.g., from rise to fall, or consecutive changes of trends, as well as combinations of onsets and changes of trends. The trend analysis of the global temperature time series actually gave evidence for two trend onsets.

Appendix A

The table below gives the empirical critical values for T_1^{C-W} and T_2^{C-W} at 5% significance level obtained from 10,000 realizations for each case. The results are for the following settings: (a) $\varepsilon_t = \varphi\varepsilon_{t-1} + u_t$, $n = 1000$ and varying φ; (b) same as (a) but for $n = 500$; (c) same as (a) but for $n = 200$; (d) ε_t is white noise and varying n. For each setting, results are shown for $u_t \sim N(0, 1)$ and $\varepsilon_t \sim U[-1/2, 1/2]$.

(a)	$n = 1000$			
	$u_t \sim N(0,1)$		$u_t \sim U[-1/2,1/2]$	
φ	T_1^{C-W}	T_2^{C-W}	T_1^{C-W}	T_2^{C-W}
0.4	0.7854	0.7527	0.7875	0.7572
0.5	0.7937	0.7446	0.8037	0.7546
0.6	0.8227	0.7487	0.8259	0.7505
0.7	0.8479	0.7379	0.8446	0.7362
0.8	0.9072	0.7264	0.9052	0.7197
0.9	1.0935	0.7056	1.0859	0.6500

(b)	$n = 500$			
	$u_t \sim N(0,1)$		$u_t \sim U[-1/2,1/2]$	
φ	T_1^{C-W}	T_2^{C-W}	T_1^{C-W}	T_2^{C-W}
0.16	0.7651	0.7606	0.7653	0.7547
0.25	0.7731	0.7597	0.7719	0.7505
0.36	0.7704	0.7402	0.7742	0.7440
0.49	0.7843	0.7367	0.7869	0.7461
0.64	0.8142	0.7279	0.8072	0.7246
0.81	0.8907	0.7037	0.8939	0.6888

(c)	$n = 200$			
	$u_t \sim N(0,1)$		$u_t \sim U[-1/2,1/2]$	
φ	T_1^{C-W}	T_2^{C-W}	T_1^{C-W}	T_2^{C-W}
0.01	0.7449	0.7470	0.7570	0.7507
0.031	0.7532	0.7480	0.7558	0.7513
0.078	0.7493	0.7372	0.7527	0.7497
0.168	0.7554	0.7490	0.7604	0.7428
0.326	0.7557	0.7301	0.7581	0.7308
0.59	0.7697	0.7043	0.7734	0.7069

(d)				
	$\varepsilon_t \sim N(0,1)$		$\varepsilon_t \sim U[-1/2,1/2]$	
n	T_1^{C-W}	T_2^{C-W}	T_1^{C-W}	T_2^{C-W}
1000	0.7965	0.7959	0.7960	0.7997
500	0.7674	0.7667	0.7681	0.7712
200	0.7471	0.7477	0.7568	0.7519

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