1. Use Euler's Formula to prove that $K_{3,3}$ is not planar.
2. Show that every connected planar graph with minimum degree at most 3 is the union of three forests.
3. Show that every planar graph contains a vertex of degree at most 5. Give an example of a planar graph G such that $\delta(G) \geq 5$.
4. A graph is called **outerplanar** if it has a drawing in which every vertex lies on the boundary of the outer (or infinite) face. Show that a graph is outerplanar if and only if it does not contain a K^4 minor or a $K_{2,3}$ minor.
5. Let G be a 2-connected plane graph. Show G is bipartite if and only if every face is bounded by an even cycle.
6. Given a plane graph G, the **dual graph,** G^***,** of G is a plane graph whose vertices correspond to the faces of G. The edges of G^* are defined as follows: for every edge $e \in E(G)$ on the boundary of faces X and Y in G, edge $\{X,Y\} \in E(G^*)$. Note that the dual graph of a simple plane graph may not be simple.
   (a) Describe the dual graphs of P^m, C_k, and K_4.
   (b) Prove that if the n-vertex plane graph G is isomorphic to its dual, G^*, then $\| G \| = 2n-2$.