Last of §1.3

<u>Prop 1.3.3</u> G is a graph w/ $rad(G) \leq k$
and $\Delta(G) \leq d$
$(d \geq 3)$,

then $|V(G)| < \frac{d}{d-2}(d-1)^k$

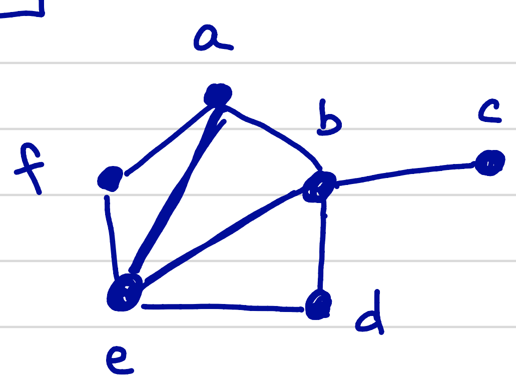
§ 1.4 Connectivity

<u>def</u>: A non-empty graph G is <u>connected</u> if

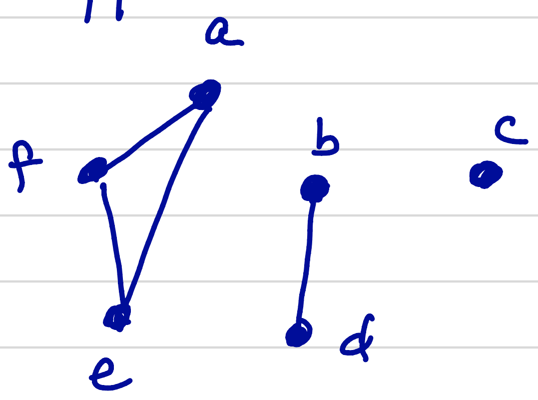
$\forall\, x, y \in V$, $\exists$ xy-path in G.

Otherwise, G is disconnected.

<u>Ex</u> G



H



- components
- <u>def</u>: $G[u_1, u_2, \dots, u_k]$ means the induced subgraph H of G w/ $V(H) = \{u_1, u_2, \dots, u_k\}$.

<u>Prop 1.4.1</u> G is a connected graph on n vertices.

It is possible to order $V(G) : (v_1, v_2, \dots, v_n)$ s.t.

Pf: (by induction on n)

Prop 1.4.2 G nontrivial (ie $|V(G)| \geq 2$)

$$\kappa(G) \leq \lambda(G) \leq \delta(G).$$

the (vertex) connectivity | the edge connectivity | minimum degree

Recall definitions: G graph

- $\delta(G) =$

- G is k-connected ($k \in \mathbb{N} \cup \{0\}$) if

- $\kappa(G)$, the (vertex) connectivity of G, is

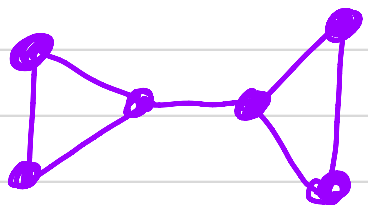
  kappa or \kappa

- G is ℓ-edge-connected if

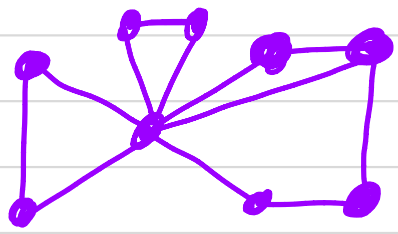
- $\lambda(G)$, the edge connectivity of G, is

Examples:

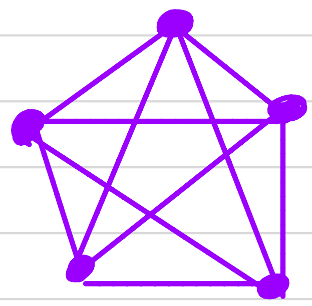
G_1



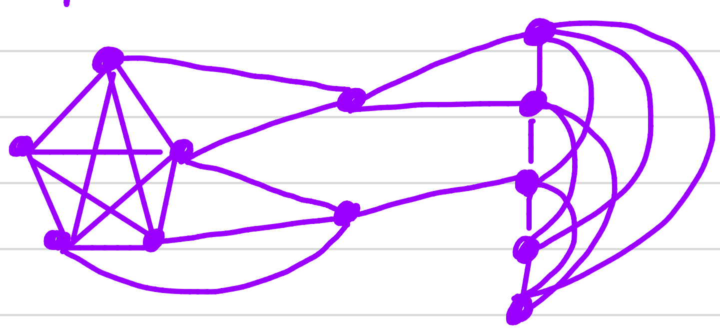
G_2



$G_3 = K^5$



G_4



Prop 1.4.2 G nontrivial (ie $|V(G)| \geq 2$)

$$\underbrace{\kappa(G)}_{\text{the (vertex) connectivity}} \leq \underbrace{\lambda(G)}_{\text{the edge connectivity}} \leq \underbrace{\delta(G)}_{\text{minimum degree}}.$$

Prop 1.4.3 (Mader)

If G graph s.t. $d(G) \geq 4k$, $k \in \mathbb{Z}^+$,

then $\exists\ H \subseteq G$ s.t.

- H is $(k+1)$-connected

  and

- $d(H) > d(G) - 2k$.