1. Give an example to show that if G is allowed to have multiple edges, then $\chi'(G)$ may exceed $\Delta(G)+1$.

2. Without using Proposition 5.3.1, show that $\chi'(G) = k$ for every k-regular bipartite graph G.

3. Give an explicit edge-coloring to prove that the n-dimensional cube, Q^n, is Class 1.

4. Prove that if G is a regular graph with a cut vertex, then $\chi'(G) > \Delta(G)$.

5. The **cartesian product** of two graph G and H, denoted $G\square H$, is the graph with vertex set $V(G\square H) = V(G)\times V(H)$. A pair of vertices $(x_1,y_1),(x_2,y_2)$ are adjacent in $G\square H$ if and only if $x_1 = x_2$ and $y_1y_2 \in E(H)$ or $x_1x_2 \in E(G)$ and $y_1 = y_2$.
   (a) Draw $P_2\square C^3$.
   (b) Prove that $\Delta(G\square H) = \Delta(G)+\Delta(H)$
   (c) Prove that if $\chi'(H) = \Delta(H)$, then $\chi'(G\square H) = \Delta(G\square H)$.

6. (a) Let G_1 be a 5-cycle with one chord. Show that there exists a graph H such that $L(H) = G_1$.
   (b) Prove that for the graph G_2 (drawn below) that there does not exist any graph H such that $L(H) = G_2$.