

1. Prove that for every graph G , there exists an order of the vertex set of G such that a greedy algorithm using this ordering will use $\chi(G)$ colors.
2. For every $n \geq 3$, construct find a bipartite graph on $2n$ vertices and an ordering of the vertex set such that the greedy algorithm will use n colors (as opposed to the optimal 2 colors).
3. A k -chromatic graph G is called **critical** if $\chi(G - v) < k$ for every vertex $v \in G$.
 - (a) Characterize critical 2-chromatic graphs.
 - (b) Find an example of a critical 3-chromatic graph.
 - (c) Prove that for $k \geq 3$ every critical k -chromatic graph is $(k - 1)$ -edge-connected.
 - (d) Characterize the set of critical 3-chromatic graphs.
4. The **clique number** of a graph, denoted by $\omega(G)$, is the largest r such that $K^r \subseteq G$. The **independence number** of a graph, denoted by $\alpha(G)$, is the largest r such that G contains an independent set of vertices of cardinality r .
 - (a) Determine $\omega(G)$ and $\alpha(G)$ for:
 - i. P^m for $m \geq 1$
 - ii. C^k
 - iii. $K_{m,n}$ where $m \leq n$
 - iv. K^n
 - (b) Prove that $\chi(G) \geq \max\{\omega(G), |G|/\alpha(G)\}$.
5. Prove or Disprove: Every k -chromatic graph G has a k -coloring in which some color class has at least $\alpha(G)$ vertices.
6. Assume that H is a k -chromatic triangle-free graph and the G is obtained from H by Mycielski's Construction.
 - (a) Prove that G is also triangle-free.
 - (b) Prove that G is $(k + 1)$ -colorable.
7. Describe the topic of your project and what source(s) you have found.