# FNCE611 Problem Set 1

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## Contents



# 1 APR and EAR

$$1 + EAR = \left(1 + \frac{APR}{m}\right)^m$$

$$EAR = \left(1 + \frac{0.15}{12}\right)^{12} - 1 = 16.08\%$$

# 2 Present Value / Future Value

$$PV = \frac{C}{(1+r)^t} = \frac{33000}{(1+0.05)^3} = \$28,506.60$$

# 3 Perpetuities

$$PV = \frac{C}{r} = \frac{875}{0.088} = 9943.18 Rupee$$

# 4 Annuities / Mortages

## 4.1 (a)

$$PV = \frac{C}{\frac{r}{m}}\left(1 - \frac{1}{(1+\frac{r}{m})^{tm}}\right) = \frac{PV\frac{r}{m}}{(1-\frac{1}{(1+\frac{r}{m})^{tm}})} = \frac{100,000\frac{0.06}{12}}{(1-\frac{1}{(1+\frac{0.06}{12})^{12\times 30}})} = \$599.55$$

## 4.2 (b) Refiance

Present value of remaining payments:

$$PV = \frac{C}{\frac{r}{m}}\left(1 - \frac{1}{(1+\frac{r}{m})^{tm}}\right) = \frac{599.55}{\frac{0.06}{12}}\left(1 - \frac{1}{(1+\frac{0.06}{12})^{25\times 12}}\right) = 93,054.30$$

At at 5% interest rate, the payments on the remaining present value for 25 years is

$$C_{25years} = \frac{PV \times \frac{r}{m}}{1-\frac{1}{(1+\frac{r}{m})^{t\times m}}} = \frac{93,054.30 \times \frac{0.05}{12}}{1-\frac{1}{(1+\frac{0.05}{12})^{25\times 12}}} = 543.99$$

and for 30 years is

$$C_{30years} = \frac{PV \times \frac{r}{m}}{1-\frac{1}{(1+\frac{r}{m})^{t\times m}}} = \frac{93,054.30 \times \frac{0.05}{12}}{1-\frac{1}{(1+\frac{0.05}{12})^{30\times 12}}} = 499.54$$

So, the NPV of the savings considering the \$2,000 refiancing fee is

$$NPV_{25years} = -2000 + \frac{C_{before} - C_{after}}{\frac{r}{m}}\left(1 - \frac{1}{(1+\frac{r}{m})^{t\times m}}\right) = -2000 + \frac{599.55 - 543.99}{\frac{0.05}{12}}\left(1 - \frac{1}{(1+\frac{0.05}{12})^{25\times 12}}\right) = 7,504$$

$$NPV_{30years} = -2000 + \frac{C_{before} - C_{after}}{\frac{r}{m}}\left(1 - \frac{1}{(1+\frac{r}{m})^{t\times m}}\right) = -2000 + \frac{599.55 - 499.54}{\frac{0.05}{12}}\left(1 - \frac{1}{(1+\frac{0.05}{12})^{30\times 12}}\right) = 16,630$$

As the NPV is positive for either scenario, refiancing is the preferred option.

# 5 U.S. Treasury Securities

## 5.1 (a) Asking Price

$$P_{ask} = \frac{\frac{Coupon}{2}}{\frac{Yield}{2}}\left(1 - \frac{1}{(1+\frac{Yield}{2})^{2\times years}}\right) + \frac{100}{(1+\frac{yield}{2})^{2\times years}} \quad (1)$$

$$= \frac{\frac{0.0215}{2}}{\frac{0.01945}{2}}\left(1 - \frac{1}{(1+\frac{0.01945}{2})^{2\times 5}}\right) + \frac{100}{(1+\frac{0.01945}{2})^{2\times 10}} = 100.85 \quad (2)$$

### 5.2 (b) Duration

$$duration = \frac{\sum_{i=1}^{N} \frac{t_i\, cf_i}{(1+y)^i}}{\sum_{i=1}^{N} \frac{cf_i}{(1+y)^i}} \tag{3}$$

$$= \frac{\frac{1\,\frac{0.02125}{2}}{(1+\frac{0.01945}{2})^1} + \frac{2\,\frac{0.02125}{2}}{(1+\frac{0.01945}{2})^2} + \cdots + \frac{10\,\frac{1.02125}{2}}{(1+\frac{0.01945}{2})^{10}}}{\frac{\frac{0.02125}{2}}{(1+\frac{0.01945}{2})^1} + \frac{\frac{0.02125}{2}}{(1+\frac{0.01945}{2})^2} + \cdots + \frac{\frac{1.02125}{2}}{(1+\frac{0.01945}{2})^{10}}} \tag{4}$$

$$= \frac{9.624}{1.009} = 9.54 \tag{5}$$

Half-year duration is 9.54, so the full year duration is $9.54 \times 0.5 = 4.77$.

# 6 Term Structure

Investing in long term bonds is riskier than investing in short term bonds, therefore interest rates for longer term bonds will increase (at a diminishing rate) and lead to an upward slope

# 7 Corporate Bonds

## 7.1 Yield to Maturity

If the coupon rate is 5.2%, then every 6 months Whole Foods will pay $1bn \times \frac{0.052}{2} = 26m$. Thus we can calculate the yield to maturity in Dec 2015 as

$$PV = \frac{coupon}{(1+\frac{y}{2})^1} + \frac{coupon}{(1+\frac{y}{2})^2} + \cdots + \frac{FV + coupon}{(1+\frac{y}{2})^{10}} \tag{6}$$

$$998.61 = \frac{52}{(1+\frac{y}{2})^1} + \frac{52}{(1+\frac{y}{2})^2} + \cdots + \frac{1000+52}{(1+\frac{y}{2})^{10}} \tag{7}$$

where

```
ytm <- function(price, coupon, term, freq, prin = 100) {
    npv <- function(yld, term, cpn, freq, prin) {
        i <- 1:(term * freq)
        payment <- (coupon / freq) * prin
        disc <- (1 + yld / freq)
        sum(payment / disc^(i)) + prin /(disc^(term * freq))
    }
    uniroot(function(yld) npv(yld, term, cpn, freq, prin) - price,
interval=c(0,100), tol=0.000001)$root
}

y = ytm(price = 998.61, coupon = 0.052, term = 5, freq = 2, prin = 1000)
```

$$y = 5.232\%$$

## 7.2 Spread

$$Spread = ytm_{Coupon} - 3\%$$

$$5.23\% - 3\% = 2.23\%$$